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Reliability Evaluation of Transmission Planetary Gears “bottom-up” approach

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Highlights

- The "bottom-up" procedure for evaluating the reliability of planetary gears.
- The Weibull model is often considered the primary reliability model.
- The competitive risk model was developed.
- The competing risk model results in more accurate reliability estimates than modeling each failure mode separately.
- Using a bottom-up approach in gear reliability modeling can provide guidance for the development and design of new planetary gear design solutions.

Abstract

The reliability study is the most important part of the engineering design process, as it is the basis of analysis and assessment of future product performance in exploitation. Since performance cannot be predicted with absolute certainty, the application of reliability theory includes probability theory and unreliability modeling. The proposed approach has been applied to assess the reliability of gear planetary power transmissions. The assessment of system reliability was determined on the basis of the block diagram method, as a function of the reliability of individual components, calculated by statistical analysis. Using the Weibull model, the reliability of the planetary gear was defined on the basis of the probability of failure of the gear teeth and the results were interpreted to assess the reliability of the component and the entire planetary train. For a more precise assessment of reliability and to avoid modeling every failure and mode of occurrence, a competitive risk model was developed. The reliability assessment study was conducted with a “bottom-up” approach. Reliability has been assessed, for instantaneous, estimated and assigned failures rate of planetary train and component.

Keywords

planetary gear, failure rate, reliability, “bottom-up”

1. Introduction

Constructions of planetary transmissions are compact structures, composed of parts with high specific load capacities. Their main disadvantages are uneven wear and destruction of the flanks of the gear teeth, small space for bearings and heating of the transmission due to the small dimensions of the elements that reduce heat radiation. The coupling of the flanks of the gear teeth in planetary transmissions is specific. The specificity consists in the fact that the sun gear is simultaneously connected to several satellite gears, so the load is divided among several teeth of the sun gear. If the sun gear is damaged, all components of the planetary gear are also damaged. The rule is that when one gear is damaged, it is impossible to replace only that gear, it is necessary to replace all of them. In the case of planetary gears, the entire set must be replaced when the sun gear is damaged. Since different types of damage are possible in planetary gears, and in order to prevent the occurrence of such damage, it is necessary to determine the distribution of damage over the failure rate of the gear teeth, as well as to estimate the failure rates for the most critical components. In this way, the reliability of the transmission can be increased, because it depends on the elemental reliability of the components. Unequal distribution of the failure rate of the flanks of the gear teeth in the planetary

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transmission will be the focus of the research presented in the following text. Gears are the most important components of planetary power transmissions. Their reliability has the greatest impact on the durability and reliability of planetary gears as mechanical systems. Therefore, researchers have done a lot of work to assess the reliability of the gear system [2, 26, 30]. For example, the influence of the stress spectrum on the tooth and the number of changes coupling the flanks of the gear teeth is widely used to assess reliability [1, 19, 20, 28]. Reliability analysis is closely related to the concepts of the required service life of the system and unreliability. Thus, the concept of reliability has a different interpretation in each step of product design, use and product service [3, 7, 13, 18, 25].

Based on reliability, it is possible to define different products, such as two-speed automotive transmission systems. The analysis of failures in the planetary gearbox on the wind turbine [5, 12, 16, 29] provides the basis for the analysis of potential damage on other types of planetary gearboxes. The working life of the new design solutions of the planetary transmission will depend on the analysis of different types of damage and the possibility of their occurrence.

The reliability of the transmission is affected by various parameters, such as the coupling of cylindrical gears [11], gear pitting fault [14, 22], assembly tolerances [21], bearing configuration [9] and can be detected by various methods such as (synchronous resample and adaptive variational mode decomposition) [31]. Detecting a large number of different parameters can increase the reliability of the system which was processed. By analyzing models of planetary gears, the authors [5] considered a large number of errors that may occur in the production process and therefore affect the functioning of planetary gears. The analysis of the dynamic model of reliability based on random lifetime [10] and disturbances [16, 24], as well as the destruction process [17] by monitoring the tests whether the tests are done in laboratory conditions or in the process of explotation [12] provide good guidelines for the development of planetary gears from the aspect of its reliability. According to the available literature, a certain number of authors dealt with predicting the reliability of wind turbines [4, 15, 23], Helicopter Main Gearbox Lubrication System using Influence Diagrams in a way that helped to analyze and define the reliability of planetary gears.

It is almost impossible for engineers to design a product which operates with absolute certainty and without failures in its working life. That is due to limitations imposed to any product engineering procedure. However, performance satisfaction during the lifecycle in accordance with the appropriate requirements and standards must be ensured using engineering criteria. The unreliability of desired performance in the working life of design and unforeseen failures can be accompanied by significant costs and followed by serious risks.

Reliability in the Mechanical Design Process is the sum of the all units failure rates. It simply means that whenever the failure rates of units are added, it is automatically assumed that the units are acting in series (i.e., if any one unit fails, the system fails).

Reliability assessment is an engineering approach to predict the performance of a product during its design cycle, the testing of which is affected by several factors, such as: test length, speed, group homogeneity, item difficulty, objectivity, test-retest interval, variation with test situations, etc. and for the successful implementation of testing, knowledge of a lot of data is necessary.

The main goal of this work is reliability modeling and assessment, to predict the desired product performance for a predetermined service life with a small amount of required product data.

In the available literature there is almost no data, or very rarely, that anyone has dealt with reliability analysis using the "bottom-up" approach on the example of planetary gears. In order to justify the application of the "bottom-up" approach, the analysis will be based on the fact that: failure modes are independent of each other, that the failure of one component causes the failure of the entire system, and that each failure mode has its own distribution failure in time. With this approach, a competitive model will be developed that component reliability represents failure mode and component failure rates represent the sum of failure rates, a "bottom-up" basis methodical approach for evaluating the reliability of gear planetary drive, which is another goal of this paper.

The work is important because it provides a different approach than many others, to define values for the reliability of components of the planetary drive train and the train as a whole using a small amount of data. And that the results obtained in this way can be applied in the development of new design solutions for the planetary power unit.

In order to achieve the goals set in this work, certain actions were preceded. For the wear period, instantaneous failure rates (IFR) and average failure rates (AFR) were calculated. Allocation methods were implemented and failure rates were assigned for all components. Reliability of planetary gear components are determined for instantaneous and average failure rate values. Mean time to failure (MTTF) values for two planetary sets are defined. The obtained reliability values of the components are tabulated and graphically presented. The failure rate functions are defined, calculated and displayed graphically. The reliability values of the first and second planetary set were calculated and graphically displayed. Reliabilities were calculated for the competitive risk model, and the values are illustrative, shown graphically.

2. Reliability modelling based on real-time systems

The reliability of planetary train, as machine systems, is a specific area based on the application of probability accounts and mathematical statistics. The legalities that are obtained by the application of these methods are of particular importance for the maintenance of machine systems but also for the design and development of new ones. The working conditions of machine systems are random, the process of failure and other types of damage are the result of random processes. In addition, other under-known processes whose impact can only be covered statistically are affected. In order to ensure complete safety in operation, without the possibility of malfunction (malfunction), the machine system would have to be either very expensive, too large dimensions and masses, irrational for exploitation, etc. Combining experiments, probability calculations and mathematical statistics make up for a whole range of under-study processes and a lack of data on the processes on which the results of the design process directly depend. In the process of constructing and maintaining machine systems, questions that theoretical science has failed to study cannot be circumvented. With the development of theoretical knowledge, the need for a statistical and experimental approach is diminished. In this sense, reliability or unreliability is a risk that can be accepted to obtain rational constructions without jeopardizing the function and quality of the system. In this regard, reliability is one of the indicators of the quality of the machine system.

Faults can be very different. Most typical are all types of failure and damage, and incorrect processes. Reliability can be defined at the level of a complex machine system and hierarchically broken down to the level of components and machine parts.

As a complex probability, the reliability of a physical machine system (Figure 1) is modeled to form a structure formed of elementary reliability (probabilities), presented in a block diagram of reliability. In the reliability structure (block diagram of reliability) of a machine system of elementary reliability, which represent the probability that a certain malfunction will not occur, can be connected in series (Fig.2a), in parallel (Fig.2b) and combined (Fig.2c.) Based on a certain probability of destruction until the occurrence of critical failure and certain failure rates of the considered groups of components and parts, their reliability was obtained.

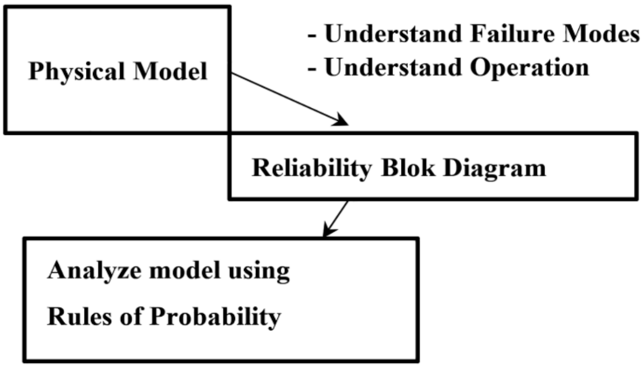

Figure 1. Reliability Block Modeling Process

For any element or component of the system, the reliability is [3, 7, 18]:

$$R_i = 1 - F_i \tag{1}$$

where F_i is unreliability. The reliability of the entire R_S system, if the components in the system are regularly connected, is equal to the product of the reliability of the components or elements:

$$R_S = \prod_{i=1}^{n} R_i \tag{2}$$

The reliability of a complex system consisting of series and parallel connected components, in any combination, is calculated on the basis of the previous equations 1 and 2, starting from the smallest groups of parallel or series connected components. The reliability of these groups is represented by one member of the next larger group of components, all the way to the final series, as a rule, of regularly bound elements of the group. In Figure 3, one such example is given arbitrarily selected connection elements - system components.

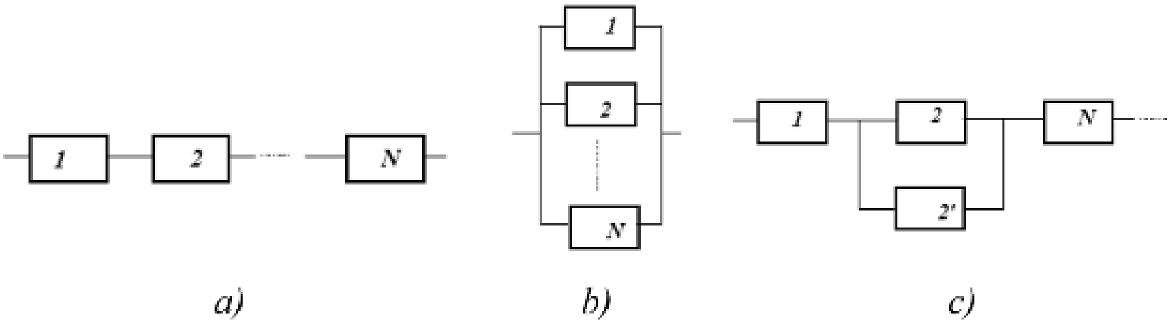

Figure 2. Series, parallel and combined structure of elementary reliability

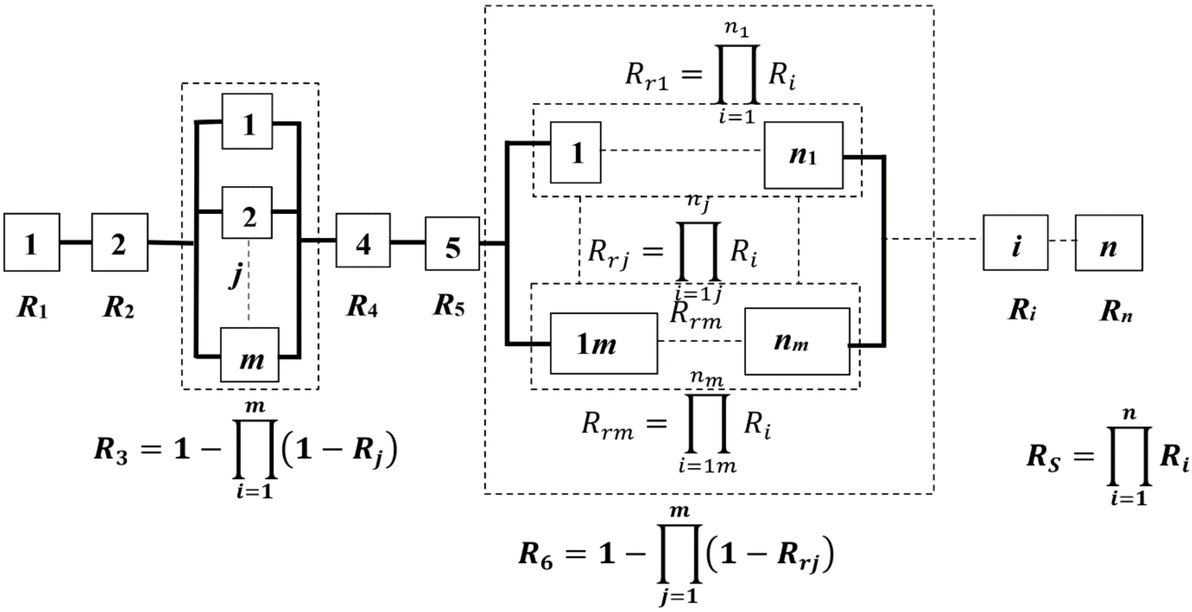

Figure 3. Example of complexly connected components of a machine system

The first task of reliability analysis is to find the value and reliability of the study. This task enables identification of properties and dependencies of a system reliability study. Reliability is defined in terms of probability, failure rate, mean time to failure (MTTF), and probability para¬meters, such as cumulative distribution functions.

Failure rate, λ(t), is defined as the ratio between the failure probability in the time interval $[t_1, t_2]$ and the product of the failure probability at t_1 and the time interval [3]:

For most machine systems, the failure rate follows a “bathtub

$$\lambda(t) = \frac{R(t_1) - R(t_2)}{(t_1 - t_2)R(t_1)} = \frac{R(t) - R(t + \Delta t)}{\Delta t R(t)} \tag{3}$$

curve”, as shown in Fig. 4 [3, 7, 18]. The “bathtub-curve” can be divided into three different areas: *I*-for the occurrence of early failure - the period of running in, *II*-for accidental failure-the period of normal work and *III*-permanent failure-the period of accelerated failure. Area I is characterized by a reduction in the failure rate. The risk of a part failing decreases with increasing time. Such early failure is mainly caused by errors in assembly, production, material or certain design defects. The instantaneous rate is constant in *II*. Thus, the risk of failure remains the same. In most cases, this risk is also relatively low. Such failure is caused, for example, by failure to work or maintenance or dirt particles. It is usually difficult to avoid such failures. The failure rate increases rapidly in the edge failure section (*III*). The risk of a part failing increases rapidly over time.

The reliability function becomes:

$$R(t) = e^{-\lambda \cdot t} \tag{4}$$

where the expected mean time to failure-*MTTF*, is:

$$MTTF = \int_0^{\infty} t \cdot f(t)\, dt = \int_0^{t} t \left[-\frac{dR(t)}{dt} \right] \cdot dt \tag{5}$$

To realize the set goal, the reliability distribution, and display modeling parts of the "bathtub-curve" is a convenient feature of Weibull distribution. The function is nonlinear and if represented as exponential, for example $\lambda(t) = \lambda_1 t^2$ as shown in Figure 4, gives a good shape.

In this model we note that if the exponent $Y = 0$ we are in the flat part of the "bathtub-curve" - the period of normal work, if is $Y < 0$, running-in period, if is $Y > 1$, a period of accele-rated failure. Weibull's distribution is presented in a similar way, but instead of Y, Weibull basically used $Y = \beta - 1$, the complete two-parameter Weibull model [7].

For machine system applications, independent events are the failure or successful execution of the intended function of each of n randomly selected, independently operating components.

If we want the probability that all of them continue to function even after t hours, we apply the multiplication rule and form the product of $n\ R(t)$ terms.

In other words, the probability that n independent identical components, with the reliability $R(t)$, all survive in the execution of functions in t hours is. $[R(t)]^n$. The probability that at least one of the n components fails is obtained by subtracting the probability that they will all remain successful in performing the intended function from 1. Applying the complement rule, the probability that at least one of n independent identical components fails to survive in time t is: $1 - [R(t)]^n = 1 - [1 - F(t)]^n$ [24].

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^{\beta}} \tag{6}$$

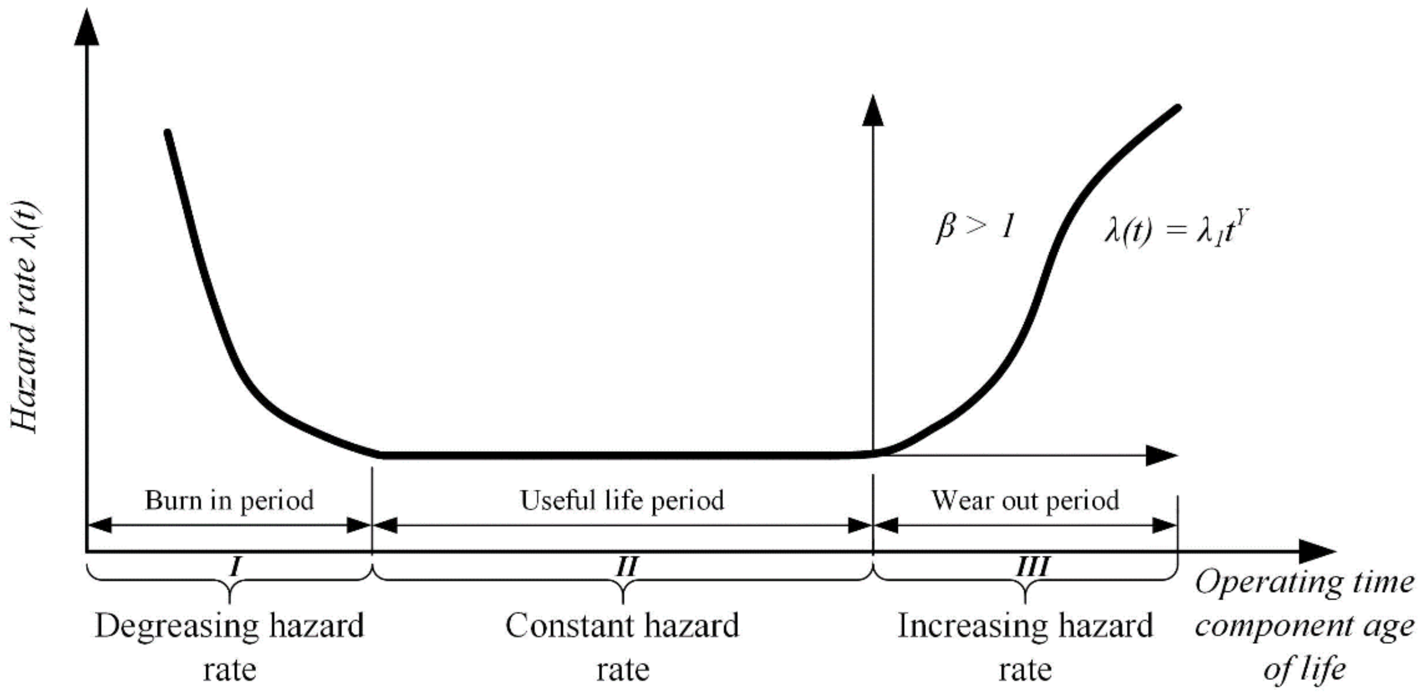

Figure 4. Bathtub curve

In practice, the instantaneous and average failure rate is defined as the failure rate limit when the time interval is infinitesimally small, as given by [18]:

$$\lambda(t) = \frac{\beta}{\eta} \cdot \left(\frac{t}{\eta}\right)^{\beta-1} \quad (7)$$

Instantaneous failure rate $\lambda(t) = \left(\frac{\beta}{t_{63}}\right) \cdot \left(\frac{t}{t_{63}}\right)^{\beta-1}$ is for the reliability engineer generally higher than the average failure rate $< \lambda >= \frac{1}{t} \cdot \left(\frac{t}{t_{63}}\right)^{\beta}$.

In Fig. 4, the wear axis is shifted to zero time, which is essentially just a time change. Any reliability distribution can have a time lag, to be included in the popular Weibull model. The Weibull model is often considered the primary reliability model. This is due to the physical significance of β: the worn region is designated $\beta > 1$, running-in region for $\beta < 1$, and steady state for $\beta = 1$, as illustrated in Fig. 4.

3. Design reliability allocation and evaluation methods

Over the years, many methods have been developed to assign reliability to components and evaluate for use during the design phase, [1, 19, 28,]. Reliability allocation and evolution, is a method and technique that is considered useful, especially in the design of new mechanical systems in order to achieve the goal set in this paper.

This method deals with assigning a failure rate to system components when the required system failure rate is known. The basics of the method are [28]:

- All system components fail independently.
- Failed component rates are constant.
- System components form a serial structure.

System failure rate:

$$\lambda_S(t) = \sum_{i=1}^{t} \lambda_i \quad (8)$$

λ_S = system failure rate (Hazard rate),
$m =$ total number of system components,
$\lambda_i =$ component failure rate i; for $i = 1,2,3, \ldots, m$.

If the system failure rate is specified λ_{Sp}, the allocated component failure rate should be $\sum_{i=1}^{m} \lambda_i^* \leq \lambda_{Sp}$Where λ_i^* failure rate, assigned to the component i, for $i = 1,2,3, \ldots, m$.

The following three steps are related to this approach:

- Estimation of the failure rate of the system components (i.e., λ_i for $i = 1,2,3, \ldots, m$) using component data.

- Calculate the relative weight, α_i, and components using the failure rate data in the previous step and the following expression:

$$\alpha_i = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i^*} \text{for} i = 1,2,3, \ldots, m \quad (9)$$

It should be noted that α_i represents the relative failure sensitivity of component i and that

$$\sum_{i=1}^{m} \alpha_i = 1 \quad (10)$$

- Assign the component failure rate also using the following equation:

$$\lambda_i^* = \alpha_i \cdot \lambda_{Sp} \text{for} i = 1,2,3, \ldots, m \quad (11)$$

4. Validation of real-time reliability modelling

The example of the selected planetary gear (Fig. 5 and Tab. 1) will show how, starting from individual models of failure regimes and using a competitive model of risk and ordinal connection as building blocks, the lower calculations of subassembly failure rates and/or system failure rates are done. (Fig. 6). Given that system-level testing is usually limited by time and cost constraints, this “bottom-up” approach is of great practical value for developed projected failure rates that can be used as indicators of gear tooth failure in the development of new shapes.

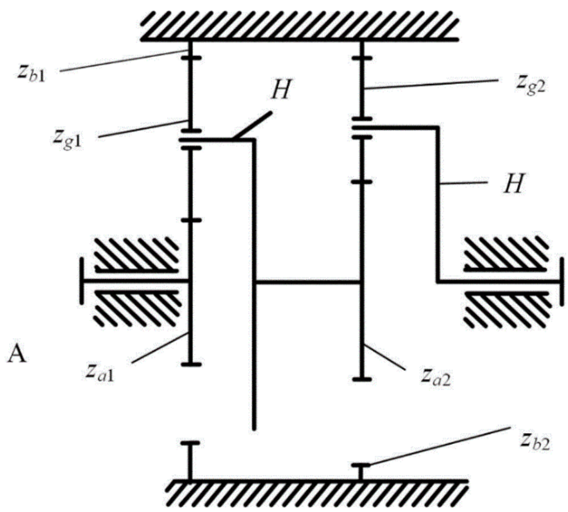

Figure 5. Kinematic scheme of the considered planetary gear train

Table 1. Experimental planetary gear train

Planetary set	I	II
Gears module	$m_{n1} = 3.5$ mm	$m_{n2} = 4$
Number of teeth of the sun gear	$z_{a1} = 12$	$z_{a1} = 13$
Number of teeth of planets gears	$z_{g1} = 30$	$z_{g2} = 26$
Number of teeth of ring gear	$z_{b1} = 72$	$z_{b2} = 65$
Torques	$T_{a1} = 538$ Nm	$T_{a2} = 3766$ Nm
Speed of pinion gears	$n_{a1} = 1108 \text{ min}^{-1}$	$n_{a2} = 158.286\text{min}^{-1}$
Planet gears speed	$n_{g1} = -221.6 \text{ min}^{-1}$	$n_{g2} = -39.573\text{min}^{-1}$
Operating stress at the flanks of the teeth	$\sigma_{Ha1} = \sigma_{Hg1} = 1421 \frac{N}{\text{mm}^2}$	$\sigma_{Ha2} = \sigma_{Hg2} = 2412 \frac{N}{\text{mm}^2}$
Number of planets gears	3	3

For planetary gear components (Fig. 5), were values determined for instantaneous failure rates (IFR), average instantaneous failure rates (AFR), assigned failure rates (λ_i^*), mean time to failure (*MTTF*), and reliability ($R_i(t)$), for all failure rates for constituent components (Table 2).

Error data sometimes cannot be modeled by a single failure time distribution. This is common in situations where the unit fails in different failure modes due to different failure mechanisms. In such situations, failure data can be modeled using competing risk models or a combination of failure rate models. Suppose that a non-repairable-replaceable component or unit *n* has different ways in which it can fail. Such failures are called cancellation regimes, and each of the cancellation modes is a failure mechanism.

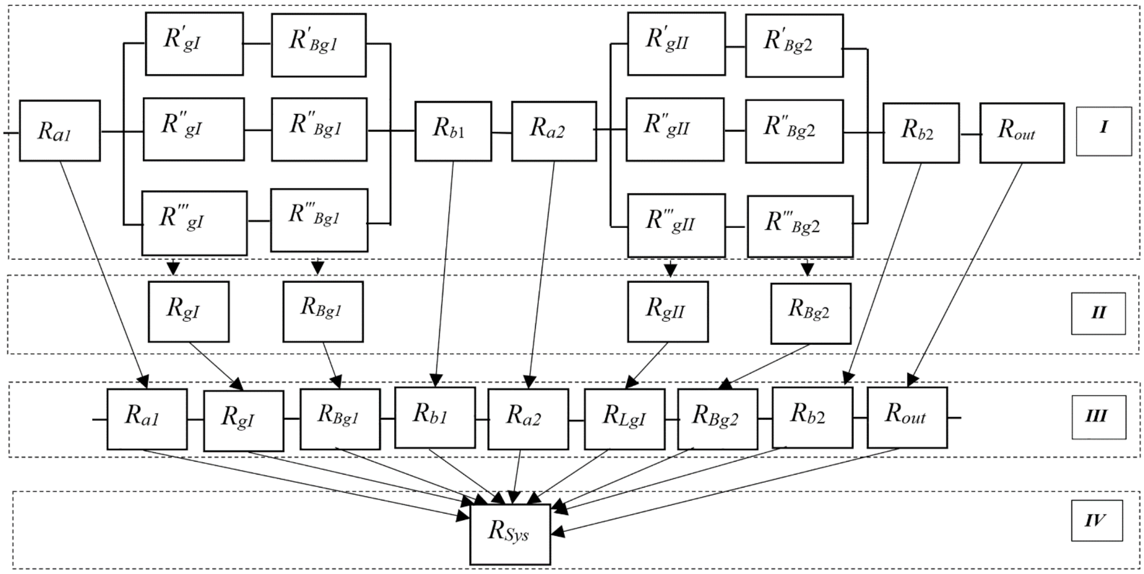

Figure 6. Decomposition of total reliability of planetary gear train $R_{a1,}R_{a2}$-reliability of pinion of *I* and *II* sets, R_{gI}, Rg_{II}-total. Reliability of planet gear of *I* and *II* sets, R_{Bg1}, R_{Bg2}-total reliability of bearings of *I* and *II* sets, R_{b1}, R_{b2}-reliability of ring gear *I* and *II* sets, R_{out}-reliability output shaft

Table 2. Determined values for instantaneous failure rates (IFR), average instantaneous failure rates (AFR), assigned failure rates (λ_i^*), mean time to failure (*MTTF*), and reliability ($R_i(t)$), for all failure rates for constituent components

		Z_{a1}	Z_{g1}	*Sum λ* Z_{g1}	Z_{b1}	Z_{a2}	Z_{g2}	*Sum λ* Z_{g2}	Z_{b2}
rpm min^{-1}		1108	443.2		92.33	158.286	79.128		15.82
$n_{\Sigma i}$		39.9e^{6}	5.32e^{6}		3.32e^{6}	5.69e^{6}	9.5e^{5}		5.69e^{5}
A	IFRλ	2.396e^{-9}	8.75e^{-10}	5.64e^{-14}	6.91e^{-10}	1.138e^{-7}	1.9e^{-8}	1.79e^{-11}	1.138e^{-8}
B	AFRλ	1.597e^{-9}	5.83e^{-10}	1.67e^{-14}	4.60e^{-10}	5.69e^{-8}	9.5e^{-9}	2.27e^{-12}	5.69e^{-9}
$\sum_{i=1}^{m} \lambda_i$	A	1.282e^{-7}		1.282e^{-7}	1.282e^{-7}	1.282e^{-7}		1.282e^{-7}	1.282e^{-7}
	B	7.415e^{-8}		7.415e^{-8}	7.415e^{-8}	7.415e^{-8}		7.415e^{-8}	7.415e^{-8}
$\alpha_i = \frac{\lambda_i}{\sum_{i=1}^{m} \lambda_i}$	A	1.868e^{-2}		4.399e^{-7}	5.393e^{-3}	0.887		1.397e^{-4}	0.088
	B	2.153e^{-2}		2.260e^{-7}	6.215e^{-3}	0.767		3.073e^{-5}	0.076
λ_{Sp},	A	1.138e^{-7}		1.138e^{-7}	1.138e^{-7}	1.138e^{-7}		1.138e^{-7}	1.138e^{-7}
	B	5.69e^{-8}		5.69e^{-8}	5.69e^{-8}	5.69e^{-8}		5.69e^{-8}	5.69e^{-8}
Assigned failure rates $\lambda_i^* = \alpha_i \cdot \lambda_{Sp}$,	A	2.125e^{-9}		5.01e^{-14}	6.13e^{-10}	1.009e^{-7}		1.59e^{-11}	1.001e^{-8}
	B	1.225e^{-9}		1.28e^{-14}	3.53e^{-10}	4.364e^{-8}		1.74e^{-12}	4.324e^{-9}
$\sum_{i=1}^{m} \lambda_i^* \leq \lambda_{Sp}$	A	1.136e^{-7}		1.136e^{-7}	1.136e^{-7}	1.136e^{-7}		1.136e^{-7}	1.136e^{-7}
	B	4.954e^{-8}		4.954e^{-8}	4.954e^{-8}	4.954e^{-8}		4.954e^{-8}	4.954e^{-8}
$R_i(t) = e^{-\lambda_i \cdot t}$	A	0.908		0.999	0.997	0.523		0.999	0.993
	B	0.938		0.999	0.998	0.723		0.999	0.996
$R_i(t) = e^{-\lambda_i^* \cdot t}$	A	0.918		0.999	0.997	0.563		0.999	0.994
	B	0.952		0.999	0.998	0.780		0.999	0.997
MTTF IFR and AFR	A	4.173e^{8}	1.142e^{9}	2.093e^{9}	1.447e^{9}	8.787e^{6}	5.263e^{7}	1.024e^{11}	8.787e^{7}
	B	6.261e^{8}	1.7152e^{9}	1.996e^{13}	2.173e^{9}	1.757e^{7}	1.052e^{8}	8.075e^{11}	1.757e^{8}
MTTF (For AFR)	A	4.705e^{8}		3.144e^{9}	1.631e^{9}	9.910e^{6}		1.152e^{11}	9.99e^{7}
	B	8.163e^{8}		1.432e^{14}	2.832e^{9}	2.291e^{7}		5.747e^{11}	2.312e^{8}

There are three prerequisites for this model:

(i) the failure modes are independent of each other,

(ii) the unit fails when the first of failure mechanisms reaches the failure state, and

(iii) each mode of cancellation has its own distribution of cancellation times.

The model considers a unit with n failure modes and distribution function $F_i(t), i = 1,2 \dots, n$ for the time to failure T_i due to failure mechanism i. The failure time of the unit is the minimum of $\{T_1, T_{2,} \dots, Tn\}$, whereas the distribution function $F(t)$ is:

$$F(t) = 1 - [1 - F_1(t)] \cdot [1 - F_2(t)] \cdots [1 - F_n(t)] \quad (12)$$

Reliability function is:

$$R(t) = \prod_{i=1}^{n} R_i\,(t) \quad (13)$$

Hazard function (rate) is:

$$h(t) = \sum_{i}^{n} h_i(t) \quad (14)$$

The competitive risk model should be understood as follows:

- All the failure mechanisms are in a race to reach failure.
- They are not allowed to see the progress of the other ones. They just go their own way as fast as they can and the first to reach "failure" causes the component to fail.
- Under these conditions the component reliability is the product of failure mode reliabilities and the component failure rate is just the sum of failure rates.

Note that this holds for any arbitrary life distribution model, if "independence" and "first mechanism failure causes the component to fail" holds. The decomposition of the total reliability of the planetary gear (Fig. 6) is based on the procedure discussed in [19].

5. Results and discussion

During the processing of reliability data in the presence of instantaneous, estimated and assigned failure rates, specific operating environments and subject to time constraints, it is necessary to determine the order of testing the activities of the planetary gear as a machine system. The “bottom-up” approach proposed in this paper attempts to control changes in the failure rate during the operational time of each planetary gear component.

Reliability model has been compiled and developed that allows to predict the evolution of the failure process when components are exposed to various changes in the number of duty cycles ($n_{\Sigma i}$) and loads. Results are shown in Tab 2.

Bearings as integral components of planetary gears are not subject to the same failure modes as gears. Their reliability has already been determined on the basis of Weibull's probability distribution of damage for the characteristic service life (η) for which 63.2% of bearings will fail. For $N_C = 10^6$ estimated Weibull distribution parameters $\eta = 817277$ rpm and $\beta = 0.7459$, i.e. Weibull distribution function is: $P_R = 1 - e^{-\left(\frac{N}{8172770}\right)^{0,7459}}$.

Values for the reliability of the bearings of the first and second planetary set $R_{Bg1} = R_{Bg2} = 1$ were estimated (They were not the subject of research in this paper).

Reliability value $R_{out} = 1$ was taken for the output shaft of the planetary gearbox, respectively. To verify the obtained results in Tab. 2, it was preceded by an experimental test of the planetary gear in a closed power circuit, according to the FZG method. The planetary gear is declared for certain nominal operating conditions, i.e., for a certain nominal torque and for a certain nominal speed of rotation. In exploitation, these conditions can appear occasionally, and as a rule, most of the working life will be easier working conditions, i.e., lower torque or lower rotation speed. Heavier operating conditions than the nominal ones are also possible, i.e., significantly higher torque or speed of rotation that occur occasionally and in shorter time intervals.

Transmission is powered directly by a 1500 min^{-1} engine. This only contributed to shorten the testing time. The torque used for the test corresponds approximately to the maximum torque that can occur during operation. The next analysis will be performed under the assumption that the frequency of rotation of the sun gear of the first planetary set is 1108 min^{-1} and that the maximum torque on this gear is equal to that in the test of 538 Nm.

Zero thickness of worn layer corresponds to zero probability. Since the process of surface destruction of the tooth flanks is long-lasting and gradual, the Weibull function is also extended over a large interval of the number of stress changes, i.e., the number of gear teeth [32].

For the stress σ_{Ha1} = 1421 N/mm^2, the pinion gear of the first planetary set, teeth number of teeth duty cycles, $n_\Sigma = 39.9\text{x}10^6$, and thickness of the worn layer of 0.2 mm (corresponds to $0.06m_n$), the estimated parameters of the Weibull distribution could be $\eta = 2.5e10^8$ and $\beta = 1.5$, i.e.the Weibull distribution function is:

$$P_{R1} = 1 - e^{-\left(\frac{N}{2,5\cdot 10^8}\right)^{1,5}}, P_{R1} = 1 - e^{-\left(\frac{39.9e^6}{2,5\cdot 10^8}\right)^{1,5}} = 0.0617.$$

Reliability is: $R_{R1} = 1 - P_{R1} = 1 - 0.0617 = 0.9383$.

For the pinion gear of the second planetary set $\sigma_{Ha2} = 2412$ N/mm^2, for the determined of teeth number of teeth duty cycles $n_\Sigma = 5.69e^6$, and the thickness of the worn layer is about 0.6 mm, i.e., $0.15m_n$. The estimated parameters of the Weibull distribution could be $\eta = 10^6$ and $\beta = 2$, i.e., the Weibull distribution function is:

$$P_{R2} = 1 - e^{-\left(\frac{N}{10^6}\right)^2}, P_{R2} = 1 - e^{-\left(\frac{5.69e^6}{10^6}\right)^2} = 0.2765.$$

Reliability is: $R_{R2} = 1 - P_{R2} = 1 - 0.2765 = 0.7235$.

Sun and satellite gears were made with level seven of manufacturing accuracy of heat treated steel Č4732 -20CrMo5 (SRPS EN ISO 683-1:2018). Bevel gears were made of steel Č4732 - 42CrMo4 (SRPS EN ISO 683-1:2018). Oil with viscosity of 41.4 – 50.6 mm^2/s was used for lubrication and cooling of the gearbox. Gears with internal toothing were made from a single piece with a cylindrical casing, [18]. In this part of the paper, several validation tests were performed.

Figures 7-9 show the results of validation tests for instantaneous (IFR) and average failure rates (AFR), for gears $z_{a1}, z_{a2}, z_{g1}, z_{g2}, z_{b1}$ and z_{b2}, whereas Figues 10-12 show mean times to damage (*MTTF*) for the same gears. Figures 13-14 show I and II sets of the reliability functions, $R(t)_I$, $R(t)_{II}$, and the reliability function of the planetary gear- $R(t)_S$.

For a system consisting of n series-connected components (or having n independent failure rates, each of which has an independent Weibull failure distribution with the form parameter β and the parameter η_i - time (cycle), the system failure rate function can be determined from [27]:

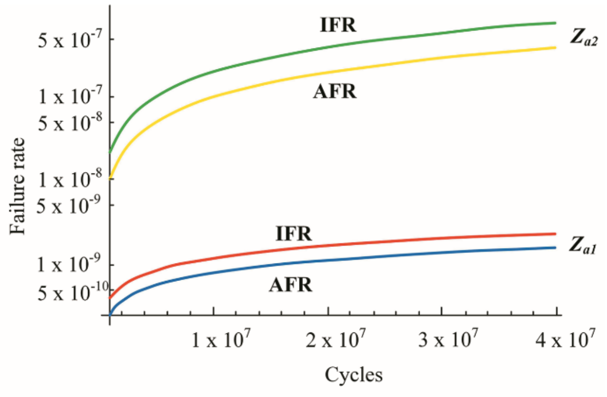

Figure 7. Instantaneous and average failure rates for pinion gears z_{a1} and z_{a2}, $n_{\Sigma} = 1e^6 - 39.9e^6$

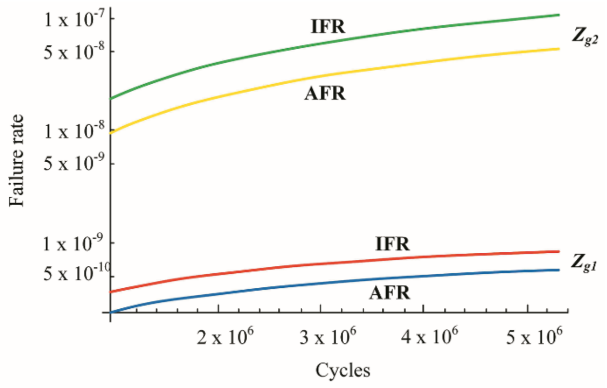

Figure 8. Instantaneous and average failure rates for planet gears z_{g1} and z_{g2}, $n_{\Sigma} = 9.5e^5 - 5.32e^6$

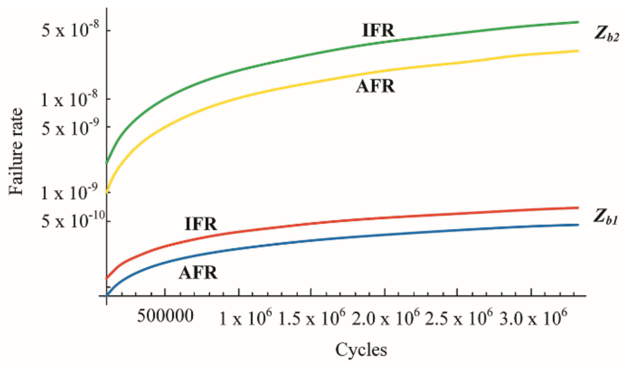

Figure 9. Instantaneous and average failure rates

$$\lambda(t) = \sum_{i=1}^{n} \frac{\beta}{\eta_i} \cdot \left(\frac{t}{\eta_i}\right)^{\beta-1} = \beta \cdot t^{\beta-1} \cdot \left[\sum_{i=1}^{n} \left(\frac{1}{\eta_i}\right)^{\beta}\right] \lambda(t) = \tag{15}$$
$$= \sum_{i=1}^{n} \frac{\beta}{\eta_i} \cdot \left(\frac{t}{\eta_i}\right)^{\beta-1} = \beta \cdot t^{\beta-1} \cdot \left[\sum_{i=1}^{n} \left(\frac{1}{\eta_i}\right)^{\beta}\right]$$

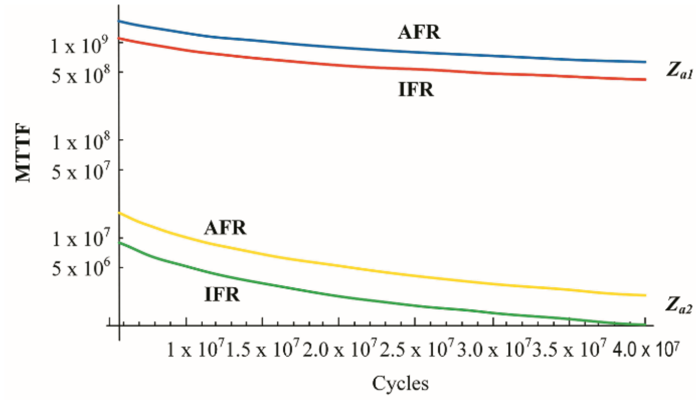

Figure 10. MTTF for pinion gears z_{a1} and z_{a2}, z_{b1} and z_{b2}, $n_{\Sigma} = 1e^5 - 3.32e^6$ $n_{\Sigma} = 5.69e^5 - 39.9e^6$

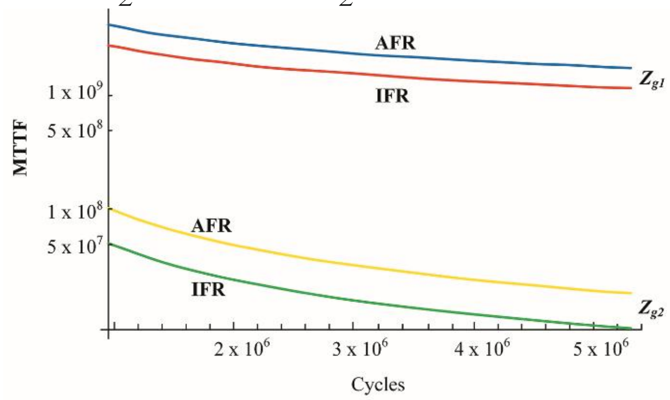

Figure 11. *MTTF* for planet gears z_{g1} and z_{g2} $n_{\Sigma} = 9.5e^5 - 5.32e^6$

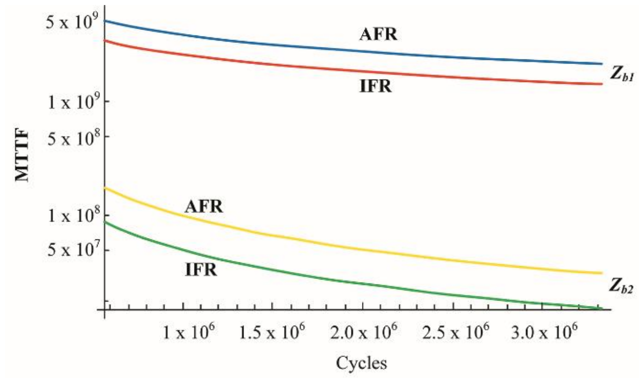

Figure 12. *MTTF* for ring gears z_{b1} and z_{b2}, $n_{\Sigma} = 5.69e^6 - 3.32e^6$

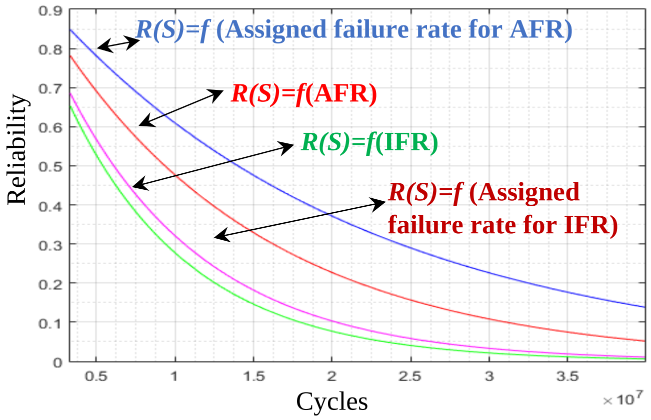

Figure 13. Reliability system for the $n_{\Sigma} = 3.32e^6 - 39.9e^6$

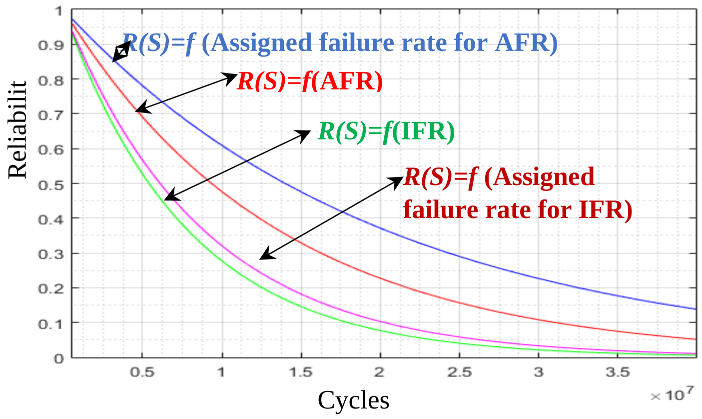

Figure 14. Reliability system for the $n_{\Sigma} = 5.69e^6 - 39.9e^6$

Figure 6 (*II*) shows the ordinal connection of the components. The system consists of $n = 9$ components, bearing reliability of the first and second planetary sets $R_{Bg1} = R_{Bg2} = 1$ and for the output shaft $R_{out} = 1$. For the reliability components R_{a1}, R_{gI} and R_{b1}, the distribution of damage probabilities is given by the Weibull cumulative function $P_{R1} = 1 - e^{-\left(\frac{N}{2,5\cdot 10^8}\right)^{1,5}}$. For reliability components R_{a2}, R_{gII} and R_{b2} function is: $P_{R2} = 1 - e^{-\left(\frac{N}{10^6}\right)^2}$.

The total failure rate of the Weibull distribution is valid only when each component has the same β form parameter and if all failure rates are Weibull but with a different β form parameter, then the system failure rate distribution will not be Weibull. This rule is characteristic for the distribution of the failure rate of the flanks of the teeth of the considered planetary gear. This demonstrates the convenience of a "bottom-up" approach for determining the reliability of planetary gears. And it leads us to the need to determine the *MTTF* and define the reliability function only on the basis of the number of stress changes- n_{Σ}, for the components $R_{a1}, R_{gI}, R_{b1}, R_{a2}, R_{gII}$ and R_{b2}. The planetary gear of the first set has Weibull distribution of the failure rate with the shape parameter $\beta = 1.5$ and the second with $\beta = 2$. Characteristic "life"- η_i (number of stress-cycle changes, number of teeth coupling changes), for the first and second planetary set, using the data from Tab. 2 are: $\eta_I = \left[\left(\frac{1}{n_{\Sigma a_1}}\right)^{\beta_I} + \left(\frac{1}{n_{\Sigma g I}}\right)^{\beta_I} + \left(\frac{1}{n_{\Sigma b_1}}\right)^{\beta_I}\right]^{\frac{-1}{\beta_I}} = \left[\left(\frac{1}{39.9e^6}\right)^{1.5} + \left(\frac{1}{5.32e^6}\right)^{1.5} + \left(\frac{1}{3.32e^6}\right)^{1.5}\right]^{\frac{-1}{1.5}}$ =2.514e^6 cycles, $\eta_{II} = \left[\left(\frac{1}{n_{\Sigma a_2}}\right)^{\beta_{II}} + \left(\frac{1}{n_{\Sigma g II}}\right)^{\beta_{II}} + \left(\frac{1}{n_{\Sigma b_2}}\right)^{\beta_{II}}\right]^{\frac{-1}{\beta_{II}}} = \left[\left(\frac{1}{5.69e^6}\right)^2 + \left(\frac{1}{9.5e^5}\right)^2 + \left(\frac{1}{5.69e^5}\right)^2\right]^{\frac{-1}{2}}$ =4.863e^5 cycles.

The mean time to failure-*MTTF* for the first and second sets are: $MTTF_I = \eta_I \cdot \Gamma\left(1 + \frac{1}{\beta_I}\right) = 2.514e^6 \cdot \Gamma\left(1 + \frac{1}{1.5}\right) = 2.514e^6 \cdot 0.90167 = 2.266e^6$ cycles, $MTTF_{II} = \eta_{II} \cdot \Gamma\left(1 + \frac{1}{\beta_{II}}\right) = 4.863e^5 \cdot \Gamma\left(1 + \frac{1}{2}\right) = 4.863e^5 \cdot 0.88623$ =4.309e^5 cycles. For the gamma function at positive integer argument, see [6].

Therefore, medians are: $t_{medI} = \eta_I \cdot (0.69315)^{\frac{1}{\beta_I}} = 2.514e^6 \cdot (0.69315)^{\frac{1}{1.5}}$ =1.969e^6 cycles, $t_{medII} = \eta_{II} \cdot (0.69315)^{\frac{1}{\beta_{II}}} = (4.863e^5) \cdot (0.69315)^{\frac{1}{2}}$ =4.048e^5 cycles.

To illustrate the application of a competitive risk model, we consider a product that has two different failure modes and for each Weibull distribution applies [27]. Product reliability is $R(t) = R_1(t) \cdot R_2(t) = e^{-\left(\frac{t}{\eta_1}\right)^{\beta_1}} \cdot e^{-\left(\frac{t}{\eta_2}\right)^{\beta_2}}$ where η_i and β_i are the scale and shape parameters, respectively, of failure mode *i*. The distribution density function is (Fig. 15):

$$f(t) = R_1(t)f_2(t) + R_2(t)f_1(t) = R(t)\left[\frac{\beta_1}{\eta_1} \cdot \left(\frac{t}{\eta_1}\right)^{\beta_1 - 1} + \frac{\beta_2}{\eta_2} \cdot \left(\frac{t}{\eta_2}\right)^{\beta_2 - 1}\right].$$

and the function of the failure rate is (Fig.16):

$$h(t) = h_1(t) + h_2(t) = \beta_1 \cdot \eta_1^{-\beta_1} \cdot t^{\beta_1 - 1} + \beta_2 \cdot \eta_2^{-\beta_2} \cdot t^{\beta_2 - 1}.$$

The characteristics of the resulting *f(t)* and *h(t)* depend on the parameter values η_1, η_2, β_1 and β_2.

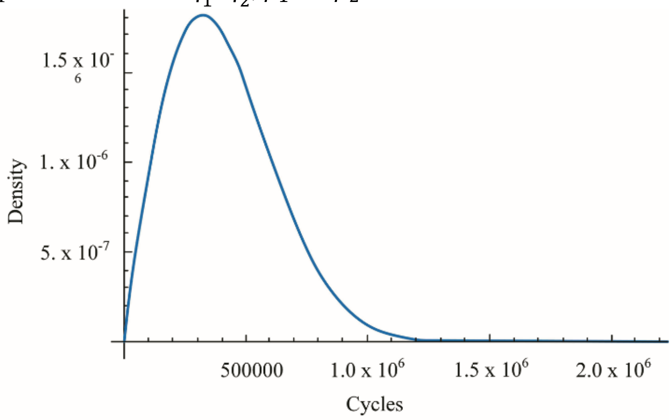

Figure 15. Distribution density function $f(t)$

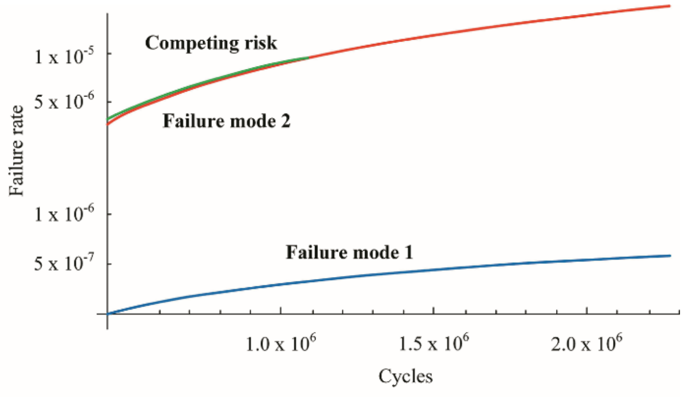

Figure 16. Failure rate functions

Hazard rate $h(t)$ shows different characteristics: decreasing, constant and increasing depending on the value and the relationship between these parameters. Reliability function (Fig. 17), for *I*, *II* set and planetary gear train, as a mechanical system based on the competing risk model, is:

$$R(t)_I = exp\left[-\left(\frac{t_1}{\eta_I}\right)^{\beta_I}\right] = exp\left[-\left(\frac{t_1}{2.514e^6}\right)^{1.5}\right],$$

$$R(t)_{II} = exp\left[-\left(\frac{t_2}{\eta_{II}}\right)^{\beta_{II}}\right] = exp\left[-\left(\frac{t_2}{4.863e^5}\right)^{2}\right],$$

$$R(t)_S = exp\left[-\left(\frac{t_1}{2.514e^6}\right)^{1.5}\right] * exp\left[-\left(\frac{t_2}{4.863e^5}\right)^{2}\right]$$

The reliability function of the planetary gear train is presented as a function of for the determined of teeth number duty cycles of gears *I* and *II* of the set. The reliability of the competitive risk model is shown in Fig.17. It is obvious that a competitive risk model results in more accurate reliability estimates than modeling each failure regime separately.

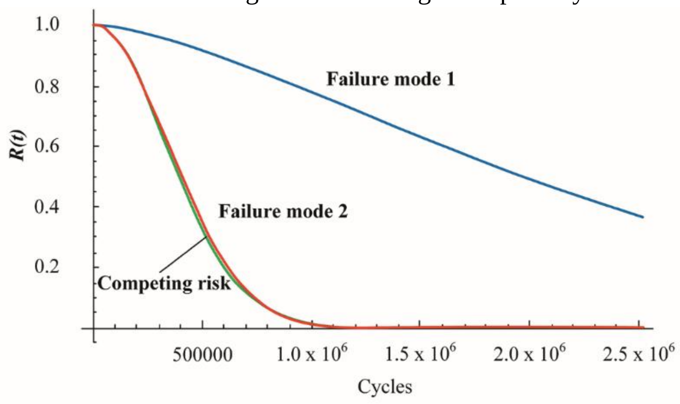

Figure 17. Reliability of the competing risk model

6. Conclusions

The applied "bottom-up" approach in this research it was shown that it is possible rationally and quickly with the use of a small set of data to arrive at the reliability of the constituent components and the reliability of complete planetary gear transmissions.

- For the realization of the set goal, by applying the Weibull function for the distribution of damage to the flanks of the gear teeth, specific values were determined for the components of the toothed planetary transmission, which was the subject of research.
- For a more precise assessment of reliability and to avoid modeling every failure and its mode of occurrence, a competitive risk model was developed.
- For established values for instantaneous failure rates (IFR), average instantaneous failure rates (AFR), assigned failure rates (λ_i^*), established are mean time to failure (*MTTF*) and the reliability for the all components and planetary gear in the whole. The obtained results were numerically and graphically interpreted.
- The characteristic "lifetime"- η_i (number of stress cycle changes, number of tooth coupling changes) was determined for the first and second planetary set.
- The results obtained in this paper provide an attempt to contribute to the wider academic community, that they can be applied and/or verified, for use in determining the reliability of planetary transmissions in development, as new construction solutions or in exploitation.

Guidelines for further research, consisting of further attempts to upgrade or supplement the work with new data, in order to confirm and/or verify the data obtained in this work.

Acknowledgements

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