Scientific Journal of Silesian University of Technology. Series Transport
Zeszyty Naukowe Politechniki Śląskiej. Seria Transport

Volume 109 **2020**

p-ISSN: 0209-3324

e-ISSN: 2450-1549

DOI: https://doi.org/10.20858/sjsutst.2020.109.13

Journal homepage: http://sjsutst.polsl.pl

Article citation information:
Pashayev, A., Sabziev, E. Refinement of the parameters of a mathematical model of quadcopter dynamics. *Scientific Journal of Silesian University of Technology. Series Transport*. 2020, **109**, 141-151. ISSN: 0209-3324.
DOI: https://doi.org/10.20858/sjsutst.2020.109.13.

Adalat PASHAYEV[1], Elkhan SABZIEV[2]

REFINEMENT OF THE PARAMETERS OF A MATHEMATICAL MODEL OF QUADCOPTER DYNAMICS

Summary. Errors in the calculation of the parameters of quadcopter control models at design stage significantly change the desired aerodynamic properties of the drone and make it difficult to control its flight along the intended path. Therefore, to calculate the adequate operation modes of the blades, it becomes necessary to refine some parameters of the mathematical model of the drone as accurately as possible. This paper shows the possibility of using control parameters (rotational speed of the blades) and information received from navigation devices of the drone to refine the values of the parameters of the mathematical model of the drone. For this purpose, a mathematical model of a quadcopter is built, and the problem of refining the parameters of its dynamic model is investigated based on the information received from navigation devices and the control parameters in the initial period of its flight. From the results obtained from several consecutive measurements, a system of equations expressing a mathematical model is solved. The mean value of the corresponding solutions of the system of three-dimensional linear equations obtained at different time intervals is the refined value of the parameters.

[1] Institute of Control Systems of Azerbaijan National Academy of Sciences, 9, B.Vahabzade St., Baku-AZ1141, Azerbaijan. Email adalat.pashayev@gmail.com. ORCID: https://orcid.org/ 0000-0001-9208-8430
[2] Institute of Control Systems of Azerbaijan National Academy of Sciences, 9, B.Vahabzade St., Baku-AZ1141, Azerbaijan. Email elkhan.sabziev@gmail.com. ORCID: https://orcid.org/0000-0001-8150-9439

Keywords: mathematical model, identification, moment of inertia, windage, quadrocopter

1. INTRODUCTION

The low cost of unmanned aerial vehicles (drones) has given impetus to their widespread use for military purposes. The possibility of equipping them with appropriate hardware makes it possible for them to perform various tasks. Quadcopters should be mentioned among such currently widely used vehicles. As a rule, quadcopters are equipped with navigation devices (gyroscope, accelerometer, etc.), which allow obtaining information related to the drone’s location and aerial orientation and control its flight.

Dynamic mathematical models describe the control and movement of the quadcopter. In this study, the drone is considered as a solid, and its mathematical model expresses the relationship between the thrust generated by the motor and acceleration, air drag, torque, rotational speed and velocity of the drone relative to the earth. There is a wide range of studies dealing with the dynamic modelling of drones. For instance, [1] is devoted to the building of a mathematical model of a drone to solve the problem of eliminating its deviation from the intended flight path. In [2], a mathematical model was built to control the stability and flight path of a quadcopter. In the paper, feedback data related to the orientation of the drone is given in the form of Euler angles. In the mathematical model built in [3], the aerodynamic forces acting on the drone are not considered, and the case of its flight at low speeds was studied. In [4], a mathematical model that defines the spatial position of the quadcopter by Euler angles was developed and used to create a flight simulation program. In [5], a mathematical model was developed, the quadcopter’s spatial position being expressed by Euler angles, and a controller stabilising the altitude and orientation of the drone was developed based on this model.

Several currently designed drone models provide for the use of MPU-9250 type devices [6]. Since this device does not measure Euler angles, it is impossible to directly apply the results of the above studies to solving problems of quadcopter control when using it. Thus, the accelerometer and gyroscope of MPU-9250 allow calculating the loads that occur during movement, as well as the rate of change of Krylov rotation angles (roll, pitch and yaw) [7, P.9], which determine the spatial position of the drone, and current orientation angles by integrating these speeds.

A mathematical model of the quadcopter control problem is built in this paper based on feedback data received from an MPU-9250 type device. The rotational speeds of the quadcopter blades are used as control parameters. It is assumed that:

- the drone has 4 symmetrically fixed identical motors that rotate its blades;
- for clarity, if we indicate the drone motors as shown in Fig. 1, blades 1 and 3 rotate clockwise, and blades 2 and 4 counterclockwise;
- due to the small angular rotational velocities, the gyroscopic forces created by the movement of the drone can be neglected;
- gravity and aerodynamic drag affect the centre of mass of the drone, so these forces do not create a torque;
- the mass of the drone is distributed only along its arms, in other words, the drone's rotary inertia matrix has a diagonal shape.

The dynamic model of a quadcopter is expressed by numerous parameters. These parameters are usually calculated using empirical formulas at the drone design stage. Stable drone control requires these parameters to be known as accurately as possible.

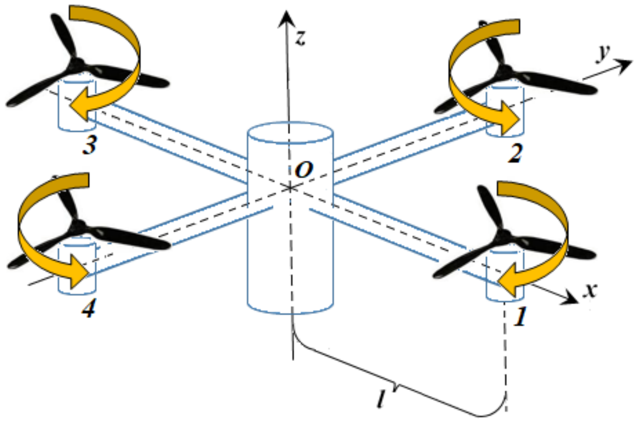

Fig. 1. Quadcopter schematic

Therefore, this paper explores the possibility of using the control parameters (rotational speed of the blades) and the data obtained from navigation devices of the drone to refine the values of the parameters of the mathematical model of the drone.

2. MATHEMATICAL MODEL OF THE DRONE

To determine the position of the quadcopter relative to the ground and build a dynamic model, a fixed-in-the-earth right-handed *normal* $O_g x_g y_g z_g$ *coordinate system* is used. For clarity, it is assumed that the $O_g z_g$ axis is directed vertically upward in the considered point, and the $O_g x_g$ and $O_g y_g$ axes are directed so that the $O_g x_g z_g$ plane is perpendicular to $O_g z_g$, forming a right-handed coordinate system.

We denote by $S_{gx}(t)$, $S_{gy}(t)$, $S_{gz}(t)$ the quadcopter's current coordinates relative to the $O_g x_g y_g z_g$ system at the considered instant t, and the components of the velocity vector by $v_{gx}(t)$, $v_{gy}(t)$, $v_{gz}(t)$. It is obvious that,

$$\begin{cases} S'_{gx} = v_{gx}(t), \\ S'_{gy} = v_{gy}(t), \\ S'_{gz} = v_{gz}(t). \end{cases} \tag{1}$$

The mathematical model of drone control provides for setting such a relationship between the flight path $\left(S_{gx}(t), S_{gy}(t), S_{gz}(t)\right)$ expressed by the coordinates in the $O_g x_g y_g z_g$ system and the angular rotational velocities of the blades $\omega_1, \omega_2, \omega_{31}, \omega_4$ that the following two problems can be solved:

- the calculation of the drone's flight path $\left(S_{gx}(t), S_{gy}(t), S_{gz}(t)\right)$ according to the angular rotational velocities $\omega_1, \omega_2, \omega_{31}, \omega_4$;

- the calculation of the angular rotational velocities $\omega_1, \omega_2, \omega_{31}, \omega_4$ for the execution of the given flight path $\left(S_{gx}(t), S_{gy}(t), S_{gz}(t)\right)$.

To specify the physical (inertial) characteristics of the quadcopter, we introduce a rectangular right-handed $Oxyz$ coordinate system fixed to it [7, P.23]. The quadcopter can be described schematically as shown in Fig. 1. Suppose that the origin of coordinates is located in the centre of the drone, the Ox and Oy axes are directed along its “arms”, and the Oz axis is directed upward perpendicular to the Oxy plane.

As mentioned above, when designing a drone control system, MPU-9250 type devices are used to provide feedback [6]. These devices allow expressing the spatial position (orientation) of the drone by the yaw angle $\psi(t)$, the pitch angle $\vartheta(t)$, the roll angle $\gamma(t)$. The definition of yaw, pitch and roll angles is given in [7, P.9]. These angles virtually indicate the spatial position of the $Oxyz$ coordinate system fixed to the drone relative to the normal $O_g x_g y_g z_g$ coordinate system. Matrix (2) can be used to find in the $Oxyz$ coordinate system the components of the vector given in the fixed-in-the-earth $O_g x_g y_g z_g$ coordinate system. For simplicity, for the angle ξ under consideration, S_ξ and C_ξ are written instead of $\sin\xi$ and $\cos\xi$ from here next.

$$\mathbf{A} = \begin{pmatrix} C_\vartheta C_\psi & -C_\vartheta S_\psi & -S_\vartheta \\ C_\gamma S_\psi - S_\gamma S_\vartheta C_\psi & C_\gamma C_\psi + S_\gamma S_\vartheta S_\psi & -S_\gamma C_\vartheta \\ S_\gamma S_\psi + C_\gamma S_\vartheta C_\psi & S_\gamma C_\psi - C_\gamma S_\vartheta S_\psi & C_\gamma C_\vartheta \end{pmatrix}. \tag{2}$$

To build the drone control model, we write the equations of its movement relative to the $Oxyz$ coordinate system [9, P.128]:

$$m\left(\frac{d\mathbf{v}}{dt} + \mathbf{w}\times\mathbf{v}\right) = \mathbf{F}, \tag{3}$$

$$\mathbf{J\varepsilon} + \mathbf{w}\times(\mathbf{Jw}) = \mathbf{M}, \tag{4}$$

where m is the mass of the drone, $\mathbf{J}$ is the rotary inertia matrix, $\mathbf{v} = \left(v_x(t), v_y(t), v_z(t)\right)$ is the velocity of the drone, $\mathbf{w} = \left(w_x(t), w_y(t), w_z(t)\right)$ is the angular rotational velocity of the drone, $\mathbf{F}$ is the sum of the forces acting on the drone, $\mathbf{M}$ is the moment created by the forces acting on the drone. Equation (3) expresses the balance of forces, and equation (4) expresses the balance of moments. The relationship between the velocities $\mathbf{v}_g$ and $\mathbf{v}$ is determined by the following equality:

$$\frac{d\mathbf{v}_g}{dt} = \frac{d\mathbf{v}}{dt} + \mathbf{w}\times\mathbf{v}. \tag{5}$$

The position of the drone relative to the $O_g x_g y_g z_g$ coordinate system can be identified with the position of the $Oxyz$ coordinate system relative to $O_g x_g y_g z_g$. Taking this into account, the angular velocity $\mathbf{w}$ and the angular acceleration $\boldsymbol{\varepsilon}$ of the drone relative to $O_g x_g y_g z_g$ can be calculated as follows:

$$\mathbf{w} = \begin{pmatrix} 0 & S_\psi & C_\psi S_\vartheta \\ 0 & -C_\psi & S_\psi S_\vartheta \\ 1 & 0 & C_\vartheta \end{pmatrix} \begin{pmatrix} \psi' \\ \vartheta' \\ \gamma' \end{pmatrix}, \quad \boldsymbol{\varepsilon} = \begin{pmatrix} 0 & S_\psi & C_\psi S_\vartheta \\ 0 & -C_\psi & S_\psi S_\vartheta \\ 1 & 0 & C_\vartheta \end{pmatrix} \begin{pmatrix} \psi'' \\ \vartheta'' \\ \gamma'' \end{pmatrix}. \tag{6}$$

Let us give the procedure of calculation of the vectors $\mathbf{F}$ and $\mathbf{M}$ on the right-hand side of the system of equations (3)-(4).

Following the above conditions for the direction of rotation of the drone’s blades, the propelling force created by its motors is always oriented in the direction of the Oz axis of the fixed $Oxyz$ coordinate system:

$$\mathbf{F}_\omega = \left(0, 0, k\sum_{j=1}^{4}\omega_j^2\right)^*, \tag{7}$$

where k is a coefficient determined experimentally, ω_j is the rotational speed of the blades of the j-th motor. Here and thereafter, the asterisk indicates the transposition operation.

The aerodynamic drag is directed opposite to the movement of the drone:

$$\mathbf{F}_a = -C_a|\mathbf{v}|\mathbf{v}, \tag{8}$$

where C_a is the drag coefficient, which is proportional to the area of the projected plane perpendicular to the direction of the drone’s movement and depends on its aerodynamic shape. In the general case, the relationship between the coefficient C_a and the aerodynamic shape is very complex and depending on the direction of movement for the drone under consideration, its value can be determined experimentally [10]. Usually, for simplicity, this coefficient is assumed to be identical in all directions. Therefore, the force $\mathbf{F}_a$ can be broken into the following components:

$$\mathbf{F}_a = -C_a\sqrt{v_x^2(t)+v_y^2(t)+v_z^2(t)}\left(v_x(t), v_y(t), v_z(t)\right)^*. \tag{9}$$

Regardless of the spatial position, gravity acting on the drone is always directed vertically to the earth. Using the transformation matrix (1), we can obtain the following formula for expressing gravity relative to the $Oxyz$ coordinate system:

$$\mathbf{F}_p = \mathbf{A}(0, 0, -g)^*. \tag{10}$$

As mentioned above, it is assumed that gravity and aerodynamic drag affect the centre of mass of the drone, so these forces do not create a torque. Since the drone's motors are fixed to it, the torque generated by the rotation of the blades can be expressed in the $Oxyz$ coordinate system as follows:

$$\mathbf{M} = \begin{pmatrix} kl\left(\omega_2^2 - \omega_4^2\right) \\ kl\left(\omega_1^2 - \omega_3^2\right) \\ b\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right) \end{pmatrix}, \tag{11}$$

where l is the length of the arms of the drone, and b is a coefficient determined experimentally.

Also, according to the condition that the drone rotary inertia matrix has a diagonal shape, it can be written as the following diagonal matrix:

$$\mathbf{J} = \begin{pmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{pmatrix}. \tag{12}$$

According to equality (6)

$$\begin{cases} w_x = S_\psi \vartheta' + C_\psi S_\vartheta \gamma', \\ w_y = -C_\psi \vartheta' + S_\psi S_\vartheta \gamma', \\ w_z = \psi' + C_\vartheta \gamma'. \end{cases} \tag{13}$$

If we write the values in equations (3)-(4) in terms of components, we can obtain the following system of differential equations for the dynamic model of the drone:

$$\begin{cases} v_x' + \dfrac{C_a}{m}\sqrt{v_x^2 + v_y^2 + v_z^2}\, v_x - w_z v_y + w_y v_z = \dfrac{g}{m} S_\vartheta, \\ v_y' + w_z v_x + \dfrac{C_a}{m}\sqrt{v_x^2 + v_y^2 + v_z^2}\, v_y - w_x v_z = \dfrac{g}{m} S_\gamma C_\vartheta, \\ v_z' - w_y v_x + w_x v_y + \dfrac{C_a}{m}\sqrt{v_x^2 + v_y^2 + v_z^2}\, v_z = \dfrac{g}{m} C_\gamma C_\vartheta + \dfrac{k}{m}\sum_{j=1}^{4}\omega_j^2, \end{cases} \tag{14}$$

$$\begin{cases} S_\psi \vartheta'' + C_\psi S_\vartheta \gamma'' + \dfrac{J_{zz} - J_{yy}}{J_{xx}} w_y w_z = \dfrac{kl}{J_{xx}}\left(\omega_2^2 - \omega_4^2\right), \\ -C_\psi \vartheta'' + S_\psi S_\vartheta \gamma'' + \dfrac{J_{xx} - J_{zz}}{J_{yy}} w_x w_z = \dfrac{kl}{J_{yy}}\left(\omega_1^2 - \omega_3^2\right), \\ \psi'' + C_\vartheta \gamma'' + \dfrac{J_{yy} - J_{xx}}{J_{zz}} w_x w_y = \dfrac{b}{J_{zz}}\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right), \end{cases} \tag{15}$$

Based on equations (4), we can write equations expressing the components of the drone's velocity vector relative to the earth:

$$\begin{cases} v_{gx} = v_x - w_z v_y + w_y v_z, \\ v_{gy} = w_z v_x + v_y - w_x v_z, \\ v_{gz} = -w_y v_x + w_x v_y + v_z. \end{cases} \tag{16}$$

To solve system (1), (6)-(16) by the functions $S_{gx}(t), S_{gy}(t), S_{gz}(t)$, $\psi(t), \vartheta(t), \gamma(t)$, their initial values for a certain moment $t = t_0$ must be given:

$$\begin{cases} S_{gx}(t_0) = S_{gx0}, \\ S_{gy}(t_0) = S_{gy0}, \\ S_{gz}(t_0) = S_{gz0}, \end{cases} \tag{17}$$

$$\begin{cases} v_{gx}(t_0) = S_{gx1}, \\ v_{gy}(t_0) = S_{gy1}, \\ v_{gz}(t_0) = S_{gz1}, \end{cases} \tag{18}$$

$$\begin{cases} \psi(t_0) = \psi_0, \\ \vartheta(t_0) = \vartheta_0, \\ \gamma(t_0) = \gamma_0, \end{cases} \tag{19}$$

$$\begin{cases} \psi'(t_0) = \psi_1, \\ \vartheta'(t_0) = \vartheta_1, \\ \gamma'(t_0) = \gamma_1. \end{cases} \tag{20}$$

System (1), (6)-(20), being a Cauchy problem written for a system of ordinary differential equations, expresses the dynamic model of a drone.

3. STATEMENT OF THE PROBLEM OF PARAMETER REFINEMENT (IDENTIFICATION)

Depending on the tasks performed, devices connected to the quadcopter (photo and video camera, radio repeater, packages of various shapes, etc.) can to some extent change their aerodynamic and technical characteristics. In this regard, the values of some parameters of the mathematical model of a quadcopter can differ from the calculated indicators.

The dynamic model of a quadcopter is expressed by several parameters, including the total mass of the drone, the components of the moment of inertia, the coefficient of proportionality between the rotational speed of the blades and the lifting force that they create, and other quantities. These values, in turn, depend on the size of the drone, the mass of its parts, the distribution of these parts relative to the centre of gravity and other factors. They can be determined experimentally or calculated analytically in the framework of hypotheses put forward in building the mathematical model. However, the errors in finding these parameters significantly change the aerodynamic properties of the drone, which makes it difficult to control its flight along the intended path.

The change in aerodynamic properties is reflected in the fact that the rotation of the blades in design modes is not enough for the drone to move along the intended path, and these modes need to be altered. In essence, it is necessary to refine some parameters of the mathematical model of the drone as accurately as possible to calculate adequate modes of operation of the blades. Mathematically, this is considered an inverse problem. Thus, the possibility of using the control parameters (rotational speed of the blades) and the data from navigation devices of the drone to refine the values of the parameters of the mathematical model of the drone is investigated in the following paragraphs.

An analysis of the dynamic model of a quadcopter written in the form of system (1), (6)-(20) above shows that the quantities in the model are divided into three groups.

The first group includes control parameters $\omega_1, \omega_2, \omega_{31}, \omega_4$ adjusted by the operator controlling the drone.

The second group includes the quantities calculated based on the loads n_x, n_y, n_z measured by navigation devices and the derivatives of Krylov orientation angles $\delta\psi, \delta\vartheta, \delta\gamma$. The application of the operations $v'_x = gn_x$, $v_x = \int gn_x dt$, $v'_y = gn_y$, $v_y = \int gn_y dt$, $v'_z = gn_z$, $v_z = \int gn_z dt$, $\psi'' = (\delta\psi)'$, $\psi' = \delta\psi$, $\psi = \int \delta\psi dt$, $\vartheta'' = (\delta\vartheta)'$, $\vartheta' = \delta\vartheta$, $\vartheta = \int \delta\vartheta dt$, $\gamma'' = (\delta\gamma)'$, $\gamma' = \delta\gamma$, $\gamma = \int \delta\gamma dt$ allows calculating all the other quantities included in the mathematical model.

The third group includes the quantities $l, m, J_{xx}, J_{yy}, J_{zz}, C_a, k, b$, which describe the physical and technical characteristics of the quadcopter and are considered unchangeable (constant) throughout the flight. Note the following two considerations concerning these variables.

I. Since the asymmetric design of the quadcopter substantially disrupts the stability of its flight, serious attention is paid to this issue, and from this point of view, we can assume with great accuracy that $J_{xx} = J_{yy}$.

II. Equations (8) that determine the quadcopter's orientation are invariant concerning the relations $\frac{J_{xx}}{l} = \frac{J_{yy}}{l}, \frac{J_{zz}}{l}$ and $\frac{b}{l}$.

With these considerations in mind, refining the values of the parameters of the dynamic model of a quadcopter implies finding the quantities $m, \frac{J_{xx}}{l} = \frac{J_{yy}}{l}, \frac{J_{zz}}{l}, C_a, k, \frac{b}{l}$ with sufficient accuracy. The initial approximate values of these quantities are assumed to be known and to solve the problem of parameter refinement, one can conduct flight experiments, adjusting the control parameters, and use measurements of navigation devices.

4. SOLUTION OF THE PROBLEM

Data coming from navigation devices is calculated at discrete instants in time, and the controller processor spends a certain amount of time on these calculations. In this regard, in the practical solution of the theoretical continuous mathematical model from the previous sections, a discrete analogue must be written.

Data received from navigation devices can be attributed with sufficiently high accuracy to the same time instants $t_i = i\Delta t$, where Δt is a time discrete, a known quantity, $i = 0, 1, 2, \ldots$. To calculate the refined value of the constants in the mathematical model, we can consider the time instants $i_1, i_2, \ldots, i_r$, which differ from each other by the control parameters of the identical flight, where $r > 3$ is a natural number.

First, let us give the calculation procedure for the quantities m, C_a, k. If we group system (14) according to the sought-for quantities and replace the coefficients with finite differences for each considered $i = i_1, i_2, \ldots, i_r$, we get:

$$\begin{cases} m\left(v'_x - w_z v_y + w_y v_z\right) + C_a\sqrt{v_x^2 + v_y^2 + v_z^2}\,v_x = gS_\vartheta, \\ m\left(v'_y + w_z v_x - w_x v_z\right) + C_a\sqrt{v_x^2 + v_y^2 + v_z^2}\,v_y = gS_\gamma C_\vartheta, \\ m\left(v'_z - w_y v_x + w_x v_y\right) + C_a\sqrt{v_x^2 + v_y^2 + v_z^2}\,v_z - k\sum_{j=1}^{4}\omega_j^2 = gC_\gamma C_\vartheta, \end{cases} \tag{21}$$

For each time instant t_i

$$\begin{cases} a_{11i}m + a_{12i}C_a = a_{14i}, \\ a_{21i}m + a_{22i}C_a = a_{24i}, \\ a_{31i}m + a_{32i}C_a + a_{33i}k = a_{34i}, \end{cases} \tag{22}$$

where

$a_{11i} = \left(gn_x - w_z v_y + w_y v_z\right)_i$, $a_{12i} = \left(\sqrt{v_x^2 + v_y^2 + v_z^2}\,v_x\right)_i$, $a_{21i} = \left(gn_y + w_z v_x - w_x v_z\right)_i$,
$a_{22i} = \left(\sqrt{v_x^2 + v_y^2 + v_z^2}\,v_y\right)_i$, $a_{31i} = \left(gn_z - w_y v_x + w_x v_y\right)_i$, $a_{32i} = \left(\sqrt{v_x^2 + v_y^2 + v_z^2}\,v_z\right)_i$,
$a_{33i} = -\left(\sum_{j=1}^{4}\omega_j^2\right)_i$, $a_{14i} = g\left(S_\vartheta\right)_i$, $a_{24i} = g\left(S_\gamma C_\vartheta\right)_i$, $a_{34i} = g\left(C_\gamma C_\vartheta\right)_i$.

The index i below indicates that the bracketed expression should be written as finite differences at the considered point t_i. It should be noted that due to errors in the measurements of the navigation devices, system (22) can degenerate at some values of i. Except in cases of degeneracy, the mean value of the solutions found for different i can be taken as the refined value of the quantities m, C_a, k.

Further, we consider the problem of calculating the relations $\frac{J_{xx}}{l} = \frac{J_{yy}}{l}$, $\frac{J_{zz}}{l}$ and $\frac{b}{l}$ using the found coefficient k. If we express the second-order derivatives of orientation angles in equations (15) by the quantities $\delta\psi, \delta\vartheta, \delta\gamma$ and rewrite them for the relations $\delta\psi, \delta\vartheta, \delta\gamma$, we obtain:

$$\begin{cases} \left(S_\psi\vartheta'' + C_\psi S_\vartheta\gamma'' - w_y w_z\right)\frac{J_{xx}}{l} + w_y w_z\frac{J_{zz}}{l} = k\left(\omega_2^2 - \omega_4^2\right), \\ \left(-C_\psi\vartheta'' + S_\psi S_\vartheta\gamma'' + w_x w_z\right)\frac{J_{xx}}{l} - w_x w_z\frac{J_{zz}}{l} = k\left(\omega_1^2 - \omega_3^2\right), \\ \left(\psi'' + C_\vartheta\gamma''\right)\frac{J_{zz}}{l} - \left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right)\frac{b}{l} = 0, \end{cases} \tag{23}$$

or for every moment t_i:

$$\begin{cases} b_{11i}\frac{J_{xx}}{l} + b_{12i}\frac{J_{zz}}{l} = b_{14i}, \\ b_{21i}\frac{J_{xx}}{l} + b_{22i}\frac{J_{zz}}{l} = b_{24i}, \\ b_{32i}\frac{J_{zz}}{l} + b_{33i}\frac{b}{l} = 0, \end{cases} \tag{24}$$

where

$$b_{11i} = \left(S_\psi (\delta\vartheta)' + C_\psi S_\vartheta (\delta\gamma)' - w_y w_z\right)_i,\ b_{12i} = \left(w_y w_z\right)_i,\ b_{21i} = \left(-C_\psi (\delta\vartheta)' + S_\psi S_\vartheta (\delta\gamma)' + w_x w_z\right)_i$$

$$,\ b_{22i} = -\left(w_x w_z\right)_i,\ b_{32i} = -\left((\delta\psi)' + C_\vartheta (\delta\gamma)'\right)_i,\ b_{33i} = -\left(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2\right)_i,\ b_{14i} = k\left(\omega_2^2 - \omega_4^2\right)_i,$$

$$b_{24i} = k\left(\omega_1^2 - \omega_3^2\right)_i.$$

It should also be noted that due to errors in the measurements of navigation devices, system (24) can degenerate for some values of i. Except in cases of degeneracy, the mean value of the solutions found for different i can be taken as the refined value of the relations $\frac{J_{xx}}{l} = \frac{J_{yy}}{l}$, $\frac{J_{zz}}{l}$ and $\frac{b}{l}$.

5. CONCLUSION

The dynamic model of a quadcopter is built in this paper, using the rotational speeds of the blades of the quadcopter motors, which are the control parameters, the loads measured by the accelerometer and the Krylov angles (yaw, pitch and roll angles) measured by the gyroscopes, which are the feedback data. The group of invariant quantities that are part of the mathematical model of the drone defining the dynamics of its movement is determined, and the possibility of determining their values as a result of flight experiments is substantiated.

References

1. Hoffmann Gabriel M., Haomiao Huang, Steven L. Waslander, Claire J. Tomlin. "Quadrotor Helicopter Flight Dynamics and Control: Theory and Experiment". *AIAA Guidance, Navigation and Control Conference and Exhibit*. 20-23 August 2007, Hilton Head, South Carolina. 20 P.
2. Teppo Luukkonen. "Modelling and Control of Quadcopter". *School of Science, Espoo.* August 22, 2011. P. 26.
3. Marcelo De Lellis Costa de Oliveira. "Modeling, Identification and Control of a Quadrotor Aircraft" *Czech Technical University in Prague. Master's Thesis*. Prague. June 2011. 75 P.
4. Гурьянов А.Е. 2014. "Моделирование управления квадрокоптером". *Инженерный вестник* 8: 523-534. [In Russian: Guryanov A.Y. 2014. "Quadcopter control modeling". *Inzhenerniy Vestnik* 8: 522-534].
5. Гэн К., Н.А. Чулин. "Алгоритмы стаблизации для автоматического управления траекторным движением квадрокоптера". 2015. *Наука и Образование. МГТУ им.Н.Э.Баумана. Электронный журнал*. 5: 218-235. [In Russian: Gen K., N.A. Chulin. 2015. "Stabilization algorithms for automatic control of the quadcopter trajectory movement". *Nauka i Obrazovaniye. N.E. Bauman MSTU. Electronic journal* 5: 218-235].
6. MPU-9250 Nine-Axis (Gyro + Accelerometer + Compass) MEMS MotionTracking™ Device. Available at: https://invensense.tdk.com/products/motion-tracking/9-axis/mpu-9250/.

7. Ефимов В.В. "Основы авиации. Часть I. Основы аэродинамики и динамики полета летательных аппаратов". Москва: МГТУ ГА, 2003. [In Russian: Yefimov V.V. "The basics of aviation. Part I. Fundamentals of aerodynamics and flight dynamics of aircraft". Moscow: MSTU GA, 2003].
8. "Динамика летательных аппаратов в атмосфере. Термины, определения и обозначения". ГОСТ 20058-80. Москва. 1980. 52 С. [In Russian: "Dynamics of aerial vehicles in the atmosphere. Terms, definitions and designations". GOST 20058-80. Moscow. 1980. 52 P.].
9. Бухгольц Н.Н. "Основной курс теоретической механики". Ч.1. М.: Наука. 1965. 468 С. [In Russian: Buchholz N.N. "The basic course theoretical mechanics". Part 1. M.: Nauka. 1965. 468 P.].
10. Бедржицкий Е.Л., Б.С. Дубов, А.Н. Радциг. "Теория и практика аэродинамического эксперимента". М.: МАИ. 1990. 216 с. ISBN: 5-7035-0003-6. [In Russian: Bedrzhitsky E.L., B.S. Dubov, A.N. Radtsig. "Theory and practice of aerodynamic experiments". M.: MAI. 1990. 216 P. ISBN: 5-7035-0003-6].

Received 11.07.2020; accepted in revised form 28.10.2020