DOI: 10.5604/01.3001.0015.2453

Volume 107 • Issue 1 • July 2021

Journal of Achievements in Materials and Manufacturing Engineering

International Scientific Journal published monthly by the World Academy of Materials and Manufacturing Engineering

Natural frequency and critical velocities of heated inclined pinned PP-R pipe conveying fluid

J.H. Mohmmed *, M.A. Tawfik, Q.A. Atiyah

Mechanical Engineering Department, University of Technology, Baghdad, Iraq

* Corresponding e-mail address: 21955@student.uotechnology.edu.iq

ORCID identifier: https://orcid.org/0000-0003-2402-9556 (Q.A.A.)

ABSTRACT

Purpose: The flow velocity and pressure of fluid flowing through a pipeline can cause the vibration of pipes, and consequently result in the modification in natural frequency via fluid-structure interaction. The value of the natural frequency of a component when approaches the excitation force to a certain degree, a severe resonance failure may occur. Hence, avoiding the resonance failure of a pipe subjected to complex conditions is an essential issue that requires to be solved urgently in the engineering field. This work treats the transverse vibration for flexible inclined heated pipe, made of polypropylene random-copolymer (PP-R), conveying fluid assuming pinned connections at the ends. The pipe was placed at different support angles and subjected to variant temperatures.

Design/methodology/approach: The inclined pipe is modelled as Euler-Bernoulli beam taking into account its self-weight, temperature variation, inclination angle, aspect ratio, and internal fluid velocity. The integral transforms method, which includes the finite Fourier sine and the Laplace transforms, was used to develop an analytic solution to the modified equation of motion and the analytical expressions for dual natural frequencies of the pipe-fluid interaction system were computed.

Findings: The proposed solution technique via finite Fourier sine and Laplace transforms offers a more convenient alternative to calculate the dynamic characteristic of pipes conveying fluid. The obtained results showed that the dynamical behaviour of pipe–fluid system is strongly affected by fluid flow velocity, degree of inclination, temperature variation, and aspect ratio of the pipe in transverse modes.

Research limitations/implications: This work focuses on fundamental (first) mode in the most discussions.

Practical implications: It was revealed that the thermal effects in the pipe are a very important factor and more significant in comparison with the internal fluid velocity and the inclination angle has a larger impact on vibration characteristics at a higher aspect ratio. The findings can be useful for the design of engineering components.

Originality/value: Determining the combining effect of inclination angle, aspect ratio, and thermal loading on vibration characteristic of the pipes conveying fluid by using an improved analytic solution to the modified equation of motion via mixed of finite Fourier sine and Laplace transforms.

Keywords: Finite Fourier sine transforms, Laplace transforms, Natural frequency, Inclined pipe conveying fluid

Reference to this paper should be given in the following way:

J.H. Mohmmed, M.A. Tawfik, Q.A. Atiyah, Natural frequency and critical velocities of heated inclined pinned PP-R pipe conveying fluid, Journal of Achievements in Materials and Manufacturing Engineering 107/1 (2021) 15-27.

DOI: https://doi.org/10.5604/01.3001.0015.2453

ANALYSIS AND MODELLING

1. Introduction

The dynamical behaviour of pipelines has attracted a lot of attention from many investigators due to their numerous applications in many engineering fields, particularly, in oil and gas installations, chemical plants, hydropower plants, reactors, cooling systems, medical equipment, and ocean mining [1,2].

Pipes conveying fluid are the simplest dynamical system of fluid-structure interaction problems, that comprises of two main sections, i.e., the external pipe and the fluid flowing through it [3]. This dynamical system not only exhibits very important and interesting dynamical behaviours, but also services big radiant meanings to other close disciplines [4]. Therefore, researches in this field have been extensively performed since the middle of the last century.

The first attempt to analyse the dynamical behaviour of pipe conveying fluid was by Ashley and Haviland. They analysed the characteristic of vibration in the Tran-Arabian pipeline [5]. Thereafter, the dynamic behaviour of pipes has been increasingly investigated by many other researchers, like, Benjamin [6], he was the first researcher who discovers that fluid friction has no effect on dynamical behaviour. Gregory and Paidoussis [7] analysed this dynamical system further, specifying that for a certain high flow speed, the pipe may be undergoing flexural oscillatory instability. Then, in the subsequent forty years, several articles on the plain pipe and its variants were published, the linear and the nonlinear dynamic behaviour of various support systems have been widely analysed. H.R. Oz [8] analysed the stability and non-linear vibrations of tensioned pipe conveying fluid with variable velocity. J. L. Hill and C. P. Swanson [9] studied the stability conditions of fluid conveying tubes and the influences of adding lumped masses on the dynamical system. Qiao Ni et. al [10] investigate the vibration characteristics and stability of pipe-fluid systems immersed in a fluid with axially moving supports. The influence of flexible (torsional spring) end conditions on the dynamics of pipe conveying fluid utilizing the Rayleigh model was studied by B.Y. Dagli and A. Ergut [11]. They found that the behaviour of the Rayleigh model becomes similar to that of the Euler-Bernoulli model at the slenderness ratio values above 100. D.B. Giacobbi, et. al [12] show that the system becomes more stable with increasing the density of fluid along the pipe, on their study about the dynamics of cantilevered and clamped end pipes conveying fluid with axially variable density. On the other hand, many researchers focused on the identification and control of pipe vibration to avoid the hazards [13-15]. However, in most of the previous work, due to the complexity of the mathematical models, the approximate and numerical methods were utilized to treat the problems of the dynamics of fluid-conveyed pipes. Researchers in Refs. [16-18], adopted the transfer matrix technique to obtain the solution of the governing equation of motion for various end conditions of pipe conveying fluid; T.A. El-Sayed and H.H. El-Mongy [19] use variational iteration methods combined with transfer matrix method; Jweeg and Ntayeesh [20] analyse the gyroscopic system for a nonlinear pipe conveying fluid by using multiple scales method; S. Chandurkar and R. Kadoli [21] use a numerical method (finite element and differential quadrature approach) to solve the governing equation for the pipe conveying fluid. Part of the interest in current work rests on presenting an exact closed-form solution via integral transform techniques to finding the natural frequencies and critical velocity of fluid-conveying pipes.

Recently, the prosperity in the world economy has led to significant development in the pipe industry, and the need for large piping systems has considerably grown. In the huge systems, the pipelines cannot all be placed vertically or horizontally, and are inevitably erected at an inclined angle for different reasons like space, material, and power supply. On the other hand, these pipes often are exposed to fluctuations in the internal or external temperatures while cyclic operational start-up and shutdown procedures cause a vibration of these pipes and propagate internal waves. Moreover, the modern petroleum industries focus on extracting oil and gas from the oceans and deep waters, many pipes, especially that carrying the petroleum or gas from the sea bed to the surface may have various aspect ratios of length to diameter. Therefore, the study of the effect of the inclination angle, aspect ratio in combination with temperature variations on natural frequencies, and critical fluid velocity of the pipeline system are imperative. A practical investigation on the vibration of horizontal pipe conveying fluid with various temperatures was conducted by Ameen et al. [22]. The influences of the gravity parameters on the stability conditions of the linear standing and hanging pipe systems were analysed by Paidoussis [23]. Wang and Q. Ni [24] investigated the stability of a standing pipe conveying fluid with elastic support and compared the results with that of a hanging system. However, most of these studies have not focus on the influence of the temperature variation, aspect ratio, and inclined pipeline systems on the natural frequencies and critical fluid velocity and did not take into consideration angle support, and thermal influences in the formulation of the governing equations, which makes the present study on this topic essential. Hence, in order to compensate and address the lack of ongoing research as mentioned above, the current work took into account the inclined pipes and the factors affecting them.

An analytic solution, by using integral transform technique, for 4th order governing equation was offered in this study. Moreover, the influence of the supported angle, aspect ratio, and thermal variation under various flow velocities on the natural frequencies and critical fluid velocity of a pinned-pinned, inclined PP-R pipe conveying fluid. The obtained results are compared with previous works in the literature.

The current paper is structured as follows. Section 1 the problem under study was introduced. In Section 2, the analysis of the inclined pipe containing an incompressible flowing fluid were presented and the governing partial differential equation of motion with appropriate assumptions was derived. In Section 3, an analytic solution by using the mixed finite Fourier sine and Laplace transform technique to solve the time-space domain governing equation was developed and the natural frequencies of the system are computed. In Section 4, results are analysed and discussed in addition to validating the adopted analytic solution against other approximate and numerical approaches. Finally, Section 5 presents conclusions of the study.

2. System model and governing equation of motion

The considered system illustrated in Figure 1 comprises of simply, inclined, uniform pipe of length L, cross-sectional area A_p, mass per unit length m_p, and bending stiffness EI, containing an incompressible flowing fluid of mass per unit length mf, and mean flow velocity υ. The cross-sectional flow area is A and the fluid pressure is p. It is subjected to the planar motion x (y, t). The axis of the pipe in its unreformed state coincides with the x-axis, which has an inclination angle θ with the horizontal axis.

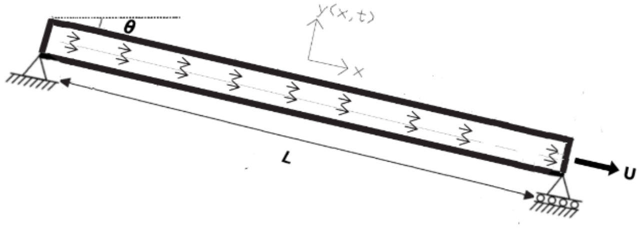

Fig. 1 Schematic of pipe with simply support

The basic assumptions made for the pipe and the fluid in the derivation of this model are as follows:

- a fully developed incompressible, non-viscous, Newtonian, fluid is flowing inside the pipe.
- The flow through the pipe is uniform across the pipe and its mean velocity along the pipe is constant.
- The pipe wall behaves elastically, i.e. the material damping is negligible.
- The material of the pipe is assumed to be isotropic and the (young modulus E, thermal conductivity k, thermal expansion coefficient α, …. etc.) are dependent on temperature.
- The ratio of the span, L, of the pipe to its outside diameter, D_o, (aspect ratio) is equal to or higher than 20, and the wavelength of its transverse deflection is large as compared to the outside diameter of the pipe, thus theory of Euler–Bernoulli theory is applicable for the representation of the transverse vibration of the pipe.
- The radial variation and the secondary effects were considered to be very small so that the plug-flow model is acceptable.
- The effect of the vibrated pipe on the velocity of flowing fluid inside it is small and can be ignored, i.e. it is assumed that the study is applied to the inside wall of the pipe so fluid velocity at these points is zero.
- No variation in the cross-section dimensions of the pipe during vibration is accounted for.
- The friction between the flowing fluid and the pipe is negligible.

Moreover, in this work, the tension force is considered to be sufficiently large in comparison to the effects arising from elongation. Also, it’s assumed that the extensional stiffness is sufficiently large so that the axial deformation causing by the pretension is negligible.

With the assumptions listed above, the model derived by Paidoussis and Li [25] is modified to accommodate the influences of temperature and support angle. For this case, infinitesimal elements for the fluid and pipe, as shown in Figure 2, were considered.

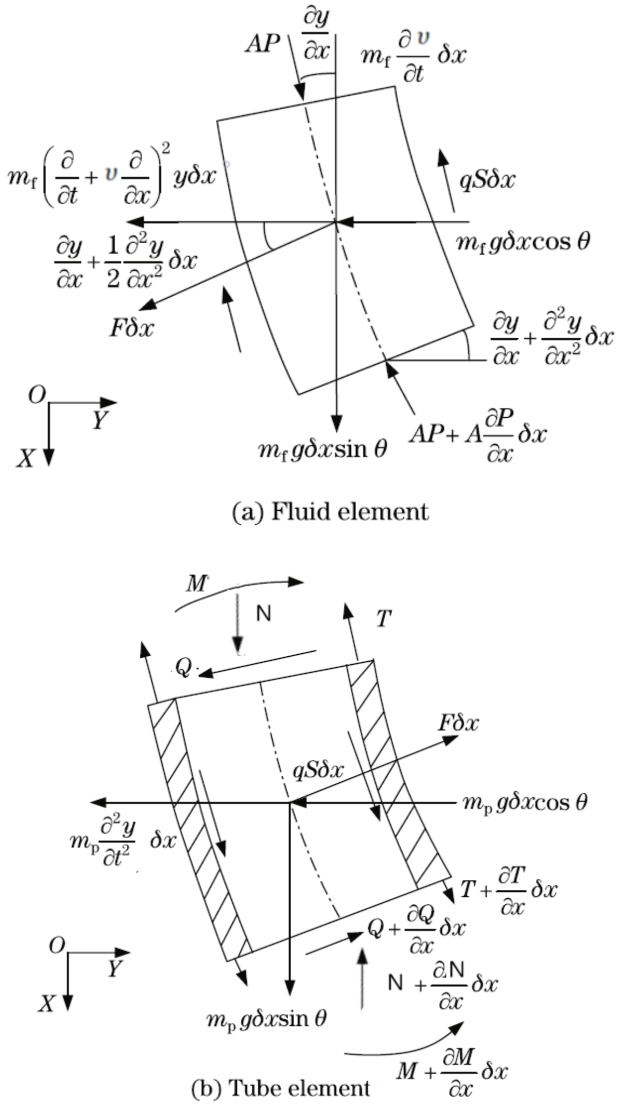

Fig. 2. Fluid and pipe elements

For the fluid element of Figure 2a, force balance in the x- and y-directions yields;

$$-A\frac{\partial P}{\partial x}+m_f g\sin\theta-qS+F\frac{\partial y}{\partial x}=0 \quad (1)$$

$$-F-A\frac{\partial}{\partial x}\left(P\frac{\partial y}{\partial x}\right)-qS\frac{\partial y}{\partial x}-m_f g\cos\theta\, dx=m_f\left(\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}\right)^2 y \quad (2)$$

where q represents the internal shear stress, S is the internal perimeter (internal section circumference) of pipe, g is the acceleration of gravity, and F represents the transverse force between the pipe wall and fluid (per unit length).

In a similar way, force balance on the pipe element shown in Figure 2b yield;

$$\frac{\partial T}{\partial x}+m_p g\sin\theta-F\frac{\partial y}{\partial x}+qS=0 \quad (3)$$

$$F+qS\frac{\partial y}{\partial x}+\frac{\partial Q}{\partial x}+\frac{\partial}{\partial x}\left(T\frac{\partial y}{\partial x}\right)-\frac{\partial}{\partial x}\left(N\frac{\partial y}{\partial x}\right)-m_p g\cos\theta=m_p\frac{\partial^2 y}{\partial t^2} \quad (4)$$

where T represents the longitudinal tension. Q represents the transverse shear force in the pipe. The transverse shear force Q in the pipe is related to the bending moment M as follows;

$$Q=\frac{\partial M}{\partial x} \quad and \quad M=-EI\frac{\partial^2 y}{\partial x^2}\ , \quad \therefore Q=-EI\frac{\partial^3 y}{\partial x^3}$$

N is the thermal force in the pipe and can be calculated as:

$N=\alpha E A_p \Delta T$, Where α: thermal expansion coefficient (1/°C), L: the length of the pipe (m). $\Delta T=\left(T_{pt}\right)_x-T_i$: the temperature change (°C). T_i: initial temperature of the pipe (°C). $\left(T_{pt}\right)_x$: instantaneous temperature of the pipe after the time (°C).

Assuming P, T and N are invariant along x and combining Eqs. (2) and (4) one obtains;

$$EI\frac{\partial^4 y}{\partial x^4}-\frac{\partial}{\partial x}\left[(T-PA)\frac{\partial y}{\partial x}\right]+\frac{\partial}{\partial x}\left(N\frac{\partial y}{\partial x}\right)+\left(m_f+m_p\right)g\cos\theta+m_p\frac{\partial^2 y}{\partial t^2}+m_f\frac{\partial^2 y}{\partial t^2}+2m_f v\frac{\partial^2 y}{\partial x\partial t}+m_f v^2\frac{\partial^2 y}{\partial x^2}=0 \quad (5)$$

By substituting equations. (1and 3), it can obtain the following equation:

$$\frac{\partial}{\partial x}(T-PA)=-\left(m_f+m_p\right)g\sin\theta \quad (6)$$

where T is axial force on the pipe.

Integrating of Eq.(6) gives;

$$(T-PA)|_L-(T-PA)|_x=-\left(m_f+m_p\right)(L-x)g\sin\theta$$

It can be noted that both T and P equal to zero at x = L, the first because there are no restrict at supports, and the latter because it is measured above the atmospheric pressure; hence this equation produces;

$$T - PA = (m_f + m_p)(L - x) g \sin\theta$$

Adding these values into Eq. 5 yields the following equation of motion of a pipe conveying fluid;

$$EI\frac{\partial^4 y}{\partial x^4} - (m_f + m_p)(L - x) g \sin\theta \frac{\partial^2 y}{\partial x^2} + m_f v^2 \frac{\partial^2 y}{\partial x^2} + N\frac{\partial^2 y}{\partial x^2} + (m_f + m_p) g \sin\theta \frac{\partial y}{\partial x} + (m_f + m_p) g \cos\theta + 2m_f v \frac{\partial^2 y}{\partial x \partial t} + (m_p + m_f)\frac{\partial^2 y}{\partial t^2} = 0 \qquad (7)$$

The components of Eq. 7 are defined as follows: the first term is the acceleration due to the restoring force, the second is the accelerations due to the gravity force, the third is the accelerations due to the centrifugal force caused by the internal flow within the pipe, after that the fourth term represent the accelerations due to the thermal force caused by the change in temperature within the pipe then the fifth and sixth terms represent the static force due to the gravity effect and the seventh term stands for acceleration due to the Coriolis forces; and following this are accelerations due to the system's inertia force. The second, third, and fourth terms represent the curvature term of the pipe. Rearrange eq. 7, gives

$$EI\frac{\partial^4 y}{\partial x^4} - [(m_f + m_p)(L - x) g \sin\theta - N - m_f v^2]\frac{\partial^2 y}{\partial x^2} + (m_f + m_p) g \sin\theta \frac{\partial y}{\partial x} + (m_f + m_p) g \cos\theta + 2m_f v \frac{\partial^2 y}{\partial x \partial t} + (m_f + m_p)\frac{\partial^2 y}{\partial t^2} = 0 \qquad (8)$$

It is convenient to rewrite Eq. 8 in the following dimensionless form;

$$\frac{\partial^4 \eta}{\partial \xi^4} - [(1 - \xi)\overline{g}\sin\theta - \overline{N} - U^2]\frac{\partial^2 \eta}{\partial \xi^2} + \overline{g}\sin\theta\frac{\partial \eta}{\partial \xi} + \overline{g}\cos\theta + \frac{\partial^2 \eta}{\partial \tau^2} + 2\beta^{\frac{1}{2}} U \frac{\partial^2 \eta}{\partial \xi \partial \tau} = 0 \qquad (9)$$

where;

$$\eta = \frac{y}{L}\ \ \xi = \frac{x}{L}, \beta = \frac{m_f}{m_f + m_p}, \overline{g} = \frac{(m_f + m_p) g L^3}{EI},$$

$$U = \left(\frac{m_f}{EI}\right)^{\frac{1}{2}} vL,\ \ \tau = \frac{1}{L^2}\left(\frac{EI}{m_f + m_p}\right)^{\frac{1}{2}},\ \overline{N} = \frac{NL^2}{EI}$$

or in another form

$$\eta'''' - [(1 - \xi)\overline{g}\sin\theta - \overline{N} - U^2]\eta'' + \overline{g}\sin\theta\,\eta' + 2\beta^{\frac{1}{2}} U\dot{\eta}' + \ddot{\eta} = -\overline{g}\cos\theta \qquad (10)$$

3. Analytic solution by mixed Fourier-Laplace transforms technique

In this part, an analytic solution for Eq. 10 via finite Fourier sine and Laplace transforms techniques will be presented. to simplify the solution of the fourth order partial differential equation, Eq. 10 can be discretized and transformed to ordinary differential equations by utilizing the separation of variables method, the term η(ξ,τ) is decomposed into space and time as follows

$$\eta(\xi, \tau) = Y(\xi) q(\tau) \qquad (11)$$

where q(τ) the generalized coordinate of the system and Y(ξ) represents a trial/comparison function that will satisfy both the geometric and natural boundary conditions. On substituting Eq. 11 into Eq. 10, it can be obtained

$$Y''''(\xi)q(\tau) - [(1 - \xi)\overline{g}\sin\theta - \overline{N} - U^2]Y''(\xi)q(\tau) + \overline{g}\sin\theta\, Y'(\xi)q + 2\beta^{\frac{1}{2}} UY'(\xi)\dot{q}(\tau) + Y(\xi)\ddot{q}(\tau) = 0 \qquad (12)$$

Suppressing the time dependency of the Eq. (12), by applying the Laplace transform

$$Y''''(\xi)q(s) - [(1 - \xi)\overline{g}\sin\theta - \overline{N} - U^2]Y''(\xi)q(s) + \overline{g}\sin\theta\, Y'(\xi)q(s) + \frac{\overline{g}}{s}\cos\theta + 2\beta^{\frac{1}{2}} UY'(\xi)sq(s) + Y(\xi)s^2 q(s) = 0 \qquad (13)$$

where q(0), and q˙(0) were assumed equal to zero. Now, consider the finite Fourier sine transform

$$\left.\begin{aligned} F_s\{\eta(\xi,\tau)\} &= \int_0^1 \eta(\xi,\tau)\sin n\pi\xi\, d\xi = \eta(n,\tau); \\ F_s^{-1}\{\eta(n,\tau)\} &= 2\sum_{n=1}^{\infty}\eta(n,\tau)\sin n\pi\xi = \eta(\xi,\tau) \end{aligned}\ n\ an\ integer\right\} \qquad (14)$$

where Fs is the Fourier transform and $F_s^{-1}$is the inverse Fourier transform.

In this case

$$F_s(\xi,\tau) = \begin{cases} 0 & when \quad -\infty \le \xi < 0 \\ \eta(\xi,\tau) & when \quad 0 \le \xi < 1 \\ 0 & when \quad 0 \le \xi < \infty \end{cases}$$

By applying the finite Fourier sine transform Eq. 13 becomes

$$\int_0^1 \Big\{Y''''(\xi)q(s) - [\overline{g}\sin\theta - \overline{N} - U^2]Y''(\xi)q(s) + \overline{g}\sin\theta\,\xi Y''(\xi)q(s) + \overline{g}\sin\theta\,Y'(\xi)q(s) + \frac{\overline{g}}{s}\cos\theta + 2\beta^{\frac{1}{2}}\,UY'(\xi)sq(s) + Y(\xi)s^2q(s)\Big\}\sin n\pi\xi d\xi = 0 \qquad (15)$$

Y(ξ) is varying according to boundary condition, in the following, the solution of motion governing equation for simply support case. By assuming Y(ξ)=φ_r (ξ), where the functions φ_r (ξ) are the beam Eigenfunctions, which are given as:

$$Y(\xi) = \sqrt{2}\,sin(\lambda_r\xi),\ \sqrt{2}\,sin(\lambda_r L) = 0 \quad \Longrightarrow \lambda_r = \frac{n\pi}{L} \qquad (16)$$

The boundary condition for simply support are

$$Y''(\xi) = Y(\xi) = 0\ \ at\ \xi = 1\ \ and\ \ \xi = 0 \qquad (17)$$

The applications of space function in current form as given above for simply support may include some difficulties and long calculations in finding the integral transformation for each term in Eq. 15, alternatively, a polynomial function can be applied for this type of support system:

$$Y(\xi) = a_o + a_1\xi + a_2\xi^2 + a_3\xi^3 + a_4\xi^4 \qquad (18)$$

Applying the boundary conditions in Eq. 18

$$Y(\xi) = (\xi^4 - 2\xi^3 + \xi)a_4 \qquad (19)$$

By applying the orthogonal rule, for $a = 1$, we have $a_4 = 4.5$ for the first mode.

On performing integration and rearrange Eq. 15

$$\Bigg[s^2\frac{24}{n^5\pi^5}(1+(-1)^{n+1}) + s\Bigg(2\beta^{\frac{1}{2}}\,U\Big(\frac{1}{n\pi}(1+(-1)^n) + \frac{12}{n^3\pi^3}(1+(-1)^n)\Big)\Bigg) + \frac{24}{n\pi}(1+(-1)^{n+1}) - [\overline{N} + \upsilon^2]\frac{24}{n^3\pi^3}(1+(-1)^{n+1}) + \overline{g}\sin\theta\Big(\frac{24}{n^3\pi^3}(1+2(-1)^n) + \frac{1}{n\pi}(1+(-1)^n) + \frac{12}{n^3\pi^3}(1+(-1)^n) + \frac{24}{n^3\pi^3}(1+(-1)^{n+1})\Big)\Bigg]q(s) = -\frac{1}{a_4}\frac{\overline{g}}{s}\cos\theta\ \ 1(\mathcal{F}) \qquad (20)$$

Let $Z_{s1} = 2\beta^{\frac{1}{2}}\,U\Big(\frac{1}{n\pi}(1+(-1)^n) + \frac{12}{n^3\pi^3}(1+(-1)^n)\Big)$ (21a)

$$Z_{s2} = \frac{24}{n\pi}(1+(-1)^{n+1}) - [\overline{N} + U^2]\frac{24}{n^3\pi^3}(1+(-1)^{n+1}) + \overline{g}\sin\theta\Big(\frac{24}{n^3\pi^3}(1+2(-1)^n) + \frac{1}{n\pi}(1+(-1)^n) + \frac{12}{n^3\pi^3}(1+(-1)^n) + \frac{24}{n^3\pi^3}(1+(-1)^{n+1})\Big) \qquad (21b)$$

$$\therefore \Big[s^2\frac{24}{n^5\pi^5}(1+(-1)^{n+1}) + Z_{s1}s + Z_{s2}\Big]q(s) = -\frac{1}{a_4}\frac{\overline{g}}{s}\cos\theta\ \ 1(\mathcal{F}) \qquad (22)$$

To make eq. 22 more concise, the following representations are used

$$\phi_{s1} = \frac{Z_{s1}}{\frac{24}{n^5\pi^5}(1+(-1)^{n+1})},\quad \phi_s^2 = \frac{Z_{s2}}{\frac{24}{n^5\pi^5}(1+(-1)^{n+1})} \qquad (23a,b)$$

$$[s^2 + \phi_{s1}s + \phi_s^2]q(s) = \frac{-\frac{1}{a_4}\frac{\overline{g}}{s}\cos\theta\ 1(\mathcal{F})}{\frac{24}{n^5\pi^5}(1+(-1)^{n+1})}$$

$$\Longrightarrow q(s) = \frac{\frac{-\frac{1}{a_4}\frac{\overline{g}}{s}\cos\theta\ 1(\mathcal{F})}{\frac{24}{n^5\pi^5}(1+(-1)^{n+1})}}{[s^2+\phi_{s1}s+\phi_s^2]} \qquad (24)$$

By invoking the finite Fourier sine and Laplace inversion, the dynamic response can be expressed as

$$\eta(\xi,\tau) = Y(\xi)\frac{-\overline{g}\cos\theta F_s(t)}{\frac{24}{n^4\pi^4}a_4} \qquad (25)$$

where,

$$F_s(t) = \Bigg(\frac{1}{\alpha_{s1}\alpha_{s2}} + \frac{1}{\alpha_{s1}\alpha_{s2}(\alpha_{s2}-\alpha_{s1})}(\alpha_{s1}e^{-\alpha_{s2}t} - \alpha_{s2}e^{-\alpha_{s1}t})\Bigg) \qquad (26)$$

$$\alpha_{s1} = \frac{\phi_{s1}}{2} + i\sqrt{\phi_s^2 - \frac{\phi_{s1}{}^2}{4}},\quad \alpha_{s2} = \frac{\phi_{s1}}{2} - i\sqrt{\phi_s^2 - \frac{\phi_{s1}{}^2}{4}} \qquad (27\ a,b)$$

In order to determine the dimensionless fundamental natural frequency and its complementary (secondary) value, the characteristic equation of the system $(s^2 + \phi_{s1}s + \phi_s^2)$ can be isolated by substituting s = ±iω into the characteristic equation

$$\omega_n{}^2 = \Big(\frac{\phi_{s1}}{2}\Big)^2 \pm \Big(\phi_s^2 - \frac{\phi_{s1}{}^2}{4}\Big)$$

$$\Longrightarrow \omega_n = \sqrt{\phi_s^2 - \frac{\phi_{s1}{}^2}{2}},\ \omega_m = \sqrt{-\phi_s^2} \qquad (28\ a,b)$$

4. Results and discussion

In this part, the results obtained from the analytic solution of the governing equation of an inclined, heated, pinned ends pipe conveying fluid, Eq. (10), are presented. Especially, the effects of support angle, aspect ratio, and temperature on the natural frequencies and dynamic deflection are emphasized. The value of the main parameters employed for the current investigation are shown in Table 1 and the results simulated in MATLAB 2019b software. Results are presented for a pipe manufactured from polypropylene random-copolymer (PP-R) material, and the fluid used in all cases was water.

Table 1.
Summary of main specifications and parameters of the numerical simulation

Specification	Value
Inner diameter D_i (m)	0.018
Thickness t (m)	0.0035
Aspect ratio (length to outer dimeter) L/D_o	(20-50) D_o
Modulus of elasticity E (GPa) at (25, 50, 70)°C	0.8, 0.38, 0.23
Density of pipe ρ_p (Kg/m^3)	909
Density of fluid ρ_f (Kg/m^3)	1000
Coefficient of expansion α (1/K)	$0.3x10^{-4}$

4.1. Solution validation

In order to verify the validity of the obtained results in the present formulation (via finite Fourier sine and Laplace transforms method) for handling the dynamic characteristic of the pipe-fluid system, the natural frequencies values obtained in Eq. 28 with its corresponding values of critical velocity are compared to the solutions obtained using a shooting method and Galerkin approximation, as described in Refs. [26,27] respectively. To establish verifications of our analysis, the following data were considered, the angle of support $\theta = 0$ and mass ratio $\beta = 0.5$. The results are presented in Table 2 for both the natural frequencies and critical velocity. It can be seen from Table 2 that the present results agree very well with the solutions obtained using the shooting and Galerkin method thus the present formulation in handling the dynamic characteristic of the pipe-fluid system was verified. This suggests that the finite Fourier sine and Laplace transforms method is sufficiently accurate and efficient and more easer compared to the numerical method and the Galerkin approximation.

Table 2.
Comparison of natural frequency and critical velocity of pinned supported pipe

Mode	Natural Frequency ω_n		Critical Velocity U_c	
No.	Ref. 18	Present	Ref. 19	Present
1	9.89011	9.8696	3.15	3.1415
2	39.5604	39.4784	6.245	6.283
3	88.8312	88.8264	9.45	9.424

4.2. Natural frequency

Continuity profile (single curve) for natural frequency will be presented in this study, instead of the separated profile of real and imaginary components of natural frequency as usually seen in literature, by introducing the absolute value of natural frequency to connect the subcritical, critical, and post-critical vibratory behaviours. This continuity profile represents the general behaviour of the system's natural frequencies. The obtained results reveal that the natural frequencies and dynamic response of the pipe system are strongly affected by factors such as the internal fluid velocity, aspect ratio, temperature, and angle support, etc. Two frequencies have been identified, the natural frequency and its complementary value. The effects of internal fluid velocity for the first three modes in each case have been investigated as shown in Figures 3 (a and b).

The displayed figures are approximately identical for major and complementary natural frequency. As expected, the profiles are similar to earlier observations where is the increase in the flow velocity leads to weakening the pipe frequency. The behaviours of the natural frequencies indicate the existence of two zones that are similar to those of ref. [26,28,29] and each zone is reminiscent of the results reported in [26,27]. In all the cases, the natural frequency profiles at flow velocities below and up to the critical velocity, are ordered such that n1<n2<n3 where n1, n2, and n3 are the first, second, and third mode respectively. The critical velocities for the major and complementary natural frequency values corresponding to the modes are so ordered i.e. U1 < U2 < U3. As the flow velocity increases, the natural frequencies are asymptotically reduced to lower values in accordance with respective critical velocities.

Whilst in Figure 4, the influence of aspect ratios on the first (fundamental) natural frequency corresponding to the n1 mode of vibration was presented. In general, the results have been shown that the aspect ratio has a significant effect on the natural frequency profiles irrespective of the zone. It is clear from results the natural frequencies are inversely proportional to the pipe aspect ratios and are observed to be decreasing in revised order to their corresponding critical velocities in the first zone before decreasing in a way become close to each other beyond the critical velocities in the second zone. This behaviour can be explained by the fact that the pipe becomes heavier in weight and weaker in its stiffness with increasing pipe aspect ratio. Meanwhile, the

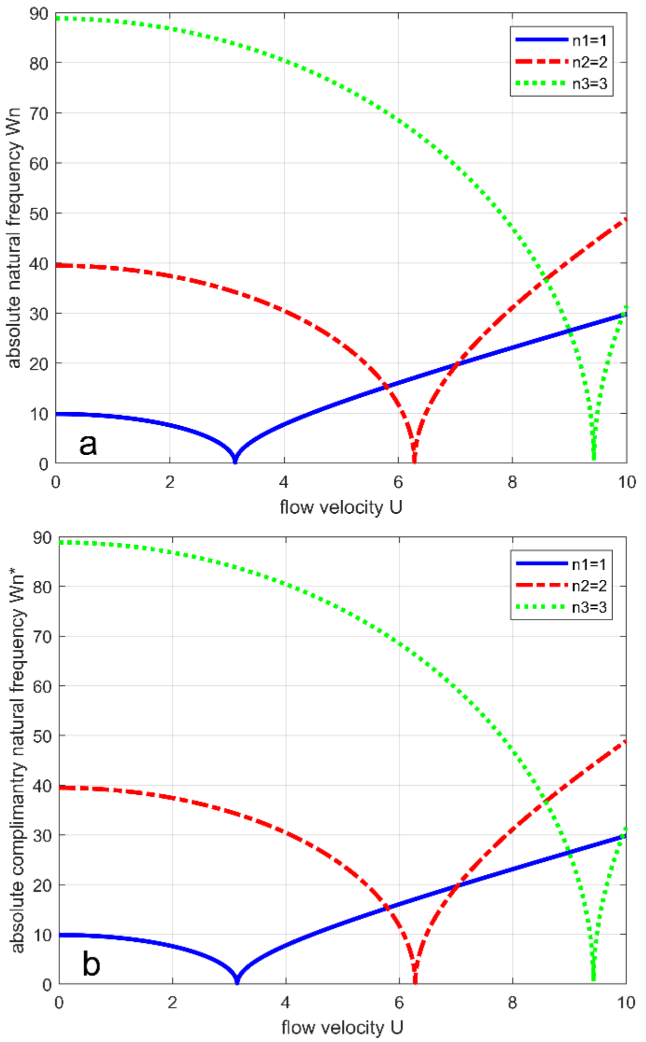

Fig. 3. Dimensionless absolute values of the transverse natural frequency vs. dimensionless flow velocity at the first three modes: (a) ω_n (b) ω_n^*. at conditions L=20D m, T=25°C, $\theta = 0°$

level of decreasing in the magnitude of natural frequencies and critical velocity for the same aspect ratio reduces with increasing the support angle. For example, illustrated in Figures 4a-c is when the internal fluid velocity equal to zero, the natural frequency reduces from 9.806 to 7.244 as the aspect ratio increase from 20 to 50 at a support angle equal to 0°, whereas the natural frequency reduces from 9.813 to 9.107 as aspect ratio increase from 20 to 50 at a support angle equal to 30°. Meanwhile, the critical flow velocity is 3.122 for the pipe model at L/D=20, and when the L/D is increasing to 50, the critical flow velocity reduces to 2.306. This behaviour can be associated with the increase in the axial tension force in the pipe bend, which plays a stiffening

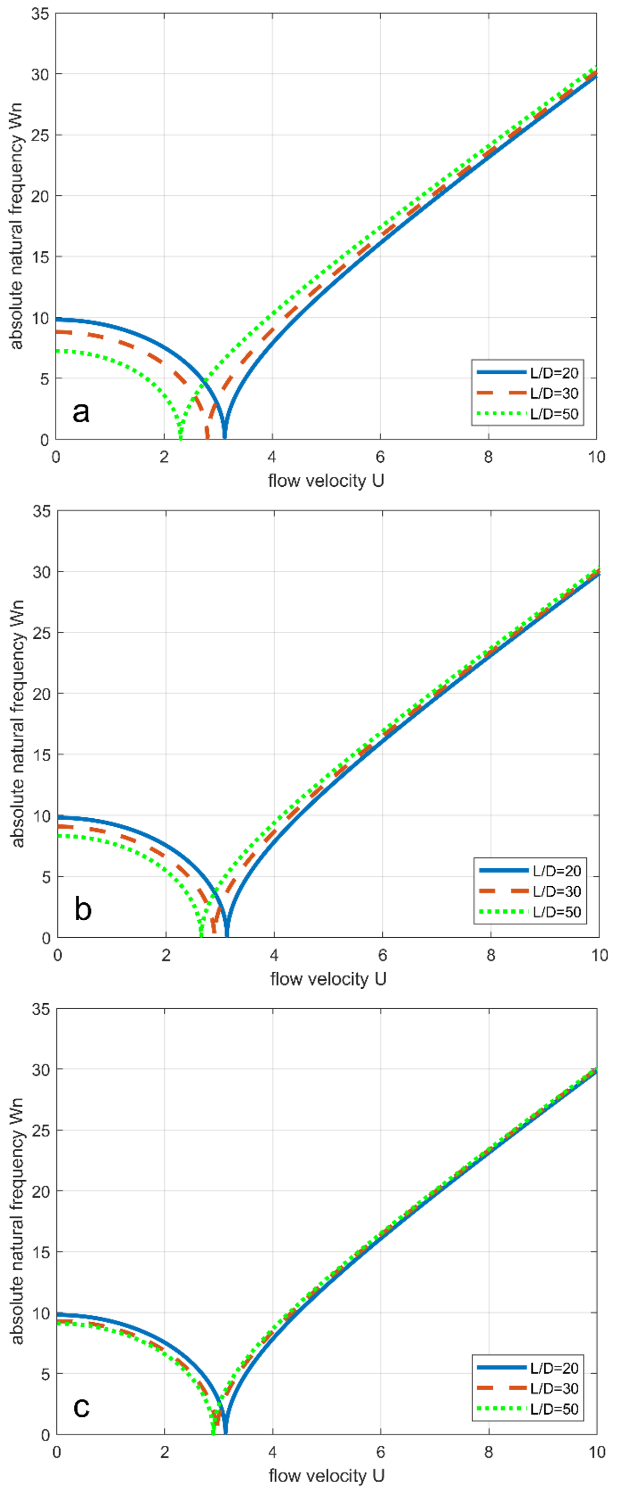

Fig. 4. Variation of dimensionless absolute values of the transverse natural frequency with dimensionless flow velocity for different aspect ratios at T=25°C: a) $\theta = 0°$, b) $\theta = 15°$, c) $\theta = 30°$

role, with increasing the angle of inclination. These are consonant with earlier results [30], which showed that any increase in aspect ratio results in reducing of the natural frequency and critical fluid velocity.

Figure 5 depicts the absolute dimensionless natural frequency versus support angles at different aspect ratios with constant flow velocity U=1 and temperature equal to 25°C. In general, increasing the inclination angle of the pipe with the horizontal axis in the range of (0°-90°) leads to enhance the pipe frequency, and then after the angle 90° this behaviour is reversed. Moreover, it is noticed that when the aspect ratio is larger, the effect of the inclination angle of the pipe on the magnitude of natural frequency becomes more noticeable. At angles between (40°-140°), the influence of the inclination angle becomes more predominant and reverses the influence of the aspect ratio. i.e. at angles lie in the range of (40°-140°) the natural frequency of the pipe increase with increasing aspect ratio. For example, at a support angle equal to 80°, when L/D=20, the natural frequency is close to 9.2071, and when L/D=50, the magnitude of the natural frequency increase to be close to 10.1303.

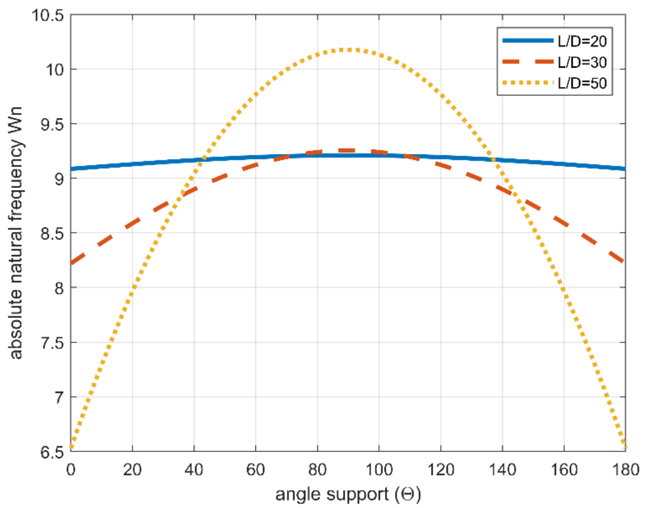

Fig. 5. Variation of dimensionless absolute values of the transverse natural frequency with support angle for different aspect ratios at U=1, T=25°C

The effect of support angle variation with different flow velocities on the fundamental natural frequency characteristics is more analysed in Figures 6(a-c). The general pattern indicates that the natural frequency is slightly increasing proportionally from θ =0° to θ =30°, irrespective of any flow velocity. This behaviour may be associated with developing an axial component of gravitational load for the

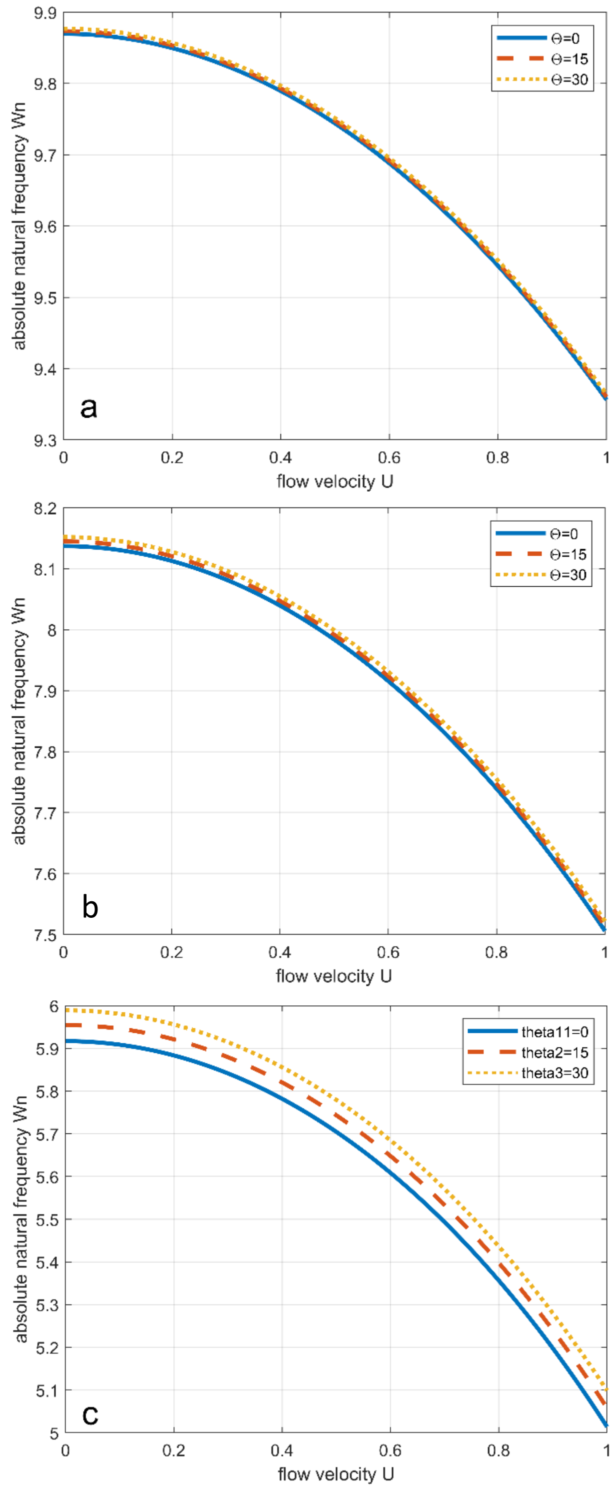

Fig. 6. Variation of dimensionless absolute values of the transverse natural frequency with dimensionless flow velocity for different support angles and different aspect ratios at T=25°C: a) *L/D* = 20, b) *L/D* = 30, c) *L/D* = 50

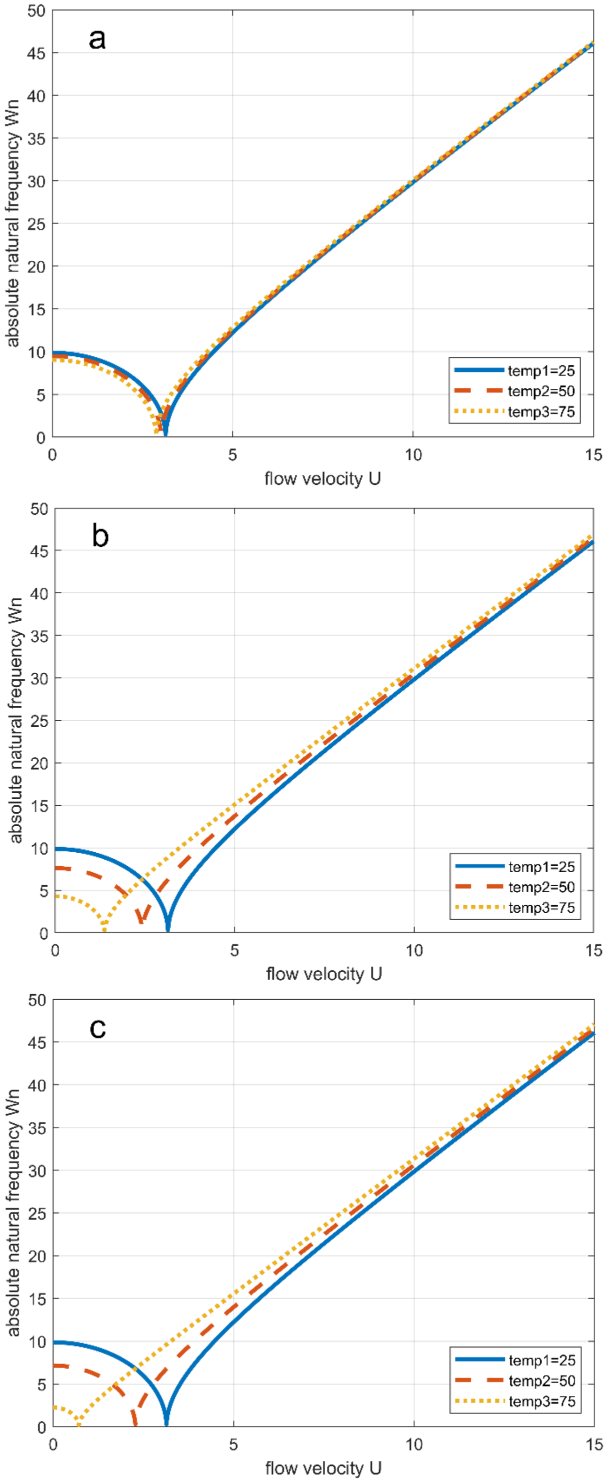

Fig. 7. Variation of the dimensionless absolute values of the transverse natural frequency with dimensionless fluid velocity with varying temperatures and different aspect ratios at T=25°C: a) *L/D* = 20, b) *L/D* = 30, c) *L/D* = 50

inclined pipe. A small increase in the axial tension of the pipe can be caused by this axial component, and it leads to an increase in the natural frequency and critical flow velocity of the pipe.

Another observation has resulted when the pipe, resting on supports that inclined to an angle with the horizontal axis, subjected to the thermal effect as it is indicated in the next results. The pipe containing flowing fluid under thermal effect is susceptible to lose its stability by the action of two types of compressive stresses; thermal stress, which introduces due to temperature variation; and the stress arising from the internal fluid velocity. However, the stress arising from the internal fluid velocity is significantly lower than the thermal stress [31], and hence the temperature variation in the pipe is a very important factor and considering more significant in comparison with the steady flow velocity in respect of its (pipe) divergence instability. So, the effect of temperature variation on the natural frequency and critical velocity of the inclined pipe is studied through the results in Figure 7, where, the effects of temperature variation on the fundamental natural frequency for different flow velocities are demonstrated.

Generally, increasing the pipe temperature leads to reduce the pipe frequency for all aspect ratios. However, this effect becomes more remarkable with a larger aspect ratio. Any further increase in internal fluid velocity after their critical value leads to converging the lines, which represent the natural frequency for each temperature, and become closer to each other. This behaviour was confirmed by the earlier works done by Refs. [32-34], which revealed that

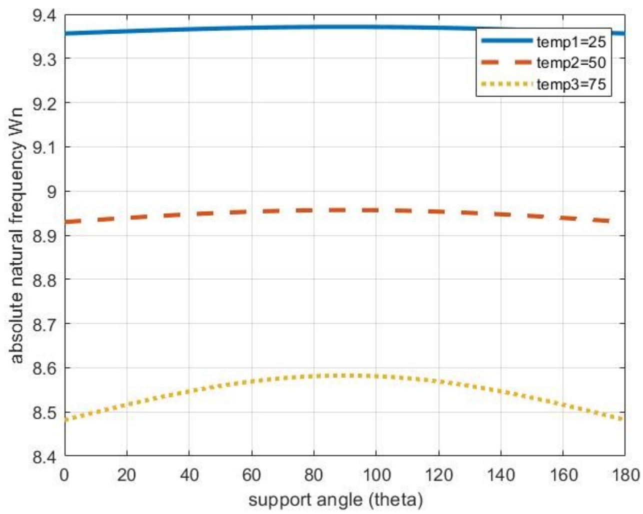

Fig. 8. Variation of the dimensionless absolute values of the transverse natural frequency with support angle for different temperatures at U = 1, *L/D* = 20

natural frequencies are inversely proportional to temperature whether of a tuning fork [32], a plate [33], or a rod [34]. Likewise, the critical flow velocity is reduced as the temperature of the pipe increases, i.e. accelerate the instability of the system. This behaviour may be associated with the softening effect, which increases at elevated temperatures, which can cause the frequency and critical flow velocity to decrease.

Figure 8 depicts the absolute dimensionless natural frequency versus support angles for varying temperatures at a constant flow velocity U=1. The figure shows that the influence of support angles has no significance at relatively low temperatures, but it becomes more noticeable at higher temperatures.

5. Conclusions

Through the theory and numerical simulations for the present study, it can be drawn the following main conclusions:

- The finite Fourier sine and Laplace transforms technique offers a more convenient alternative to calculate the dynamic characteristic of pipes conveying fluid.
- The natural frequencies and critical velocity were inversely proportional to the internal fluid velocity, aspect ratio, temperature variation, and mode number.
- Increasing the inclination angle with the horizontal axis from 0° up to 90° will lead to an increase to the natural frequency and critical fluid velocity. This behaviour is reversed at inclined angles larger than 90° and below 180°.
- There is an optimum value for the aspect ratio with the angle of inclination with the horizontal axis that gives the larger natural frequency and critical velocity of the fluid.
- When the aspect ratio is larger, the corresponding natural frequency reduces more dramatically with increasing the internal fluid velocity.
- At aspect ratio equal to 50, the natural frequency of the pipe increases by 27% when the angle of inclination is increased to 40°, while it decreases by 75% when the temperature is increased to 75°C.

Acknowledgements

We sincerely thank Mechanical Engineering staff at University of Technology for constructive comments which led to serious improvement of the article.

References

[1] M.P. Païdoussis, Fluid-Structure Interactions, Vol. 1, Academic Press Publications, 2014.

[2] R.S. Reddy, S. Panda, A. Gupta, Nonlinear dynamics of an inclined FG pipe conveying pulsatile hot fluid, International Journal of Non-Linear Mechanics 118 (2020) 103276. DOI: https://doi.org/10.1016/j.ijnonlinmec.2019.103276

[3] H.L. Dai, L. Wang, Q. Ni, Dynamics of a fluid-conveying pipe composed of two different materials, International Journal of Engineering Science 73 (2013) 67-76. DOI: https://doi.org/10.1016/j.ijengsci.2013.08.008

[4] H.L. Dai, L. Wang, Dynamics and Stability of Magnetically Actuated Pipes Conveying Fluid, International Journal of Structural Stability and Dynamics 16/6 (2015) 1550026. DOI: https://doi.org/10.1142/S0219455415500261

[5] H. Ashley, G. Haviland, Bending vibrations of a pipe line containing flowing fluid, Journal of Applied Mechanics 17/3 (1950) 229-232. DOI: https://doi.org/10.1115/1.4010122

[6] T.B. Benjamin, Dynamics of a system of articulated pipes conveying fluid. I: theory, Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences 261/1307 (1962) 457-486. DOI: https://doi.org/10.1098/rspa.1961.0090

[7] R.W. Gregory, M.P. Paidoussis, Unstable oscillation of tubular cantilevers conveying fluid. I: theory, Proceedings of the Royal Society of London, Series A, Mathematical and Physical Sciences 293/1435 (1966) 512-527. DOI: https://doi.org/10.1098/rspa.1966.0187

[8] H.R. OzÖz, Non-linear vibrations and stability analysis of tensioned pipes conveying fluid with variable velocity, International Journal of Non-Linear Mechanics 36/7 (2001) 1031-1039. DOI: https://doi.org/10.1016/S0020-7462(00)00065-2

[9] J.L. Hill, C.P. Swanson, The Effects of Lumped Masses on the Stability of Fluid Conveying Tubes, Journal of Applied Mechanics 37/2 (1970) 494-497. DOI: https://doi.org/10.1115/1.3408533

[10] Q. Ni, Y. Luo, M. Li, H. Yan, Natural frequency and stability analysis of a pipe conveying fluid with axially moving supports immersed in fluid, Journal of Sound and Vibration 403 (2017) 173-189. DOI: https://doi.org/10.1016/j.jsv.2017.05.023

[11] B.Y. Dagli, A. Ergut, Dynamics of Fluid Conveying Pipes Using Rayleigh Theory Under Non-Classical

Boundary Conditions, European Journal of Mechanics – B/Fluids 77 (2019) 125-134. DOI: https://doi.org/10.1016/j.euromechflu.2019.05.001

[12] D.B. Giacobbi, C. Semler, M.P. Païdoussis, Dynamics of Pipes Conveying Fluid of Axially Varying Density, Journal of Sound and Vibration 473 (2020) 115202. DOI: https://doi.org/10.1016/j.jsv.2020.115202

[13] M.J. Mohammed, I.Z.M. Darus, Active vortex induced vibration controller and neuro identification for marine risers, Journal of Theoretical and Applied Information Technology 70/1 (2014) 152-162.

[14] M.J. Mohammed, I.Z.M. Darus, N.M.R. Shaharuddin, A.A.M. AL-Khafaji, Open Loop Active Control Technique on Segmented Marine Riser Vibration Using Electromechanical Actuator", ranian Journal of Science and Technology, Transactions of Mechanical Engineering 43 (2019) 799-813.
DOI: https://doi.org/10.1007/s40997-018-0229-y

[15] M.J. Mohammed, I.Z.M. Darus, PID controller for NARX and ANFIS models of marine pipe cylinder undergoes vortex induced vibration, Journal of Theoretical and Applied Information Technology 66/1 (2014) 359-367.

[16] D. Yu, M.P. Païdoussis, H. Shen, L. Wang, Dynamic Stability of Periodic Pipes Conveying Fluid, Journal of Applied Mechanics 81/1 (2013) 011008. DOI: https://doi.org/10.1115/1.4024409

[17] M.A. Tawfik, Z.K. Kadhim, R.Y. Hammoudi, Vibration Analysis of Sudden Enlargement Pipe Conveying Fluid with Presence of Heat Flux, Engineering and Technology Journal 27/3 (2009) 533-557.

[18] M.J. Jweeg, Z.I. Mohammad, Vibration Characteristics of Different Cross-Section Pipes with Different End Conditions, Engineering and Technology Journal 28/8 (2010) 1634-1654.

[19] T.A. El-Sayed, H.H. El-Mongy, Free vibration and stability analysis of a multi-span pipe conveying fluid using exact and variational iteration methods combined with transfer matrix method, Applied Mathematical Modelling 71 (2019) 173-193.
DOI: https://doi.org/10.1016/j.apm.2019.02.006b

[20] M.J. Jweeg, T.J. Ntayeesh, Dynamic analysis of pipes conveying fluid using analytical, numerical and experimental verification with the aid of smart materials, International Journal of Science and Research 4/12 (2015) 1594-1605.

[21] S. Chandurkar, R. Kadoli, Finite Element and Differential Quadrature Solution for Natural Frequency of a Clamped-Free Pipe Conveying Fluid, AIP Conference Proceedings 2134/1 (2019) 040006. DOI: https://doi.org/10.1063/1.5120214

[22] K.A. Ameen, M.J. Al-Dulaimi, A.A. Hatem, Experimental Study of Vibration on Pipe Conveying Fluid at Different End Conditions for Different Fluid Temperatures, Engineering and Technology Journal 37/12 (2019) 512-515.

[23] M.P. Paidoussis, Dynamics of tubular cantilevers conveying fluid, Journal of Mechanical Engineering Science 12/2 (1970) 85-103. DOI: https://doi.org/10.1243%2FJMES_JOUR_1970_012_017_02

[24] L. Wang, Q. Ni, A note on the stability and chaotic motions of a restrained pipe conveying fluid, Journal of Sound and Vibration 296/4-5 (2006) 1079-1083. DOI: https://doi.org/10.1016/j.jsv.2006.03.016

[25] M.P. Paidousiss, G. Li, Pipes Conveying Fluid :a Model Dynamical Problem, Journal of Fluids and Structures 7/2 (1993) 137-204.
DOI: https://doi.org/10.1006/jfls.1993.1011

[26] R.H. Plaut, Post buckling and Vibration of End-Supported Elastic Pipes Conveying Fluid and Columns Under Follower Loads, Journal of Sound and Vibration 289/1-2 (2006) 264-277.
DOI: https://doi.org/10.1016/j.jsv.2005.02.032

[27] Y. Huang, Y. Liu, B. Li, Y. Li, Z. Yue, Natural Frequency Analysis of Fluid Conveying Pipeline with Different Boundary Conditions, Nuclear Engineering and Design 240/3 (2010) 461-467.
DOI: https://doi.org/10.1016/j.nucengdes.2009.11.038

[28] D. Jung, J. Chung, In-plane and out-of-plane motions of an extensible semi-circular pipe conveying fluid, Journal of Sound and Vibration 311/1-2 (2008) 408-420. DOI: https://doi.org/10.1016/j.jsv.2007.09.011

[29] C.A. Osheku, Non-Linear Flow Induced Vibration with Respect to an Offshore Pipeline in Deep Ocean, European Journal of Scientific Research 66/4 (2011) 541-562.

[30] N. Haidar, S. Obaid, M. Jawad, Instability of angled pipeline arising from internal fluids flow, The Iraqi Journal for Mechanical and Material Engineering 12/2 (2012) 222-237.

[31] Q. Qian, L. Wang, Q. Ni, Instability of simply supported pipes conveying fluid under thermal loads, Mechanics Research Communications 36/3 (2009) 413-417.
DOI: https://doi.org/10.1016/j.mechrescom.2008.09.011

[32] Z.L. Greer, Temperature, frequency, and young's modulus of an aluminum tuning fork, International School Bangkok Journal of Physics 5/1 (2011) 1-5.

[33] P. Lee, J. Wang, Frequency-temperature relations of thickness-shear and flexural vibration of contoured quartz resonators, Proceedings of IEEE International Frequency Control Symposium, Honolulu, HI, USA, 1996, 632-639.
DOI: https://doi.org/10.1109/FREQ.1996.559944

[34] X.Q. Shi, H.L. Pang, W. Zhou, Z.P. Wang, Low cycle fatigue analysis of temperature and frequency effects on eutectic solder alloy, International Journal of Fatigue 22/3 (2000) 217-228.
DOI: https://doi.org/10.1016/S0142-1123(99)00124-3