Experimental Validation of the 3D Dynamic Unicycle-Unicyclist Model

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Abstract

The problem of motion of a unicycle – unicyclist system in 3D is studied. The equations of motion of system have been derived using the Boltzmann-Hamel equations. A description of the unicycle – unicyclist system dynamical model, simulation results and experimental validation of the system are presented in the paper.

Keywords: unicycle, 3D dynamic model, Boltzmann-Hamel equations

1. Introduction

1.1. Unicycle – one-wheel vehicle

Unicycle, one-wheel vehicle, is a specific type of single track, which is a bicycle. It has only one road wheel. Unicycle is shown in the figure below [1].

Figure 1. Typical unicycle [2]

The main feature of unicycle is fixed gear. Therefore, the rotation of the cranks directly controls the rotation of the wheel, and positions of unicyclist's legs. Riding without pedalling is impossible. Riding a unicycle is more difficult than on regular bicycle, due to the fact that there is only one point of support. For this reason, a balance must be simultaneously maintained in two planes, transverse and parallel to the direction of moving, so that the centre of gravity oscillates above the fulcrum of the wheel.

In technical aspect unicycle-unicyclist system, can be considered as a moving double inverted spherical pendulum with follow-up control system.

1.2. Boltzmann-Hamel equations

The Boltzmann-Hamel equations are rarely used because of complicated formulae containing Hamel coefficients and complex relationships for the determination of these coefficients [3, 4, 5, 6, 7]. The classic form of the Boltzmann-Hamel equations for a system with the number of coordinates equal to k is as follows [3, 4]

$$\frac{d}{dt}\left(\frac{\partial T^*}{\partial w_n}\right)-\frac{\partial T^*}{\partial \pi_n}+\sum_{m=1}^{m=k}\sum_{l=1}^{l=k}\sum_{i=1}^{j=k}\sum_{j=1}^{j=k} b_{li}b_{mj}\left(\frac{\partial a_{im}}{\partial q_l}-\frac{\partial a_{il}}{\partial q_m}\right)\frac{\partial T^*}{\partial w_i}w_j=\Pi_n\,,\quad (n=1,\ldots k) \tag{1}$$

Matrix form of Boltzmann-Hamel equations

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{T}^*}{\partial \mathbf{w}}\right)+\mathbf{B}^{\mathbf{T}}\left(\dot{\mathbf{A}}^{\mathbf{T}}-\mathbf{D}^{\mathbf{T}_0}\mathbf{B}\,\mathbf{w}\right)\frac{\partial T^*}{\partial \mathbf{w}}-\mathbf{B}^{\mathbf{T}}\frac{\partial \mathrm{T}^*}{\partial \mathbf{q}}=\mathbf{B}^{\mathbf{T}}\left(\mathbf{f}\;-\frac{\partial \mathrm{V}}{\partial \mathbf{q}}\right) \tag{2}$$

allows to automate generation of Hamel coefficients and eliminates all difficulties associated with a determination of these quantities [8].

2. Description of the analysed model

For the unicycle-unicyclist model description we use fixed inertial frame *Oxyz* (Fig. 2). We also use no inertial frames $x_iy_iz_i$, inertial frames $\xi_i\eta_i\zeta_i$ and parallel frames $x_i'y_i'z_i'$ or $\xi_i'\eta_i'\zeta_i'$ related to each link (i=1,…,7), attached at the end of it.

Table 1. Model of the unicycle-unicyclist system

i	1	2	3	4	5	6	7
mark	w	f	b	tir	thr	til	thl
link	wheel	frame	body	tibia right	thigh right	tibia left	thigh left

To consider motion of the system, we introduce the following generalized coordinates

$$\mathbf{q}=[x_w,\,y_w,\,z_w,\alpha_w,\,\beta_w,\gamma_w,\alpha_f,\alpha_b,\beta_b]^T\,, \tag{3}$$

where x_w, y_w, z_w are the coordinates of the wheel contact point, and the remaining ones are the Euler angles describing spatial orientation with respect to the particular frame, Fig. 2.

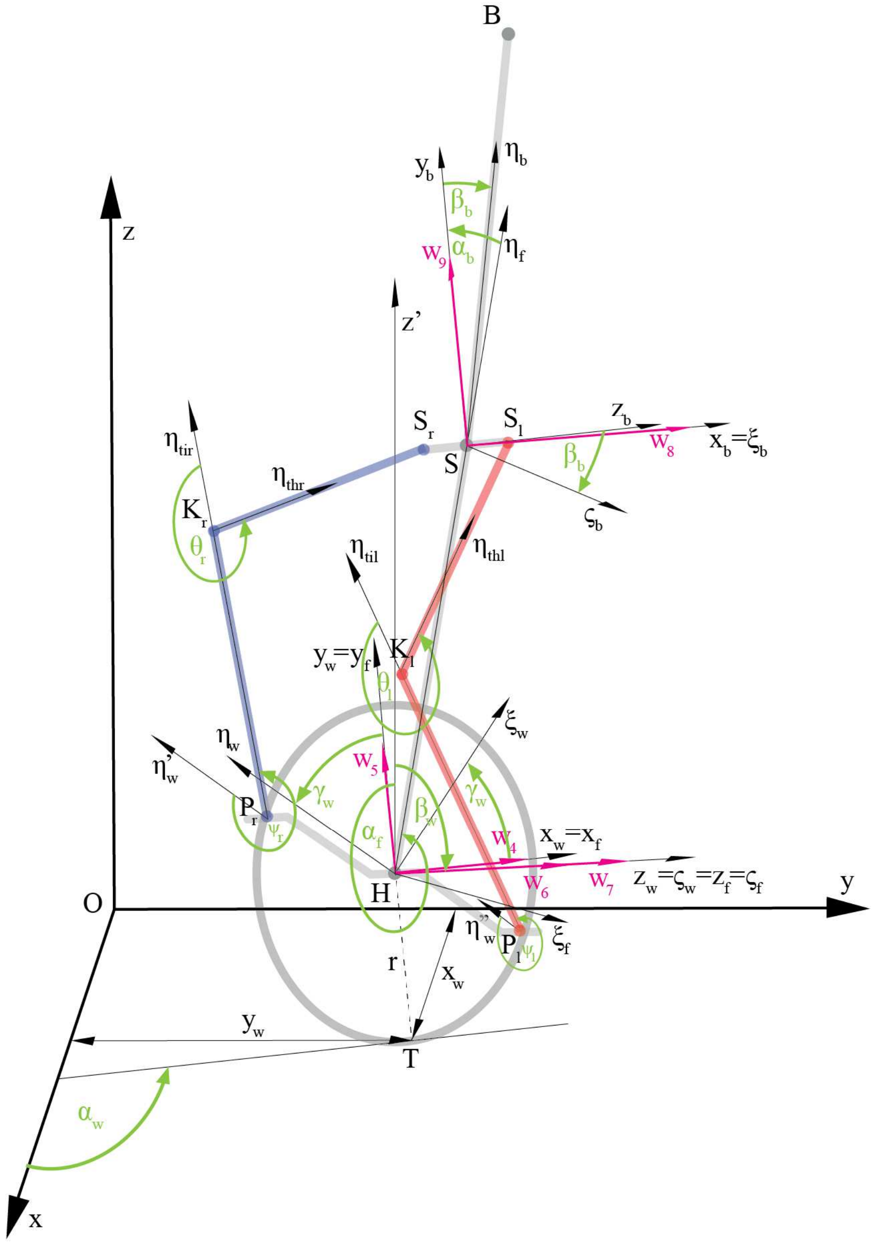

Figure 2. Model of the system (some axes are omitted for reasons of clarity)

A unicyclist leg which is used in this model consists of thigh and tibia. Foot is omitted due to the specific and complex motion in one rotational cycle, which does not aspect significantly in a ride. Thereby, pedal axes are covered up with ankle. Therefore, the leg can be treated as a crank mechanism and the leg position is clearly defined by γ_w and α_f [9].

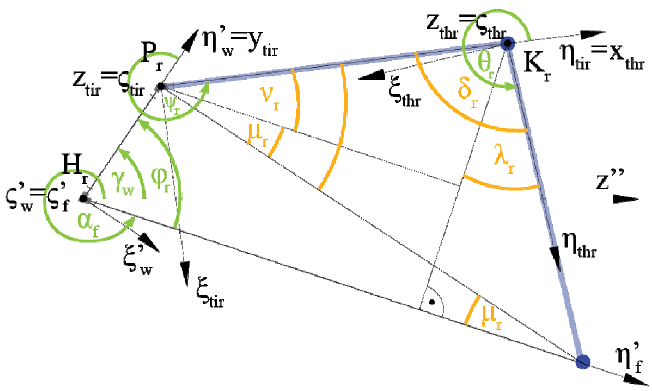

Figure 3. Leg positions and coordinates, on an example of the right leg

Quasi-velocities (Fig. 2) defining the model velocities are assumed in the form:

$$
\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \\ w_7 \\ w_8 \\ w_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & r\cos\alpha_w & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & r\sin\alpha_w & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sin\beta_w & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\beta_w & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\beta_w & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_w \\ \dot{y}_w \\ \dot{z}_w \\ \dot{\alpha}_w \\ \dot{\beta}_w \\ \dot{\gamma}_w \\ \dot{\alpha}_f \\ \dot{\alpha}_b \\ \dot{\beta}_b \end{bmatrix} = \mathbf{A}\,\dot{\mathbf{q}}. \tag{4}
$$

where r is the radius of the wheel. Equations (4) are valid under assumption that the wheel is a rigid hoop making point contact with the road and it rolls without longitudinal slip on a flat surface. It means that the constraint equations for the wheel are: w_1=0, w_2=0 and w_3=0. Kinetic energy, with respect to mass canters of the system is obtained using the formula

$$
T = \frac{1}{2}\sum_{i=1}^{n} \mathbf{v}_i^T \mathbf{M}_i \mathbf{v}_i + \frac{1}{2}\sum_{i=1}^{n} \boldsymbol{\omega}_i^T \mathbf{I}_i \boldsymbol{\omega}_i\,, \quad (n = 1,\ldots,7). \tag{5}
$$

where $\mathbf{v}_i$ is the vector of linear velocities, $\mathbf{M}_i$ is the mass matrix, $\boldsymbol{\omega}_i$ is the vector of angular velocities and $\mathbf{I}_i$ are the moments of inertia standing in the mass matrix.

The equations of model dynamics based on Boltzmann-Hamel equation (2) were generated automatically and solved using *Wolfram Mathematica*.

3. Simulation results

Results of numerical simulation for the unicycle-unicyclist model motion are shown in Figs. 4–6. The most important initial conditions for simulations are the vertical position and the constants of wheel velocity. It is a wire model; which means that every link is a rigid rod, except the wheel regarded here as a rigid circular hoop. Appropriate damping in the nodes provides that the system does not immediately collapse and small values of masses of legs epitomize control of the unicycle by a unicyclist.

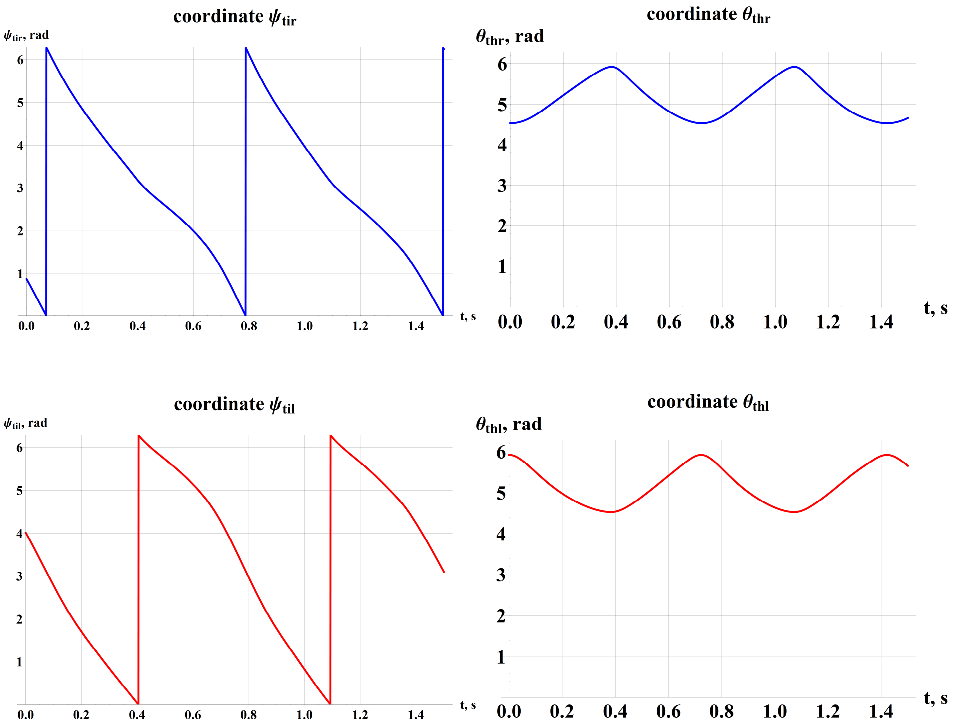

Figure 4. Time histories of legs coordinates. Right leg (blue) and left leg (red)

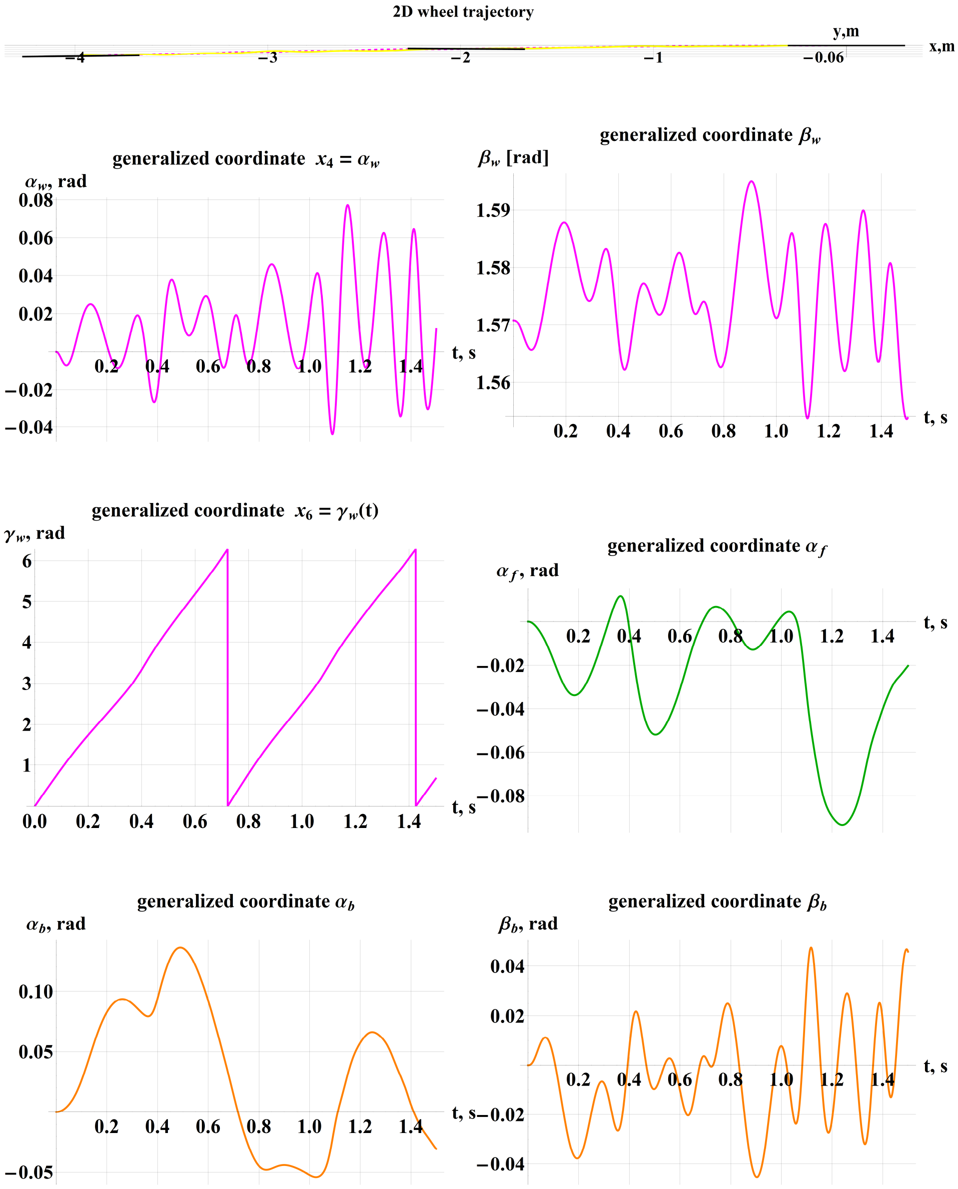

Figure 5. Wheel 2D trajectory and time histories of the system Euler angels

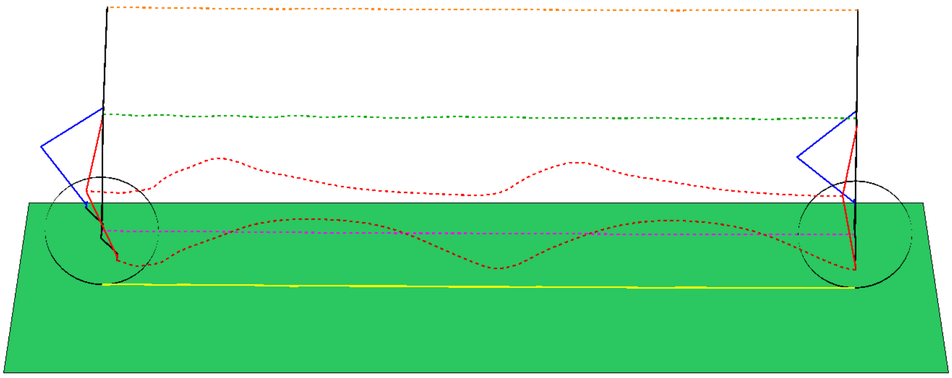

Figure 6. 3D trajectories of the system

4. Experimental validation

To capture motion of the real object, a high speed camera was used. A duration of single attempt is about two seconds. The quadrant symmetry markers were used. To process the movies, the *TEMA* software was used. An experiment in 2D was made in order to check, if the way of modelling is correct. Below there are shown the parametric plots of positions of the characteristics point of the model.

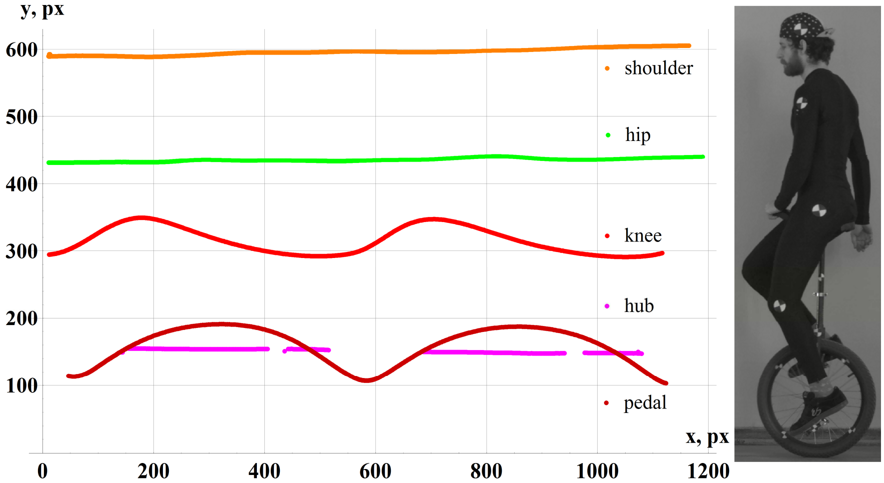

Figure 7. 2D trajectories of the motion capture of the real object

By comparing Fig. 6. with Fig. 7. it can be seen, that trajectories of characteristic points have very similar courses. Dissimilarities may be due to the fact that the experiment was made in 2D, while the real object moves in 3D.

5. Conclusions

The matrix notation of Boltzmann-Hamel equations eliminates drawbacks occurring with the classical formulation of these equations. Its application allows an automation of generation process of motion equations.

It is clearly shown that the model during movements swings around an unstable equilibrium. Because of unbalance caused by legs and cranks with pedals, the wheel moves in a “snake style”. To sum up, our model behaves like a real object. It is confirmed by a comparison of the trajectory of characteristic points, by 2D motion capture of the real object.

In the future, in this model also a tire will be taken into consideration as well as and a system control method are going to be introduced. Upon those steps, the 3D motion capture will be made to validate the final model.

References

1. M. Niełaczny, J. Grabski, J. Strzałko, *3D dynamic model of the unicycle – unicyclist system*, Journal of Vibrations in Physical Systems, Poznan, 2014.
2. [online] [access: 18-12-2011] http://qu-ax.com/en/products
3. R. Gutowski R, *Mechanika analityczna*, PWN Warszawa 1971.
4. J. I. Nejmark, N. A. Fufajew, *Dynamika układów nieholonomicznych*, PWN Warszawa 1971.
5. W. Blajer, *Metoda projekcyjna - teoria i zastosowania w badaniu nieswobodnych układów mechanicznych*, WSI Radom, 13/1994.
6. W. Blajer, Projective formulation of Lagrange's and Bolzmann-Hamel equations for multibody systems, ZAMM, **75** (1995) 107 – 108.
7. J. M. Maruskin, A. M. Bloch, *The Boltzmann's Hamel equations for the optimal control of mechanical systems with nonholonomic constraints*, Int. J. Robust. Nonlinear Control, **21** (2011).
8. J. Grabski, J. Strzałko, *Automatic Generation of Boltzmann-Hamel Coefficients for Mechanical Problems.* Mechanics and Mechanical Engineering International Journal, **1** (1997) 61 – 77.
9. JI. B. Берестов, *Сравнительный анализ реакций в кинематических парах механизма шарнирног четырехзвенника для различных схем уравновешивания*, Механика машин. – М.: Наука, (1977) 61 – 70.