

Modeling and Parameter Estimation of Radar Sea-Clutter with Trimodal Gamma Population

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Abstract—Real radar data often consist of a mixture of Gaussian and non-Gaussian clutter. Such a situation creates one or more inflexion points in the curve of the empirical cumulative distributed function (CDF). In order to obtain an accurate fit with sea reverberation data, we propose, in this paper, a trimodal gamma disturbance model and two parameter estimators. The non-linear least-squares (NLS) fit approach is used to avoid computational issues associated with the maximum likelihood estimator (MLE) and moments-based estimator for parameters of the mixture model. For this purpose, a combination of moment fit and complementary CDF (CCDF) NLS fit methods is proposed. The simplex minimization algorithm is used to simultaneously obtain all parameters of the model. In the case of a single gamma probability density function, a $\text{zlog}(z)$ method is derived. Firstly, simulated life tests based on a gamma population with different shape parameter values are worked out. Then, numerical illustrations show that both MLE and $\text{zlog}(z)$ methods produce closer results. The proposed trimodal gamma distribution with moments NLS fit and CCDF NLS fit estimators is validated to be in qualitative agreement with different cell resolutions of the available IPIX database.

Keywords—CCDF, estimation, least squares, MLE, modeling, trimodal Gamma model, $\text{zlog}(z)$.

1. Introduction

Modeling of unknown and non-stationary radar sea-clutter statistics is a serious research subject for target detection with a constant false alarm rate (CFAR). Compound Gaussian class distributions are useful models for sea-clutter observed by high resolution radars [1]. It is shown that gamma, inverse gamma, lognormal, and inverse Gaussian are efficient disturbances for the texture component characterizing the variability of both sea surface conditions and selected radar parameters [2]. A two-parameter family of continuous probability distributions on the positive real line is such a class of models.

Popular K distribution is widely applied in many disciplines of radar signal processing and is obtained from a gamma distributed texture component. The well-known Pareto type II model has been shown to occur as intensity distribution of the compound Gaussian process with an inverse gamma texture [3]. The compound Gaussian inverse Gaussian (CGIG) distribution is constructed if the modulation component follows the inverse Gaussian law [4]. However, in situations when we have a sequence of sea clutter with two or more distributions, compound-Gaussian models cited above fail to fit in with empirical data. This is particularly true for intelligent pixel X-band (IPIX) backscatter obtained from a small grazing angle and/or a low-range cell surface, using horizontal antenna polarization for transmit and receive [5], [6].

Various distributions that are probabilistic mixtures of other distributions have been proposed in the available literature [7]–[9]. Rosenberg *et al.* [7] analyzed the KK distribution for modeling the Ingara radar database with different scenarios. The addition of multiple looks and a thermal noise component is considered to produce greater accuracy of the mean and underlying shape parameters. In [8], a mixture of K and lognormal distributions is proposed to model the clutter data, the target data, or the mix of clutter and target data. The ML method using the expectation-maximization approach is presented for estimating the parameters of the mixture model. Experiments including synthetic aperture radar (SAR) data are conducted to show the effectiveness of the mixture model against KK and lognormal-lognormal distributions. In [9], a trimodal discrete (3MD) radar clutter model is utilized for modeling radar sea-clutter. A six-parameter mixture model is considered in paper [9], involving a multi-look Gaussian clutter scenario.

Parameter estimation of clutter models involving shape and scale parameters is an essential task, particularly if coherent or non-coherent detection processors are required to comply with the CFAR property. In [10], the authors presented moment-based methods including higher order moments,

fractional order moments and log-moments for estimating K distribution parameters. A method for synthesizing correlated K -distributed random fields is also reported. The Gauss quadrature method based on Legendre and Laguerre polynomials is applied to constrained non-integer order moments, $\text{zlog}(z)$ and MLE approaches for the estimation of Pareto type II, K and CGIG clutter parameters [11], [12]. Convergence is verified by using the first two integer order moments, in which the estimation problem is reduced to one dimension. Computer simulations are illustrated with known and unknown clutter-to-noise ratio (CNR).

In [9], four estimation methods based on a mixture of Gaussian distributed parameters are derived: two method-of-moments estimators (i.e. integer order moments and fractional order moments) and two other, based on NLS fit to the CCDF and MLE procedures. Accuracy and the computational time of the estimators is compared in terms of sample size, number of pulses and clutter parameter values. In [13], the Wilson-Hilferty normal-based approximation method is proposed to estimate the parameters of a gamma mixture model. The methodology uses a popular Gaussian mixture clustering algorithm, namely the CLUSTring (MCLUST) method and a confidence interval-based search approach to obtain the estimates. Performance comparisons with the existing expectation maximization (EM) approach are performed using both simulated and real-life datasets.

In this paper, we extend the recent work presented in [9] by using a mixture gamma distribution in a single-look transmission. It is an alternative solution proposed as a more accurate model in some scenes of the IPIX data (low range resolution) labeled trimodal gamma disturbance. Because gamma and incomplete gamma functions are presented in this model, the NLS fit approach is applied in this work to avoid computational issues associated with ML and moments estimators of the parameters of mixture models. For this purpose, the matching of moments and the CCDF approach is proposed, in which simplex minimization is used to obtain all model parameters simultaneously.

In the case of single gamma PDF, the $\text{zlog}(z)$ method is derived. Simulated life tests performed with the use of the gamma population with different values of the shape parameters are worked out at first. Numerical illustrations show that both MLE and $\text{zlog}(z)$ methods produce closer results. The proposed trimodal gamma distribution with moment NLS fit and CCDF NLS fit estimators is validated to be in qualitative agreement with different cell resolutions of the available IPIX database.

The paper is organized as follows. In Section 2, we initially review the mixture gamma distribution, the mixture gamma CDF and the mixture r -th raw moment's expression. Then, in Section 3, we present the MLE- and $\text{zlog}(z)$ -based estimators for a single gamma distribution. After that, estimation procedures based on NLS CCDF fit and NLS moments fit are presented for the mixture of two and three gamma distributions. A series of numerical illustrations is given in Section 4. Finally, conclusions are outlined in Section 5.

2. Mixture Gamma Distribution

Analysis of high-resolution surveillance radar shows that statistical mixture models fit accurately with land and sea reverberation data. Gamma distribution is a two-parameter family of continuous probability distributions. Exponential distribution, Erlang distribution, and chi-square distribution are special cases of the gamma model. The gamma PDF of the random variable x is given by [14], [15]:

$$f(x; \beta, \alpha) = \frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)} e^{-\beta x}, \quad (1)$$

where x denotes clutter intensity (power), $\Gamma(\cdot)$ is the gamma function, α is the shape parameter and β is the scale parameter. Its CDF is:

$$F(x; \alpha, \beta) = \gamma(\beta x, \alpha), \quad (2)$$

where $\gamma(x, a) = \frac{1}{\Gamma(a)} \int_0^x t^{a-1} e^{-t} dt$ is the lower incomplete gamma function [16]. The r -th raw moment can be defined as:

$$E[x^r] = \beta^{-r} \frac{\Gamma(\alpha + r)}{\Gamma(\alpha)}. \quad (3)$$

As discussed, many models, including gamma distribution, fail to fit in well with real data that follow two or more densities. This occurs when radar echoes are observed from small range cells. To this effect, a general mixture gamma distribution could be capable of describing the majority of data scenarios given by [13]:

$$f(x; p_i, \beta_i, \alpha_i) = \sum_{i=1}^n p_i \frac{\beta_i^{\alpha_i} x^{\alpha_i-1}}{\Gamma(\alpha_i)} e^{-\beta_i x}, \quad (4)$$

where $p_i, i = 1, \dots, n$ is the probability, and n is the number of gamma distributions. The corresponding CDF of Eq. (4) is written as:

$$F(x; p_i, \alpha_i, \beta_i) = \sum_{i=1}^n p_i \gamma(\beta_i x, \alpha_i). \quad (5)$$

The r -th raw moment is given as a function of the gamma function with fractional variables:

$$E[x^r] = \sum_{i=1}^n p_i \beta_i^{-r} \frac{\Gamma(\alpha_i + r)}{\Gamma(\alpha_i)}. \quad (6)$$

The mixture model has $3n - 1$ parameters, n discrete scale parameters, β_i , n discrete shape parameters α_i and $n - 1$ probabilities p_i . Note that, Eqs. (4)–(6) reduce to the mixture expressions given by Bocquet *et al.* [9] with $\alpha_i = 1$, $\beta_i = \frac{1}{b_i}$ and a single look scenario transmission.

3. Parameter Estimation

In this section, we consider some estimators based on MLE, log-moments, CCDF fit and moments fit of parameters for gamma distribution and a mixture of two and three gamma distributions.

3.1. MLE and zlog(z) Estimators of Gamma PDF

The MLE method has been used to estimate shape and scale parameters from a finite number of independent samples, x_1, x_2, \dots, x_N observed [18]. This was achieved by maximizing the likelihood function (LF), so that:

$$\begin{cases} \ln(\hat{\alpha}) - \ln(\langle x \rangle) = \frac{\partial}{\partial \hat{\alpha}} \ln [\Gamma(\hat{\alpha})] + \langle \ln(x) \rangle = 0 \\ \hat{\beta} = \frac{\hat{\alpha}}{\langle x \rangle} \end{cases} \quad (7)$$

Simplifying (7) allows to numerically determine $\hat{\alpha}$ as:

$$\begin{cases} \ln(\hat{\alpha}) - \ln(\langle x \rangle) - \psi(\hat{\alpha}) + \langle \ln(x) \rangle = 0 \\ \hat{\beta} = \frac{\hat{\alpha}}{\langle x \rangle} \end{cases} \quad (8)$$

where $\langle \cdot \rangle$ denotes the empirical mean and $\psi(\cdot)$ is the psi function [16]. From formula (8), it can be easily seen that the resulting MLE for estimating α is non-linear and has no closed form solutions. Many papers reveal that MLE and zlog(z) estimators have approximate results, i.e. [17]. To obtain a short execution time, a closed form zlog(z) estimator is derived below, but with some mathematical manipulations of log-moments. Using the fact that:

$$\Gamma(x+1) = x\Gamma(x), \quad \frac{\partial \Gamma(x)}{\partial x} = \Gamma(x)\psi(x)$$

and

$$\psi(x+1) = \psi(x) + \frac{1}{x} \quad [16],$$

the derivative of Eq. (3) with respect to the moment order r is found to be:

$$\begin{aligned} \frac{\partial \langle x^r \rangle}{\partial r} &= \langle x^r \ln(x) \rangle \\ &= \beta^{-r} \ln\left(\frac{1}{\beta}\right) \frac{\Gamma(\alpha+r)}{\Gamma(\alpha)} + \beta^{-r} \frac{\Gamma(\alpha+r)}{\Gamma(\alpha)} \psi(\alpha+r). \end{aligned} \quad (9)$$

Using the first integer moment, $\langle x \rangle$ and Eq. (9) using $r = 0$ and $r = 1$, we write:

$$\begin{cases} \langle x \rangle = \beta^{-1} \alpha \\ \langle \ln(x) \rangle = \ln\left(\frac{1}{\beta}\right) + \psi(\alpha) \\ \langle x \ln(x) \rangle = \beta^{-1} \ln\left(\frac{1}{\beta}\right) \alpha + \beta^{-1} \alpha \psi(\alpha+1) \end{cases} \quad (10)$$

Manipulating formula (10) finally produces the closed form zlog(z) estimator:

$$\begin{cases} \hat{\alpha} \left[\frac{\langle x \ln(x) \rangle}{\langle x \rangle} - \langle \ln(x) \rangle \right]^{-1} \\ \hat{\beta} = \frac{\hat{\alpha}}{\langle x \rangle} \end{cases} \quad (11)$$

Equation (11) has no special functions making it easier to implement compared to (8).

3.2. CCDF Fit and Moments Fit Estimators

Recurrence relations of gamma and incomplete gamma functions with real variables given in Eqs. (5) and (6)

are not easily tractable analytically. For the case of $n = 2$, the corresponding CCDF and moments expressions are:

$$\begin{cases} CCDF(T; p_i, \alpha_i, \beta_i) = p_i [1 - \gamma(\beta_1 T, \alpha_1)] \\ \quad + (1 - p_1) [1 - \gamma(\beta_2 T, \alpha_2)] \\ E[x^r] = p_1 \beta_1^{-r} \frac{\Gamma(\alpha_1 + r)}{\Gamma(\alpha_1)} + p_2 \beta_2^{-r} \frac{\Gamma(\alpha_2 + r)}{\Gamma(\alpha_2)} \end{cases} \quad (12)$$

where T is the normalized detection threshold. Note that the manipulation of equations in formula (12), having five parameters, could not reduce the complexity of the estimation in one or two dimensions. Moreover, the use of log-moments does not resolve such an estimation issue. Therefore, the curve fitting technique is the most suitable approach for simultaneously estimating all model parameters. Its convergence is related to the application of an effective optimization algorithm. Based on expressions (5), (6) and (12), the fitness functions considered in this work are written as the sum of quadratic errors between empirical and theoretical quantities:

$$\begin{cases} Fitness_{CCDF} = \sum_{j=1}^{m_1} \left[real_{CCDF_j} - \sum_{i=1}^n p_i [1 - \gamma(\beta_i T_j, \alpha_i)] \right]^2 \\ Fitness_{Moment} = \sum_{j=1}^{m_2} \left[\langle x^{r_j} \rangle - \sum_{i=1}^n p_i \beta_i^{-r_j} \frac{\Gamma(\alpha_i + r_j)}{\Gamma(\alpha_i)} \right]^2 \end{cases} \quad (13)$$

subject to $\alpha_i > 0$, $\beta_i > 0$ and $\sum_{i=1}^n p_i = 1$. m_1 and m_2 denote numbers of points used in the curves of objective functions. To reduce the research space dimension to $3n - 2$, β_1 expression may be inserted in (13) as a function of the empirical mean $\langle x \rangle$ and the model parameters, so that:

$$\beta_1 = \frac{p_1 \alpha_1}{\langle x \rangle - \sum_{i=2}^n p_i \frac{\alpha_i}{\beta_i}} \quad (14)$$

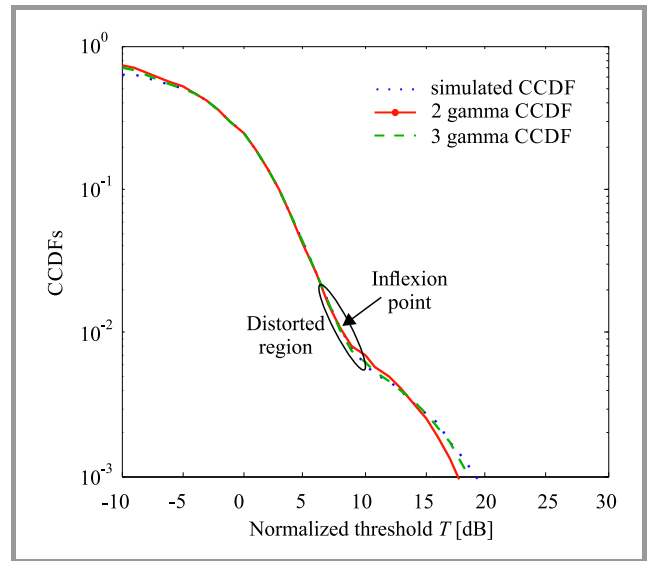


Fig. 1. Simulated CCDF from mixture of gamma distributed samples for: $p_1 = 0.3$, $p_2 = 0.5$, $p_3 = 0.2$, $\alpha_1 = 0.01$, $b_1 = 0.006$, $\alpha_2 = 1$, $b_2 = 1$, $\alpha_3 = 1.2$, and $b_1 = 0.48$.

To show the incapability of modeling radar sea-clutter with a single gamma distribution, we simulate, in Fig. 1, a mixture of Gaussian (i.e. $\alpha = 1$) and gamma (i.e. $\alpha \neq 1$) distributed clutter using the composition method [20]. The distorted region of the CCDF curve exhibited by one or more inflexion points is observed with the presence of Gaussian and non-Gaussian samples in the data sets.

From [5], it has been shown that several data scenes of the IPIX database have a similar nature as the CCDF curve shown in Fig. 1, i.e. a mixture of two or three distributions with different parameter values. Some of these scenarios will be presented in Section 4 by means of two estimators. Based on this, expression (13) is treated as a non-linear optimization problem involving four or seven parameters with a mixture of two and three gamma distributions, respectively.

Many optimization algorithms have been proposed in the literature [14], [19]. Some of them include genetic algorithms, biogeography-based optimization, simplex minimization, and particle swarm optimization [23]. For the purpose of fast convergence, one has to resort to iterative simplex minimization search based on the Nelder-Mead (NM) algorithm [19].

3.3. Simplex Minimization Algorithm

Simplex minimization was widely used for solving a variety of optimization problems. The NM simplex algorithm [21], [22] is an enormously popular search method for multidimensional unconstrained optimization. No derivative of the cost function is required, which makes the algorithm interesting for noisy problems. The NM algorithm falls in the more general class of direct search algorithms. It maintains simplexes being approximations of the optimal point. The vertices are sorted according to objective function values. The algorithm attempts to replace the worst vertex with a new point which depends on the worst point and the center of the best vertices. The goal of this part is to provide an NM direct search optimization method to solve the above constrained optimization problem given by expression (13).

This algorithm is based on the iterative update of a simplex made up of $m + 1$ points $S = \{V_i\}$, $i = 1, 2, \dots, m + 1$. Each point in the simplex is called a vertex and is associated with a function value $f_i = f(V_i)$. It uses four parameters: coefficient of reflection $\rho > 0$, expansion $\chi > 1$ with $\chi > \rho$, contraction $0 < \gamma < 1$ and shrinkage $0 < \sigma < 1$. The standard values of these coefficients are $\rho = 1$, $\chi = 2$, $\gamma = 0.5$ and $\sigma = 0.5$. Moves of the NM simplex are executed according to five main operators: reflection, expansion, inside contraction, outside contraction, shrink after inside contraction, and shrink after outside contraction. These components are interpreted by mathematical equations that are detailed in [21], [22].

The cost function of each estimator is given in expression (13). In the NM optimizer, the search of unknown parameters is carried out with constraints, because all model parameters are real positive. In particular, the two probabilities of trimodal gamma distribution must be limited be-

tween 0 and 1. In the estimation procedure, scale and shape parameters in (13) are restricted, in the following manner, to the range of $[0, 10]$:

$$\begin{cases} \alpha_i = \max[\min(\alpha_i, 10), 0] \\ \beta_i = \max[\min(\beta_i, 10), 0] \\ p_1 = \max[\min(p_1, 1), 0] \\ p_2 = \max[\min(p_2, 1), 0] \\ p_2 = \max[\min(p_2, 1 - p_1), 0] \end{cases}, \quad i = 1, \dots, n. \quad (15)$$

The constraint of the probabilities, p_1 and p_2 in the interval of $[0, 1]$ is also considered in (15). Figure 2 summarizes different steps of the optimization of formula (13) with the constraint procedure given in (15) using an SM based on the NM algorithm.

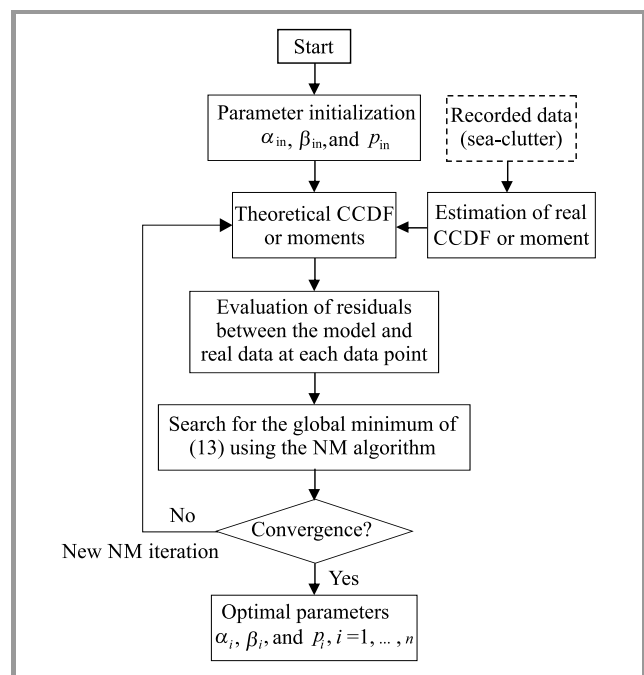


Fig. 2. Flowchart of the simplex minimization algorithm of expression (13).

4. Numerical Illustrations

In this section, we assess the above MLE, $\text{zlog}(z)$, SM CCDF fit and SM moments fit estimators with the use of both simulated and real IPIX databases. One, two and three component gamma distributions are considered for estimation and modeling of some scenarios of the radar IPIX database.

4.1. Gamma Model Case

Consider a random sample of size $N = 1000$ from a finite gamma distribution with its PDF as defined in Eq. (1). A number $n = 1000$ of Monte Carlo runs is used to average MSE and bias criterion tests. MLE and $\text{zlog}(z)$ methods

given by formulas (8) and (11) are executed to estimate gamma PDF parameters.

Through this simulation study, the performance of $\text{zlog}(z)$ estimator is assessed vis-à-vis the MLE method in terms of MSE and bias estimates of the shape parameter as depicted in Fig. 3. From these plots, a comparison of estimates reveals that both approaches produce closer results.

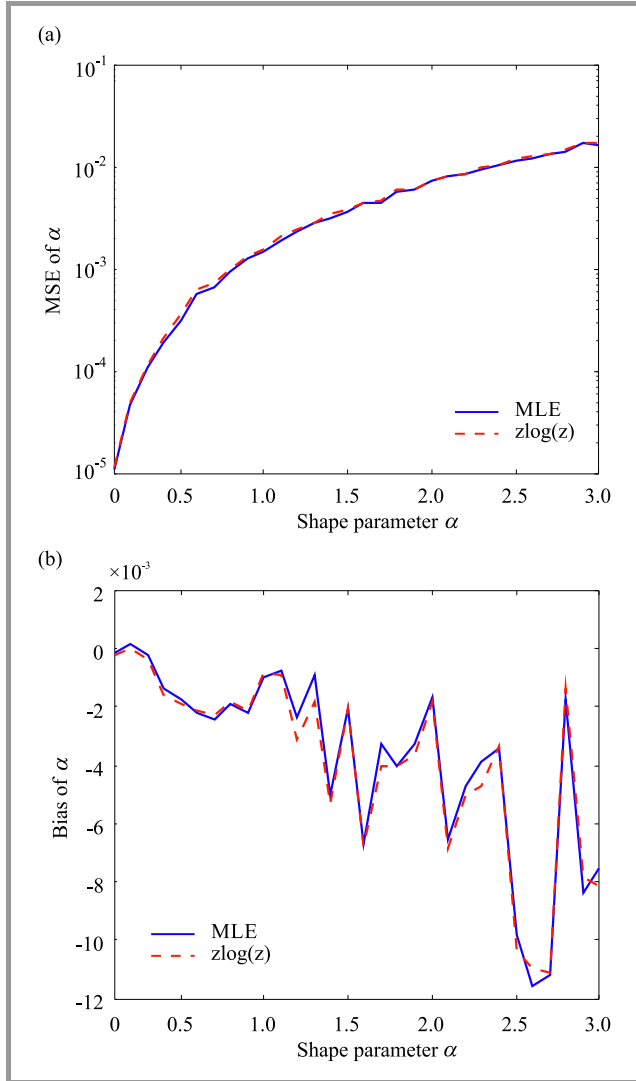


Fig. 3. Estimation comparison of MLE and $\text{zlog}(z)$ methods of α for $N = 1000$ and $n = 1000$: (a) MSE metric test, (b) bias metric test.

An implementation of the $\text{zlog}(z)$ methodology will provide estimates with a shorter execution time than the MLE method. The $\text{zlog}(z)$ procedure may be attributed to the fact that it uses, in the computation of the adaptive CFAR detection, a threshold for radar targets embedded in gamma distributed clutter with unknown parameters. The modeling of IPIX data with a resolution of 30 m, VV polarization and 13th range cell is checked, as shown in Fig. 4, using the two estimators. Figure 4a shows a comparison of empirical and estimated moments with the moments' orders between $r = 0.1$ and $r = 2$. Figure 4b highlights the fit of CCDF with real data always using the two estimators.

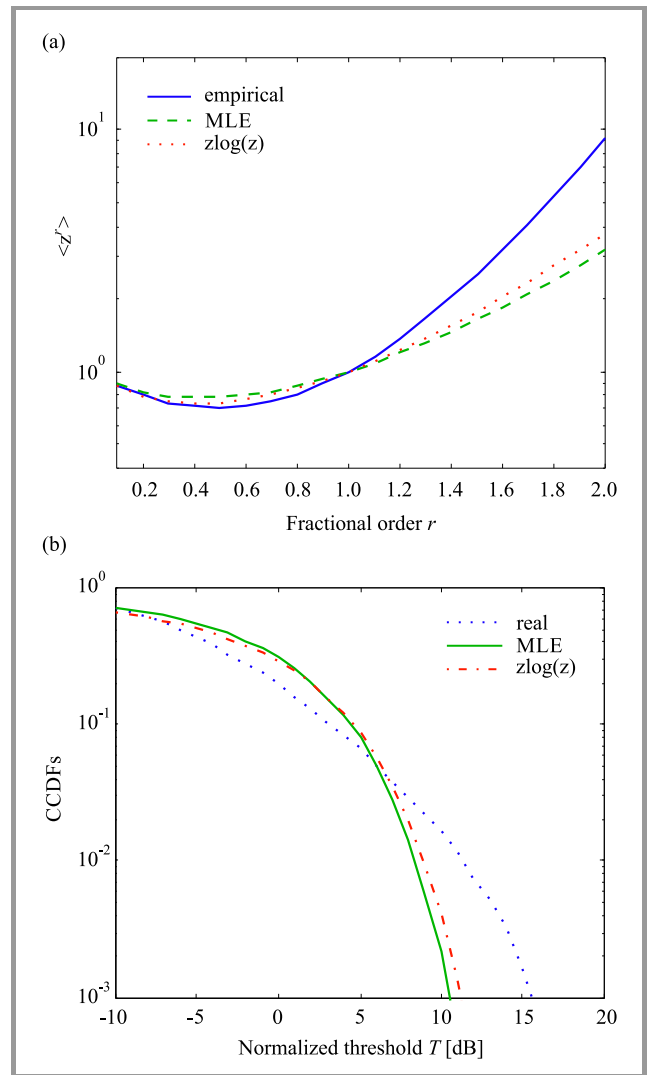


Fig. 4. Modeling of IPIX data with a resolution of 30 m, VV polarization and 13th range cell: (a) moments fit test, (b) CCDF fit test.

It is clearly seen that a single gamma PDF is not capable of modeling such a data scene. Unfortunately, several scenarios of IPIX data cannot be fitted by a single gamma PDF as well. For the purpose of a goodness-of-fit with real data, we consider in the following subsection a mixture of two and three gamma distributions.

4.2. Mixture Gamma Model Case

In this subsection, SM moment fit and SM CCDF fit approaches for the estimation of a mixture of gamma distributions are implemented on IPIX real-life datasets, as described in [12], [24]. The IPIX radar experimental data we processed were collected at Grimsby, Ontario, Canada, from the Communications Research Laboratory, McMaster University.

Sea clutter sets to be used here are measured with the use of the McMaster IPIX radar, a fully coherent X-band radar, with advanced features, such as dual transmit/receive polarization, frequency agility, and stare/surveillance mode.

It is extremely versatile, as each feature is highly adjustable through software in the control computer. Originally, the IPIX radar was shorthand for “ice multi-parameter imaging X-band” radar, called that way as the radar was designed for the detection of growlers, i.e. small pieces of ice breaking away from icebergs. After major upgrades introduced between 1993 and 1998, the high-resolution data collected by the IPIX radar became a benchmark for testing intelligent detection algorithms. Accordingly, the adjustable meaning of the IPIX acronym was changed to “intelligent pixel processing X-band” radar, where the term “pixel” refers to a picture element [24]. The IPIX radar is equipped with computer control and digital data acquisition capabilities. In 1998, a database of high-resolution radar measurements was collected in Grimsby, on the shore of Lake Ontario, between Toronto and Niagara Falls, Canada. The 222 data sets in this database focus specifically on the presence of

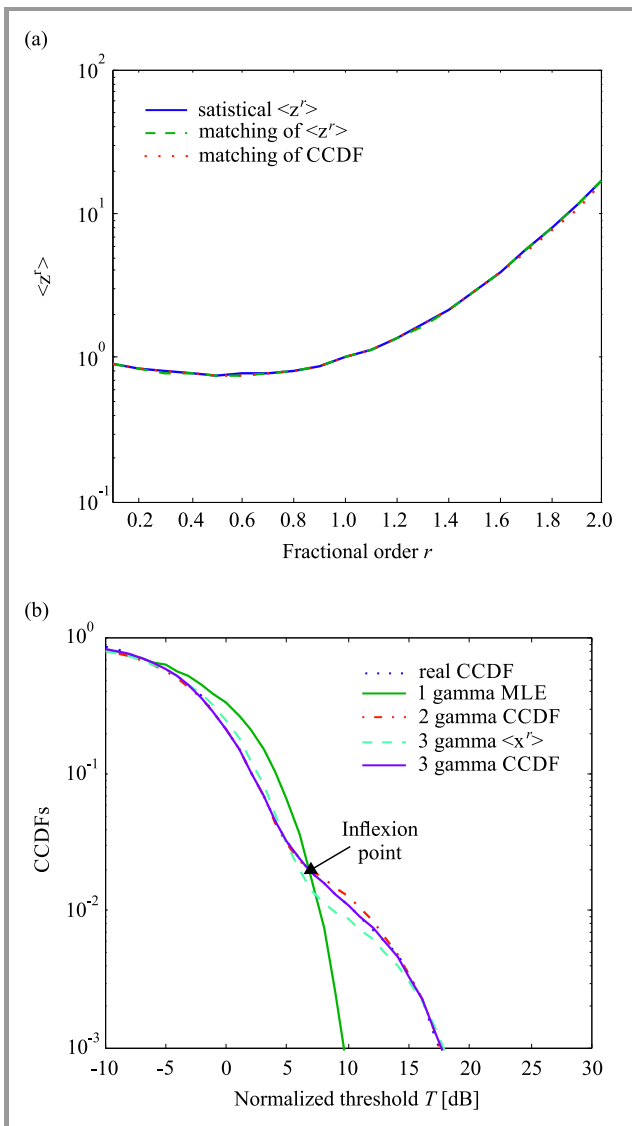


Fig. 5. Modeling of IPIX data with a resolution of 3 m, HH polarization and 17th range cell: (a) moments fit test for the case of 3 gamma PDFs, (b) CCDF fit test of 1 gamma, 2 gamma and 3 gamma PDFs.

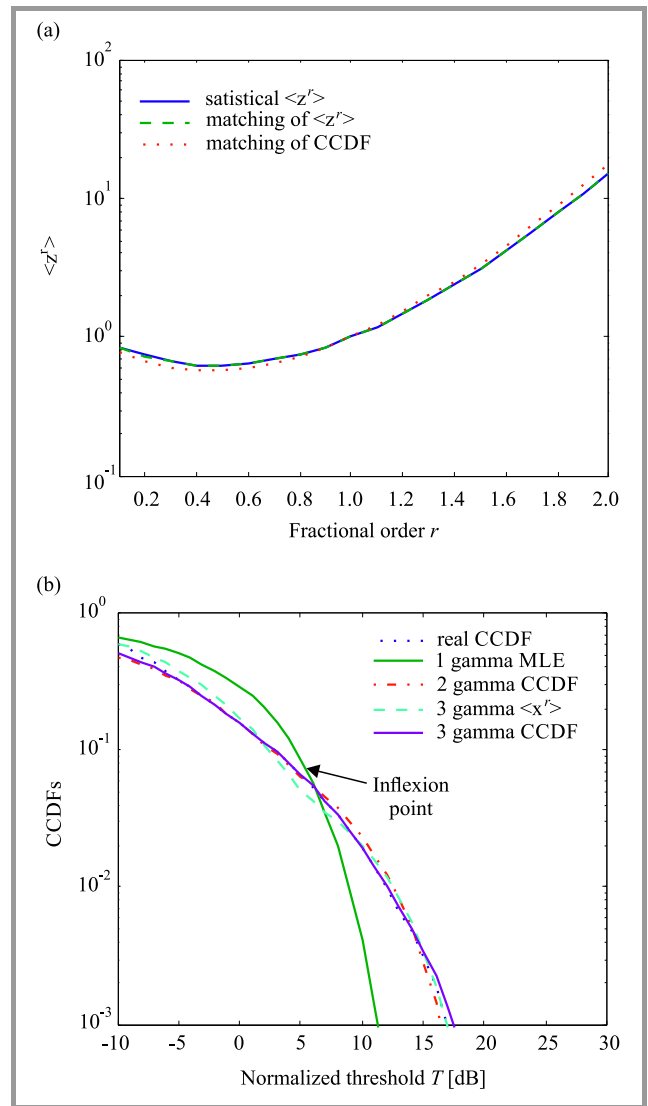


Fig. 6. Modeling of IPIX data with a resolution of 15 m, HH polarization and 5th range cell: (a) moments fit test for the case of 3 gamma PDFs, (b) CCDF fit test of 1 gamma, 2 gamma and 3 gamma PDFs.

floating objects (targets) of varying size, observed under varying weather conditions. A graphical representation of all 222 datasets and images including radar return plots and time Doppler spectra are available in [24]. The characteristic features of the IPIX radar and the environmental conditions under which the radar data was collected are also presented in [24].

The above estimators are executed according to the flowchart presented in Fig. 2. To start the optimization, CCDF values in (13) are considered between $10^{-0.55}$ and 10^{-3} . On the other side, 200 moments values with orders between $r = 0.1$ and $r = 2$ are taken in formula (13). Our first study concerns modeling of IPIX data with a high resolution of 3 m, HH (horizontal-horizontal) antennas polarization and 17th range cell using one gamma, two gamma and three gamma PDFs defined by Eqs. (1) and (4), as shown in Fig. 5. Moment fit curves are depicted for the case of 3 gamma PDFs – see Fig. 5a – and CCDF fit curves

are presented for the case of 1 gamma, 2 gamma and 3 gamma PDFs (see Fig. 5b). From Fig. 5a, it is observed that the mixture of gamma PDF moments has smaller errors between empirical and estimated moment curves. This indicates that this scenario of IPIX data has probably a mixture of Gaussian and non-Gaussian clutter. From Fig. 5b, the best tail fitting with such data is obtained by a mixture of three gamma distributions using the CCDF fit estimator.

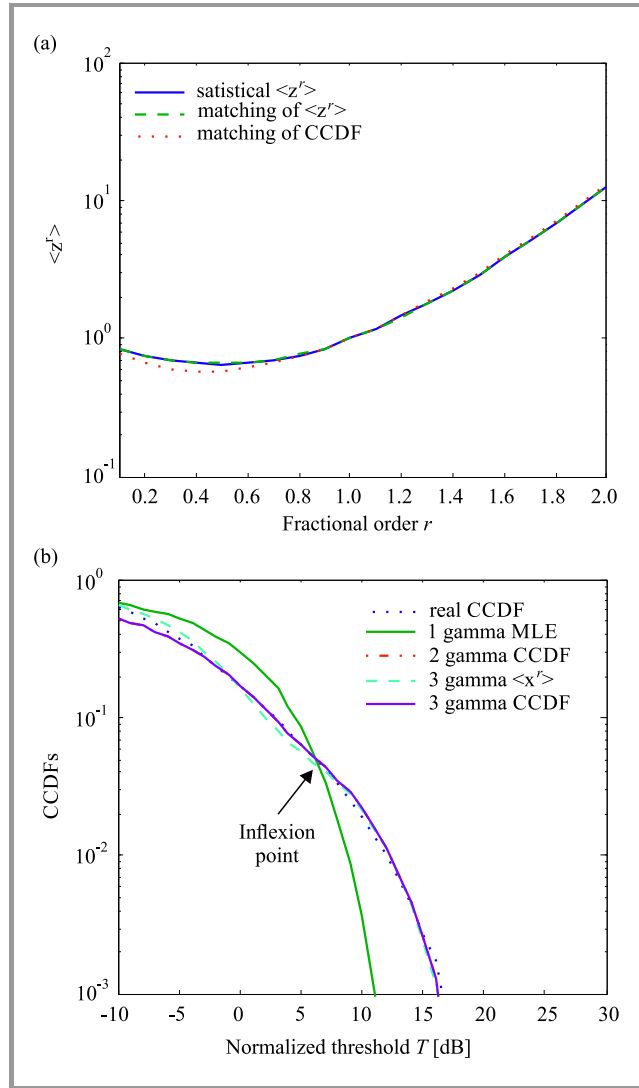


Fig. 7. Modeling of IPIX data with a resolution of 30 m, HH polarization and 11th range cell: (a) moments fit test for the case of 3 gamma PDFs, (b) CCDF fit test of 1 gamma, 2 gamma and 3 gamma PDFs.

Moreover, it is clearly seen in Fig. 6 that the IPIX data with a resolution of 15 m, HH polarization and 5th range cell are well modeled by a mixture of three gamma components with the moment fit estimator. This means that the proposed estimators converge to the best estimates of respective 7 parameters.

Figure 7, taking into consideration the modeling study of real data with a low resolution of 30 m, HH polariza-

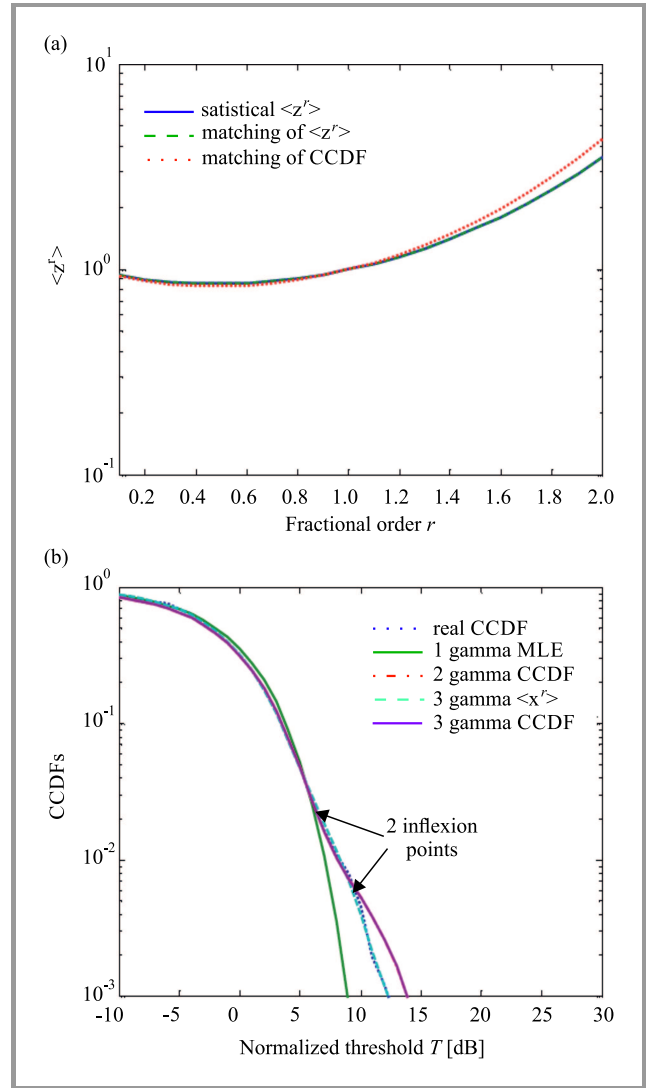


Fig. 8. Modeling of IPIX data with a resolution of 3 m, VV polarization and 21st range cell: (a) moments fit test for the case of 3 gamma PDFs, (b) CCDF fit test of 1 gamma, 2 gamma and 3 gamma PDFs.

Table 1
Comparison of complexity (number of FLOP) and performance of different model structures

Estimator IPIX data	SQE using 2 gamma CCDF	SQE using 3 gamma CCDF	SQE using 3 gamma $\langle x^r \rangle$
HH, 3 m and 17th cell	1.98×10^{-4}	2.32×10^{-5}	0.0195
HH, 15 m and 5th cell	8.48×10^{-4}	3.57×10^{-5}	0.0086
HH, 30 m and 11th cell	5.34×10^{-4}	5.86×10^{-4}	1.98×10^{-4}
VV, 3 m and 21st cell	5.40×10^{-4}	5.37×10^{-4}	5.87×10^{-7}

tion and 11th range cell, shows the efficiency of both SM CCDF fit and SM moment fit estimators. Here, two and three gamma component models offer almost similar results. For the case of modeling IPIX data with a resolution of 3 m, VV (vertical-vertical) antenna polarization and 21st range cell, Fig. 8 depicts the different curves of moments and CCDFs respectively. It is remarkable that the mixture of three gamma PDFs additionally ensures goodness-of-fit with real data. This model has the ability to track empirical CCDF in the presence of different inflexion points. From the above, it is shown that the SM moment fit and SM CCDF fit ensure proper estimation results.

From the above results, the trimodal gamma distribution fits IPIX data in most cases in terms of different range resolutions. Finally, the comparison of fitness function values, i.e. the sum of quadratic errors (SQE) given in (13) is highlighted in Table 1. Residuals corresponding to the CCDF NLS fit method are calculated for $10^{-0.55} < CCDF < 10^{-3}$. On the other hand, residuals corresponding to the moments, i.e. $\langle x^r \rangle$, the NLS fit method, are calculated for $0.01 < r < 2$. From Table 1, improved results are offered by the CCDF NLS fit method for a mixture of 3 gamma populations.

5. Conclusion

Two methods for estimating parameters of a mixture of gamma PDFs were introduced in this paper. One, two and three gamma component models were considered for the purpose of parameter estimation and modeling tasks. The proposed $\text{zlog}(z)$ approach does not involve the use of numerical calculi, but it is used for the case of a single gamma PDF.

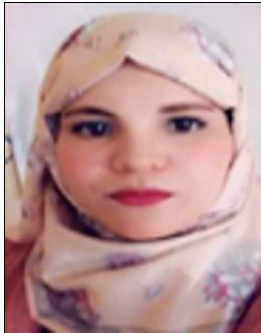
Experiments showed that the latter is not suitable for modeling IPIX data. In order to show the efficiency of the proposed SM moment fit and SM CCDF fit, modeling of several data scenes have been tested by means of two and three gamma component models. From numerical illustrations, it was shown that the best fit with radar echoes was achieved by a mixture of three gamma PDFs.

The computational time associated with the proposed moment and CCDF fits is relatively high owing to the fact that they search seven dimensional spaces. To conclude, the application of the proposed mixture of three gamma PDFs with SM CCDF fit and SM moments fit methods is a novel approach to modeling IPIX radar echoes. In fact, this model produces excellent tail fitting of the recorded data.

References

- [1] V. Anastassopoulos, G. A. Lampropoulos, A. Drosopoulos, and M. Rey, "High resolution radar clutter statistics", *IEEE Trans. on Aerosp. and Electron. Syst.*, vol. 35, no. 1, 1999 (DOI: 10.1109/7.745679).
- [2] I. Chalabi and A. Mezache, "Estimators of compound Gaussian clutter with log-normal texture", *Remote Sensing Lett.*, vol. 10, no. 7, pp. 709–716, 2019 (DOI: 10.1080/2150704X.2019.1601275).
- [3] V. G. Weinberg, S. D. Howard, and C. Tran, "Bayesian framework for detector development in Pareto distributed clutter", *IET Radar Sonar & Navig.*, vol. 13, no. 9, pp. 1548–1555, 2019 (DOI: 10.1049/iet-rsn.2018.5635).
- [4] H. Yu, P. L. Shui, and Y. T. Huang, "Low-order moment-based estimation of shape parameter of CGIG clutter model", *Electron. Lett.*, vol. 52, no. 18, pp. 1561–1563, 2016 (DOI: 10.1049/el.2016.2248).
- [5] A. Mezache, F. Soltani, M. Sahed, and I. Chalabi, "Model for non-Rayleigh clutter amplitudes using compound inverse Gaussian distribution: An experimental analysis", *IEEE Trans. on Aerosp. and Electron. Syst.*, vol. 51, no. 1, 2015 (DOI: 10.1109/TAES.2014.130332).
- [6] A. Farina, F. Gini, M. V. Greco, and L. Verrazzani, "High resolution sea clutter data: statistical analysis of recorded live data", *IEE Proc. – Radar, Sonar & Navig.*, vol. 144, no. 3, pp. 121–130, 1997 (DOI: 10.1049/ip-rsn:19971107).
- [7] L. Rosenberg, D. J. Crisp, and N. J. Stacy, "Analysis of the KK-distribution with medium grazing angle sea-clutter", *IET Radar, Sonar & Navig.*, vol. 4, no. 2, pp. 209–222, 2010 (DOI:10.1049/iet-rsn.2009.0096).
- [8] X. Zhou, R. Peng, and C. Wang, "A two-component K-lognormal mixture model and its parameter estimation method", *IEEE Trans. on Geosci. and Remote Sens.*, vol. 53, no. 5, 2015 (DOI: 10.1109/TGRS.2014.2363356).
- [9] S. Bocquet, L. Rosenberg, and C. H. Gierull, "Parameter estimation for a compound radar clutter model with trimodal discrete texture", *IEEE Trans. on Geosci. and Remote Sens.*, vol. 58, no. 10, 2020 (DOI: 10.1109/TGRS.2020.2979449).
- [10] J. R. Nicholas, "Estimating the parameters of the K distribution in the intensity domain", Rep. DSTO-TR-0839, DSTO Electronics and Surveillance Research Laboratory, pp. 1–76, 1999, South Australia [Online]. Available: <https://apps.dtic.mil/sti/pdfs/ADA368069.pdf>
- [11] S. Bocquet, "Parameter estimation for Pareto and K distributed clutter with noise", *IET Radar, Sonar & Navig.*, vol. 9, no. 1, pp. 104–113, 2015 (DOI: 10.1049/iet-rsn.2014.0148).
- [12] A. Mezache, A. Gouri, and H. Oudira, "Parameter estimation of CGIG clutter plus noise using constrained NIOME and MLE approaches", *IET Radar, Sonar & Navig.*, vol. 12, no. 2, pp. 176–185, 2018 (DOI: 10.1049/iet-rsn.2017.0234).
- [13] R. Vani Lakshmi and V. S. Vaidyanathan, "Parameter estimation in gamma mixture model using normal-based approximation", *J. of Statis. Theory and Appl.*, vol. 15, no. 1, pp. 25–35, 2016 (DOI: 10.2991/jsta.2016.15.1.3).
- [14] Q. Xianxiang, Z. Shilin, Z. Huanxin, and G. Gui, "A CFAR detection algorithm for generalized gamma distributed background in high-resolution SAR images", *IEEE Geosci. and Remote Sens. Lett.*, vol. 10, no. 4, 2013 (DOI: 10.1109/LGRS.2012.2224317).
- [15] É. Magraner, N. Bertaux, and P. Réfrégier, "A new CFAR detector in gamma-distributed non homogeneous backgrounds", in *Proc. of 16th Eur. Sig. Process. Conf. EUSIPCO 2008*, Lausanne, Switzerland, 2008 [Online]. Available: <https://www.eurasip.org/Proceedings/Eusipco/Eusipco2008/papers/1569104566.pdf>
- [16] M. Abramowitz and I. A. Stegun, Ed., *Handbook of Mathematical Functions*. New York: Dover Publications, Inc., 1970 (ISBN: 978-0486612720).
- [17] D. Blacknell and R. J. A. Tough, "Parameter estimation for the K-distribution based on $[z \log(z)]$ ", *IEE Proc. – Radar, Sonar & Navig.*, vol. 148, no. 6, 309–312, 2001 (DOI:10.1049/ip-rsn:20010720).
- [18] T. P. Minka, "Estimating a Gamma distribution", 2002 [Online]. Available: <https://tminka.github.io/papers/minka-gamma.pdf>
- [19] S. A. Hamadi, A. Chouder, M. M. Rezaoui, S. Motahhir, and A. M. Kaddouri, "Improved hybrid parameters extraction of a PV module using a moth flame algorithm", *Electronics*, vol. 10, 2021 (DOI: 10.3390/electronics10222798).
- [20] D. P. Kroese, T. Taimre, and Z. I. Botev, *Handbook of Monte Carlo Methods*. New York, NY, USA: Wiley, 2011 (ISBN: 9780470177938).
- [21] J. A. Nelder and R. Mead, "A simplex method for function minimization", *The Computer J.*, vol. 7, no. 4, pp. 308–313, 1965 (DOI: 10.1093/comjnl/7.4.308).
- [22] M. Baudin, "Nelder-Mead user's manual" [Online]. Available: <https://www.scilab.org/sites/default/files/neldermead.pdf>

- [23] Shu-Kai, S. Fan, and E. Zahara “A hybrid simplex search and particle swarm optimization for unconstrained optimization”, *European J. of Oper. Res.*, vol. 181, no. 2, pp. 527–548, 2007 (DOI:10.1016/j.ejor.2006.06.034).
- [24] R. Bakker and B. Currie, “The McMaster IPIX radar sea clutter database”, 2001 [Online]. Available: <http://soma.mcmaster.ca/ipix/>



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