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Experimental study and mathematical modeling of the residence time distribution in magnetic mixer

Rafał Rakoczy*, Marian Kordas, Przemysław Grądzik, Maciej Konopacki, Grzegorz Story

West Pomeranian University of Technology, Szczecin, Institute of Chemical Engineering and Environmental Protection Process, al. Piastów 42, 71-065 Szczecin, Poland
**Corresponding author: e-mail: rrakoczy@zut.edu.pl*

This study reports on research results in the field of a mixing process under the action of a transverse rotating magnetic field (TRMF). The main objective of this paper is to present the effect of this type of a magnetic field on residence time distribution (RTD) measurements. This paper evaluates the performance of a magnetic mixer by comparing the results of an experimental investigations in a pilot set-up and theoretical values obtained from mathematical model. This model consisting of the set of ideal continuous stirred tank reactors (CSTR) fitted well the experimental data.

Keywords: mixing model, residence time distribution, modelling, magnetic field.

INTRODUCTION

The simulation of chemical engineering processes is connected with the knowledge about hydrodynamics, transfer processes and chemical reactions. The residence time distribution (RTD) is one of the most informative characteristics to obtain hydrodynamic information. This systemic approach is allowed to establish the flow patter in the various types of mixers or chemical reactors. The RTD technique may be treated as a quantitative method to derive simple hydrodynamic models in the case of complex engineering systems. For design and scale-up purposes, it is essential to have information on the RTD. Many experimental investigations and theoretical considerations are reported on the RTD of impinging chemical engineering devices[1–13].

The interpretation of RTD measurements may be realized by using compartmental models which are composed of the branches containing the elementary ideal reactors connecting all together by the nodes. The application of a mixing model (combination of a plug flow reactor (PRF) and a continuous stirred tank reactor (CSTR) with circulation flow) to study of RTD measurements is widely presented in the relevant literature[14–17].

From the experimental point of view, the non-ideal flow in the reactor may be estimated by means of the stimulus-response technique. This method is based both on the analysis of the response curve (the response is the recording tracer leaving the vessel) and physical information about the process itself. The non-ideal flow in the reactor may be caused by channeling of fluid, by recycling of fluid, or by creation of stagnant regions in the vessel[18]. An accurate description of this type of reactors is still difficult and mathematical model for non-ideal mixing process may be obtained by using the compartment models. These type of models are provided a basis for understanding the complex nature of the velocity field in the gross sense. Moreover, the compartment models have important advantages over more fundamental modelling (e.q. CFD technique), particularly in simplicity and cost.

Several types of reactors or mixers are used in chemical engineering industries[19–21]. Static or alternating might be applied to augment the process intensity instead of mechanical mixing. The novel approach to a mixing process is based on the application of a transverse rotating magnetic field[22]. It is obvious that this kind of magnetic field may be induced a complex flow[23]. There is, therefore, a need to investigate the influence of a transverse rotating magnetic field (TRMF) on the hydrodynamic conditions. In this study, a mathematical model based on a compartment model is developed to simulate the mixing process under the action of a TRMF.

EXPERIMENTAL DETAILS

Experimental apparatus

The schematic diagram of experimental set-up is graphically presented in Figure 1. The experimental apparatus is consisted of a cylindrical plexiglas column (1) (inner diameter – 0.1 m; outer diameter – 0.11 m; length – 1 m); inlet (2) and outlet (3) spouts; a spout for introduction of trace into the fluid entering the vessel (4); inlet (5a) and outlet (5b) conductivity electrodes; temperature sensors (6a and 6b); generator of a TRMF (7); a.c. transistorized inverter (8); multifunction computer meter CyberScan PCD 6500 Eutech (9) and personal computer (10).

Investigations were realised with the temperature of the working liquid (tap water) variation between 20 and 25°C. The volumetric flow rate of liquid were varied from 0.0084 to 0.0167 $dm^3 \cdot s^{-1}$. The working volume (volume of TRMF generator) was equal to $5 \cdot 10^{-3}$ m^3.

Mixing was provided by means of the modified three-phase stator of a induction squirrel cage electric motor, which parameters are compatible with Polish Standard PN-72/E-06000. The magnetic induction of TRMF was controlled by mans of the alternating current frequency equal to the frequency of the TRMF. In these measurements, this stator was supplied with 50 Hz three-phase alternating current. The a.c. transistorized inverter was used to change the frequency of the TRMF. In this experimental procedure, this frequency was varied in the range of f = 1–20 Hz. The more detailed description of this kind of magnetic field measurements can be found in the relevant literature[24]. Moreover, the obtained values in this article suggest that the averaged values of magnetic induction may be analytically described by the following relation:

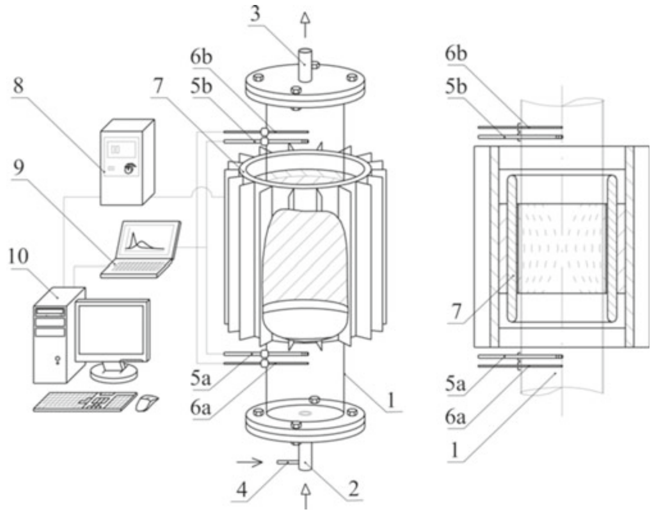

Figure 1. The scetch of experimental set-up: 1 – plexiglas column, 2 – inlet spout, 3 – outlet spout, 4 – spout for introduction of trace, 5 – conductivity electrodes (a – inlet and b – outlet), 6 – temperature sensors (a – inlet and b – outlet), 7 – generator of TRMF, 8 – a.c. transistorized inverter, 9 – multifunction computer meter CyberScan PCD 6500 Eutech, 10 – personal computer

$$B_{avg} = 14.05\left[1 - e^{-0.05 f_{TRMF}}\right] \tag{1}$$

Experimental procedure

The mixing process under the action of a TRMF was evaluated from residence time distribution measurement using the stimulus-response technique. This technique is the simplest and most direct way of finding the residence time distribution (RTD) curves. In the case of this experimental work, the RTD is obtained by injecting a tracer instantaneously (a pulse input) at the inlet of flow system, and then measuring the tracer concentration, $c(t)$, at the outlet as a function of time. The stimulus-response technique demands injecting the tracer within a very short time compared to the flow residence-time scale. This impulse can be described by the Dirac delta function (δ-Dirac function), that is $\delta\left(t-\tau_0\right)\begin{cases} = 0 & t \neq \tau_0 \\ \neq 0 & t = \tau_0 \end{cases}$ such that $\int_{-\infty}^{\infty} \delta\left(t-\tau_0\right) dt = 1$. RTD experiments were realized with a saturated solution of brine (25 wt % NaCl).

The typical examples of experimental results obtained in this work are graphically illustrated in the Figure 2.

In order to describe the dynamic behaviour of the magnetic mixer the experimental measurements were performed for different sections of the installation. The continuous measurements of conductivity were collected for the liquid phase by introduction of the tracer solution under the effect of TRMF (conductivity electrode – 5b) and without the effect of this kind of magnetic field (conductivity electrode – 5a). Once the operating conditions are stable (or considered so by constant value of B_{avg} and $\dot{V}$), the tracer is introduced into the inlet of the plexiglas column (spout for introduction of trace – 4). Samples are then collected every 1 s until the tracer disappearance in the tested magnetic mixer.

The mean time of duration, t_d, is equal about 3000 s. The typical examples of conductivity measurements under the effect of TRMF and without the effect of TRMF for $t_d \in (0, 1000)$ are presented in Figure 3.

RESULTS AND DISCUSSION

Time-lag of residence time distribution (RTD) measurements

In the development of the dynamic model for the magnetic mixer plant, the results of the obtained RTD measurements are taken into account. Based on the experimental curves, the normalized time-lag was obtained at different volumetric flow rates, $\dot{V}$, and the normalized and the averaged values of magnetic induction, B_{avg}. The variation of this time-lag with operating parameter is presented in Figure 4.

To establish the effect of the working parameters on the time-lag in the analysed set-up, we propose the following relationship to work out the experimental database

$$\left(\tau_0\right)^* = p_1 + p_2 \exp\left[-p_3\left(B_{avg}\right)^*\right] \tag{2}$$

In the above relationship (2) the parameter $\left(\tau_0\right)^*$ is the normalized value of the time lag $\left(\left(\tau_0\right)^* = \frac{\tau_0}{\left(\tau_0\right)_{\max}} \Rightarrow \left(\tau_0\right)^* = \frac{\tau_0}{331}\right)$ and the parameter $(B_{avg})^*$ is the normalized value of the averaged magnetic induction $\left(\left(B_{avg}\right)^* = \frac{B_{avg}}{\left(B_{avg}\right)_{\max}} \Rightarrow \left(B_{avg}\right)^* = \frac{B_{avg}}{8.88}\right)$. The obtained values of the coefficient p_1, p_2 and p_3 are given in Table 1.

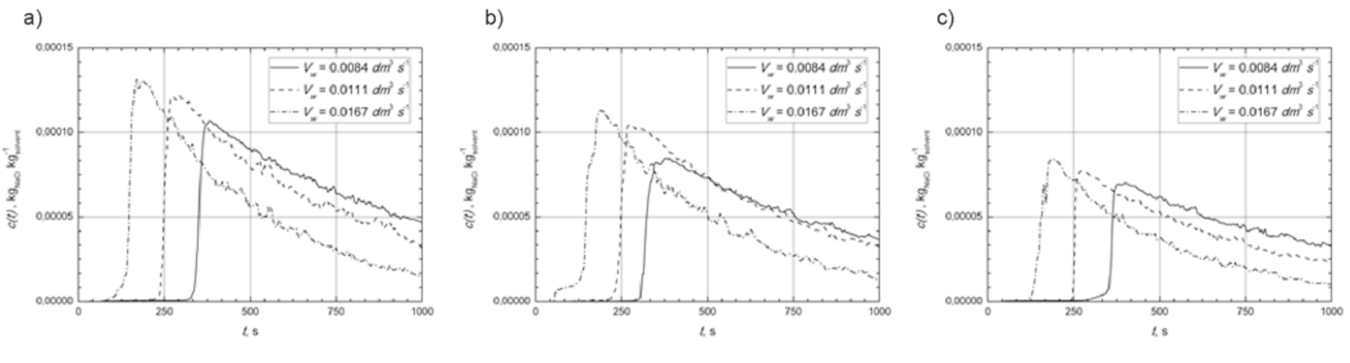

Figure 2. The typical examples of experimental results: a) B_{avg} = 0.69 mT (f_{TRMF} = 1 Hz); b) B_{avg} = 5.53 mT (f_{TRMF} = 10 Hz); B_{avg} = 8.89 mT (f_{TRMF} = 20 Hz)

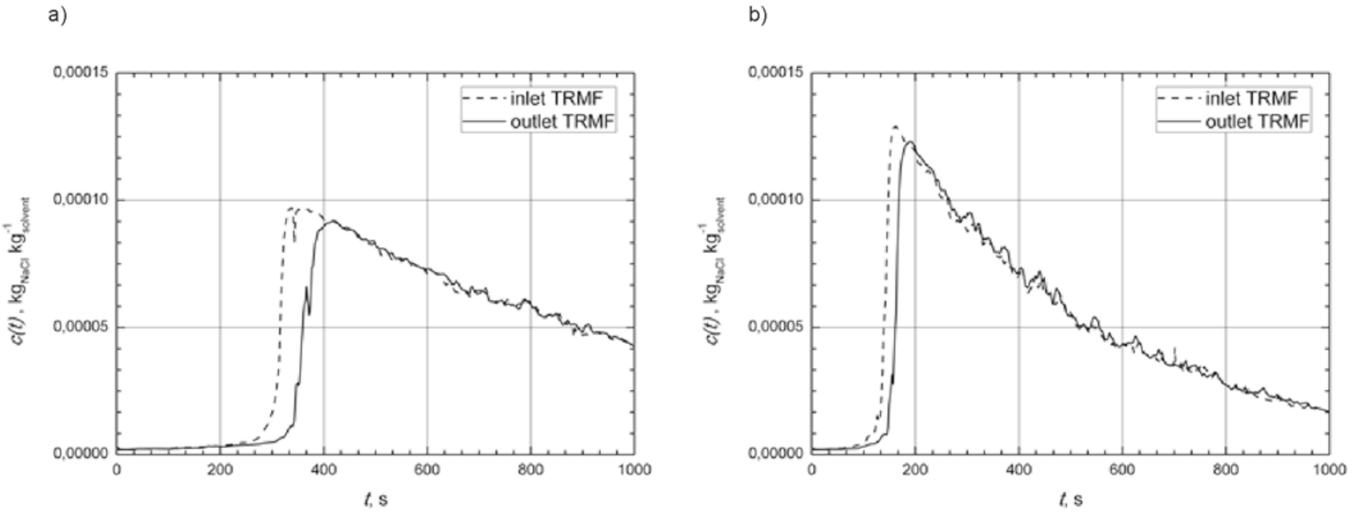

Figure 3. The typical examples of conductivity measuremnets under the action of TRMF (outlet TRMF) and without the effect of TRMF (inlet TRMF): a) B_{avg} = 0.69 mT, $\dot{V}$ = 0.0111 dm^3s^{-1}; b) B_{avg} = 8.89 mT, $\dot{V}$ = 0.0167 dm^3s^{-1}

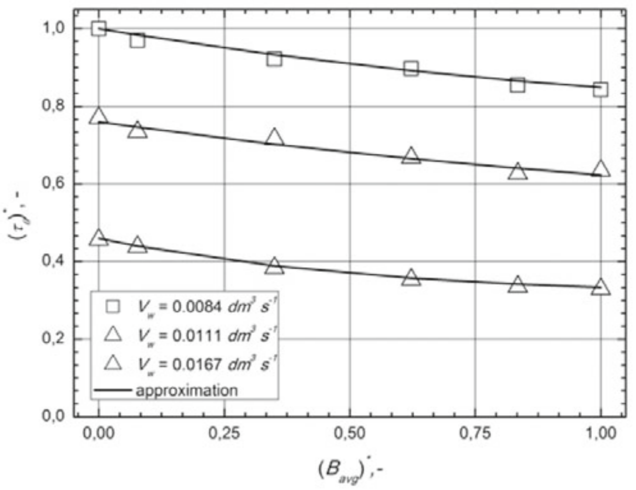

Figure 4. The influence of the averaged values of magnetic induction on parameter τ_0

As can be clearly seen (see Figure 4) the normalized time-lag, τ_0, decrease with increasing values of the volumetric flow rates. It is found that as the intensity of the TRMF increases, the values of delay time decrease. It may be concluded that the TRMF strongly influenced on the RTD measurements.

Mathematical model for the magnetic mixer

A mathematical model must be based on the flow characteristic in the mixer. Every compartment model is composed of two parts, a structural part, which takes into account the architecture of the model, and a parametric part, which contains the extrinsic parameters of the model[25]. It is clear that the compartment model is composed of the branches containing the elementary reactors (e.g. cascade of CSTR). It should be noticed that the proposition of the mathematical model structure may be treated as the master problem. The generation of the structure consists in the choice of the elementary ideal flow models and the determination of their connections. By comparing the experimental RTD curve with the theoretical curves for various combinations of compartments and throughflow, we can find which model best fits the real mixer.

The correct model of the magnetic mixer must be based on considering it as the sum of different regions, each with a characteristics flow pattern, therefore, the tested set-up can be modelled by dividing it into sections as shown in Figure 5.

An ideal continuous stirred tank reactor (CSTR) model used for the each sections in the magnetic mixer. The hydrodynamic of this type of mixer is represented by the

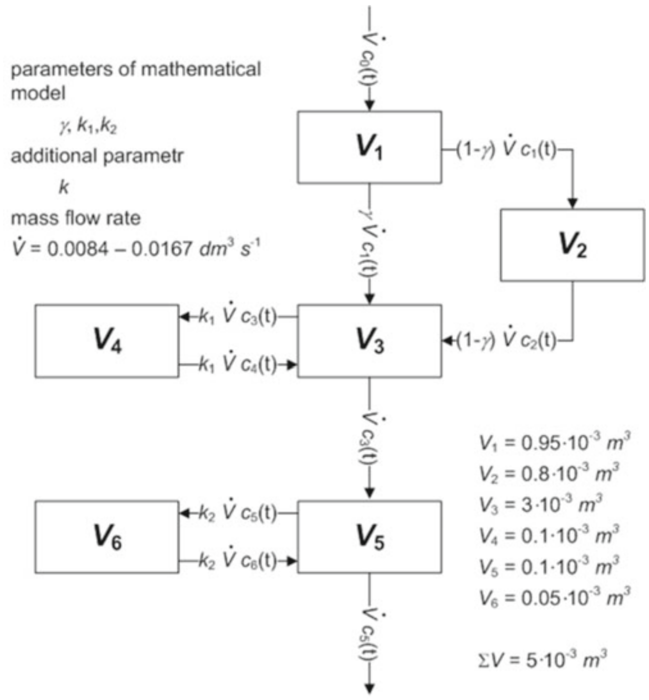

Figure 5. The model structure of liquid mixing in magentic mixer

Table 1. The parameters of Eq.(2)

$\dot{V}$, $dm^3 s^{-1}$	p_1	p_2	p_3	R	σ_{aprox}	MPE
0.0084	0.73	0.27	1.21	0.99	0.06	-0.61
0.0111	0.47	0.29	1.57	0.97	0.06	0.27
0.0167	0.31	0.15	0.54	0.99	0.05	-1.13

following system of differential equations describing the transfer mass balances in all compartments:

$$\begin{cases} V_1 \dfrac{dc_1(t)}{dt} = \dot{V} c_0(t) - (1-\gamma)\dot{V} c_1(t) - \gamma \dot{V} c_1(t) \\ V_2 \dfrac{dc_2(t)}{dt} = (1-\gamma)\dot{V} c_1(t) - (1-\gamma)\dot{V} c_2(t) \\ V_3 \dfrac{dc_3(t)}{dt} = \gamma \dot{V} c_1(t) + (1-\gamma)\dot{V} c_2(t) + k_1 \dot{V} c_4(t) - k_1 \dot{V} c_3(t) - \dot{V} c_3(t) \\ V_4 \dfrac{dc_4(t)}{dt} = k_1 \dot{V} c_3(t) - k_1 \dot{V} c_4(t) \\ V_5 \dfrac{dc_5(t)}{dt} = \dot{V} c_3(t) + k_2 \dot{V} c_6(t) - k_2 \dot{V} c_5(t) - \dot{V} c_5(t) \\ V_6 \dfrac{dc_6(t)}{dt} = k_2 \dot{V} c_5(t) - k_2 \dot{V} c_6(t) \end{cases} \quad (3)$$

For the solution of the set of differential equations (3), compartment volumes, $V_{i\in\overline{1,6}}$, and the flow rates, $\dot{V}$, must be known. It should be noticed that the compartment volumes are here unequal (see Figure 5). The proposed model, as depicted in Figure 5, is a four-parameter model (k, k_1, k_2, γ). In the case of this experimental work, the parameter k is treated as a gain constant.

From the practical point of view, the analysed magnetic mixer may be described by the following general relationship:

$$outlet = inlet * mathematical\ operator \;\Rightarrow\; y(t) = x(t-\tau_0)A(t) \quad (4)$$

According to the above relation (Eq.(4)), the diagram of magnetic mixer is shown in Figure 6.

Laplace transform functions may be used in derivation of RTD equation for simplifying the analytical procedure leading to the solution. Based on the Laplace transform functions, the Eq.(4) may be given by (see Fig. 6b):

$$Y(s) = X(s)e^{-\tau_0 s}W(s) \quad (5)$$

A transfer function is a mathematical representation of the relation between the input and output of the

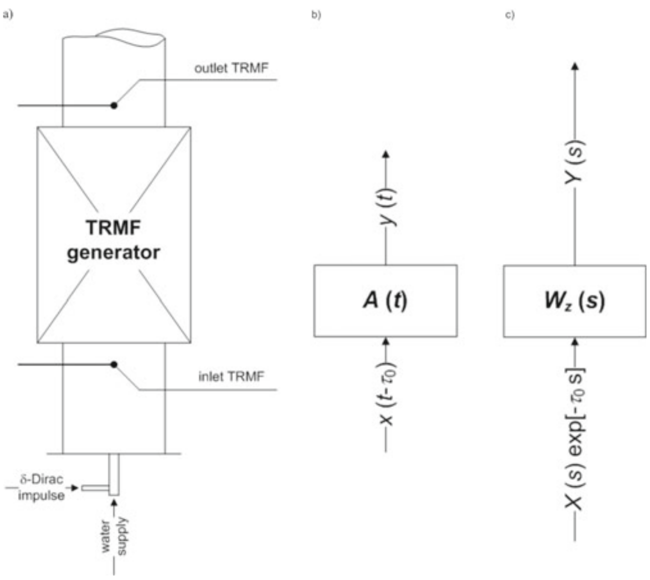

Figure 6. The diagram of magentic mixer

considered system. The transfer function of a linear system with initial conditions set to zero is defined as a ratio between the system output and the perturbation input in the Lapalce domain. In the case of this work, the transfer function may be expressed as follows:

$$W(s) = \frac{Y(s)}{X(s)} e^{-\tau_0 s} \quad (6)$$

In the above relation (Eq.6), the term, $e^{-\tau_0 s}$, is represented the time-lag in RTD measurements, and it may be expressed in the following form:

$$e^{-\tau_0 s} \equiv \frac{12 - 6\tau_0 s + \tau_0^{22} s}{12 + 6\tau_0 s + \tau_0^{22} s} \quad (7)$$

By taking the Lapalce transform of Eq.(3), the mathematical model of the magnetic mixer can be written as:

$$\begin{cases} T_1 s c_1(s) = c_0(s) - (1-\gamma)c_1(s) - \gamma c_1(s) \\ T_2 s c_2(s) = (1-\gamma)c_1(s) - (1-\gamma)c_2(s) \\ T_3 s c_3(s) = \gamma c_1(s) + (1-\gamma)c_2(s) + k_1 c_4(s) - k_1 c_3(s) - c_3(s) \\ T_4 s c_4(s) = k_1 c_3(s) - k_1 c_4(s) \\ T_5 s c_5(s) = c_3(s) + k_2 c_6(s) - k_2 c_5(s) - c_5(s) \\ T_6 s c_6(s) = k_2 c_5(s) - k_2 c_6(s) \end{cases} \quad (8)$$

where the time-constant for compartment volumes is given by:

$$T_{i\in\overline{1,6}} = \frac{V_{i\in\overline{1,6}}}{\dot{V}} \quad (9)$$

Taking into account the system of algebraical equations (see Eq.(8)), we obtain the following relationships:

$$W_1(s) = \frac{c_1(s)}{c_0(s)} \;\Rightarrow\; W_1(s) = \frac{1}{T_1 s + 1} \quad (10a)$$

$$W_2(s) = \frac{c_2(s)}{c_1(s)} \;\Rightarrow\; W_2(s) = \frac{1-\gamma}{T_1 s + 1 - \gamma} \quad (10b)$$

$$c_3(s) = \frac{1}{T_3 s + k_1 + 1}\left[\gamma c_1(s) + (1-\gamma)c_2(s) + k_1 c_4(s)\right] \quad (10c)$$

where:

$$c_1(s) = W_1(s)c_0(s);\; c_2(s) = W_2(s)c_1(s);\; c_4(s) = W_4(s)c_3(s)$$

$$W_4(s) = \frac{c_4(s)}{c_3(s)} \;\Rightarrow\; W_4(s) = \frac{k_1}{T_4 s + k_1} \quad (10d)$$

$$c_5(s) = \frac{1}{T_5 s + k_2 + 1}\left[c_3(s) - k_2 c_6(s)\right] \quad (10e)$$

where: $c_6(s) = W_6(s)c_5(s)$

$$W_6(s) = \frac{c_6(s)}{c_5(s)} \;\Rightarrow\; W_6(s) = \frac{k_2}{T_6 s + k_2} \quad (10f)$$

Taking into account the relations (10a), (10b) and (10d), we find the following form of Eq.(10c):

$$W_3(s) = \frac{c_3(s)}{c_0(s)} \;\Rightarrow\; W_3(s) = \frac{W_1(s)\left[\gamma + (1-\gamma)W_2(s)\right]}{T_3 s + k_1\left[1 - W_4(s)\right] + 1} \quad (11)$$

Applying the Eq.(10f) and introducing Eq.(9) one finds that Eq.(10e) can be expressed as:

$$W_5(s) = \frac{c_5(s)}{c_0(s)} \;\Rightarrow\; W_5(s) = \frac{W_3(s)}{T_5 s + k_2\left[1 + W_6(s)\right] + 1} \quad (12)$$

Based on the above equations, the transfer function of the magnetic mixer can be expressed as follows:

$$W(s) = e^{-\tau_0 s}\frac{c_5(s)}{c_0(s)} \quad (13)$$

It should be noticed that the above Eq.(13) may be rewritten in the following form which is useful for the simulation analysis:

$$W(s) = \left\{\frac{12 - 6\tau_0 s + \tau_0^{22} s}{12 + 6\tau_0 s + \tau_0^{22} s}\right\}\left\{\frac{W_1(s)\left[\gamma + (1-\gamma)W_2(s)\right]}{\left[T_3 s + k_1\left[1 - W_4(s)\right] + 1\right]\left[T_5 s + k_2\left[1 + W_6(s)\right] + 1\right]}\right\} \quad (14)$$

The above mathematical model in the form of the complex transfer function may be effectively analysed by means of the Matlab software. By taking the inverse transform of Eq.(14), the RTD function of the proposed model can be written as:

$$c(t)= p_1 e^{-p_2 t} + p_3 e^{-p_4 t} + p_5 e^{-p_6 t}\left[\cosh(p_7 t)+ p_8 \cosh(p_7 t)\right] + p_9 e^{-p_{10} t}\left[\cosh(p_{11} t)+ p_{12} \cosh(p_{11} t)\right] \quad (15)$$

The proposed mathematical model was used to model the dynamic behaviour of the magnetic mixer, and Eq.(15) was fitted according to the experimental data. An example of model predicted curve with experimentally measured RTD is shown in Figure 7. As shown in the figure, experimental data fit the proposed mathematical model satifactionary.

With the assumed model, the objective of the optimization problem was to obtain the values of parameters that minimize the sum of squares of deviation, ε, between the measured and the predicted outlet concentration curves. The objective function minimized here is given by the following equation:

$$\varepsilon = \frac{1}{N}\sum_i \left[c(t_i)\big|_{\exp} - c(t_i)\big|_{\mathrm{mod}} \right]^2 \quad (16)$$

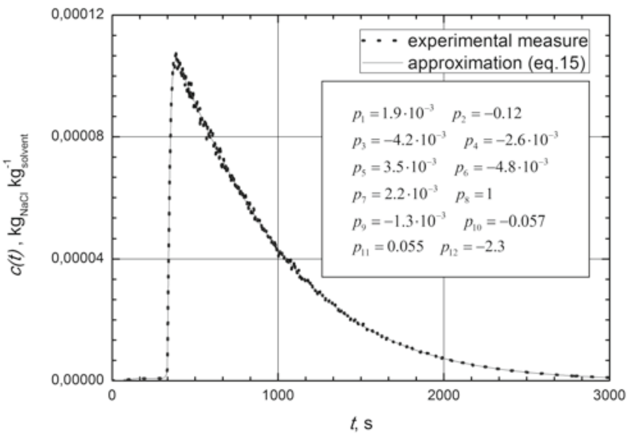

Figure 7. The typical example of experimental data fit by using the proposed mathematical model (Eq.15)

In order to work out the above objective function and the resulting optimization problem for the experimental database, a code was written, validated and implemented by using some programming function available in Matlab software.

According to the calculated results of the model parameters (k, k_1, k_2, γ), the effect of the operating parameters is investigated. The influence of the volumetric flow rate and the magnetic field intensity on these parameters is graphically presented in Figure 8.

The obtained values of parameters shown in this figure as points could be analytically described by the relationships presented in Table 2.

The RTD technique is served as a reasonable indicator of the type and extent of mixing. Response curves from the RTD experiment may be treated as the quantitative description of the mixing system. For the tracer input method the RTD (or exit age-distribution) function, $E(t)$, may be related to the outler tracer concentration, $c(t)$, by the following function of time:

$$E(t)=\frac{c(t)}{\int_0^\infty c(t)\,dt} \quad or \quad E(t)=\frac{c(t_i)}{\sum_{i=0}^{\infty} c(t_i)\Delta t_i} \quad (17)$$

Taking into account the RTD function (Eq.(17)), statistical parameters such as the mean residence time, t_m, and the variance σ^2 can be determined experimentally by using the following relationships:

$$t_m\big|_{exp} = \frac{\int_0^\infty t\,E(t)\,dt}{\int_0^\infty E(t)\,dt} \quad (18a)$$

$$\sigma^2\big|_{exp} = \int_0^\infty (t-t_m)\,E(t)\,dt \quad (18b)$$

The mean residence time, which gives the average time the exiting fluid element spend in the low system, may be also defined as follows (because $\int_0^\infty E(t)\,dt=1$):

$$t_m\big|_{exp} = \int_0^\infty t\,E(t)dt \quad or \quad t_m\big|_{exp} = \sum_0^\infty t_i\,E(t_i)\Delta t_i \quad (19)$$

For a constant volumetric rate, the mean residence time (often also called reactor holding time) in a reactor is equal

$$t_m\big|_{CSTR} = \frac{V}{\dot{V}} \quad (20)$$

It should be noticed that the above equation is valid for an ideal continuous stirred tank reactor (without the dead and stagnant zones within the reactor). The mean residence time are equal to 595, 450 and 300 s for the volumetric flow rates equal to 0.0084, 0.0111 and 0.0167 $dm^3 \cdot s^{-1}$, respectively.

Table 2. The relationships defining the parameters of the model parameters (k, k_1, k_2, γ)

Parameter of Eq.(15)	Relation	R	σ_{aprox}	MPE
k	$k\left(B_{avg},\dot{V}\right)=\left(0.041\dot{V}-0,002\right)B_{avg}+\left(-0.563\dot{V}+0,075\right)$	0.99	0.009	0.22%
k_1	$k_1\left(B_{avg},\dot{V}\right)=\left(0.118\dot{V}-0,002\right)B_{avg}+\left(0.517\dot{V}+0,073\right)$	0.98	0.005	0.12%
k_2	$k_2\left(B_{avg},\dot{V}\right)=\left(-1.579\dot{V}+0,029\right)B_{avg}+\left(-34.429\dot{V}+0,632\right)$	0.99	0.151	0.28%
γ	$\gamma\left(B_{avg},\dot{V}\right)=\left(0.445\dot{V}+0,003\right)B_{avg}+\left(12.392\dot{V}+0,636\right)$	0.98	0.058	0.17%

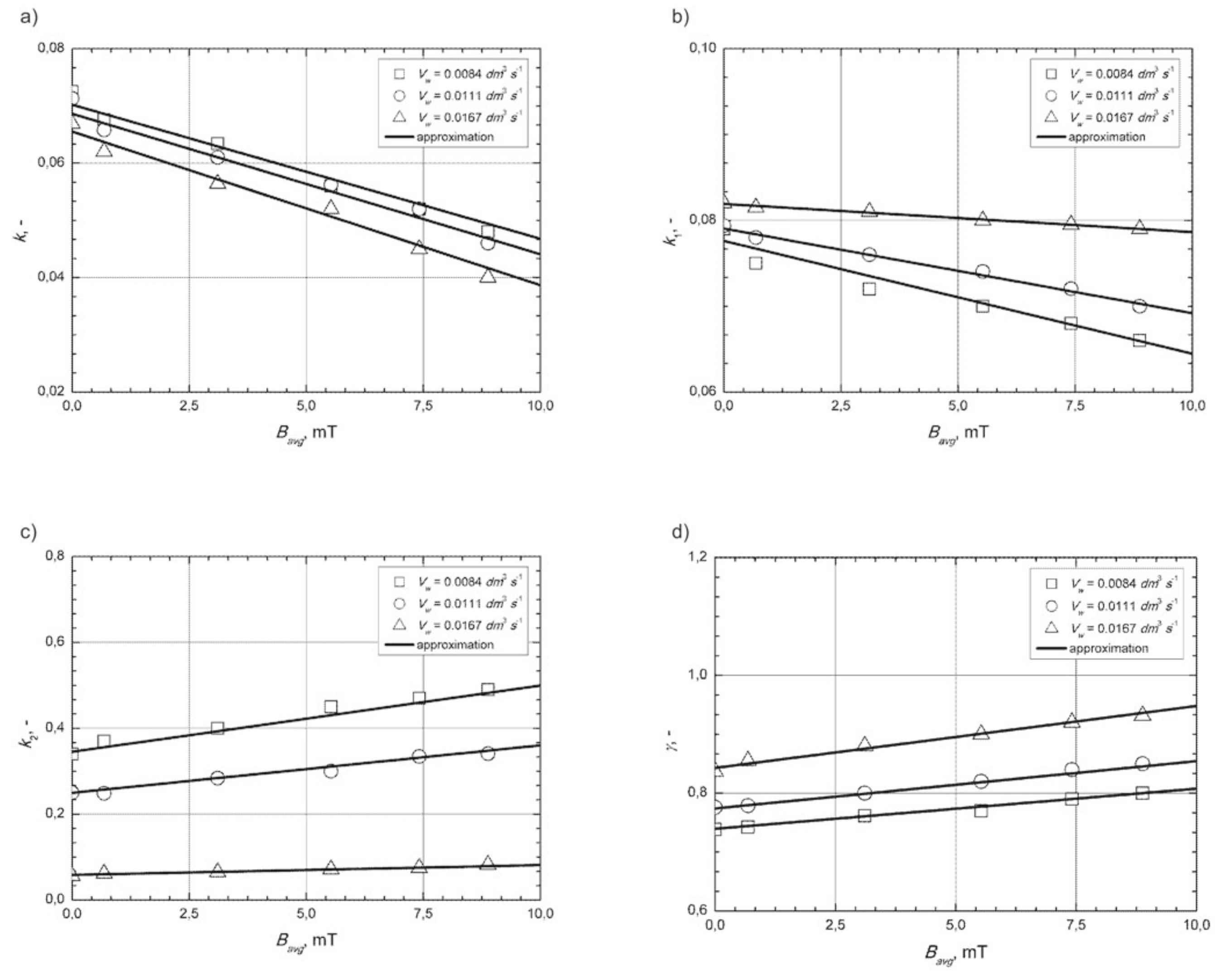

Figure 8. The influence of the volumetric flow rate and the magnetic field intensity on parameters of mathematical model

Theoretically the experimental mean residence time (Eq. 19) and the mean residence time (Eq. 20) should be identical[7]

$$t_m\big|_{exp} \cong t_m\big|_{CSTR} = t_m \tag{21}$$

The variance is a measure of the spread of the distribution of time and it may be expressed in the form:

$$\sigma^2\big|_{exp} = \sum_0^{\infty}\left(t_i - t_m\right)^2 E(t_i)\Delta t_i \tag{20}$$

Basing on the definition of the mean residence time and the variance, the coefficient of variation, CV, may be defined in the following form:

$$CV = \frac{\sqrt{\left(\sigma^2\big|_{exp}\right)}}{t_m} \tag{21}$$

This coefficient is a measure used to assess the variability of the standard deviation with the respect to the mean residence time. In the context of the mixing process, a CV of zero would imply complete plug-flow mixing while a non-zero CV implies that there is a axial dispersion or mixing caused by other mechanism. It should be noticed that the smaller the variance or the CV, the narrower is the RTD, the closer is the distribution to the mean residence time, and the better the mixing quality[1].

From the practical point of view, the TRMF may be characterized by means of the following numbers:

– dimensionless Chandrasekhar number

$$\mathrm{Q} = \frac{B^2 l^2 \sigma_e}{\rho \nu} \tag{22}$$

– dimensionless field frequency based Reynolds number

$$\mathrm{Re}_{\omega} = \frac{\omega_{\mathrm{TRMF}} l^2}{\nu} \tag{23}$$

The interaction between of the TRMF and the used liquid may be also described by means of the dimensionless Hartmann number

$$\mathrm{Ha} = Bl\sqrt{\frac{\sigma_e}{\rho\nu}} \tag{24}$$

where l is the characteristic dimension (e.g. diameter of TRMF generator).

It should be noticed that the Chandrasekhar number is the square of the Hartman number. The product of the squared Hartmann number, as the acting force of the magnetic field, and the filed frequency based Reynolds number is known as the dimensionless Taylor number

$$\mathrm{Ta} = \left(Bl\sqrt{\frac{\sigma_e}{\rho\nu}}\right)^2\left(\frac{\omega_{\mathrm{TRMF}} l^2}{\nu}\right) \Rightarrow \mathrm{Ta} = \frac{\omega_{\mathrm{TRMF}} B^2 l^4 \sigma_e}{\rho \nu^2} \tag{25}$$

In the presence of a steady or a rotating magnetic field, the relative influence of the magnetic Lorentz force vs. viscous force is determined by using of the Hartman number. It should be noticed that the Hartman number (and indirectly Taylor number) is proportional to the strength of the produced magnetic field and its square is a measure of the relative importance of the electromagnetic to the viscous force. Therefore, the obtained values the coefficient of variation, *CV*, should be expressed as the function of the dimensionless Taylor number. The influence of the TRMF on the coefficient of variation, *CV*, for the different flow rates is graphically presented in Figure 9.

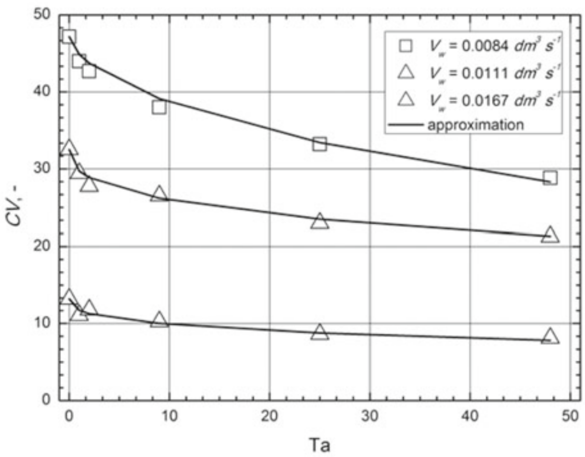

Figure 9. The graphical demonstration of the influence of dimensionless Taylor number on the mean residence time

Figure 9 shows how *CV* values change with respect to the dimensionless Taylor number values for the tested magnetic mixer. The effect of the TRMF may be described by means of the relatively simple relationship as follows:

$$CV(\mathrm{Ta}) = p_1 \exp\left[-p_2\, \mathrm{Ta}^{p_3}\right] \quad (26)$$

The graphical presentation of the above Eq.(26) is given in Figure 9. The values of the parameters for the proposed Eq.(26) are presented in Table 3.

It should be noticed that the coefficient of variation, *CV*, may be used to evaluate the degree of mixing. The narrower the RTD, the closer it approaches ideal plug-flow behaviour. The moderate increase in *CV* with increase in the Taylor number implies a reasonable decrease in mixing performance of magnetic mixer.

CONCLUSIONS

In the present report, the influence of the TRMF on the RTD was studied systematically. RTD of magnetic mixer was determined experimentally by impulse method. Based on these investigations, the compartment model is developed in order to simulate the mixing process under the action of the TRMF. Form tests it was found that the RTD of magnetic mixer is well described by the proposed mathematical model. Moreover, the characteristics of RTD such as the mean residence time and the coefficient of variation was used to as indirect measurements to quantify mixing performance in the tested mixing device.

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NOMENCLATURE

B – magnetic field, kg · A^{-1} · s^{-2}
c – tracer concentration, $kg_{NaCl} \cdot (kg_{solvent})^{-1}$
CV – coefficient of variation
d – diameter, m
E – residence time-distribution function obtained directly, s^{-1}
f – frequency of transverse rotating magnetic field, s^{-1}
k, k_1, k_2 – parameters of mathematical model (see Eq.(3))
l – characteristic dimension, m
N – number of data points
p_{var} – parameter of equation (Eq.(2). Eq.(15) or Eq.(26))
R – correlation coefficent
s – Laplace domain
t – time, s
t_d – time duration, s
t_m – mean residence time distribution, s
T – time-constant for compartment volumes, s
V – compartment volume, dm^3
$\dot{V}$ – volumetric flow rate, $dm^3 \cdot s^{-1}$
$W(s)$ – transfer function
x, X – inlet signal
y, Y – outlet signal

Greek letters

δ – Dirac delta function
ε – sum of squares of deviation between measured and predicted values
γ – parameter of mathematical model (see Eq.(3))
ν – kinematic viscosity, $m^2 \cdot s^{-1}$
ρ – density, $kg \cdot m^{-3}$
σ_e – electrical conductivity, $A^2 \cdot s^3 \cdot kg^{-1} \cdot m^{-3}$
σ – standard deviation of residence time distribution, s
σ^2 – variance of residence time distribution, s^2
τ – time, s
τ_0 – time-lag, s
ω – angular velocity of transverse rotating magnetic field-lag, rad · s^{-1}

Superscript

* – normalized value of parameter

Table 3. The parameters of Eq.(26)

$\dot{V}$, $dm^3 s^{-1}$	p_1	p_2	p_3	R	σ_{aprox}	MPE
0.0084	47.18	0.05	0.6	0.98	7.32	0.18
0.0111	32.54	0.09	0.4	0.99	4.18	–1.02
0.0167	12.21	0.12	0.38	0.97	1.99	0.63

Subscripts
avg – averaged
exp – experimental
mod – model

Dimensionless numbers

$$Ta = \frac{\sigma_e \left(B_{avg}\right)^2 \omega_{TRMF} d^4}{\rho \nu^2}$$ – Taylor number

$$\mathrm{Ha} = Bl\sqrt{\frac{\sigma_e}{\rho\nu}}$$ – Hartmann number

$$\mathrm{Re}_\omega = \frac{\omega_{\mathrm{TRMF}} l^2}{\nu}$$ – field frequency based Reynolds number

$$\mathrm{Q} = \frac{B^2 l^2 \sigma_e}{\rho\nu}$$ – Chandrasekhar number

Abbreviations
CSTR – continuous stirred tank reactor
MPE – mean percentage error
PRF – plug flow reactor
RTD – residence time distribution
TRMF – transverse rotating magnetic field

LITERATURE CITED

1. Adeosun, J. & Lawal, A. (2009). Numerical and experimental studies of mixing characteristics in a T-junction microchannel using residence time distribution. *Chemical Engineering Science*, 64, 2422–2432. DOI: 10.1016/j.ces.2009.02.013.

2. Christensen, D., Nijenhuis, J., van Ommen, J. & Coppens, M.-O. (2008). Residence times in fluidized beds with secondary gas injection. *Powder Technology*, 180, 321–331. DOI: 10.1016/j.powtec.2007.02.021.

3. Gao, Y., Vanarase, A., Muzzio, F. & Ierapetritou, M. (2011). Characterizing continuous powder mixing using residence time distribution. *Chemical Engineering Science*, 66, 417–425. DOI: 10.1016/j.ces.2010.10.045.

4. García-Sera, J., García-Verdugo, E., Hyde, J.R., Fraga-Dubreuil, J., Yan, C., Poliakoff, M. & Cocero, M.J. (2007). Modelling residence time distribution in chemical reactors: A novel generalised *n*-laminar model. Application to supercritical CO_2 and subcritical water tubular reactors. *The Journal of Supercritical Fluids*, 41, 82–91. DOI: 10.1016/j.supflu.2006.08.016.

5. Guo, Q., Liang, Q., Ni, J., Xu, S., Yu, G. & Yu, Z. (2008). Markov chain model of residence time distribution in a new type entrained-flow gasifier. *Chemical Engineering and Processing*, 47, 2061–2065. DOI: 10.1016/j.cep.2007.10.017.

6. Harris, A., Thorpe, R. & Davidson, J. (2002). Stochastic modelling of the particle residence time distribution in circulating fluidised bed risers. *Chemical Engineering Science*, 57, 4779–4796. DOI: 10.1016/S0009-2509(02)00278-6.

7. Hornung, Ch. & Mackley, M. (2009). The measurements and characterisation of residence time distribution for laminar liquid flow in plastic microcapillary arrays. *Chemical Engineering Science*, 64, 3889–3902. DOI: 10.1016/j.ces.2009.05.033.

8. Madhurabthakam, Ch., Pan, Q. & Rempel, G. (2009). Residence time distribution and liquid holdup in kenics KMX static mixer with hydrogenated nitrile butadiene rubber solution and hydrogen gas system. *Chemical Engineering Science*, 64, 3320–3328. DOI: 10.1016/j.ces.2009.04.001.

9. Melo, P.A., Carlos Pinto, J. & Biscaia Jr., E. (2001). Characterization of the residence time distribution in loop reactors. *Chemical Engineering Science*, 56, 2703–2713. DOI: 10.1016/S0009-2509(00)00517-0.

10. Mizonov, V., Berthiaux, H., Gatumel, C., Barantseva, E. & Khokhlova, Y. (2009). Influence of crosswise non-homogeneity of particulate flow on residence time distribution in a continuous mixer. *Powder Technology*, 190, 6–9. DOI: 10.1016/j.powtec.2008.04.052.

11. Nikitine, C., Rodier, E., Sauceau, M. & Fages, J. (2009). Residence time distribution of a pharmaceutical grade polymer melt in a single screw extrusion process. *Chemical Engineering Research and Design*, 87, 809–816. DOI: 10.1016/j.cherd.2008.10.008.

12. Pröll, T., Todinca, T., Şuta, M. & Friedl, A. (2007). Acid gas absorption in trickle flow columns – Modelling of the residence time distribution of a pilot plant. *Chemical Engineering and Processing*, 46, 262–270. DOI: 10.1016/j.cep.2006.06.006.

13. Zhang, T., Wang, T. & Wang, J. (2005). Mathematical modelling of the residence time distribution in loop reactors. *Chemical Engineering and Processing*, 44, 1221–1227. DOI: 10.1016/j.cep.2005.05.001.

14. Buso, A., Giomo, M., Boaretto, L. & Paratella, A. (1997). New electrochemical reactor for wastewater treatment: mathematical model. *Chemical Engineering and Processing*, 36, 411–418. DOI: 10.1016/S0255-2701(97)00008-1.

15. Cocero, M.J. & Garcia, J. (2001). Mathematical model of supercritical extraction applied to oil seed extraction by CO_2 + saturated alcohol – II. Shortcut methods. *Journal of Supercritical Fluids*, 20, 245–255. DOI: 10.1016/S0896-8446(01)00069-9.

16. Yianatos, J.B., Bergh, L.G., Díaz, F. & Rodríguez, J. (2005). Mixing characteristics of industrial flotation equipment. *Chemical Engineering Science*, 60, 2273–2282. DOI:10.1016/j.ces.2004.10.039.

17. Znad, H., Báleš, V. & Kawase, Y. (2004). Modeling and scule up of airlift bioreactor. *Computers and Chemical Engineering*, 28, 2765–2777. DOI: 10.1016/j.compchemeng.2004.08.024.

18. Levenspiel, O. (1962). Chemical Reactor Engineering, Wiley, New York.

19. Masiuk, S. & Rakoczy, R. (2006). The entropy criterion for the homogenization process in a multi-ribbon blender. *Chemical Engineering and Processing*, 45, 500–506. DOI: 10.1016/j.cep.2005.11.008.

20. Masiuk, S. & Rakoczy, R. (2007). Power consumption, mixing time, heat and mass transfer measurements for liquid vessel that are mixed using reciprocating multiplates agitator. *Chemical Engineering and Processing*, 46, 89–98. DOI: 10.1016/j.cep.2006.05.002.

21. Rakoczy, R., Masiuk, S., Kordas, M. & Grądzik, P. (2011). The effects of power characteristics on the heat transfer process in various types of motionless mixing devices. *Chemical Engineering and Processing. Process Intensification*, 50, 959–969. DOI:10.1016/j.cep.2011.07.001.

22. Rakoczy, R. & Masiuk, S. (2011). Studies of a mixing process induced by a transverse rotating magnetic field. *Chemical Engineering Science*, 66, 2298–2308. DOI: 10.1016/j.ces.2011.02.021.

23. Hristov, J. (2009). Magnetic field assisted fluidization – a unified approach. Part 7. Mass transfer: Chemical reactors, basic studies and practical implementations thereof. *Review in Chemical Engineering*, 25, 1–254. DOI: 10.1515/REVCE.2009.25.1-2-3.1.

24. Rakoczy, R. & Masiuk, S. (2009). Experimental study of bubble size distribution in a liquid column exposed to a rotating magnetic field. *Chemical Engineering and Processing. Process Intensification*, 48, 1229–1240. DOI: 10.1016/j.cep.2009.05.001.

25. Claudel, S., Fonteix, C., Leclerc, J.P. & Lintz, H.G. (2003). Application of the possibility theory to the compartment modelling of flow pattern in industrial processes. *Chemical Engineering Science*, 58, 4005–4016. DOI: 10.1016/S0009-2509(03)00269-0.