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Availability analysis of vertical milling centre using Markov approach and Monte Carlo simulations

Keywords

availability, reliability, Markov approach, Monte Carlo, redundancy

Abstract

The purpose of this article is to investigate the availability of vertical machining centre using a Markovian technique and Monte Carlo simulation (MSC). Availability is a critical performance metric for industrial systems. Conventional methodologies focus for steady-state availability evaluation of mechanical systems. The research analyses transient availability assessment for four different system configurations. Monte Carlo simulation modelling is used to compare the results and future scope is suggested to use the developed MCS based algorithms/codes for non-exponential (time dependant) failure and repair time distributions. The research also investigates the influence of active and passive redundancy on availability, indicating that for the vertical machining centre, parallel architecture with standby redundancy outperforms active load sharing. The chapter includes a sensitivity study that modifies the repair rates of the ball screw and sub-assembly to make the component selection process easier for engineers. The authors believe that this chapter will be useful to maintenance and practising engineers because it will assist them in making informed decisions about system availability, developing maintenance/replacement policies, and determining the repair level required to achieve the desired system availability.

1. Introduction

In today’s world, innovative technologies are used to increase system productivity while decreasing manufacturing costs. The optimal technologies should be highly dependable, conveniently accessible, and maintainable in order to return the system to its original condition in the event of a manufacturing failure. Lowering the system's load can also improve its availability. The significance of system availability is clear in today's culture, as the globe grows increasingly reliant on modern technology systems that need intricate operations and revolutionary administration.

A balance must be struck between availability and other resources, such as cost, volume, and weight, when building a system for dependability and availability. From home to industrial uses, automated systems have become an increasingly important part of our everyday lives, increasing our reliance on them.

Critical system failures, such as those in air traffic control, nuclear reactors, or hospital patient monitoring, can have devastating repercussions. As a result, enterprises are always working to decrease the likelihood of failure and boost system availability using quantitative analytical methodologies based on industrial engineering and operational

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research ideas.

This evaluation needs precise knowledge of constant characteristics, such as the failure and repair rates of systems/subsystems prone to random failures caused, among other things, by bad design, a lack of operating skills, or manufacturing technologies.

2. Literature review

In this section, literature review of problem solved using Monte Carlo simulation (MCS) or Markovian approaches (MA) are described.

2.1. Application of Monte Carlo simulation and Markovian approach

Ahmed et al. (Ahmed et al., 1989) suggested a semi-Markov modelling method to assess performance measures in queuing networks during the design of Computer Integrated Manufacturing Systems. This technique lowers complexity by grouping states together and only observing states during transition periods. Mean residence durations are determined using probability distribution functions of conditional state occupancy periods. Bounds of performance metrics are met, resulting in a straightforward and effective computational method. Alexander (Alexander, 2003) proposed the concepts of Monte Carlo simulations and demonstrated their application in determining the reliability of pump systems. Attar et al. (Attar et al., 2017) suggested a simulation-based optimisation technique for multi-objective joint availability redundancy allocation issues in multi-component series-parallel setups using active, cold, and hot backup strategies.

Borgonovo et al. (Borgonovo et al., 2000) introduced a Monte Carlo method for assessing plant maintenance strategies and working routines while adhering to economic limitations. Cadini et al. (Cadini et al., 2017) developed a repair model that combines an extreme weather stochastic model with a genuine cascading failure emulator to measure the effect of extreme weather events on power system reliability/availability. They also presented a Monte Carlo model-based approach for predicting the failure likelihood of degraded components incorporated within an optimum condition-based component replacement strategy, with an application to fatigue degradation. Çekyay and Özekici (Çekyay & Özekici, 2015) investigated the system dependability, mean time to failure, and steady-state availability of coherent systems, as well as the series connection of k-out-of-n standby subsystems with exponentially dispersed component lives. Garg et al. (Garg et al., 2010) used probability analysis and the extra variable method to explain the availability of a crankcase production system in the car business. Ge et al. (Ge et al., 2014) used Monte Carlo modelling to address the reliability evaluation issue of low and high Distribution Generation (DG) penetration levels in active distribution systems and discovered that DG incorporation can enhance system reliability when run actively. Lin and Donaghey (Lin & Donaghey, 1993) suggested using Monte Carlo simulation to determine the minimum cut sets and system reliability based on the reliability block diagram and the life distribution of each component. Based on Monte Carlo simulation and statistical analysis, Maciejewski and Caban (Maciejewski & Caban, 2008) provided an efficient method for predicting the availability of a repairable system using a Beta distribution. Chawla and Kumar (Chawla & Kumar, 2013) proposed a transient availability evaluation for a mechanical system with condition based maintenance strategy using Markov method. Varghese and Kumar (Varghese & Kumar, 2014) also used Markov approach for availability assessment for a mechanical system with opportunistic maintenance scenario.

Okafor et al. (Okafor et al., 2016) used a Markovian method to evaluate the availability of a thermal power station’s steam and gas turbine units. Sharma and Kumar (Sharma & Kumar, 2008) presented a paper to examine the need for maintenance practices that reduce both sudden sporadic failures and operation and maintenance costs. Kumar et al. (Kumar et al., 2013) suggested a semi-Markov process-based analytical approach for steady-state availability evaluation.

Sharma and Vishwakarma (Sharma & Vishwakarma, 2014) analyzed the performance measures of complex repairable systems involving reduced states using time-homogeneous Markov processes. Lognathan et al. (Lognathan et al., 2015) attempted steady-state availability evaluation using semi-Markov analytical approach for a vertical milling centre. Kumar et al. (Kumar et al., 2019) suggested simulative approach for reliability and availability assessment for repairable mechanical systems.

2.2. Gaps in literature

The literature on evaluating the availability of mechanical systems was examined, but the findings derived from the different methods proposed by researchers were having several limitations. Due to the assumptions made in the modelling the results were significantly different from what were anticipated.

Majority of the research work was focussed for steady-state availability evaluation. In some of the papers non-exponential distributions were approximated to exponential distribution so that Markov approach can be applied. Monte Carlo simulation approach was applied to simple mechanical systems with limited number of states. Monte Carlo simulation is given rare focus in spite of being one of the most powerful approaches in reliability and availability evaluation which can handle time-dependent failure and repair rates.

2.3. Problem statement

This chapter suggests that effective maintenance planning and management can return failed systems to their original functioning state by performing numerous fixes and component changes in the least amount of time. The authors suggest a quantitative method for assessing system availability that employs a stochastic modelling namely Markov model. The Markov model creates transition states, which are then solved by algorithms. The results of these findings are confirmed using Monte Carlo simulation, a technique that was not widely used for availability assessment of complex systems.

This chapter is an extension of the work of Loganathan et al. (Loganathan et al., 2016) wherein the availability evaluation was carried using semi-Markov approach in a case study of vertical milling centre (VMC). However, only active redundancy is considered in their work. Moreover, the work was done for steady-state assessment. In the present work, system is evaluated both in active and standby redundancy by using Markovian approach which is more of a transient system approach and result obtained are more real life compliant. Also, the use of Monte Carlo simulation which is being used here to back the result by further relaxing norms and assumptions.

The chapter approach is defined in the next section with the methodology being employed. A stepped account of work is presented below:

- modelling a series configuration and parallel configuration (passive redundancy) system as an example by using Markovian approach and results are compared with MCS,
- modelling case study system of VMC in active and passive redundancy using Markov model and MCS,
- sensitivity analysis to acknowledge dependence on repair rates, and also identifying critical components to availability.

3. Methodology

In this section, methodology used for availability evaluation for three different cases is presented using Markovian approach and Monte Carlo simulation. Following 4 cases are considered:

- two components in parallel configuration (passive redundancy) – Markov approach and MCS,
- two components in series configuration – Markov approach and MCS,
- VMC (3 components) – series-parallel case (active redundancy) – Markov approach,
- VMC (3 components) – series-parallel case (passive redundancy) – Markov approach and MCS.

3.1. Two components in parallel configuration-passive redundancy

Availability evaluation for system containing two components in parallel configuration (Figure 1) is discussed.

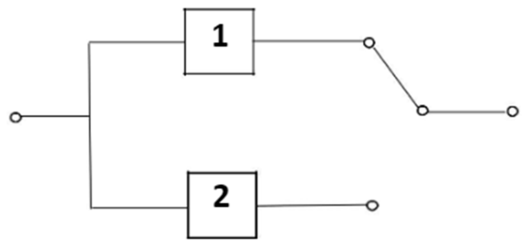

Figure 1. A system containing two components in parallel connection.

3.1.1. Markovian approach

System availability evaluation using Markov model is performed as following.

Step i: First all the possible states are identified to develop the model.

Step ii: Next the Markov model transition diagram is constructed.

Step iii: Transition rates are gathered at the component level.
Step iv: Rate equations are formulated in the form of Ordinary differential equations.
Step v: State probability solutions are evaluated using MATLAB and used for availability evaluation.
Table 1 presents all the possible states, transitions among states and system level status in various states. Using Table 1, a state transition diagram is constructed for Markov model and the same is given in Figure 2. Bold line in Figure 2 represents transition from state when one of the components fails, while dotted line is used to represent transition from one state to another when one of the failed components is repaired to original working condition.

Table 1. Development of system states (2-component in passive redundancy)

State	Sub-Assembly 1	Sub-Assembly 2	System Response	Transition to State
1	O	S	W	3
2	S	O	W	4
3	F	O	W	2, 5
4	O	F	W	1, 5
5	F	F	NW	4, 3

O – operating, S – standby, F – failed, W – working, NW – non-working.

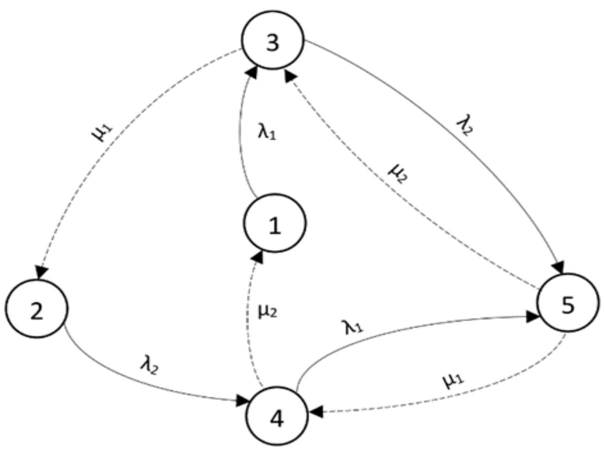

Figure 2. Transition state diagram for system containing two components in parallel arrangement (passive redundancy).

For the system sub-assemblies the failure time and the repair time distributions are Weibull and Lognormal respectively. The Weibull and Lognormal parameters values are given in Table 2. As Markov approach is based on constant failure rate (λ) and constant repair rate (μ). The equations (1), (2) and (3) are used to evaluate distributions parameters.

$$\lambda = \frac{1}{\mathrm{MTTF}} \text{ and } \mu = \frac{1}{\mathrm{MTTR}}. \tag{1}$$

The values of MTTF and MTTR are evaluated from the following results:

$$\mathrm{MTTF} = \theta\, \Gamma(1 + 1/\beta) \tag{2}$$

$$\mathrm{MTTR} = e^{(\mu+\frac{\sigma^2}{2})}. \tag{3}$$

The data presented in Table 2 is taken from (Loganathan et al., 2016).

Table 2. Weibull and Lognormal distribution parameters (Loganathan et al., 2016)

Distribution	CDF	Parameters
$W(\beta_{13}, \theta_{13})$	1→3	β_{13} = 1.5; θ_{13}= 2438
$W(\beta_{24}, \theta_{24})$	2→4	β_{24} = 1.5; θ_{24} = 2438
$W(\beta_{35}, \theta_{35})$	3→5	β_{35} = 1.5; θ_{35} = 2438
$Ln(\mu_{32}, \sigma_{32})$	3→2	μ_{32} = 3.4; σ_{32} = 1.79
$W(\beta_{45}, \theta_{45})$	4→5	β_{45} = 1.5; θ_{45} = 2438
$Ln(\mu_{41}, \sigma_{41})$	4→1	μ_{41} = 1.5; σ_{41} = 1.79
$Ln(\mu_{53}, \sigma_{53})$	5→3	μ_{53} = 3.4; σ_{53} = 1.79
$Ln(\mu_{54}, \sigma_{54})$	5→4	μ_{54} = 3.4; σ_{54} = 1.79

The failure rate and the repair rate are evaluated using equations (1), (2) and (3). The same are given here:

λ_1 = failure rate of Sub-Assembly 1 = 0.00061648,
λ_2 = failure rate of Sub-Assembly 2 = 0.00061648,
μ_1 = repair rate of Sub-Assembly 1 = 0.00672,
μ_2 = repair rate of Sub-Assembly 2 = 0.00672.

State transition equations for the given system are derived as following:

$$\frac{dy_1}{dt} = -\lambda_1 y_1 + \mu_2 y_2 \tag{4}$$

$$\frac{dy_2}{dt} = -\lambda_2 y_2 + \mu_1 y_3 \tag{5}$$

$$\frac{dy_3}{dt} = -(\mu_1 + \lambda_2) y_3 + \lambda_1 y_1 + \mu_2 y_5 \tag{6}$$

$$\frac{dy_4}{dt} = -(\lambda_1 + \mu_2) y_4 + \mu_1 y_5 + \lambda_2 y_2 \tag{7}$$

$$\frac{dy_5}{dt} = -(\mu_1 + \mu_2) y_5 + \lambda_2 y_3 + \lambda_1 y_4. \tag{8}$$

System availability (series configuration) is given as:

$$A(t) = P(1) + P(2) + P(3) + P(4). \quad (9)$$

3.1.2. Monte Carlo simulation

MCS approach is presented here which will be used to compare the result obtained from MA. Steps involved are as follows:

- step i: identification of working/ non-working states,
- step ii: developing an algorithm for implementation in MATLAB,
- step iii: generation of MATLAB codes based on algorithm developed.

Algorithm steps

(i) Identify the working states and non-working states at system level.
(ii) Start by generating a random number and use them to generate TTF (time to failure) and TTR (time to repair).
(iii) First develop TTF value for component 1 and then similarly for component 2.
(iv) Compare the values of two and decide which component fails first, which leads to transition from working states to non-working states.
(v) Store the down time of machine in some variable, say DT.
(vi) Similarly, repeat the steps till all possible cases are accounted for and total down time (TDT) is measured.
(vii) Putting the values in Availability equation as given below, Availability of system is calculated.
(viii) Running the algorithm in MATLAB for a number of iterations, till value of Availability converges towards a constant value.

Formulae used in MCS evaluation are given below. These are derived from Billinton and Allan (Billinton & Allan, 2007).

$$\mathrm{TTF} = -\left(\frac{1}{\lambda}\right) \cdot \log(\mathrm{YF}) \quad (10)$$

$$\mathrm{TTR} = -\left(\frac{1}{\mu}\right) \cdot \log(\mathrm{YR}) \quad (11)$$

where:
λ = corresponding failure rate,
μ = corresponding repair rate,
YF/YR = random variables in interval (0,1).

In a parallel system, as described in Markovian approach above, the system is down only when both the components in parallel fails simultaneously. The system is functional whenever one of the components is operating. As can be seen from the Figure 3 that system is down when the component failures coincide during the mission time, rest of the time it is up and working.

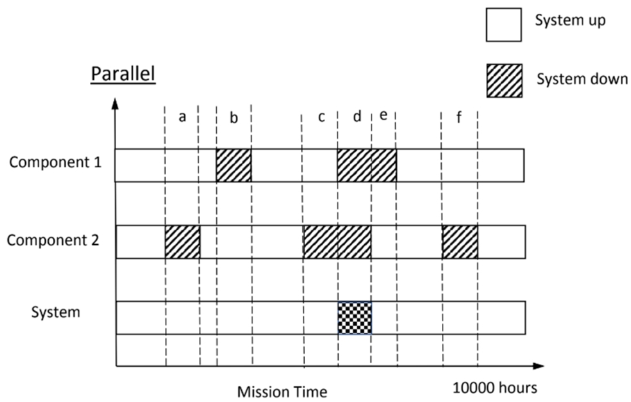

Figure 3. Systems working/non-working states (two component in passive redundancy).

Using MCS approach availability is evaluated as:

$$A(t) = \frac{\text{total uptime}}{\text{mission time}}. \quad (12)$$

3.2. Two components in series configuration

System availability evaluation using Markov model is performed, with similar methodology followed for parallel configuration.

3.2.1. Markovian approach

Table 3 shows the transition possible along with the definition of state and system response. All these represent the possible states and their transitions to one another.

Table 3. Development of system states (2 component in series configuration)

State	Sub-Assembly 1	Sub-Assembly 2	System Response	Transition to State
1	O	O	O	3, 2
2	O	F	F	1
3	F	O	F	1

Using Table 3 a Markov model diagram is constructed as given in Figure 4.
Table 4 shows the distribution used along with the

transition of state and the parameters associated. The data presented in Table 4 is derived taken from Loganathan et al. (Loganathan et al., 2016).

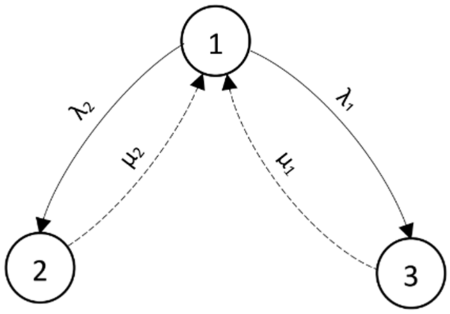

Figure 4. Transition state diagram for system with two components in series connection.

Table 4. Weibull and Lognormal distribution parameters (Loganathan et al., 2016)

Distribution	CDF	Parameters
$W(\beta_{12}, \theta_{12})$	1→2	$\beta_{12} = 2.72$; $\theta_{12} = 3315$
$W(\beta_{13}, \theta_{13})$	1→3	$\beta_{13} = 1.9$; $\theta_{13} = 1828$
$Ln(\mu_{21}, \sigma_{21})$	2→1	$\mu_{21} = 2.88$; $\sigma_{21} = 1.55$
$Ln(\mu_{31}, \sigma_{31})$	3→1	$\mu_{31} = 3.4$; $\sigma_{31} = 1.79$

For this work, the failure rate and the repair rate for the components are evaluated using the equations (1), (2) and (3). The rates are given here:

λ_1 = failure rate of Sub-Assembly 1 = 0.0003391,
λ_2 = failure rate of Sub-Assembly 2 = 0.00061648,
μ_1 = repair rate of Sub-Assembly 1 = 0.01688,
μ_2 = repair rate of Sub-Assembly 2 = 0.00672.

State transition equations for the given system are as following:

$$\frac{dy_1}{dt} = -(\lambda_1 + \lambda_2)\, y_1 + \mu_2 y_2 + \mu_1 y_3 \quad (13)$$

$$\frac{dy_2}{dt} = -(\mu_2) y_2 + \lambda_2 y_1 \quad (14)$$

$$\frac{dy_3}{dt} = -(\mu_1) y_3 + \lambda_1 y_1. \quad (15)$$

System availability (series configuration) is given as:

$$A(t) = P(1). \quad (16)$$

MCS approach is presented below which will be used to compare the result obtained from MA. Steps followed are similar to previous sections.

TTF and TTR are calculated using equations (10) and (11).

The working states are easily identifiable in series system. The only possibility being both the component up and working. Even when one of them fails the whole system goes to non-working state and system is down. Refer Figure 5, whenever either component fails, the system is down.

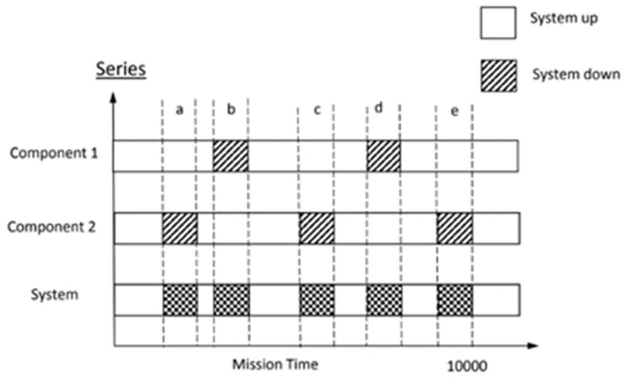

Figure 5. Systems working/non-working states (two components in series).

The availability is evaluated using the equation (12).

3.3. Vertical milling centre (3 components in series-parallel arrangement)

In this section, availability evaluation of the vertical milling centre (Figure 6) is done taking both cases of active and passive redundancy. First availability assessment is done MA and in the next MCS is employed to compare the results. However, MCS is carried out for passive redundancy only as dynamic variation in failure rate in case of active redundancy couldn't be easily estimated.

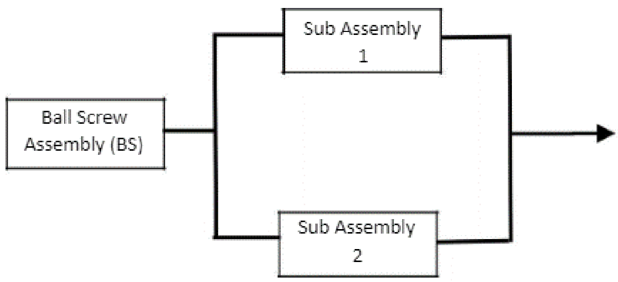

Figure 6. Vertical milling centre system.

3.3.1. Availability modelling – active redundancy (Markovian approach)

System availability evaluation using Markov Model is performed. Table 5 presents possible states, its transitions and system response. Using Table 5, a state transition diagram for Markov model is generated.

Table 5. Development of system states (vertical milling centre system – active redundancy)

State	Ball Screw	Sub-Assembly 1	Sub-Assembly 2	System Response	Transition to state
1	O	O	O	W	2, 3, 5
2	O	O	F	W	1, 4, 6
3	O	F	O	W	1, 4, 7
4	O	F	F	NW	2, 3
5	F	O	O	NW	1
6	F	O	F	NW	2
7	F	F	O	NW	3

The same is shown in Figure 7. Bold line in Figure 7 represents transition from state when one of the components fails, while dotted line is used to represent transition from one state to another when one of the failed components is repaired to original working conditions.

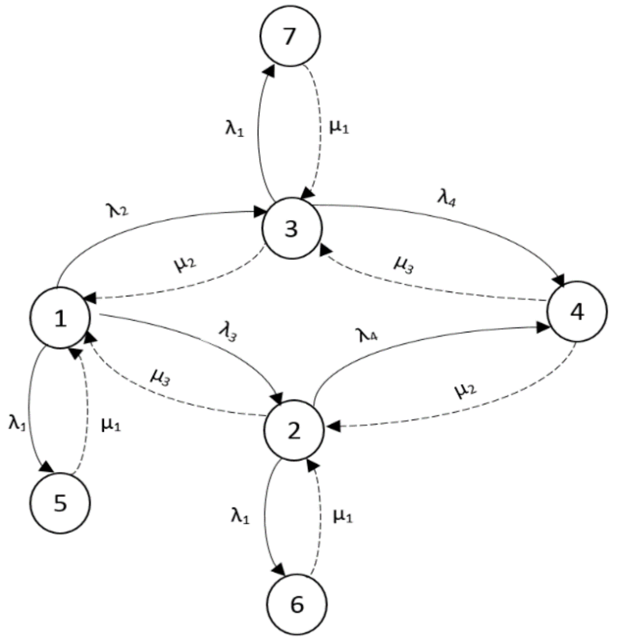

Figure 7. Markov model transition diagram for mechanical subsystem of twin-spindle VMC in load sharing (active redundancy).

Table 6 shows the distribution used along with the transition of state and the parameters associated. The data presented in Table 6 is derived from Loganathan et al. (Loganathan et al., 2016).

Table 6. Weibull and Lognormal distribution parameters (Loganathan et al., 2016)

Distribution	CDF	Parameters
$W(\beta_{12}, \theta_{12})$	1→2	$\beta_{12} = 1.5; \theta_{12} = 2438$
$W(\beta_{13}, \theta_{13})$	1→3	$\beta_{13} = 1.5; \theta_{13} = 2438$
$W(\beta_{15}, \theta_{15})$	1→5	$\beta_{15} = 2.72; \theta_{15} = 3315$
$Ln(\mu_{21}, \sigma_{21})$	2→1	$\mu_{21} = 3.4; \sigma_{21} = 1.79$
$W(\beta_{24}, \theta_{24})$	2→4	$\beta_{24} = 1.9; \theta_{24} = 1828$
$W(\beta_{26}, \theta_{26})$	2→6	$\beta_{26} = 2.72; \theta_{26} = 3315$
$Ln(\mu_{31}, \sigma_{31})$	3→1	$\mu_{31} = 3.4; \sigma_{31} = 1.79$
$W(\beta_{34}, \theta_{34})$	3→4	$\beta_{34} = 1.9\ \theta_{34} = 1828$
$W(\beta_{37}, \theta_{37})$	3→7	$\beta_{37} = 2.72\ \theta_{37} = 3315$
$Ln(\mu_{42}, \sigma_{42})$	4→2	$\mu_{42} = 3.4; \sigma_{42} = 1.79$
$Ln(\mu_{43}, \sigma_{43})$	4→3	$\mu_{43} = 3.4; \sigma_{43} = 1.79$
$Ln(\mu_{51}, \sigma_{51})$	5→1	$\mu_{51} = 2.88; \sigma_{51} = 1.55$
$Ln(\mu_{62}, \sigma_{62})$	6→2	$\mu_{62} = 2.88; \sigma_{62} = 1.55$
$Ln(\mu_{73}, \sigma_{73})$	7→3	$\mu_{73} = 2.88; \sigma_{73} = 1.55$

The failure rate and the repair rate for the components are evaluated and given here:

λ_1 = failure rate of Ball Screw = 0.0003391,
λ_2 = failure rate of Sub-Assembly 1 with load sharing = 0.0004544,
λ_3 = failure rate of Sub-Assembly 2 with load sharing = 0.0004544,
λ_4 = failure rate of Sub-Assemblies without load sharing = 0.00061648,
μ_1 = repair rate of Ball Screw = 0.01688,
μ_2 = repair rate of Sub-Assembly 1 = 0.00672,
μ_3 = repair rate of Sub-Assembly 2 = 0.00672.

State transition equations for the given system are derived as following:

$$\frac{dy_1}{dt} = -(\lambda_2 + \lambda_3 + \lambda_1)\, y_1 + \mu_1 y_5 + \mu_3 y_2 + \mu_2 y_3 \quad (17)$$

$$\frac{dy_2}{dt} = -(\lambda_1 + \lambda_4 + \mu_3)\, y_2 + \lambda_3 y_1 + \mu_2 y_4 + \mu_1 y_6 \quad (18)$$

$$\frac{dy_3}{dt} = -(\lambda_1 + \mu_2 + \lambda_4) y_3 + \mu_1 y_7 + \lambda_2 y_1 + \mu_3 y_4 \quad (19)$$

$$\frac{dy_4}{dt} = -(\mu_3 + \mu_2) y_4 + \lambda_4 y_3 + \lambda_4 y_2 \quad (20)$$

$$\frac{dy_5}{dt} = -(\mu_1) y_5 + \lambda_1 y_1 \quad (21)$$

$$\frac{dy_6}{dt} = -(\mu_1) y_6 + \lambda_1 y_2 \quad (22)$$

$$\frac{dy_7}{dt} = -(\mu_1) y_7 + \lambda_1 y_3. \quad (23)$$

Availability of system is defined as:

$$A(t) = P(1) + P(2) + P(3). \qquad (24)$$

3.3.2. Availability modelling – passive redundancy (Markovian approach)

System availability evaluation using Markov Model is performed. Table 7 presents possible states, its transitions and system response.

Table 7. Development of system states (VMC – passive redundancy)

State	Ball Screw	Sub-Assembly 1	Sub-Assembly 2	System Response	Transition to state
1	O	O	S	W	2, 7
2	F	O	S	NW	1
3	O	O	F	W	9, 4, 1
4	F	O	F	NW	3
5	O	S	O	W	3, 6
6	F	S	O	NW	5
7	O	F	O	W	5, 8, 9
8	F	F	O	NW	7
9	O	F	F	NW	7, 3

Using Table 7, a state transition diagram for Markov model is generated. The same is shown in Figure 8. Bold line in Figure 8 represent transition from state when one of the component fails, while dotted line is used to represent transition from one state to another when one of the failed component is repaired to original working condition.

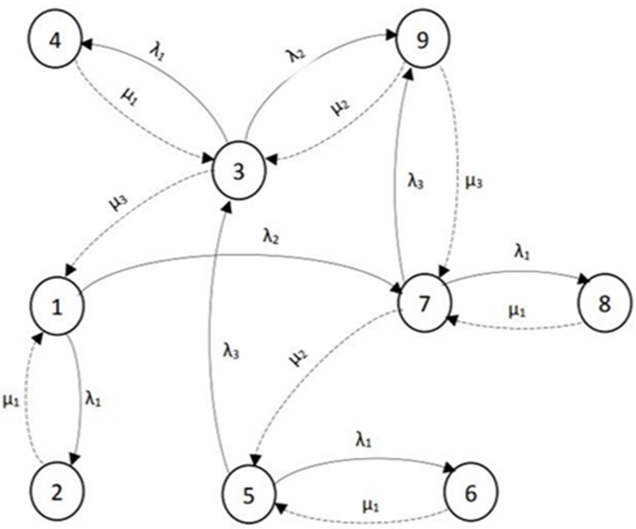

Figure 8. Markov model transition diagram for twin-spindle VMC (passive redundancy).

Table 8 shows the distribution used along with the transition of state and the parameters associated. The data presented in Table 8 is derived from Loganathan et al. (Loganathan et al., 2016).

Table 8. Weibull and Lognormal distribution parameters (Loganathan et al., 2016)

Distribution	CDF	Parameters
$W(\beta_{12}, \theta_{12})$	1→2	$\beta_{12} = 2.72$; $\theta_{12} = 3315$
$W(\beta_{17}, \theta_{17})$	1→7	$\beta_{17} = 1.5$; $\theta_{17} = 2438$
$Ln(\mu_{21}, \sigma_{21})$	2→1	$\mu_{21} = 2.88$; $\sigma_{21} = 1.55$
$Ln(\mu_{31}, \sigma_{31})$	3→1	$\mu_{31} = 3.4$; $\sigma_{31} = 1.79$
$W(\beta_{34}, \theta_{34})$	3→4	$\beta_{34} = 2.72$; $\theta_{34} = 3315$
$W(\beta_{39}, \theta_{39})$	3→9	$\beta_{39} = 1.5$; $\theta_{39} = 2438$
$Ln(\mu_{43}, \sigma_{43})$	4→3	$\mu_{43} = 2.88$; $\sigma_{43} = 1.55$
$W(\mu_{53}, \sigma_{53})$	5→3	$\beta_{53} = 1.5$; $\theta_{53} = 2438$
$W(\beta_{56}, \theta_{56})$	5→6	$\beta_{56} = 2.72$; $\theta_{56} = 3315$
$Ln(\mu_{65}, \sigma_{65})$	6→5	$\mu_{65} = 2.88$; $\sigma_{65} = 1.55$
$Ln(\mu_{75}, \sigma_{75})$	7→5	$\mu_{75} = 3.4$; $\sigma_{75} = 1.79$
$W(\beta_{78}, \theta_{78})$	7→8	$\beta_{78} = 2.72$; $\theta_{78} = 3315$
$W(\beta_{79}, \theta_{79})$	7→9	$\beta_{79} = 1.5$; $\theta_{79} = 2438$
$Ln(\mu_{87}, \sigma_{87})$	8→7	$\mu_{87} = 2.88$; $\sigma_{87} = 1.55$
$Ln(\mu_{93}, \sigma_{93})$	9→3	$\mu_{93} = 3.4$; $\sigma_{93} = 1.79$
$Ln(\mu_{97}, \sigma_{97})$	9→7	$\mu_{97} = 3.4$; $\sigma_{97} = 1.79$

The failure rate and the repair rate for the components are evaluated and given here:

λ_1 = failure rate of Ball Screw = 0.0003391,
λ_2 = failure rate of Sub Assembly 1 = 0.00061648,
λ_3 = failure rate of Sub Assembly 2 = 0.00061648,
μ_1 = repair rate of Ball Screw = 0.01688,
μ_2 = repair rate of Sub Assembly 1 = 0.00672,
μ_3 = repair rate of Sub Assembly 2 = 0.00672.

State transition equations for the given system are derived as following:

$$\frac{dy_1}{dt} = -(\lambda_1 + \lambda_2)\, y_1 + \mu_1 y_2 + \mu_3 y_3 \qquad (25)$$

$$\frac{dy_2}{dt} = -\mu_1 y_2 + \lambda_1 y_1 \qquad (26)$$

$$\frac{dy_3}{dt} = -(\lambda_1 + \lambda_2 + \mu_3) y_3 + \mu_2 y_9 + \lambda_3 y_5 + \mu_1 y_4 \qquad (27)$$

$$\frac{dy_4}{dt} = -\mu_1 y_4 + \lambda_1 y_3 \qquad (28)$$

$$\frac{dy_5}{dt} = -(\lambda_1 + \lambda_3)\, y_5 + \mu_1 y_6 + \mu_2 y_7 \qquad (29)$$

$$\frac{dy_6}{dt} = -(\mu_1) y_6 + \lambda_1 y_5 \qquad (30)$$

$$\frac{dy_7}{dt} = -(\lambda_1 + \lambda_3 + \mu_2)\,y_7 + \lambda_2 y_1 + \mu_1 y_8 + \mu_3 y_9 \quad (31)$$

$$\frac{dy_8}{dt} = -\mu_1 y_8 + \lambda_1 y_7 \quad (32)$$

$$\frac{dy_9}{dt} = -(\mu_2 + \mu_3)\,y_9 + \lambda_2 y_3 + \lambda_3 y_7. \quad (33)$$

Availability of system is defined as:

$$A(t) = P(1) + P(3) + P(5) + P(7). \quad (34)$$

3.3.3. Availability modelling – passive redundancy (Monte Carlo simulation)

MCS approach is presented to compare the results obtained from MA. Steps followed are similar to previous sections. TTF and TTR are calculated using equations (10) and (11).
The working states are easily identifiable in series-parallel combination system. The possibility being both the subsystems/subassembly in series being up and working. Even when one of the subsystem/subassembly fails the whole system goes to non-working state and system is down. As seen in the Figure 9, whenever either subsystem fails, the system is down.

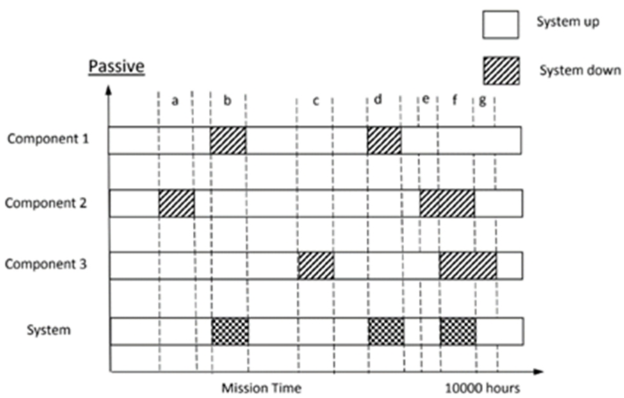

Figure 9. Possible states in series-parallel combination system.

4. Results and analysis

In this section, the result are obtained from study of 4 different systems using Markov model and Monte Carlo simulation and the same are discussed.

4.1. Transient availability results

Based on the Markov model, availability evaluation is done for the 4 different system models presented in previous section. The set of state equations are solved using MATLAB software (ODE 45 function) at time interval of 10,000 hrs and these state probability values are used to evaluate system availability. Also the result obtained using MCS approach except the case of VMC active redundancy case. All the results are presented in the Table 9. Also the results are presented in graphical form in Figure 10 to Figure 16. From the results under Table 9, it is clear that the Markov and MCS availability results are close. Therefore, MCS algorithm are giving the results comparable with closed form results of the Markovian approach.

Table 9. Transient availability results for 10,000 hrs

System type	System availability (Markov approach)	System availability (MCS approach)
two components in parallel configuration (passive redundancy)	0.9962	0.9937
two components in series configuration	0.8994	0.9051
vertical milling centre (3 components series-parallel case – active redundancy)	0.9751	NA
vertical milling centre (3 components series-parallel case – passive redundancy)	0.9766	0.9731

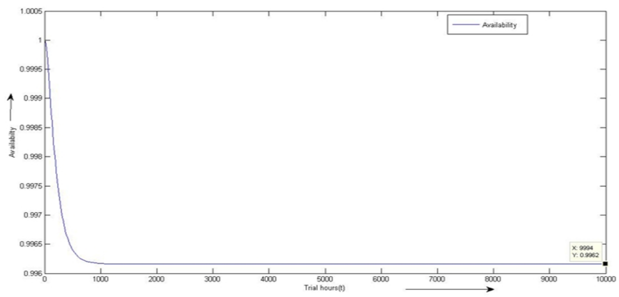

Figure 10. Availability Markov result of two components in parallel configuration (passive redundancy).

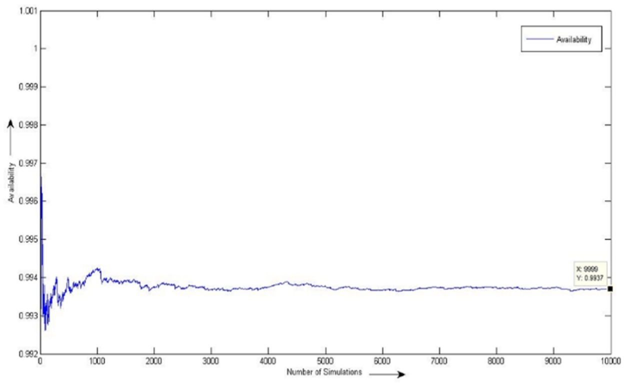

Figure 11. Availability MCS result of two components in parallel configuration (passive redundancy).

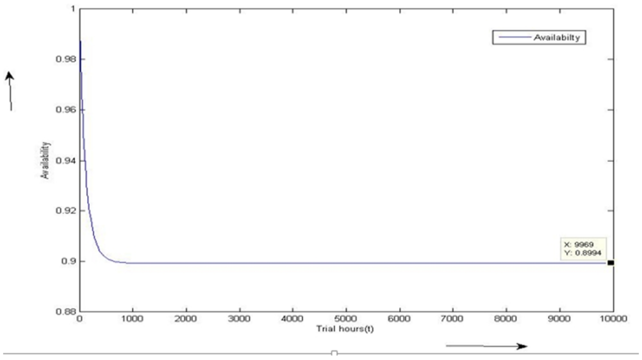

Figure 12. Availability Markov result of two components in series configuration.

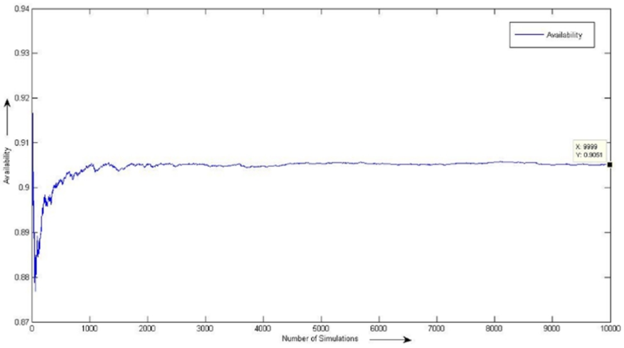

Figure 13. Availability MCS result of two components in series configuration.

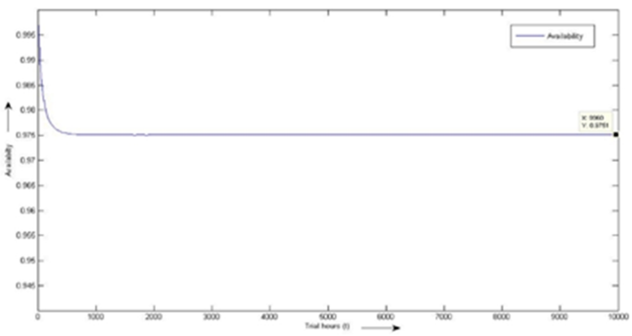

Figure 14. Availability Markov result of vertical milling centre (3 components series – parallel case – active redundancy).

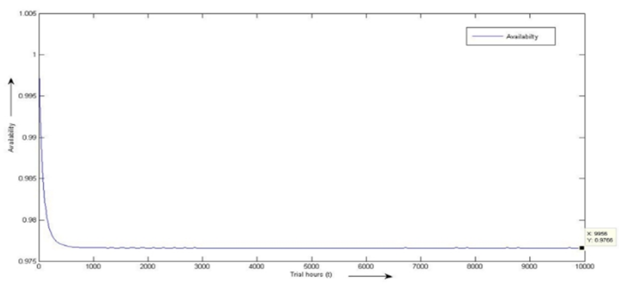

Figure 15. Availability Markov result of vertical milling centre (3 components – series –parallel case – passive redundancy).

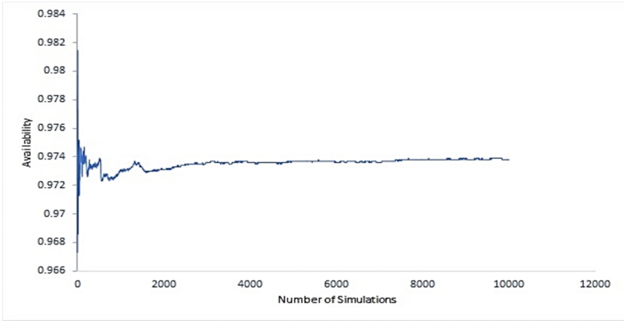

Figure 16. Availability MCS result of vertical milling centre (3 components) – series-parallel case (passive redundancy).

4.2. Sensitivity analysis – active redundancy

Sensitivity analysis is done by varying the value of failure and repair rate. At different value of failure rate and repair rate availability value changes. Here as only repair rate is in our control we vary the same. Sensitivity analysis is done as under taking λ_1 = 0.0003391, λ_2 = 0.0004544, λ_3 = 0.0004544, λ_4 = 0.00061648, μ_2 = 0.00672, μ_3 = 0.00672 as constant and changing the value of μ_1, the availability assessment is done and results presented in Table 10.

Table 10. Variation of availability with change in repair rate of ball screw assembly

μ_1	0.01688	0.02	0.04	0.08
Availability	0.9751	0.9781	0.9863	0.9904

Similarly, taking λ_1 = 0.0003391, λ_2 = 0.0004544, λ_3 = 0.0004544, λ_4 = 0.00061648, μ_1 = 0.01688, μ_3 = 0.00672 as constant and changing the value of μ_2, the availability values are evaluated and shown in Table 11.

Table 11. Variation of availability with change in repair rate of sub assembly

μ_2	0.00672	0.008	0.02	0.04
Availability	0.9751	0.9759	0.9785	0.9794

On the similar lines, taking λ_1 = 0.0003391, λ_2 = 0.0004544, λ_3 = 0.0004544, λ_4 = 0.00061648, μ_1 = 0.01S688, μ_2 = 0.00672 as constant and changing the value of μ_3 availability values are obtained. The same are shown in Table 12.

Table 12. Variation of availability with change in repair rate of sub assembly 2

μ_3	0.00672	0.008	0.02	0.04
Availability	0.9751	0.9759	0.9785	0.9794

4.3. Sensitivity analysis – passive redundancy

Sensitivity analysis is done by varying the value of failure and repair rate. At different value of failure rate and repair rate availability value changes. Here as only repair rate is can be controlled, we vary the same. Sensitivity analysis is done as under taking λ_1 = 0.0003391, λ_2 = 0.0004544, λ_3 = 0.0004544, μ_2 = 0.00672, μ_3 = 0.00672 as constant and changing the value of μ_1 the availability is assessed. The same are shown in Table 13.

Table 13. Variation of availability with change in repair rate of ball screw assembly

μ_1	0.01688	0.02	0.04	0.08
Availability	0.9766	0.9796	0.9878	0.992

Similarly, taking λ_1 = 0.0003391, λ_2 = 0.0004544, λ_3 = 0.0004544, μ_1 = 0.01688, μ_3 = 0.00672 as constant and changing the value of μ_2 the availability values are evaluated and the same are given in Table 14.

Table 14. Variation of availability with change in repair rate of sub assembly 1

μ_2	0.00672	0.008	0.02	0.04
Availability	0.9766	0.9772	0.979	0.9797

On the similar lines, taking λ_1 = 0.0003391, λ_2 = 0.0004544, λ_3 = 0.0004544, μ_1 = 0.01688, μ_2 = 0.00672 as constant and changing the value of μ_3 the availability values are obtained. The same are presented in Table 15.

Table 15. Variation of availability with change in repair rate of sub assembly 2

μ_3	0.00672	0.008	0.02	0.04
Availability	0.9766	0.9772	0.979	0.9797

5. Discussion

The availability evaluation of a Vertical Milling Centre is done using Markov model considering constant failure and repair rates during period of operation and further Monte Carlo simulation is used for the same system to compare the results. Standby redundancy is used to further increase the availability of the system. It was found out that out of 10,000 hours of operation, system in passive (standby) redundancy would be available for 9,766 hours. While in active (load sharing) redundancy it will be available for 9,751 hours.

Based on above results it can be easily pointed out that using passive redundancy over active redundancy will result in an increase of 15 hours of available life, or an increase of 0.15% of available life over active redundancy system. Further, Monte Carlo simulation method is used for the same system under passive redundancy to compare the results obtained from Markovian approach. Next, sensitivity Analysis is performed under active redundancy and it can be seen an increase in repair rate of the Ball screw and Sub-assemblies in VMC, availability increases.

Table 10 shows that by increasing the repair rate of Ball Screw from 0.01688 to 0.08, availability increases by 1.5691%. Table 11 shows that by increasing the repair rate of Sub-Assembly-1 from 0.00672 to 0.04, availability increases by 0.4409%. Table 12 shows that by increasing the repair rate of Sub Assembly-2 from 0.00672 to 0.04, availability increases by 0.4409%. Again, sensitivity analysis is performed under passive redundancy and it can be seen an increase in repair rate of the Ball screw and Sub-assemblies in VMC, availability increases. Table 13 shows that by increasing the repair rate of Ball Screw from 0.01688 to 0.08, availability increases by 1.577%. Table 14 shows that by increasing the repair rate of Sub-Assembly-1 from 0.00672 to 0.04, availability increases by 0.31%. Table 15 shows that by increasing the repair rate of Sub-Assembly-2 from 0.00672 to 0.04, availability increases by 0.31%. Hence, it can be easily pointed out, by increasing repair rate of ball screw sub assembly, availability gain for overall system is higher and hence recommended.

6. Limitations

Assumptions taken during availability analysis restrict the result to specified condition wherein results are found to be valid and in conformance to actual observed. These limitations need to be relaxed to form a much universally acceptable result. Some of these limitations as present in the proposed models are given below:

- in this work, Markovian approach is limited to constant repair and failure rates which needs to be relaxed,
- the model assumes that their cannot be any failure in standby components, but in real scenario the probability of standby components to fail is > 0%,

- constants and factors should be multiplied to availability obtained as in present context smooth operation of machine without any kind of shock or dynamic loading is considered for evaluation of failure rates (which in real world would be greater than as obtained),
- further the initial wear in period effects are neglected, which can substantially decrease availability.

7. Conclusion

In order to make the approach more effective and applicable to the real system, the following aspects are proposed under the future scope.

- Costs are not considered in this study for the availability analysis. It will be relevant if it is connected to the cost of the various maintenance activities and an analysis is performed in terms of availability benefits vs. cost of replacement.
- Due to the limitations of the Markovian method, an exponential distribution is considered for failure and repair time. For a more practical analysis, this premise can be dropped and suitable non-exponential distributions such as Weibull for failure time and Log-normal for restoration time may be considered.
- Integration of availability evaluation with decision-making algorithms that take various variables into consideration.
- Extension of use of developed MCS algorithms and codes for systems with non-exponential (time dependent) failure/repair time distributions just by replacing the TTF and TTR calculation in terms of non-exponentials distribution parameters.
- Monte Carlo simulation approach can be used for availability evaluation and validation of MA for the system in case of active redundancy where changes in failure rates due to load sharing occurs.

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