

# Mathematical Model of Plain Weave Fabric at Various Stages of Formation

## Abstract

*This article proposes a system of equations for forecasting grey fabric structure parameters and the parameters of fabric in different loom zones. The system of equations suggested is solved by the numerical method in various applications. This article is based on the nonlinear bending moment theory of thin elastic plate bending developed by E.P. Popov. The analysis results of fabric micro cross-sections from different loom zones are presented for comparison of calculated and experimental data. First as computed parameters are the density of the fabric in the warp (knowing the density of the fabric, defined by the reed) and weft (knowing the density of the fabric, which determines the cloth beam regulator) in different areas of the machine, which makes it possible to design a more accurate process of the fabric forming process and predict the yarn axis in different loom zones.*

**Key words:** fabric structure, bending stiffness, yarn curvature, fabric model, wave height of yarn curvature.

## ■ Introduction

Many scientists have been engaged in the issues of forecasting fabric structure parameters. The most famous works by Peirce and Olofsson [1, 2] contain the foundations of the fabric structure theory. The work [2] focuses on the theory of forecasting fabric structure parameters enabling to define fabric structure parameters to high precision. In [4] the author applies the theory of elasticity. Similar methods were applied in work [3]. The works mentioned were executed a long time ago, and the authors applied table values of elliptic functions. Lomov in his works [5 - 7] used an approach based on the minimum energy principle when calculating multi-layer fabric structure parameters. Also noteworthy is the work [8] by Stepanov. The author uses a system of nonlinear differential equations with regard to single-texture fabric; however, the yarn bending stiffness is assumed to be constant.

Based on calculated values of fabric structure parameters, researchers in [9 - 15] use various methods of approach for modelling the shapes of yarn in fabric, thus defining the calculated model structure for subsequent calculations (for example, for finite element method analysis [16, 17]). Work [9] used b-splines, and in [10, 11, 13] various trigonometric functions were applied. There are also simplified methods of submission of form threads in fabric [12, 14] using curves consisting of straight lines and arcs of an ellipse. Interesting studies presented in [15] use the modelling of the yarn axis by an elastica curve. It should be noted that most of the approaches (including [10 - 15] above) to form yarn modelling use known data about the structure of the

fabric formed, which makes it impossible to forecast the structure parameters.

A very important parameter defining the simulation of the woven material destruction process is the yarn length in the weave cell, which must be calculated prior to creating a geometrical model.

Therefore the issue of forecasting fabric structure parameters is a matter of topical interest. However, quite a number of parameters should be calculated to define any weave fabric cell:

- length of yarn in spaces between yarns,
- pressure force between yarns,
- warp/weft density,
- height of yarn bend waves,
- yarn bending stiffness,
- yarn curvature,
- bending moment in cross-sections,
- additional parameters for calculating the shape of yarn in the fabric.

Fabric densities on the loom and in a free condition must be assumed to be different.

Regarding conventional weaving technology, the values calculated should be spaces between picks in the formation zone, and spaces between warp yarns in the breast beam zone (i.e. forecasting of the fabric width in the breast beam zone), as well as the density of fabric collected from the weaving machine.

This was taken as a basis for our theory of elasticity [4], which was successfully applied in the design of the structure of fabric [1 - 3, 15, 20] and in the study of the shape of fabric [18].

This article presents a mathematical model of the plain weave fabric structure in different loom zones as well as the structure of fabric collected from the weaving loom. Experimental data is given for pure linen fabric (50 tex linen yarn for warp and weft). Linen fabric was chosen as the most difficult to calculate. This mathematical model can also be applied for calculation of the yarn elastic line based on the numerical solution of the fabric structure mathematical model. The article we published before [20] serves as a basis for this work.

## ■ Mathematical model

In this article we introduce a mathematical model in a more compendious form, considerably corrected with provisions for variable bending stiffness. We also produce calculation results of all the mathematical model parameters.

The mathematical model suggested in [20] is based on work [19].

The author of [19] suggests the nonlinear bending moment theory of large displacements for elastic rod plane bending based on precise solutions of differential equations of elastic curves. See **Figure 1** (page 44), for a schematic overview of the yarn in a cross-woven fabric. Where  $F_{cp}$  – warp yarn tension, N;  $P_o$  – resultant force, N;  $N$  – unknown force that puts pressure from weft yarn side and activates vertical components of warp/weft yarn tension, N;  $L_y$  – space between picks, mm;  $h_o/2$  – half of bend wave height of warp yarn in fabric, mm;  $M_c$  – bending moment, N·mm;  $M_A$  – reactive moment, N·mm;  $\delta o$  – angle defining direction of X axis towards  $P_o$  force direction at initial

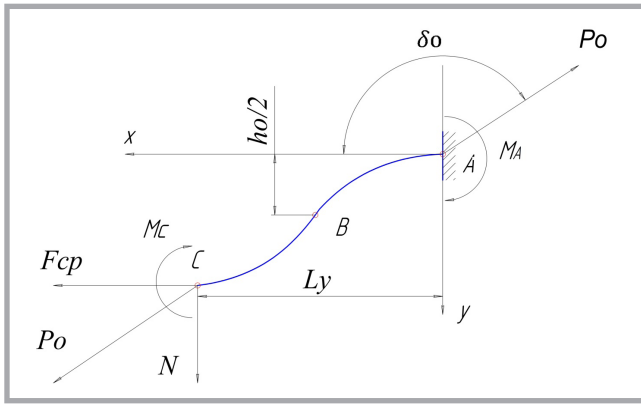


Figure 1. Schematic overview of warp yarn tension in cross-woven fabric.

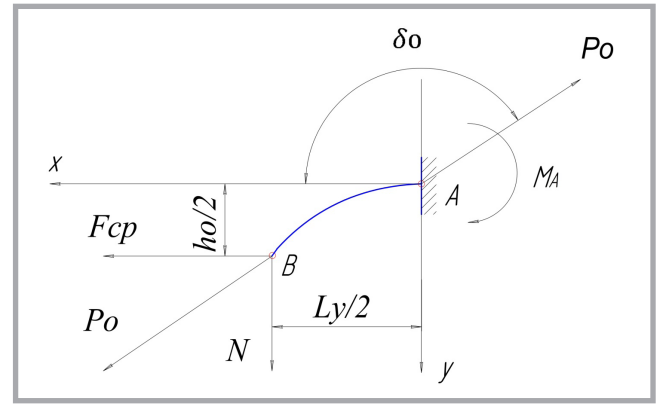


Figure 2. Diagram defining bend wave height of warp yarn in fabric.

point A [17, page 40], rad; ABC – warp yarn middle line.

The yarn shape is symmetrically relative to point “B” as the yarn tension diagram corresponds to a periodic elastic curve of the bending kind [17, page 60].

Therefore a diagram which will help us look for displacement is shown in **Figure 2**:

The following **Equations** [17] define the fabric structure:

$$\sqrt{\frac{P_o \cdot l_o^2}{H_o}} = \int_{\varphi_A}^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - k^2 \cdot \sin^2(\varphi)}} = F\left(\frac{\pi}{2}, k\right) - F(\varphi_A, k) \quad (1)$$

where:

$l_o$  - half warp yarn length in fabric, mm.

$\varphi_A$  - elliptic amplitude at point “A”,

$k$  - elliptic modulus,

$H_o$  - warp yarn bending stiffness N·mm<sup>2</sup>,  
 $F(\pi/2, k)$  - complete Legendre elliptic integral of the first kind;

$F(\varphi_A, k)$  - incomplete Legendre elliptic integral of the first kind

The elliptic amplitude value at point “B” is  $\pi/2$ .

Let us introduce the following notation:

$$F\left(\frac{\pi}{2}, k\right) - F(\varphi_A, k) = \Delta F\left(\frac{\pi}{2}, \varphi_A, k\right) \quad (2)$$

The following equation is applied to define the force leading to yarn bending:

$$P_o = \sqrt{F_{cp}^2 + N^2} \quad (3)$$

Angle  $\delta_o$  from the triangle formed by forces  $P_o$ ,  $F_{cp}$ ,  $N$  is defined as follows:

$$\delta_i = \frac{\pi}{2} + \arctg\left(\frac{F_{cp}}{N}\right) \quad (4)$$

The inclination angle of tangent ( $\zeta_A$ ) at point “A” of the curved yarn towards the line of action of force  $P_o$  is equal to angle  $\delta_o$ , then, according to [17]:

$$\zeta_A = \delta_i = 2 \arcsin(k \cdot \sin(\varphi_A)) \quad (5)$$

The formula for endpoint displacement “B” along the Y-axis is:

$$\frac{h_o}{2l_o} = \left( \frac{2k \cdot \cos(\varphi_A)}{\sqrt{\frac{P_o \cdot l_o^2}{H_o}}} \right) \cdot \cos(\delta_o) + \left( \frac{2 \cdot \int_{\varphi_A}^{\frac{\pi}{2}} \sqrt{1 - k^2 \cdot \sin^2(\varphi)} d\varphi}{\sqrt{\frac{P_o \cdot l_o^2}{H_o}}} - 1 \right) \cdot \sin(\delta_o) \quad (6)$$

$$\int_{\varphi_A}^{\frac{\pi}{2}} \sqrt{1 - k^2 \cdot \sin^2(\varphi)} d\varphi = E\left(\frac{\pi}{2}, k\right) - E(\varphi_A, k) \quad (7)$$

where  $E(\pi/2, k)$  is a complete elliptic Legendre integral of the second kind, and  $E(\varphi_A, k)$  is an incomplete elliptic Legendre integral of the second kind.

$$E\left(\frac{\pi}{2}, k\right) - E(\varphi_A, k) = \Delta E\left(\frac{\pi}{2}, \varphi_A, k\right) \quad (8)$$

The formula for “B” endpoint displacement along the X-axis is:

$$\frac{L_y}{2l_o} = \left( \frac{2 \cdot \Delta E\left(\frac{\pi}{2}, \varphi_A, k\right)}{\Delta F\left(\frac{\pi}{2}, \varphi_A, k\right)} - 1 \right) \cdot \cos(\delta_o) + \frac{2k \cdot \cos(\varphi_A)}{\Delta F\left(\frac{\pi}{2}, \varphi_A, k\right)} \cdot \sin(\delta_o) \quad (9)$$

Therefore the unknown parameters are  $\delta_o$ ,  $\varphi_A$ ,  $l_o$ ,  $P_o$ ,  $N$ ,  $h_o$  and  $k$ .

Similar equations must be set up for the weft yarn to define the height of yarn

bend waves. The diagram will be similar to that in **Figures 1** and **2**. Therefore:

$$P_y = \sqrt{F_y^2 + N^2} \quad (10)$$

$$\delta_y = \frac{\pi}{2} + \arctg\left(\frac{F_y}{N}\right) \quad (11)$$

$$\zeta_{Ay} = \delta_y = 2 \arcsin(ky \cdot \sin(\varphi_Ay)) \quad (12)$$

$$\sqrt{\frac{P_y \cdot l_y^2}{H_y}} = \int_{\varphi_Ay}^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - ky^2 \cdot \sin^2(\varphi)}} \quad (13)$$

where  $l_y$  is the length of the curved weft yarn in the fabric, mm (in this case corresponding to half of the curved yarn line in the fabric);

$\varphi_Ay$  – elliptic amplitude at point “A”,

$ky$  – elliptic modulus,

$H_y$  – weft yarn bending stiffness, N·mm<sup>2</sup>.

For further understanding: the subscript “y” is for the weft yarn and  $l_o$  is the space between warp yarns, mm.

$$\frac{h_y}{2l_y} = \left( \frac{2ky \cdot \cos(\varphi_Ay)}{\Delta F\left(\frac{\pi}{2}, \varphi_Ay, ky\right)} \right) \cdot \cos(\delta_y) + \left( \frac{2 \cdot \Delta E\left(\frac{\pi}{2}, \varphi_Ay, ky\right)}{\Delta F\left(\frac{\pi}{2}, \varphi_Ay, ky\right)} - 1 \right) \cdot \sin(\delta_y) \quad (14)$$

$$\frac{L_o}{2l_y} = \left( \frac{2 \cdot \Delta E\left(\frac{\pi}{2}, \varphi_Ay, ky\right)}{\Delta F\left(\frac{\pi}{2}, \varphi_Ay, ky\right)} - 1 \right) \cdot \cos(\delta_y) + \frac{2ky \cdot \cos(\varphi_Ay)}{\Delta F\left(\frac{\pi}{2}, \varphi_Ay, ky\right)} \cdot \sin(\delta_y) \quad (15)$$

$$\frac{L_o}{2l_y} = \left( \frac{2 \cdot \Delta E\left(\frac{\pi}{2}, \varphi_Ay, ky\right)}{\Delta F\left(\frac{\pi}{2}, \varphi_Ay, ky\right)} - 1 \right) \cdot \cos(\delta_y) + \frac{2ky \cdot \cos(\varphi_Ay)}{\Delta F\left(\frac{\pi}{2}, \varphi_Ay, ky\right)} \cdot \sin(\delta_y) \quad (15)$$

Therefore the unknown parameters are  $\delta_y$ ,  $\varphi_Ay$ ,  $l_y$ ,  $P_y$ ,  $N$ ,  $h_y$  and  $ky$ .

The main geometrical relationship for the warp and weft yarn is as follows:

$$h_o + h_y = d_{oe} + d_{ye} \quad (16)$$

where  $d_{oe}$  – warp yarn vertical diameter, mm;  $d_{ye}$  – weft yarn vertical diameter, mm;

The displacement of weft yarn axial line points in the formation zone is associated

with the weft yarn tensile deformation [7]. This article indicates that the receipt of additional weft reserve from neighbouring elements on account of sliding on the warp yarn is not possible [7, page 79 - 81]. Therefore one more relationship may be written for the weft. Relative weft yarn deformation in the fabric formation zone during insertion of weft yarn into a fabric is:

$$\Delta = \frac{ly - \left( \frac{Lo}{2} - \frac{\frac{Lo}{2} \cdot \Delta ini}{100} \right)}{\frac{Lo}{2} - \frac{\frac{Lo}{2} \cdot \Delta ini}{100}} \cdot 100 \quad (17)$$

where  $\Delta$  is the relative longitudinal deformation of weft yarn in a fabric, in %,  $\Delta ini$  – initial relative longitudinal deformation of weft yarn laid in the shed, in %,

The initial relative longitudinal deformation of the weft can be defined by weft yarn tension at the moment of its propulsion in the shed.

Knowing the functional relationship between the relative deformation and tension ( $f(\Delta)$ ), we can obtain a formula to define the weft yarn tension:

$$Fy = f(\Delta) \quad (18)$$

The bending stiffness of the yarn will be affected by

- yarn curvature,
- the bending moment,
- the tensile load affecting the yarn.

Knowing the experimental dependence between them, it is possible to define the bending stiffness. Let us write the known equations to calculate the bending stiffness through the bending moment and curvature of yarn axes in the fabric:

$$H_o = \frac{fo(\rho_o, Fcp)}{\rho_o} \quad (19)$$

$$H_y = \frac{fy(\rho_y, Fy)}{\rho_y} \quad (20)$$

where,  $fo$ ,  $fy$  are functional relationships between the curvature of the axis line of curved warp/weft yarn, tensile forces and bending moments emerging in yarns;  $\rho_o$ ,  $\rho_y$  – curvature of axial curved lines of warp/weft yarn accordingly,  $mm^{-1}$ .

Let us use the ratios known from [17]:

$$\rho_o = 2k \frac{\cos(\varphi A) \cdot \sqrt{Po}}{\sqrt{H_o}} \quad (21)$$

$$\rho_y = 2k \frac{\cos(\varphi Ay) \cdot \sqrt{Py}}{\sqrt{H_y}} \quad (22)$$

Therefore the system consists of 19 equations (1, 3 - 6, 9 - 22). The unknown values, depending on the zone for which the calculation is made, are  $\delta o$ ,  $\varphi A$ ,  $lo$ ,  $Po$ ,  $N$ ,  $ho$ ,  $k$ ,  $\delta y$ ,  $\varphi Ay$ ,  $ly$ ,  $Py$ ,  $hy$ ,  $ky$ ,  $\Delta$ ,  $Fy$ ,  $Ho$ ,  $Hy$ ,  $\rho_o$ ,  $\rho_y$ ,  $Ly$  &  $Lo$ , i.e. 21 parameters. This system can be solved by numerical methods in customised software packages.

Several equations to determine the vertical diameters of warp yarn depending on the strength of the normal pressure between them can be introduced into the equation system. However, it is easier to use coefficients of yarn crumpling in the fabric or instrumentally defined values of yarn vertical diameters for conducting primary experiments.

It is possible to simplify the system by substituting the values of resulting forces  $Po$ ,  $Py$  and their inclination angles  $\delta o$   $u$   $\delta y$ , as well as yarn curvatures  $\rho o$  and  $\rho y$  into corresponding equations. The number of equations can be reduced to 13 (the number of variables to be defined is up to 15). However, in the fabric formation simulation process, the parameters mentioned may have complex expressions that are appropriate to leave in the equation system.

Let us define what structure parameters are required to be calculated in the various loom zones.

It is common knowledge that in the zone of formation, a fabric must have a weft density different from the one preset by the beam regulator. The fabric in the doffer/breast beam zone has a weft density nearest to the preset density. The warp density is also different in the formation zone and at the breast beam. The preset values are spaces between picks at the doffer and spaces between warp yarns in the zone close to the formation zone. Therefore the fabric structure at the breast beam and doffer must be taken into account in order to define fabric structure parameters in the zone adjacent to the formation zone (the warp yarn length in the weave cell is a parameter required for calculation of the structure in the zone adjacent to the formation zone).

Let us assume that weft yarn tension is missing in the doffer/breast beam zone. The preset parameters (if no relationships for definition of yarn vertical diameters are available) are:

- values of yarn vertical diameters,
- spaces between picks,
- functional relationships between the curvature, bending moment and tensile force,
- warp yarn tension (it is assumed that the fabric structure is formed from the cycle average of the warp yarn tension),
- weft yarn length,
- initial deformation of weft yarn in the shed that serves as a basis for definition of the weft yarn length in the fabric element (affected by the warp density preset by the reed).

The unknown parameters are  $\delta o$ ,  $\varphi A$ ,  $lo$ ,  $Po$ ,  $N$ ,  $ho$ ,  $k$ ,  $\delta y$ ,  $\varphi Ay$ ,  $hy$ ,  $ky$ ,  $Ho$ ,  $Hy$ ,  $\rho_o$ ,  $\rho_y$ , &  $Lo$ , i.e. 16 parameters of the fabric structure (parameters  $\Delta$  &  $Fy$  are equal to 0;  $Py$  can be substituted by  $N$ ) subject to definition. This number of unknown parameters corresponds to the number of equations in the above system of equations, except for (10, 17, 18). The structure parameters at the breast beam can be calculated with a different tension parameter and warp yarn length already known to us from spaces between weft yarns.

The solution of the equation system in the doffer/breast beam zone allows us to define the fabric structure at the formation zone.

The unknown parameters for fabric structure calculation in the formation zone are  $\delta o$ ,  $\varphi A$ ,  $ly$ ,  $Po$ ,  $N$ ,  $ho$ ,  $k$ ,  $\delta y$ ,  $\varphi Ay$ ,  $Py$ ,  $hy$ ,  $ky$ ,  $\Delta$ ,  $Fy$ ,  $Ho$ ,  $Hy$ ,  $\rho_o$ ,  $\rho_y$  &  $Ly$ .

The parameters of fabric collected from the weaving machine must be based on the yarn length preserved in the spaces between adjacent yarns after the removal of loads (tensions along the warp/weft).

The unknown parameters in this zone are  $\varphi A$ ,  $N$ ,  $ho$ ,  $k$ ,  $\varphi Ay$ ,  $hy$ ,  $ky$ ,  $Ho$ ,  $Hy$ ,  $\rho_o$ ,  $\rho_y$ ,  $Ly$  &  $Lo$ .

Knowing the yarn length of a fabric in the warp/weft and in spaces between adjacent yarns as well as the load/tension ratio, we can define lengths after the loads have been removed. The system of equations (1, 3 - 6, 9 - 22) can be simplified considerably because angles  $\delta o$  and

**Table 1.** Result of calculation of the mathematical model.

System parameters	Parameter value			
	Doffer/Breast beam (1)	Breast beam zone (2)	Formation zone (3)	Grey fabric (4)
<i>k</i>	0.9926	0.9980	0.9973	0.8743
<i>ky</i>	0.8247	0.8473	0.9959	0.7979
<i>ky1</i>	0.7699	0.7828	0.9995	0.7551
<i>ho</i>	0.2165	0.1948	0.2664	0.2903
<i>hy</i>	0.1001	0.1218	0.0386	0.0756
<i>hy1</i>	0.1001	0.1218	0.0386	0.0756
$\varphi A$	1.2744	1.3411	1.2661	0.9421
$\varphi Ay$	1.0302	0.9873	1.4398	1.089
$\varphi Ay1$	1.1640	1.1275	1.5129	1.2124
$\delta o$	2.5020	2.6650	2.5155	1.5708
$\delta y$	1.5708	1.5708	2.8232	1.5708
$\delta y1$	1.5708	1.5708	3.0104	1.5708
<i>Po</i>	0.2992	0.5064	0.5553	0.1224
<i>Py</i>	0.3328	0.4294	1.8982	0.2324
<i>Py1</i>	0.0244	0.0353	0.4325	0.0124
<i>N</i>	0.3328	0.4294	0.5942	0.2324
<i>N1</i>	0.0244	0.0353	0.0566	0.0124
<i>Ly</i>	0.7692	0.7763	0.7516	0.7378
<i>Lo</i>	0.3976	0.3902	0.4125	0.4041
<i>Lo1</i>	0.7996	0.7960	0.8107	0.8028
<i>Ho</i>	0.0181	0.0206	0.0195	0.0157
<i>Hy</i>	0.0182	0.0188	0.0599	0.0173
<i>Hy</i>	0.0105	0.0124	0.0366	0.0071
$\rho o$	2.3561	2.2532	3.1895	2.8669
$\rho y$	3.625	4.4682	1.4653	2.7067
$\rho y1$	0.9284	1.1328	0.3980	0.6992
<i>Fy</i>	0	0	1.8028	0
<i>Fy1</i>	0	0	0.4288	0
$\Delta$	0	0	0.5195	0
$\Delta 1$	0	0	0.136	0
<i>Lo</i>	0.4022	0.4025	0.4025	0.4019
<i>Ly</i>	0.2062	0.2062	0.2073	0.2062
<i>ly1</i>	0.4035	0.4035	0.4041	0.4035

**Table 2.** Comparison of calculated and experimental data.

Loom zone	ho, mm	Deviation in %	Measurement error, %	hy, mm	Deviation in %	Measurement error, %
Doffer/breast beam zone	0,216	-	-	0.100	-	-
Breast beam zone	0.195/0.191	-2.0	3.3	0.122/0.126	3.0	4.0
Formation zone	0.266/-	-	-	0.0386/0.0405	4.7	10.0
Grey fabric	0.290/0.282	-2.9	2.5	0.0756/0.0839	9.9	5.0

$\delta y$  are  $90^\circ$ , and no tensile forces will be present.

Below is a calculation example of the fabric structure in different weaving machine zones (50 tex linen fabric in the warp and weft). The warp density in the formation zone is 164 yarns/10cm, and that of the weft at the doffer amounts to 130 picks/10 cm, with the warp yarn tension average of the machine cycle constituting 0.45 N/yarn (assuming that the fabric tension in the breast beam/fell zone approximately corresponds to that

value, whereas in the doffer/breast beam zone, the fabric tension is assumed to be smaller according to Euler’s formula by a value of  $e f \cdot \alpha$ , where  $f$  is the coefficient of friction (fabric/breast beam), and  $\alpha$  is the wrap angle of the breast beam to the fabric). The initial tension of weft yarns before the battening was assumed to be absent.

Stress-strain curves obtained using an Uster Tenzorapid (elongation rate - 150 mm per minute):

The curve “load – deformation” for the weft:

$$f(Fy) = -0.0172 \cdot Fy^2 + 0.3143 \cdot Fy \quad (23)$$

The curve “load – deformation” for the pick:

$$f(Fcp) = -0.0542 \cdot Fcp^2 + 0.3451 \cdot Fcp \quad (24)$$

The relationship for definition of the weft yarn bending moment:

$$fo(\rho_y, Fy) = 0.021 \cdot \rho_y + 0.37 \cdot Fy - 0.01 \quad (25)$$

The relationship for definition of the warp yarn bending moment:

$$fo(\rho_y, Fcp) = 0.017 \cdot \rho_y + 0.026 \cdot Fcp - 0.0036 \quad (26)$$

In so doing, the yarn axis curvature values must not be smaller than  $0.35 \text{ mm}^{-1}$ .

Dependence (25 - 26) is obtained by rounding a thread on another thread, imitating the opposite strands of the system using the nonlinear theory of bending, by way of the formulas presented above. At the same time, it takes into account the longitudinal forces acting on the thread. More details on this method of determining the bending stiffness will be presented in the following publications.

This model is true if the warp yarn is equally distributed on the reed. However, during the manufacture of medium density fabric, warp yarns often come in pairs when entering one reed blade/dent (gaps between warp yarns are different). We examined the structure of such fabric, in which we introduced additional equations for the weft yarn (10 - 15, 17 - 18, 20 - 22) into the mathematical model proposed and for the condition of equality of weft yarn curvatures at the warp yarn intervals. Different forces act at different intervals of the weft yarn, resulting in yarn curvatures; the warp yarn is affected by a force equal to half of the sum of the said forces.

The calculation result of fabric structure parameters is presented in **Table 1**. The structure tested was linen fabric made from 50 tex flax yarn in the weft and warp, with a warp density of 164 yarns/10 cm, and the weft density, set by the cloth beam regulator, is 130 picks/10 cm. The fabric was produced on a STB-2-175 machine (projectile type PLC “Textilmash”, Russia). Immediately after stopping the machine, the middle of the fabric was filled with epoxy resin, hardening up to 10 minutes (only one band after each stop of the machine)



in the formation zone till 3 mm from the edge of the fabric examined and in a micro cross-section region 3 mm from the edge of the fabric zone. The breast beam zone was defined as the distance from the breast beam to the zone of formation i.e. less than 20 mm and exploited an area no more than 10 mm from the side edge of the breast beam fabric. Grey fabric was filled immediately after removal from the machine in the center and a width not exceeding 100 mm exploited. The sum of yarn vertical diameters was based on the results of fabric micro section analysis from various loom zones (except the doffer/breast beam zone). The gap between warp yarns was measured on the frame. At the formation zone, it was 0.41 mm and 0.81 mm ( data for this gap are given by index "1").

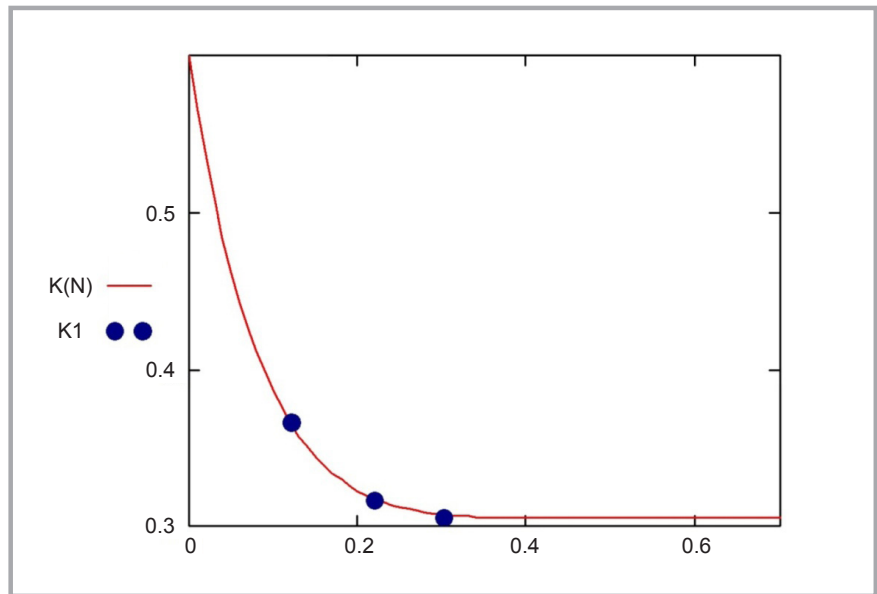
A comparison of calculated and experimental data obtained by micro sections of the fabric is shown in **Table 2**. The micro sections of the fabric in the breast beam/doffer zone were not made because of access difficulties to the fabric in the area mentioned. About 100 measurements of yarn curvature height were taken (except the formation zone). In the formation zone, 50 measurements of weft yarn curvature heights were made. The height of the warp yarn curvature was not measured. The sum of vertical diameters was estimated by the sum of yarn vertical diameters, amounting to 0.305 mm.

The numerator shows calculated data, and the denominator stands for experimental data.

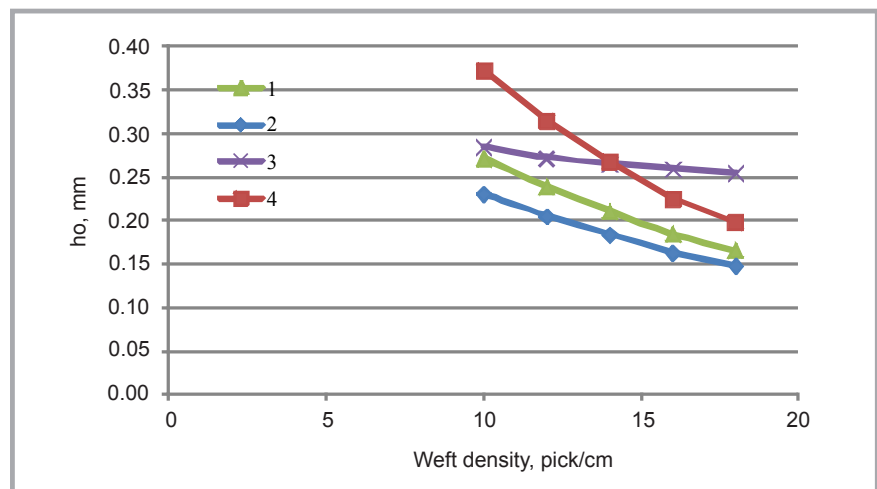
Of practical interest is the study of various fabric feeding parameters and their influence on the fabric structure and its non-structural characteristics (for example, normal pressure forces between yarns). Here is an example of a theoretical study. We will study the influence of weft density preset by the fabric beam regulator at the bend wave height of the warp yarn (ho). The relationship "Normal pressure force - Sum of yarn vertical diameters" based on the data of **Tables 1 & 2** may also be taken into account.

The relationship is as follows (see **Figure 3**):

$$f(N) = e^{(0.624 - \frac{0.624(\frac{N+N_1}{2}) - 0.12}{0.43})^6} - 0.695$$



**Figure 3.** Relationship "Normal pressure force - Sum of yarn vertical diameters".



**Figure 4.** Relationship between the bend wave height of warp yarn and weft density.

The approximation specified for small values  $N + N_1$  provides an overestimate value. However, it is sufficient to give a good quality description (in our calculations  $N + N_1 > 0.145 N$ .)

The calculation result is shown in **Figure 4**. The number of the row corresponds to the zone number in **Table 1**.

## Conclusion

A mathematical model was developed based on the nonlinear bending moment theory for forecasting the structure parameters of grey fabric and fabric on a weaving machine. It was taken into account that the weft density in the formation zone differs from that preset by the fabric beam regulator. The average discrepancy between calculated and ex-

perimental data is 4.5%. With the aid of this method of calculation, it is possible to obtain data on the structure of fabric at different stages of technological parameters in view, which is important for the prediction process for a given fabric structure. It is possible to calculate the density of weft in the formation zone of a fabric, which is known to be very different from the one that sets the cloth beam regulator. This will allow the presentation of a mathematical model to predict parameters of the beating up process. The parameters of the structure of grey fabric calculated can be used as a basis for calculating the deformation properties of fabrics on the basis of the theory presented in this article, or to calculate by the finite element method.



## References

1. Peirce FT. The Geometry of Cloth Structure. *Journal of the Textile Institute Transactions* 1937; 28, 3: 45-96.
2. Olofsson B. A General Model of a Fabric as a Geometric-Mechanical Structure. *Journal of the Textile Institute Transactions* 1964; 55, 11: 541-557.
3. Meliachenko GV, Nikolaev SD. Intercommunication of weaving technological parameters and fabric structure. *Technologiya Textilnoy Promyshlennosti* 1991; 1: 47-50.
4. Levien R. The elastica: a mathematical history. Technical Report No. UCB/EECS 2008 103, University of California, Berkeley, 2008.
5. Lomov SV, Barbarski M, Truong T, Verpoest I. Deformability and internal geometry of textile reinforcements and laminates. In: 14th International Conference on Composite Materials. San Diego, CA, USA, 2003.
6. Lomov SV, Huysmans G, Verpoest I. Hierarchy of Textile Structures and Architecture of Fabric Geometric Models. *Textile Research Journal* 2001; 71: 534.
7. Lomov SV. Description of the form of the multifilament yarn in the fabric with arbitrary weave with spline – functions. *Technologiya Textilnoy Promyshlennosti* 1991; 6: 49-52.
8. Stepanov SG. The development of the theory of the formation and structure of the fabric on the basis of non-linear mechanics of flexible filaments. D. Sc. Thesis, Ivanovo, ISTA, 2007, p. 443.
9. Turan RB, Bager G. Three-dimensional computer simulation of 2/2 twill woven fabric by using B-splines. *Journal of The Textile Institute* 2010; 101: 10: 870-881.
10. Szablewski P. Numerical modeling of geometrical parameters of textile composites. *Fibres & Textiles in Eastern Europe* 2008; 16, 6, 71: 49-52.
11. Szablewski P. Sinusoidal Model of Fiber-reinforced Plastic Composite. *Journal of Industrial Textiles* 2009; 38, 4: 277-288.
12. Kolčavová Sirková B, Vyšanská M. Methodology for Evaluation of Fabric Geometry on the Basis of the Fabric Cross-Section. *Fibres & Textiles in Eastern Europe* 2012; 20, 5, 94: 41-47.
13. Zeman J, Šejnoha M. Homogenization of balanced plain weave composites with imperfect microstructure. Part I – Theoretical formulation. *Int. J. of Solids and Struct.* 2004: 6549–6571.
14. Lomov SV. Computer-Aided design of multi-layer woven structures”, Proceedings of Higher Institutions. *Technologiya Textilnoy Promyshlennosti* 1993; 1: 40-45.
15. Hakan Özdemir, Güngör Baser. Computer Simulation of Woven Fabric Appearances Based on Digital Video Camera Recordings of Moving Yarns. *Textile Research Journal* 2008; 78, 2: 148–157.
16. Hivet G, Boisse P. Consistent 3D geometrical model of fabric elementary cell. Application to a meshing preprocessor for 3D finite element analysis. *Finite Elements in Analysis and Design* 2005: 25-42.
17. Barbero EJ, Trovillion J, Mayugo JA, Sikkil KK. Finite element modeling of plain weave fabrics from photomicrograph measurements. *Composite Structures* 2006;73: 41-52.
18. Szablewski P. Analysis of the Stability of a Flat Textile Structure. *Autex Research Journal* 2006; 6, 4.
19. Popov EP. Theory and Design of flexible elastic rods. Publisher Nauka, Main Edition of physical and mathematical literature, USSR, 1986, p. 296.
20. Grechukhin, AP, Zaitsev DV. Improving method of forecasting the parameters of the fabric structure based on nonlinear theory of bending. *Nauchn. Vestn. Kostromskogo Gos. Tehnol. Univ.*, p. 12, 2011; <http://vestnik.kstu.edu.ru/>.

Received 07.11.2013 Reviewed 26.12.2013

## 18<sup>th</sup> International Papermaking Conference & Exhibition 'PROGRESS14' INNOVATIONS & COMPETITIVENESS

23-25 September 2014 r., Hotel Andel's, Łódź, Poland

### Topics:

- Main factors which create the development of the world's papermaking industry: globalization, protection of the environment, informative technologies, the increasing use of waste-paper
- Actual state of art and tendencies of technologies development for manufacturing the fibrous pulp, paper and board
- Raw materials and auxiliary products for the production of fibrous pulp and paper – wood and non-wood raw materials, waste-paper, fillers, pigments, paper sizes and other products
- Machines, devices and equipment
- New technologies and equipment for the manufacturing of packaging from corrugated board
- Quality of paper and paper products
- Energy problems

### Exhibition and poster session

Simultaneously with the two-day conference activity a poster session presenting research works will be held, as well as a technical exhibition of the manufacturers and suppliers, who will present machines, equipment, control and measuring devices, informative systems, as well as raw materials and auxiliary agents.

### International programme committee:

Małgorzata Michniewicz, Ph.D. Eng. (IBWCH) – **chairperson**  
Katarzyna Godlewska Ph.D. Eng. (SPP) – **secretary**  
Konrad Olejnik Ph.D. Eng. (IPiPŁ)  
Prof. Jean-François Bloch (Grenoble INP-Pagora)  
Hannes Vomhoff Ph. D. (INNVENTIA AB)  
Prof. Miloslav Milichovsky (TU Pardubice)  
Prof. Samuel Schabel (TU Darmstadt)

### Organising committee:

Andrzej Głębowski – **chairperson**  
Zbigniew Fornalski – **organising secretary**  
Katarzyna Godlewska  
Michał Jastrzębski  
Rafał Kadyrow  
Małgorzata Michniewicz  
Jolanta Tybuś  
Agnieszka Werner

### For more information please contact:

Association of Polish Papermakers

Plac Komuny Paryskiej 5a, skr. poczt. 200, 90-007 Łódź, Poland, Tel. (42) 6300117, Fax (42) 6324365, E-mail: [info@spp.pl](mailto:info@spp.pl)