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Optimisation of Pad Thicknesses in Ironing Machines During Coupled Heat and Mass Transport

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Abstract
The main goal of the paper is to determine the physical and mathematical models of coupled heat and mass transport within pads during ironing as well as optimize the pads thicknesses. Introducing the Fick's diffusion in fibers, heat and mass transport equations are formulated and accompanied by a set of boundary and initial conditions. Optimization of thickness is gradient oriented, the necessary objective functionals are analyzed. Numerical examples of optimization concerning the pads thickness are presented.

Key words: *ironing machine, coupled heat and mass transport, optimization, thickness.*

Nomenclature

- **A** matrix of heat conduction coefficients, $Wm^{-1}K^{-1}$
- **b** vector of design parameters, m
- c thermal heat capacity, $J\,kg^{-1}K^{-1}$
- c_v volumetric heat capacity of textile material, $J\,m^{-3}K^{-1}$
- **D** matrix of water vapour diffusion coefficients within fibres, m^2s^{-1}
- $div_{\Gamma}\mathbf{v}^p$ tangent divergence of vector $\mathbf{v}^p$ on external boundary Γ, -
- e water vapour pressure, Pa
- F objective functional, -
- F' Lagrange functional (the auxiliary function), -
- $g_p = Dg/Db_p$ global (material) derivative of g in respect of design parameter b_p, -
- $g^p = \partial g/\partial b_p$ partial (local) derivative of g in respect of design parameter b_p, -
- H mean curvature of the external boundary Γ, m^{-1}
- H_a relative humidity of air, -
- H_f relative humidity of fibres, -
- h surface film conductance, $Wm^{-2}K^{-1}$
- h_w mass transport coefficient of water vapour in air, $m\,s^{-1}$
- N number of functionals during the sensitivity analysis, -
- **n** unit vector normal to external boundary Γ, directed outwards to the domain Ω bounded by this boundary, -
- P number of design parameters during the sensitivity analysis, -
- p proportion between the sorption rates, -
- **q** vector of heat flux density, Wm^{-1}
- $\mathbf{q_w}$ vector of mass flux density, $kg\,m^{-2}s^{-1}$
- $q_n = \mathbf{n}\cdot\mathbf{q}$ heat flux density normal to the external boundary, Wm^{-1}
- $q_{nw} = \mathbf{n}\cdot\mathbf{q_w}$ mass flux density normal to the external boundary, $kg\,m^{-2}s^{-1}$
- R_1; R_2 first- and second-stage sorption rates, $kg\,m^{-3}\,s^{-1}$
- s_1; s_2 adjustable parameters of the sorption process, $kg\,m^{-3}\,s^{-1}$
- T temperature, K/°C
- t real time in the primary and additional structures, s
- t_{eq} time to reach quasi-equilibrium during the sorption process, s
- $\mathbf{v}^p(\mathbf{x},\mathbf{b},t)$ transformation velocity field associated with design parameter b_p, -
- $v_n^p = \mathbf{n}\cdot\mathbf{v}p$ transformation velocity normal to the external boundary Γ, -
- w_a water vapour concentration in the air filling the interfibre void space, $kg\,m^{-3}$
- w_f water vapour concentration within the fibres, $kg\,m^{-3}$
- $W_C = w_f/\rho$ fractional water content on fibre surface, -
- Γ external boundary of the structure, -
- β approximation coefficient of sorption / desorption on the boundary of fibres, -
- γ boundary integrand of the objective functional, -
- ε effective porosity of the textile material, -
- η absorption coefficient, -
- λ_w cross coefficient described as the heat sorption of water vapour by fibres, $J\,kg^{-1}$
- Σ discontinuity line between adjacent parts of piecewise smooth boundary Γ, -
- ρ density of fibres, $kg\,m^{-3}$
- τ transformed time in the adjoint structure, s
- χ Lagrange multiplier, -
- Ω domain of the structure, m^2
- ∇ gradient operator, -

Introduction

Ironing machines are used to improve the quality of clothing components as well as the process of heat treatment (i.e. compression effects, working conditions and optimisation of the time). Each iron-

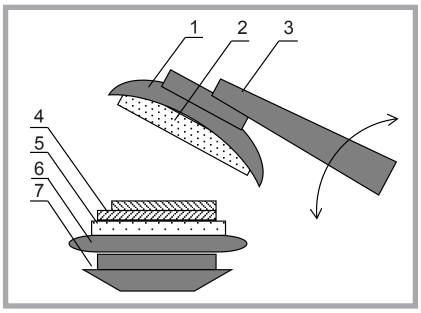

***Figure 1.** Scheme of ironing device with mobile upper and stationary lower plate: 1 – upper plate; 2 – upper pad; 3 – turning arm; 4 – ironed clothing; 5 – lower pad; 6 – lower plate; 7 – base of lower plate.*

ing machine has two plates provided with pads of different shapes, ***Figure 1***. The lower plate is fixed to the base, whereas the upper one is mobile. Ironing machines can be divided into the following: (i) universal general application – all textiles are ironed on pads of the same shape, and (ii) specialised, particular application – each part of clothing is ironed on a pad of suitable shape.

Both plates secure an adequate sequence of ironing and indispensable pressure. The base of the upper plate is usually made of aluminum, whereas the lower one is of aluminum and cast iron. The steam presses are additionally equipped with devices to drain the steam from the operations area to the surroundings. Pads are designed according to the base of the ironing machine, the operation sequence, the product line and the type of ironing (i.e. inter-operational or final ironing). Materials are characterised by: adequate elasticity, equivalent air permeability, uniform porosity, suitable temperature resistance, impurity resistance, moisture resistance, wear resistance, mechanical durability, optimal heat and moisture transport to the surroundings, maintainability and washability. Consequently the pad consists of a few different textile layers of different requirements. The universal pads are usually rectangular, while specialised pads have a shape corresponding to the clothing element. Hard pads are applied during inter-operational ironing, whereas it is soft material during the finishing procedure.

Analysis of available literature showed that publications relating to the theoretical description of heat and mass transport within elements of ironing machines (i.e. textile pads) as well as ironed textiles are so far unknown. The existing publications describe some practical aspects concerning different materials, change of material characteristics under pressure and temperature etc. The specific solutions of ironing machines are offered by particular companies [1 - 3]. Some interesting information concerning the design of ironing machines, textile pads, applied materials etc. can be found in catalogues and spare parts lists [4, 5]. There are some general works where the authors provide information on recommended ironing technology, the technological regime, time, temperature, humidity etc. [6, 7]. The pads and ironed material are subjected to heat and water vapour transport. Coupled heat, moisture and liquid water transfer is discussed by Li, Zhu [8]; Li, Zhu, Yeung [9]. Thus the moisture is supplied as water vapour during ironing, and the precise description of coupled heat and water vapour transport is analysed by Li [10]; Li, Luo [11]. The authors introduce the essential assumptions and the third equation according to David, Nordon [12]. A Mathematical model is described by second-order differential equations with a set of conditions.

The main goal of the work was to determine the physical and mathematical models of coupled heat and mass transport within pads during ironing as well as optimise pad thicknesses. Optimisation of the thickness is gradient oriented and needs the first-order sensitivities of the objective functional. The direct and adjoint approaches to sensitivity analysis were previously discussed by Dems, Mróz [13] & Korycki [14 - 16].

The gradient oriented optimisation of textiles during heat and mass transport is an extension of a previous work concerning textile engineering optimisation [14 - 16]. The novelty elements are as follows: (i) compact modelling of heat and moisture transport in ironing plates, (ii) thickness optimisation of textile pads, and (iii) application of moisture-dependent material parameters. The results obtained will be verified in the next paper using a sweating guarded hotplate, which simulates the coupled emission of heat and moisture from the skin.

Physical and mathematical model of coupled heat and water vapour transport

Let us introduce a physical model of the couped heat and mass transfer within an ironing machine equipped with two plates and two pads. Textile pads are usually made of two or three material layers of different characteristics. The components of an optimal ironing process are as follows: the prescribed moisture flux density distributed from the upper and lower plates, the draining of moisture, the specified temperature and the relaxation time. Pads are filled by water vapour because the moisturising time is relatively short and the moisture flux density is relatively insignificant. Thus volume changes in textiles caused by moisture diffusion are negligible in respect of dry material, Li [10]. The main problem also is to optimise the material thickness of the textile pad to secure the optimal technological parameters.

The most general case is 3D formulation of the problem. However, the finite element model is complicated and the calculations are time-consuming. Let us introduce the general application of the ironing machine and plates of a rectangular shape and same dimensions of the cross-section. To simplify the calculations, the spatial problem can be reduced to an optional cross-section of the pad i.e. 2D plane material thickness is analysed.

Coupled heat and water vapour transport is characteristic for textile structures, cf. Li [10], Li, Luo [11]. A part of heat is transported with moisture, whereas water vapour is transported with heat at a molecular level. Heat is transported by conduction within the textile material as well as by convection and radiation from the external surfaces to the void spaces between fibres. Moisture is transported by diffusion within fibres and spaces between the textile material.

The extile material of the pad is inhomogeneous, irrespective of the structure (i.e. woven fabric, knitted fabric, nonwovens), and should be homogenised. The textile structure consists of fibres and free spaces between them, which requires a complex two-stage homogenisation procedure. Fibres are heterogeneous as a result of the spinning process and are homogenised in respect of internal inhomogeneity during the "internal" homogenisation. The whole structure is homogenised as a composition of textile material and void spaces during the "external" homogenisation. The fibrous material is characterised by instantaneous thermal equilibrium on contact surfaces between fibres and free spaces. The material characteristics are porosity-dependent and can be determined according to [17].

The physical and mathematical state of a dynamical system subjected to coupled heat and mass transport is described by a set of state variables, which are, for instance, temperature, moisture concentration, entropy, pressure, internal energy etc. These variables should be representative for the dynamical system and depend on problem formulation. Coupled heat and water vapour transport is determined by the heat and moisture balances accompanied by the correlation of moisture diffusion within the fibre. Thus the state of the fibrous structure is described by second-order differential correlations of the temperature T and water vapour

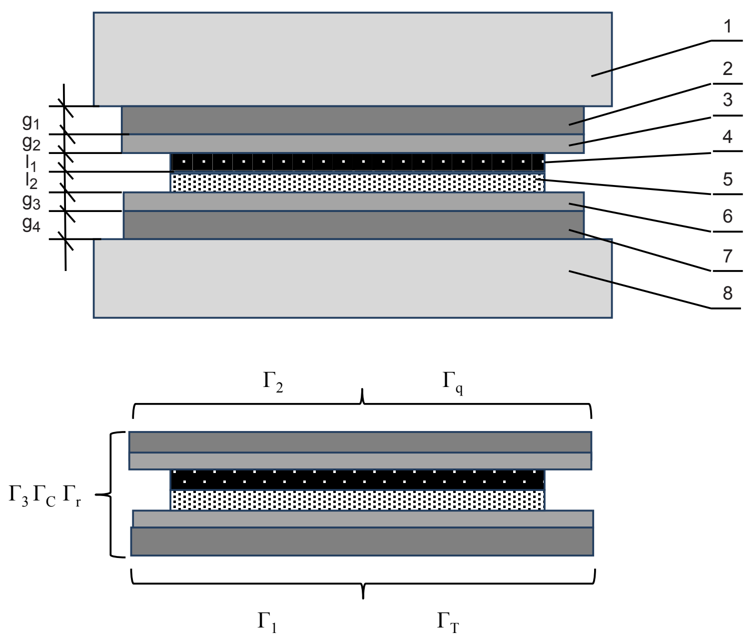

***Figure 2.** Structure of ironing plates and pads; a) structure of plates, pads and design variables, b) external boundary portions of ironing system, $g_1 – g_4$ pads thicknesses (design variables), l_1, l_2 – thicknesses of textiles subjected to ironing, 1 – upper plate, 2 – filling pad of upper plate, 3 – protective fabric of upper plate, 4, 5 – ironed textiles, 6 – protective fabric of lower plate, 7 – filling pad of upper plate, 8 – lower plate.*

concentrations w_a; w_f [14, 16], which are consistently the state variables.

Mathematically speaking, coupled heat and mass transport is determined by the following: (i) heat and mass balances, (ii) constitutive equations of the material, (iii) relations between the state variables, and (iv) physical and chemical correlations defining all phases of the material. A typical ironing machine does not contain internal heat and mass sources (i.e. the source capacities are equal to zero $f = f_w = 0$) *nor* the initial heat and mass fluxes $\boldsymbol{q}^* = \boldsymbol{q}_w{}^* = 0$. The heat and mass transport equations in the *i*-th layer have the form [14] presented in the set of ***Equations 1.***

To solve the problem, let us introduce the experimental relationship, Li [10] show as ***Equation 2***.

$$\begin{cases} \left(1-\varepsilon^{(i)}\right)\dfrac{dw_f^{(i)}}{dt}+\varepsilon^{(i)}\dfrac{dw_a^{(i)}}{dt}=-\mathrm{div}\mathbf{q}_w^{(i)}; \qquad \mathbf{q}_w^{(i)}=\mathbf{D}^{(i)}\cdot\nabla w_f^{(i)}; \\ \rho^{(i)}c^{(i)}\dfrac{dT^{(i)}}{dt}-\lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\dfrac{dw_f^{(i)}}{dt}=-\mathrm{div}\mathbf{q}^{(i)}; \qquad \mathbf{q}^{(i)}=\mathbf{A}^{(i)}\cdot\nabla T^{(i)} \end{cases} \tag{1}$$

$$\frac{dw_f}{dt}=\left(1-p\right)R_1+pR_2 \qquad \begin{array}{l} p=0 \quad \text{for } W_C<0.185 \text{ and } t<t_{eq}; \\ p=0.5 \quad \text{for } W_C\geq 0.185 \text{ and } t<t_{eq}; \\ p=1 \quad \text{for } t>t_{eq}. \end{array} \tag{2}$$

$$R_1(\mathbf{x},t)=\frac{dw_f\left(\mathbf{x},R_f,t\right)}{dt}=\frac{d\left\lfloor\rho W_C\left(H_f\right)\right\rfloor}{dt}=\frac{1}{r}\frac{d\left(rD\,dw_f\right)}{dr^2} \tag{3}$$

$$\begin{cases} \eta^{(i)}\left(1-\varepsilon^{(i)}+\dfrac{\varepsilon^{(i)}}{\beta^{(i)}}\right)\dfrac{dw_f}{dt}=-\mathrm{div}\mathbf{q}_w^{(i)}; \qquad \mathbf{q}_w^{(i)}=\mathbf{D}^{(i)}\cdot\nabla w_f^{(i)}; \\ \rho^{(i)}c^{(i)}\dfrac{dT^{(i)}}{dt}+\lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\dfrac{dw_f^{(i)}}{dt}=-\mathrm{div}\mathbf{q}^{(i)}; \qquad \mathbf{q}^{(i)}=\mathbf{A}^{(i)}\cdot\nabla T^{(i)}. \end{cases} \tag{4}$$

Equations 1, 2, 3, and 4.

The equilibrium time t_{eq} is defined experimentally for some textiles, cf. Li [10]; Haghi [17]. The ironing time equal to 15 s is considerably shorter than the equilibrium time t_{eq} = 540 s. Moreover the fractional water content is relatively low because the liquid water is technologically inconvenient. Thus the first stage of sorption is determined by the proportion $p = 0$ and Fick's diffusion in a dry textile because the water vapour diffuses into the relaxed material. The general shape of fibres can be assumed as cylindrical. By reason of instantaneous thermodynamic equilibrium on contact between fibres and void spaces within the material, the following correlation can be introduced, see ***Equation 3***.

Global description of heat and mass transport equations during the first sorption phase can be determined by ***Equation 1*** and the third equation according to David, Nordon (3). The sorption and desorption of water vapour on the fibre surface can be approximated by coefficient β, cf. Crank [18], Li [10] $w_f = \rho\beta w_a$ consequently we denote for the ***i***-th layer, cf. Korycki [14] - see the set of ***Equations 4***.

The transport equations are accompanied by the set of conditions shown in ***Figure 2***. The upper portion of the upper filling pad contacts the heating and moistening devices within the plate, which creates the prescribed value of temperature and moisture flux density. The model is characterised by the second-kind conditions of heat and water vapour transport and the adequate boundary portions are Γ_q and Γ_2. Side boundaries are open to the surroundings. Heat is lost by convection (third-kind condition, part Γ_C) and radiation (part Γ_r) and moisture by convection (third-kind condition, part Γ_3). Heat and water vapour are /transported from the lower part of the lower filling plate to the surroundings. Thus the model is subjected to the first-kind conditions for portions Γ_T and Γ_1. Temperature distribution is determined according to ***Figure 3***. The closed ironing machine secures the pressure of textiles prescribed. The model is described by the fourth-kind conditions on the common surfaces between materials Γ_i and Γ_4. The initial conditions determine the temperature and water vapour concentrations at the beginning of ironing. The particular boundaries as well the initial conditions are presented in the relationships (5).

Upper boundary:

$$q_n(\mathbf{x},t) = 0.200\cos\left(\frac{\Pi t}{150}\right);\ \mathbf{x}\in\Gamma_q;$$

$$q_w(\mathbf{x},t) = 5\cdot 10^{-0.1t}e^{-x};\ \mathbf{x}\in\Gamma_2;$$

$$t\in\langle 0;15\rangle s$$

Side boundaries:

$$q_n(\mathbf{x},t) = h\left[T(\mathbf{x},t) - T_\infty\right]\ \mathbf{x}\in\Gamma_c;$$

$$q_w(\mathbf{x},t) = h_w\left\lfloor w_f(\mathbf{x},t) - w_{f\infty}\right\rfloor\ \mathbf{x}\in\Gamma_3;$$

$$q_n^r(\mathbf{x},t) = \sigma T^4 \quad \mathbf{x}\in\Gamma_r$$

Lower boundary:

$$T(\mathbf{x},t) = \begin{cases} 100t & t\in\langle 0;1\rangle s \\ 100 + \frac{34}{15}(t-1) & t\in\langle 1;15\rangle s \end{cases} \quad \mathbf{x}\in\Gamma_T;$$

$$w_f(\mathbf{x},t) = 0.08\sin\left(t\frac{\Pi}{30}\right)\quad t\in\langle 0;15\rangle s;\ \ \mathbf{x}\in\Gamma_1;$$

Internal boundaries:

$$T^{(i)}(\mathbf{x},t) = T^{(i+1)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_i;$$

$$w_f^{(i)}(\mathbf{x},t) = w_f^{(i+1)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_4; \qquad (5)$$

Initial conditions:

$$T(\mathbf{x},0) = T_\infty \quad \mathbf{x}\in(\Omega\cup\Gamma);$$

$$w_f(\mathbf{x},0) = w_{f\infty} \quad \mathbf{x}\in(\Omega\cup\Gamma).$$

Application of first-order sensitivity approach

Optimal thicknesses of pads are determined using the sensitivity approach. The material derivative of the objective functional in respect of design parameters is defined as the first-order sensitivity $DF/Db_p = F_p$. The objective functional is generally described by variables w_f; q_w.

$$F = \int_0^{t_f}\left[\int_{\Gamma(\mathbf{b})}\gamma(w_f, q_w)d\Gamma\right]dt; \qquad (6)$$

Integrand γ is the continuous and differentiable function on the adequate boundary part.

The direct approach [14 - 16] is convenient to optimise the shapes described by a small number of design variables, for example pad thicknesses. The approach is characterised by *P* problems of additional heat and mass transfer associated with each design parameter and *1* primary i.e. *(P+1)* problems. State variables are the temperature T^p and the moisture concentrations w_f^p, w_f^p. The shape, material parameters and transport conditions are the same in primary and additional structures, but the distribution of heat and moisture fields is different [13, 14]. The direct and additional problems are defined in real time. The correlations are determined by material differentiation of the appropriate ***Equations 5***. Some components are now design parameter-independent, $\frac{DT^0}{Db_p} = 0; \mathbf{x}\in\Gamma_T$; $\frac{Dw_f^0}{Db_p} = 0; \mathbf{x}\in\Gamma_1$;

$\frac{Dq_n^0}{Db_p} = 0; \mathbf{x}\in\Gamma_q$; $\frac{Dq_w^0}{Db_p} = 0; \mathbf{x}\in\Gamma_2$;

$\frac{Dw_{f0}}{Db_p} = 0; \mathbf{x}\in(\Omega\cup\Gamma)$ [14]. The final correlations are the presented in ***Equation 7***.

The first-order sensitivity correlation can be determined as presented in ***Equation 8*** [14].

The alternative adjoint approach introduces the set of adjoint problems associated with each objective functional i.e. when defining *N* objective functionals, we must solve *N* adjoint problems and

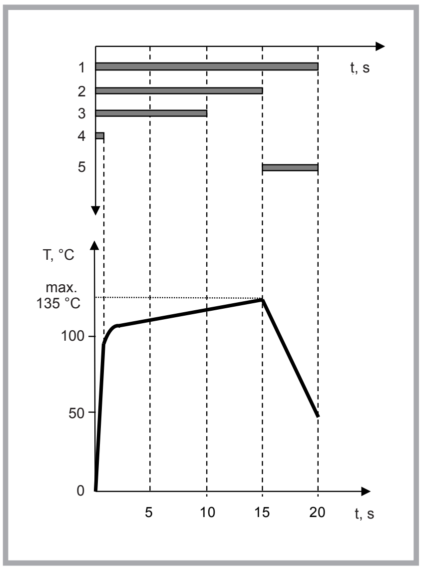

Figure 3. *Example of phases of ironing cycle; T – temperature on the lower surface of lower pad; t – time, 1 – total working time; 2 – closure of plates; 3 – moisture from the upper plate; 4 – moisture from the lower plate; 5 – partially open upper plate, draining of moisture.*

$$\begin{cases} \eta^{(i)}\left(1-\varepsilon^{(i)}+\frac{\varepsilon^{(i)}}{\beta^{(i)}}\right)\frac{dw_f^p}{dt} = -\mathrm{div}\mathbf{q}_w^{(i)p}; & \mathbf{q}_w^{(i)p} = \mathbf{D}^{(i)}\cdot\nabla w_f^{(i)p}; \\ \rho^{(i)}c^{(i)}\frac{dT^{(i)}}{dt} + \lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\frac{dw_f^{(i)p}}{dt} = -\mathrm{div}\mathbf{q}^{(i)p}; & \mathbf{q}^{(i)p} = \mathbf{A}^{(i)}\cdot\nabla T^{(i)p}. \end{cases}$$

$$T^p(\mathbf{x},t) = -\nabla T^0\cdot\mathbf{v}^p \quad \mathbf{x}\in\Gamma_T; \quad w_f^p(\mathbf{x},t) = -\nabla w_f^0\cdot\mathbf{v}^p \quad \mathbf{x}\in\Gamma_1;$$

$$q_n^p(\mathbf{x},t) = h\left(T^p - T_\infty^p\right) + \mathbf{q}_\Gamma\cdot\nabla_\Gamma v_n^p \quad \mathbf{x}\in\Gamma_c;$$

$$q_w^p(\mathbf{x},t) = h_w\left(w_f^p - w_{f\infty}^p\right) + q_{w\Gamma}\cdot\nabla_\Gamma v_n^p \quad \mathbf{x}\in\Gamma_3; \qquad \mathbf{(7)}$$

$$q_n^{rp} = 4\sigma T^3 T^p \quad \mathbf{x}\in\Gamma_r;$$

$$q_n^p(\mathbf{x},t) = \mathbf{q}_\Gamma^0\cdot\nabla_\Gamma v_n^p - \nabla_\Gamma q_n^0\cdot\mathbf{v}_\Gamma^p - q_{n,n}^0 v_n^p \quad \mathbf{x}\in\Gamma_q;$$

$$q_w^p(\mathbf{x},t) = \mathbf{q}_{w\Gamma}^0\cdot\nabla_\Gamma v_n^p - \nabla_\Gamma q_w^0\cdot\mathbf{v}_\Gamma^p - q_{w,n}^0 v_n^p \quad \mathbf{x}\in\Gamma_2;$$

$$T^{p(i)}(\mathbf{x},t) = T^{p(i+1)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_i; \quad w_f^{p(i)}(\mathbf{x},t) = w_f^{p(i+1)}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_4;$$

$$T_0^p(\mathbf{x},0) = -\nabla T_0\cdot v^p \quad \mathbf{x}\in(\Omega\cup\Gamma); \quad w_{f0}^p(\mathbf{x},0) = -\nabla w_{f0}\cdot v^p \quad \mathbf{x}\in(\Omega\cup\Gamma).$$

$$F_P = \int_0^{t_f}\left\{\int_{\Gamma_1}\left[-\gamma_{,w_f}\left(\nabla_\Gamma w_f^0\cdot\mathbf{v}_\Gamma^p + w_{f,n}^0 v_n^p\right) + \gamma_{,q_w}\left(q_w^p - \mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right)\right]d\Gamma_1 + \right.$$
$$+\int_{\Gamma_2}\left\{\gamma_{,w_f} w_f^p - \gamma_{,q_w}\left(\nabla_\Gamma q_w^0\cdot\mathbf{v}_\Gamma^p + q_{w,n}^0\cdot v_n^p\right)\right\}d\Gamma_2 +$$
$$+\int_{\Gamma_3}\left[\gamma_{,w_f} w_f^p + \gamma_{,q_w} h_w\left(w_f^p - w_{f\infty}^p\right)\right]d\Gamma_3 + \qquad \mathbf{(8)}$$
$$+\int_\Gamma\left(\gamma_{,n} - 2H\gamma\right)v_n^p\, d\Gamma +$$
$$\left. +\int_\Sigma\left]\gamma\mathbf{v}^p\cdot\boldsymbol{\upsilon}\right[\right\}dt \qquad p = 1, 2...P.$$

$T^{p(i)}($

Equations 7 & 8.

$$
\begin{cases}
\eta^{(i)}\left(1-\varepsilon^{(i)}+\dfrac{\varepsilon^{(i)}}{\beta^{(i)}}\right)\dfrac{dw_f^a}{dt}=-\mathrm{div}\mathbf{q}_w^{(i)a}; \quad \mathbf{q}_w^{(i)a}=\mathbf{D}^{(i)}\cdot\nabla w_f^{(i)a}; \\
\rho^{(i)}c^{(i)}\dfrac{dT^{(i)}}{dt}+\lambda_w^{(i)}\left(1-\varepsilon^{(i)}\right)\dfrac{dw_f^{(i)a}}{dt}=-\mathrm{div}\mathbf{q}^{(i)a}; \quad \mathbf{q}^{(i)a}=\mathbf{A}^{(i)}\cdot\nabla T^{(i)a}.
\end{cases}
$$

$$
\begin{aligned}
& w_f^a(\mathbf{x},\tau=0)=0 \quad \mathbf{x}\in(\Omega\cup\Gamma); \qquad q_w^{*a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Omega; \\
& w_f^{0a}(\mathbf{x},\tau)=\gamma,_{q_w}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_1; \qquad q_w^{0a}(\mathbf{x},\tau)=-\gamma,_{w_f}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_2; \\
& w_{f\infty}^a(\mathbf{x},\tau)=\frac{1}{h_w}\gamma,_{w_f}(\mathbf{x},t)+\gamma,_{q_w}(\mathbf{x},t) \quad \mathbf{x}\in\Gamma_3. \\
& T^a(\mathbf{x},\tau=0)=0 \quad \mathbf{x}\in(\Omega\cup\Gamma); \\
& f^a(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Omega \qquad T^{0a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_T \qquad q_n^{ar}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_r; \\
& q^{*a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Omega; \quad q_n^{0a}(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_q; \quad T_\infty^a(\mathbf{x},\tau)=0 \quad \mathbf{x}\in\Gamma_c;
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
F_p=-\int_0^{t_f}\Bigg\{ & \int_{\Gamma_1}\left[\left(\gamma,_{w_f}+q_{nw}^a\right)\left(\nabla_\Gamma w_f^0\cdot\mathbf{v}_\Gamma^p+w_f^0,_n v_n^p\right)+\gamma,_{q_w}\mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right]d\Gamma_1+ \\
& -\int_{\Gamma_2}\left[\left(\gamma,_{q_w}-w_f^a\right)\left(\nabla_\Gamma q_w^0\cdot\mathbf{v}_\Gamma^p-q_w^0,_n v_n^p\right)-w_f^a\mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right]d\Gamma_2+ \\
& +\int_{\Gamma_3}\left[\left(w_f^a-\gamma,_{q_w}\right)h_w w_{f\infty}^p-w_f^a\mathbf{q}_{w\Gamma}\cdot\nabla_\Gamma v_n^p\right]d\Gamma_3+\int_\Gamma\left(\gamma,_n-2H\gamma\right)v_n^p\,d\Gamma+ \\
& +\int_\Gamma \gamma,_{w_{f\infty}}w_{f\infty}^p\,d\Gamma+\int_\Sigma \left]\gamma\mathbf{v}^p\cdot\boldsymbol{\upsilon}\right[\Bigg\}dt
\end{aligned} \tag{10}
$$

Equations 9, 10 &13.

the *1* primary problem. The smaller the number of functionals, the greater the advantage of the method. Adjoint problems have the same geometry, material parameters and transport conditions as the primary one, although some fields of state variables are different. The state variables are the temperature T^a and e water vapour concentrations w_f^a, w_a^a in the adjoint structure. Let us introduce the transport equations and boundary conditions of the form analogous to the primary structure, cf. [14], see set of relations (9).

Typical of the adjoint structure is time reversal between the time τ of the adjoint approach and the time t of the primary approach. A bit of trouble makes the information storage of time because the final time $t=t_f$ is predefined as the starting time $\tau=0$ according to the rule $\tau = t_f - t$.

The first-order sensitivity expression has the form in presented in ***Equation 10*** respect of [14].

Problem of optimal design

The problem of optimal design can be determined as a minimisation of the objective functional F accompanied by the constant temperature or heat flux density on the external boundary (see ***Equation 11***).

$$
\begin{cases}
F\rightarrow\min; \\
\displaystyle\int_0^{t_f}\left(\int_{\Gamma_{ext}}Td\Gamma_{ext}\right)dt-T^0=0.
\end{cases}
\qquad
\begin{cases}
F\rightarrow\min; \\
\displaystyle\int_0^{t_f}\left(\int_{\Gamma_{ext}}q_n d\Gamma_{ext}\right)dt-q_n^0=0.
\end{cases}
\tag{11}
$$

The equations are rearranged using the Lagrangian functional and its stationarity conditions [14], which allows to formulate adequate optimality conditions.

The variational approach to sensitivity analysis necessitates the unique physical application of the objective functional. Therefore, clear physical definitions of the optimisation problems should be classified. The important technological criterion is correct moisture content within the pads during ironing. Hence the moisture flux density should be minimised on the external boundary portion of pads during the technological process, which is described as presented by ***Equation 12***.

$$
F=\int_0^{t_f}\left[\int_{\Gamma_{ext}}q_w d\Gamma_{ext}\right]dt\rightarrow\min. \tag{12}
$$

The correct draining of moisture requires the equalizsed distribution of water vapour on the external boundary. The functional is the global measure of water vapour concentration within fibres w_f on the external surface of the pads. Numerically speaking, the optimisation criterion provides the minimal global measure which minimizes the local maxima of state variables. Locations of the maxima can change because the maximal values of concentrations are time-dependent. On the other hand, the locations are of low importance during optimization of constant pad thickness (see ***Equation 13***).

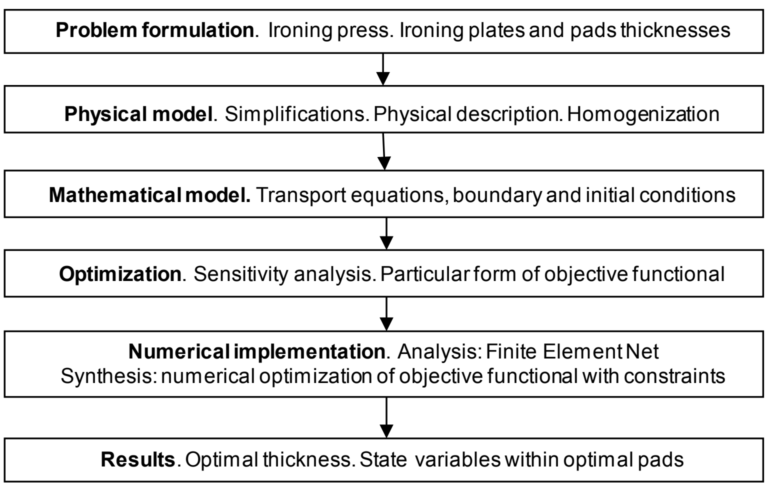

Figure 4. *Algorithm of thickness optimization of textile pads within ironing press.*

An algorithm of thickness optimisation of textile pads within an ironing machine is shown in ***Figure 4***.

Thickness optimisation of textile pads in an ironing machine

The main goal of the paper presented is to optimise pad thicknesses. Let us assume an ironing machine of general application provided with rectangular pads. Optimisation can be reduced to an optional cross-section of the structure i.e. the plane problem is analysed. A fundamental part of the technological process is closure of the ironing machine and moistening of textiles by both the upper and lower plates, cf. ***Figure 3***. Optimal parameters of the pads are determined during this part of the ironing procedure. Water vapour is transported from the upper and lower plates during the time sequence prescribed. The structure of ironing plates, boundary portions and design variables are shown in ***Figure 2***, ***Figure 3***. Let us introduce two-layer pads made of different materials. The fillings are made of industrial felt FTK [19], which is polyester fabric of surface mass 2160×10^{-3} kg/m^2. The material is inexpensive, relatively durable, thermal and its moisture characteristics are stable, although difficult to dry. External protection is woven fabric of high mechanical resistance and air permeability, of surface mass 228×10^{-3} kg/m^2. Design variables are the thicknesses $g_1 - g_4$. The material parameters depend on the fractional water content on the fibre surface i.e. water vapour concentration in fibres and the material density. An orthotropic matrix of the diffusion coefficients of water vapour in fibres can be determined in respect of [17]. Let us assume that the textile material subjected to ironing is cotton. Thus we can denote for the fibrous material of the *i-th* layer ($i = 1$ is protective woollen fabric, $i = 2$ polyester felt, $i = 3$ cotton), see ***Equation 14***.

Both textiles are homogenised using the most efficient and simple 'rule of mixture' [20]. The diffusion coefficient of water vapour in air is equal to D_a = 2.5e^{-5}. The material porosity and sorption coefficient of water vapour in fibres are introduced as constant in the form of ***Equation 15***.

Heat conduction coefficients in the orthotropic material are defined as ***Equation 16*** [17].

$$F = \int_0^{t_f} (F_w)^{\frac{1}{n}}\, dt = \int_0^{t_f} \left\{ \left[\int_{\Gamma_{ext}} \left(\frac{w_f}{w_{f0}} \right)^n d\Gamma_{ext} \right]^{\frac{1}{n}} \right\} dt \;\rightarrow \min\;;\; n \rightarrow \infty. \quad (13)$$

$$\mathbf{D}^{(i)} = \begin{vmatrix} D_{11}^{(i)} & 0 & 0 \\ 0 & D_{22}^{(i)} & 0 \\ 0 & 0 & D_{33}^{(i)} \end{vmatrix};$$

$$\begin{aligned}
&\text{wool}: D_{11}^{(1)} = D_{22}^{(1)} = \left(1.04 - 68.20W_C - 1342.59W_C^2\right)\cdot 10^{-14}, \\
&\quad D_{33}^{(1)} = \left(0.98 - 66.80W_C - 1380.59W_C^2\right)\cdot 10^{-14}, \\
&\text{polyester felt}: D_{11}^{(2)} = D_{22}^{(2)} = \left(1.12 - 410W_C - 8200W_C^2\right)\cdot 10^{-13}; \\
&\quad D_{33}^{(2)} = \left(1.09 - 406W_C - 8250W_C^2\right)\cdot 10^{-13}. \\
&\text{cotton}: D_{11}^{(3)} = D_{22}^{(3)} = \left(0.8481 + 50.6W_C - 1100W_C^2\right)\cdot 10^{-14}; \\
&\quad D_{33}^{(3)} = \left(0.8081 + 49.0W_C - 1150W_C^2\right)\cdot 10^{-14}.
\end{aligned} \quad (14)$$

$$\begin{aligned}
&\text{wool}: \varepsilon^{(1)} = 0.850;\; \eta^{(1)} = 0.650; \\
&\text{polyester felt}: \varepsilon^{(2)} = 0.950;\; \eta^{(2)} = 0.310; \\
&\text{cotton}: \varepsilon^{(3)} = 0.750;\; \eta^{(3)} = 0.750
\end{aligned} \quad (15)$$

$$\mathbf{A}^{(i)} = \begin{vmatrix} \lambda_{11}^{(i)} & 0 & 0 \\ 0 & \lambda_{22}^{(i)} & 0 \\ 0 & 0 & \lambda_{33}^{(i)} \end{vmatrix};$$

$$\begin{aligned}
&\text{wool}: \lambda_{11}^{(1)} = \lambda_{22}^{(1)} = \left(38.49 - 0.720W_C + 0.113W_C^2 - 0.002W_C^3\right)\cdot 10^{-3}; \\
&\quad \lambda_{33}^{(1)} = \left(36.49 - 0.700W_C + 0.109W_C^2 - 0.002W_C^3\right)\cdot 10^{-3}; \\
&\text{polyester felt}: \lambda_{11}^{(2)} = \lambda_{22}^{(2)} = 28.8\cdot 10^{-3};\quad \lambda_{33}^{(2)} = 33.0\cdot 10^{-3}. \\
&\text{cotton}: \lambda_{11}^{(3)} = \lambda_{22}^{(3)} = \left(44.1 + 63.0W_C\right)\cdot 10^{-3};\; \lambda_{33}^{(3)} = \left(40.1 + 62.0W_C\right)\cdot 10^{-3}.
\end{aligned} \quad (16)$$

$$\begin{aligned}
&\text{wool}: \lambda_w^{(1)} = 1602.5\exp\left(-11.72W_C\right) + 2522.0; \quad c^{(1)} = 373.3 + 4661.0W_C + 4.221T; \\
&\text{polyester felt}: \lambda_w^{(2)} = 2522; \quad c^{(2)} = 1610.9; \\
&\text{cotton}: \lambda_w^{(3)} = 1030.9\exp\left(-22.39W_C\right) + 2522.0; \quad c^{(3)} = \frac{1663.0 + 4184.0W_C}{\left(1 + W_C\right)1610.9}.
\end{aligned} \quad (17)$$

Equations 13, 14, 15, 16 & 17.

The cross-transport coefficient λ_w and heat capacity c are assumed according to ***Equation 17*** [17].

Heat and mass transport correlations for the primary problem are determined by ***Equations 4***. Let us optimise the material thickness in respect of the minimal moisture flux density on the lower boundary portions in the lower plate $\Gamma_I \cup \Gamma_T$, cf. ***Equation 12***. The continuous constraint is the time-dependent temperature. The problem is solved by means of direct and adjoint approaches to sensitivity analysis. The primary problem is defined using ***Equations 4*** & ***Equations 5***, and the direct approach by ***Equations 7***. Time changes from the initial t_0 = 0 to final t_k = 15 s and the discrete increase is assumed as Δt = 1 s. Convection is characterised by h = 8 W/(m^{2}K) and the surrounding conditions by T_∞ = 20 °C, $w_{f\infty}$ = 0.06 kg/m^3. Introducing the linear external boundary i.e. the mean curvature $H = 0$, the optimisation problem and final form of the sensitivity expression are described as follows, compare with ***Equation 8***, ***Equations 11*** & ***Equation 12*** show in ***Equation 18*** (see page 134).

The adjoint approach is defined by ***Equations 9*** and sensitivity expression by

$$\begin{cases} F = \int_0^{t_f} \left[\int_{\Gamma_{ext}} q_w d\Gamma_{ext} \right] dt \to \min; \\ \int_0^{t_f} \left(\int_{\Gamma_{ext}} T d\Gamma_{ext} \right) dt - T^0(\mathbf{x}, t) = 0; \quad \Gamma_{ext} = \Gamma_1 \cup \Gamma_T. \end{cases} \qquad F_P = \int_0^{t_f} \left\{ \int_{\Gamma_3} \left[h_w \left(w_f^p - w_{f\infty}^p \right) + q_{w},_n v_n^p \right] d\Gamma_3 + \int_{\Sigma} \left] q_w \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[\right\} dt; \quad p = 1, ..., 4 \tag{18}$$

$$F_p = \left[\int_{\Omega} \eta \left(1 - \varepsilon + \frac{\varepsilon}{\beta\rho} \right) w_f^a \left[\nabla w_f \cdot \mathbf{v}^p \right] d\Omega \right]_{t=0} + \\ + \int_0^{t_f} \left\{ - \int_{\Gamma_{ext}} \left[q_w^a \left(\nabla_{\Gamma} w_f^0 \cdot \mathbf{v}_{\Gamma}^p + w_f^0,_n v_n^p \right) - \mathbf{q}_{w\Gamma} \cdot \nabla_{\Gamma} v_n^p \right] d\Gamma_{ext} + \int_{\Gamma_{ext}} q_{w,n} v_n^p \, d\Gamma_{ext} + \int_{\Sigma} \left] q_w \mathbf{v}^p \cdot \boldsymbol{\upsilon} \right[\right\} dt. \tag{19}$$

$$\begin{cases} F = \int_0^{t_f} (F_w)^{\frac{1}{n}} \, dt = \int_0^{t_f} \left\{ \left[\int_{\Gamma_{ext}} \left(\frac{w_f}{w_{f0}} \right)^n d\Gamma_{ext} \right]^{\frac{1}{n}} \right\} dt \to \min; \quad n = 25; \\ \int_0^{t_f} \left[q_n(\mathbf{x}, t) d\Gamma_{ext} \right] dt - 0.200 \cos\left(t \frac{\Pi}{150} \right) = 0; \quad \Gamma_{ext} = \Gamma_2 \cup \Gamma_q. \end{cases} \tag{20}$$

$$\frac{DF}{Db_p} = F_p = \int_0^{t_f} \left(\frac{1}{n} (F_w)^{\frac{1-n}{n}} \frac{D(F_w)}{Db_p} \right) dt \\ (F_w)_p = \int_0^{t_f} \left\{ \int_{\Gamma_{ext}} \left[\frac{n}{w_{f0}} \left(\frac{w_f}{w_{f0}} \right)^{n-1} w_f^p + \left(\frac{w_f}{w_{f0}} \right)^n ,_n v_n^p \right] d\Gamma_{ext} + \int_{\Sigma} \left] \left(\frac{w_f}{w_{f0}} \right)^n \mathbf{v}^{\mathrm{p}} \cdot \boldsymbol{\upsilon} \right[d\Sigma \right\} dt; \quad p = 1, ..., 4 \tag{21}$$

Equations 18, 19, 20 & 21.

Equation 10, having the form of ***Equation 19***.

Each step of the iterative procedure consists of a synthesis and analysis stage. The thickness at the analysis stage is approximated during the calculations of heat and moisture transport by the same finite elements of the serendipity family according to Zienkiewicz [21]. Let us introduce the plane rectangular 4-nodal elements of the nodes in the corners. All pads are approximated by 600 elements of 1600 nodes. State fields for the primary set of the additional and adjoint problems are calculated by solution of the basic finite element equation. The directional minimum at the synthesis stage is calculated by means of the second-order Newton procedure and alternatively y the first-order method of steepest descent. The initial and optimal thicknesses of the pads are listed in ***Table 1***, the thicknesses of ironed cotton layers are constant $l_1 = l_2 = 0.03$ m. The minimal objective functional is obtained in 13 steps and reduced to 15.71% in relation to the initial value. Each optimal layer made of polyester felt is thicker than the initial (39.8% and 53.7%), whereas the optimal protective woolen fabrics are thinner than the initial (17% and 22%). The sum of the optimal is 33.5% greater than that of the initial thicknesses.

Table 1. *Initial and optimal thicknesses of pads during minimization of moisture flux density.*

Design variable $\cdot 10^{-2}$m	g_1	g_2	l_1	l_2	g_3	g_4
Initial value	4.00	1.00	3.00	3.00	1.00	4.00
Optimal value	5.59	0.83	3.00	3.00	0.78	6.15

Table 2. *Initial and optimal thicknesses of pads during equalized distribution of moisture.*

Design variable $\cdot 10^{-2}$m	g_1	g_2	l_1	l_2	g_3	g_4
Initial value	4.00	1.00	3.00	3.00	1.00	4.00
Optimal value	4.50	0.93	3.00	3.00	0.95	4.28

The other technological criterion is the equalised distribution of moisture on the upper surface of the upper pad. The optimisation functional is a global measure of local moisture concentration in fibres w_f on the surface $\Gamma_2 \cup \Gamma_q$. The continuous constraint is the time-dependent heat flux density. The problem is solved using the direct approach, cf. ***Equations 11***, ***Equation 13*** see ***Equation 20***.

The primary problem is defined using ***Equations 4*** & ***Equations 5***, and the direct approach by ***Equations 7***. The sensitivity expression of the direct approach can be simplified by ***Equation 8*** to the form presented in ***Equation 21***.

The cross-section of the structure is approximated at the analysis stage using the same finite element net for the primary and the set of additional problems. The directional minimum at the synthesis stage is calculated using the Newton procedure and alternatively the method of steepest descent. The initial and optimal values are listed in ***Table 2***. The minimal objective functional is obtained in 11 steps and reduced of 10.11% in relation to the initial value. It follows that the optimal polyester layers are again thicker and the woolen one thinner; however, the optimal thicknesses are comparable to the initial values. The maximum difference in dimensions is 12.5%.

Conclusions

The optimisation of pad thicknesses in an ironing machine subjected to coupled heat and mass transport is a multidisciplinary engineering problem. It is necessary to introduce the physics of heat and mass transfer, description by means of second-order differential equations with boundary and initial conditions as well as determine an optimal solution. In order to describe the mathematical model, we have to analyse the heat and mass balances, formulate transport equations accompanied by the set of conditions as well as introduce a physical description of moisture diffusion in fibres according to Fick's Law. The objective functionals applied help to determine technological improvements i.e. minimal moisture flux density during ironing and the equalised distribution of moisture. It follows that thickness optimization helps to solve the number of practical problems concerning the ironing. In conclusion, the problem seems to be a promising tool for generating the optimal thickness of pads during the ironing process. Numerical optimisation is cheaper and more universal than complex analysis of finished prototypes, and provides practical benefits.

The optimization functionals applied minimize the moisture flux density as well as the global measure of moisture concentration, whereas constraints are the temperature prescribed or heat flux density. The alternative integrands can describe some phenomena of the heat transfer, whereas the constraints can determine the moisture concentrations prescribed or moisture flux density. Generally speaking, the optimal solutions can be limited to permissible increases in dimensions, minimal material thicknesses, minimal parameters of heat and mass transport etc. The optimisation procedure can be also extended to the design process of plates, draining channels of optimal dimensions, the transport of media within heat and moisture channels etc. The consecutive novelty elements are the newly introduced physical problems as well as optimisation techniques implemented to develop the analysis.

The paper presented is a theoretical optimisation of pad thicknesses by means of computer analysis. The other approach is practical analysis, where the necessary parameters are temperature, pressing time as well as the minimal time of the operation cycle together with the minimal energy necessary to realise the product. A significant factor is also the repetition time. On the other hand, nomograms can be applied to determine the operational parameters required.

The theoretical results of optimal pad thicknesses obtained should be compared with corresponding tests, which is beyond the scope of the paper presented. The initial results allow to conclude that practical verification should be discussed in the consecutive paper.

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