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CONTENTS

PART I: THEORY (TEORIA)

PART II: GRAPHICS EDUCATION (DYDAKTYKA)

PART III: APPLICATIONS (ZASTOSOWANIA)

PART IV: HISTORY OF DESCRIPTIVE GEOMETRY (HISTORIA GEOMETRII WYKREŚLNEJ)

PART V: INFORMATION AND NEWS (WYDARZENIA I INFORMACJE)

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IMAGE AND SPATIAL MEANING OF THE OCTAGON IN ARCHITECTURE

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Abstract. In this article, we investigated on the interest shown by architecture in a regular polygon: the octagon, which represents a mediation between the square and the circle. In order to investigate the image and the spatial meaning of the regular octagon in architecture, we compared monuments from different historical periods as Ancient Roman buildings, Christian baptisteries, Islamic architectures and decorations, Renaissance central plans and Baroque fountains.

Keywords: geometry, architecture, decoration, octagon

1 Introduction

The interest shown by architecture for the regular octagon is probably linked to its multiple geometric and symbolic meanings. A recent study connects the Byzantine octagon domes of the 11th century with the Roman construction tradition [1]. Another study deal with projects for 19th century houses in America [2], with insights related to the constructive and distributive significance of the adoption of the octagonal plan.

This study highlights some episodes throughout the history of architecture that are linked by the use of the octagonal shape in two different ways: one which assimilates it to the circular shape, with its properties related to centrality, and the other that generates it through the rotation of two squares, which constitutes a development of the form of the so-called Greek cross. To examine the subject, we compared monuments from different historical periods and sites, and they were redrawn in order to investigate the subtended geometric rules.

The regular octagon is the geometrical and symbolic mediation between the circle (sky) and the square (earth). In fact, it could be defined as an approximation to a circumference, interpreted as a regular polygon with an infinite number of sides, and it could be generated by the 45° rotation of two squares. Geometrical characteristics are matched by numeric properties: eight is the first perfect cube after one, and takes on important symbolic meanings for many cultures. The group of eight primigenial gods of Ancient Egypt, as we shall see, is echoed by the numerology of monotheistic religions, which finds physical manifestation in the architecture dedicated to them.

2 The centrality of the octagon

The octagon is the sum of the four cardinal directions plus four intermediate ones: this is why it was adopted in the plan of the Tower of Winds (Athens, Greece, around 300 years before Christ), designed as an instrument for evaluation of winds and positions of the Sun. The building, transformed over the centuries and still partially existing, was an articulated instrument for measuring the wind (with a statue of Triton with a rotating rod) and time (with sundials and a water clock that made it independent from weather conditions). Surveying these two elements of nature was its first function and recent studies [2] relate it to some residential structures built in America by Orson Squire Fowler (1809-1887) in the 19th century and the more recent Pego Guesthouse (Sintra, Portugal) nicknamed *Mickey Mouse House*, built by Álvaro Siza and António Madureira (2007). Reference can also be made to Jeremy Bentham's *Panoptikon*, conceived in the 18th century as a central building useful for effective control. The basic form was circular, but there were also polygonal variants, which retained the preeminent centrality of the structure.

Among the many examples of octagonal layouts, one of the most important in ancient Roman architecture is the octagonal room in the Domus Aurea of Nero (65-68 AD), built after the great fire and then covered by the Trajan's Baths (104-109 AD). The great residence was spread out over several buildings located on the Palatine, Caelian and Oppian hills. The last of these hosts the pavilion, designed by the architects Severus and Celer, with about one hundred and fifty rooms, including the round room, used as *triclinium.* In the secondary wing of the building, the same function was provided by the octagonal hall [3, p. 253], covered by a cement cloister vault (diameter about 14 meters), which approximated a semi-spherical dome near the central eye (Fig. 1a). The hall, perhaps designed by Leonidas of Alexandria, presented a mechanism which made the ceiling revolve, representing the mutability of the Universe (Svetonius, Nero, XXXI, 2) [3, p. 260]. The building was inspired by the myth of Apollo and the Sun, evidenced by the colossal statue of *Nero Sol* (about 35 meters high) and by a profusion of gold and the use of radiant Figures [3, p. 246] like the octagon. The layout and materials of the vault of Nero's octagonal hall also reminds us of the Pantheon (120-125 AD), which represents the mature expression of the use of cement to build circular domes.

From then on, we find an increasing presence of central plans of Oriental inspiration, and of octagonal structures as well, for example the hall and fountain of the peristyle of the Domus Flavia (81-98 AD), built for Emperor Domitian on the Palatine by the architect Rabirius.

The octagonal shape often recurs in the plans of Christian baptisteries, the prototype of which was probably the Lateran Baptistery in Rome (early 4th century), which became the model for similar coeval buildings in the West (San Giovanni alle Fonti in Milan, Baptistery of Neon in Ravenna) and in the East (in Ephesus and Constantinople). In the following centuries, the octagonal plan spread further, as can be seen from many Italian and French examples, including the Baptistery of San Giovanni in Florence (Fig. 1b). Its shape, however, probably harks back to a church of the 7th century, built over Roman structures and which became a baptistery in the 12th century [4] with reference on established cases of baptisteries built on Roman thermal structures. The floor also reflects the geometry of the octagon and the sacred cut [5].

While not all baptisteries are octagonal, it is true that they usually have a central plan, which can also be circular, often being confused, in descriptions, with an octagonal plan [4, pp. 92-93]. We also find some octagonal religious buildings with non-baptismal functions, which confirm the great importance taken on by the number eight in Christianity, associated not only with the purifying rite of Baptism, but also as a symbol of the New Testament and hence of Christ's redeeming function. This importance is shared by Judaism, where the eighth day marks a new beginning after the creation, and by Islam.

There are Byzantine examples of octagonal churches, such as San Vitale (Ravenna, 530-547) with an octagonal core surrounded by an octagonal ambulatory (Fig. 1c). The diffusion of octagonal domes relates to the tradition of places of worship with a central plan

[6] and to Roman imperial architecture, in particular with the Nero's octagonal hall and the tradition of the mausoleums between the 3rd and 5th centuries [1, p. 279].

The Byzantine tradition finds an echo in Orthodox churches and in Islamic religious buildings, as in the Dome of the Rock (Jerusalem, 687-691), a shrine built by Byzantine masters which has an octagonal perimeter with a dome in the center (Fig. 1d). The same shape was used for mosques and mausoleums, for example the octagonal mausoleum of Shah-i-Zinda (Samarkand, 15th century).

Other octagonal forms associated with water – apart from baptisteries – are provided by many examples of basins and fountains. The tradition started in antiquity (e.g. the aforementioned fountain of Domus Flavia or the *frigidarium* in the Baths of Villa del Casale, in Villa Armerina, Sicily, 4th century), but it was also frequent in the Middle Ages and in modern times, showing a continuous relationship among water, the origin of life and the meaning of purification [4, p. 20]. Among countless examples, we may remember Giacomo Della Porta’s fountain, formerly in Piazza del Popolo (1572), later moved to within the ellipse of Saint Peter’s Colonnade designed by Gian Lorenzo Bernini (started in 1656).

Another renowned architectural octagon stands in front of the Baptistery of San Giovanni in Florence: the dome of the Church of Santa Maria del Fiore, designed by Filippo Brunelleschi in 1418. Brunelleschi himself chose the Baptistery of San Giovanni to illustrate his method of perspective [7], probably precisely because of its octagonal shape, that provides symmetry features which can easily be matched with the rigors of the geometrical linear perspective.

Brunelleschi also designed an octagonal plan for the unfinished Rotonda di Santa Maria degli Angeli, work on which began in 1436 but was cut off in 1437 (Fig. 1e), to be recommenced in later years. Brunelleschi’s plan, as reproduced by Giuliano da Sangallo (Vatican Library, Cod. Lat. Barb. 4424, c. 15v.), is also found in drawings by Filarete, Leon Battista Alberti, Donato Bramante [8] and Leonardo da Vinci (Institut de France, Ms. B, c. 95r). In Leonardo’s work, the model of the octagonal plan with chapels radiating out finds a vast echo (Fig. 1f), as can be seen from various drawings of churches with a central plan, which also feature an original technique of representation [9]. The significant presence of octagon plans is also inspired by the cited Florentine examples, in which Leonardo found “a geometric expression of grandeur and practicality in spatial organization, design, and development.” [10].

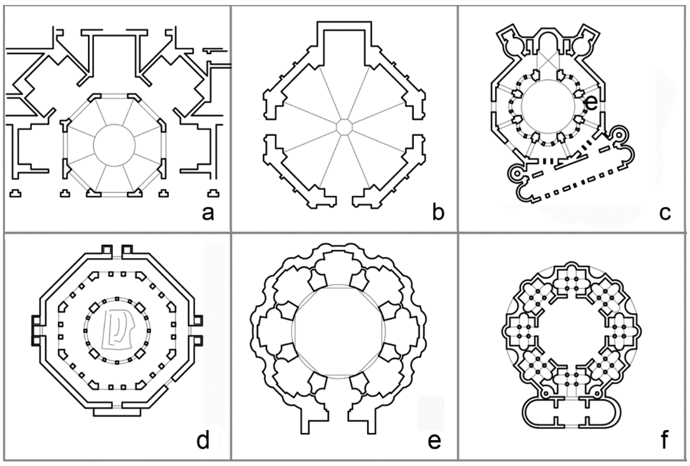

Figure 1: Octagonal plans (schemes by the author): a) Domus Aurea, Rome; b) San Giovanni, Florence; c) San Vitale, Ravenna; d) Dome of the Rock, Jerusalem; e) Santa Maria degli Angeli, Florence; f) Central plan church by Leonardo da Vinci

Circle and octagon, sometimes confused with one another, are often used for the same typologies and they are able to generate spaces characterized by the indistinctness of the different parts that compose them, in order to generate continuity and fluidity.

3 Octagonal star and combination of squares

Islamic architecture is inseparable from the history of its decoration, with its frequent recourse to ceramic tiles with geometrical motifs, the themes of which were developed through the centuries with the use of circles and regular polygons, repeated and superimposed (for example in the interior of the Jāmeh Mosque, Isfahān, Iran, began in the 8th century) (Fig. 2a). A connection has been found between Islamic decorators and issues dealt with in practical geometry; this has led to the acknowledgement that the octagon was probably obtained by rotating two equal squares by 45° to each other, an operation which does not require a compass and which can easily be performed by master decorators [11, p. 181].

The octagon and the eight points star – the *khatam* obtained through reciprocal rotation of two squares – recur frequently, and start to dominate during the late 9th century [12, p. 250]. Variations on geometrical themes [13] [14] are also documented by illustrated scrolls such as those of Topkapi, made between the late 15th and early 16th century (Topkapı Palace Library, Istanbul, Ms. H. 1956). The scroll (29.5 meters long and 33-34 centimeters high, containing 114 drawings) comes from the imperial collection and was made putting together two scrolls which probably represented unapproved drawings submitted to the sultan for the city of Tabriz [15, p. 29]. It contemplates a vast repertoire of geometrical decorations including architectural designs and projections of vaults which sometimes take on the shape of *muqarnas* (a vault obtained combining round and square geometries) [15] (Fig. 2b) or in the form of the dome, as the one of the aforementioned Jāmeh Mosque in Isfahān (Fig. 2c).

We can find a eastern influence in the Castel del Monte in Andria (Italy, around 1240) with an eight-sided plan echoed in the octagon of the inner yard and in the eight octagonal often occur, as the ratio between the side and the diagonal of a square characteristic in Islamic architecture. The Cistercian mason who built the castle was probably aware of that tradition, or there may have been a suggestion by the emperor himself, interested in mathematics and architecture, or by some of Arab mathematicians at his court, such as Theodorus of Antioch [16].

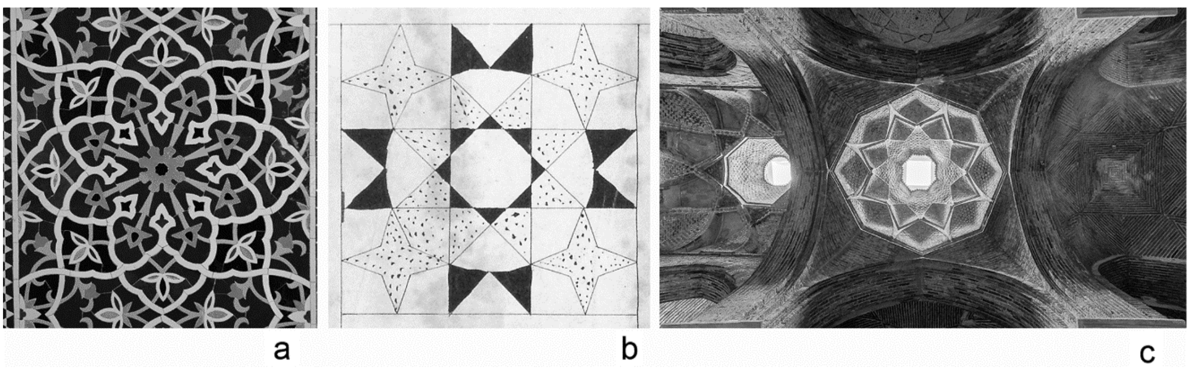

Figure 2: Islamic octagons: a) Jāmeh Mosque (Isfahān, Iran): decoration. Photo by آرش -, CC BY-SA 3.0; b) Muqarnas vault for a square bay with a central star octagon. Topkapi scroll, tav. 23, wiki/File:Topkapi_Scroll_P297.JPG; c) Jāmeh Mosque (Isfahān, Iran): dome. Photo by Diego Delso, delso.photo, License CC-BY-SA

Decorative geometrical taste applied to architecture also appears in the West, as can be seen in the cloister of Saint John in Lateran (13th century), evidence of the architectural and mathematical culture of its creators, the *maestri cosmateschi* who were also influenced by Byzantine culture of which they adopted a strict geometrical manner of expression.

The 16th century saw the spread of the central plan in religious buildings, also in relation to the designs for the church of Saint Peter's in Rome, and in residential buildings, as well clearly appeared in the work of Andrea Palladio. As for the Michelangelo Buonarroti's plan for Saint Peter's in Rome (around 1546, but derived from the plan by Donato Bramante and other architects involved in San Pietro design) [17] [18] we can recognize a resemblance with the described scheme generated by rotating squares (Fig. 3b).

Palladio's work was probably one of the reasons for the great spread of the central plan, also thanks to the repertoire of his projects published in his *Quattro libri dell'architettura* (Venice 1570). The plan of Villa Almerico Capra (also known as la Rotonda, near Vicenza), which Andrea Palladio began building in 1566, is based on a square with four monumental stairways [19] preceded by as many projecting porticos which extend along each axis (Fig. 3c). It is a Greek cross plan; the square taken up by the villa with its four porticos rotated by 45° has its vertices coinciding with the total area that also includes the stairways [20]. The octagonal geometrical layout generated by rotated squares governs the whole design, as in the aforementioned examples described, where it adds articulation in the architectural spaces.

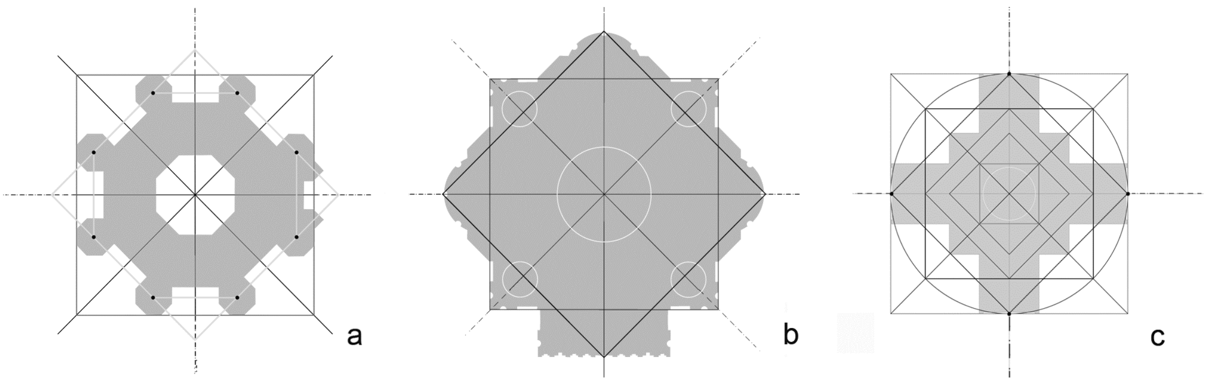

Figure 3: The matrix of rotated squares and octagonal star: a) Castel del Monte in Andria; b) Michelangelo's project for Saint Peter; c) Palladio's project for Villa Almerico Capra (La Rotonda)

4 Conclusion

In this paper, we have studied the interest shown by architecture in projects generated by one regular polygon, the octagon, with its multiple geometric and symbolic meanings. To examine the subject, we have compared architectural plans and spaces from different historical periods and sites. We have found recurrence of some meanings linked with life and its main element - the water of fountains and baptisteries - and we have observed that the octagonal-based prisms are often conceived as a variant of the cylindrical spaces, as shown, for example, by the simultaneous presence of circular and octagonal Roman hall and baptisteries. A different spatial articulation is obtained with the creation of the octagon by two rotated squares and the consequent generation of octagonal stars able to introduce a hierarchical system of spaces, as was observed in Islamic architecture and in some Renaissance plants.

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References

[1] Freze A.: *Byzantine Octagon Domed Churches of the 11th Century and the Roman Imperial Architecture*. Actual Problems of Theory and History of Art, 2015, Vol. 5, 277-286. doi: 10.18688/aa155-2-28.
[2] Gonçalves E.: *The Octagon in the Houses of Orson Fowler,* Nexus Network Journal 13, 2, 2011, 337-349.
[3] Carandini A. Fraioli F. Bruno D.: *Le case del potere nell'antica Roma*. Laterza, Roma 2010.
[4] Longhi A.: *L'architettura del battistero: storia e progetto*. Skira, Milano 2003.
[5] Williams K.: The sacred cut revisited: the pavement of the baptistery of San Giovanni, Florence. *The Mathematical Intelligencer* 16, 2, 1994, 18–24.
[6] Krautheimer R. *Introduction to an "Iconography of Mediaeval Architecture". Journal of the Warburg and Courtauld Institutes* 5, 1942, 1–33.
[7] Camerota F.: *La prospettiva del Rinascimento. Arte, architettura, scienza.* Electa, Milano 2006.
[8] Wittkover R.: *Architectural Principles in the Age of Humanism*, Warburg Institute, London 1949.
[9] Xavier J.: *Leonardo's Representational Technique for Centrally-Planned Temples*. Nexus Network Journal 10, 1, 2008, 77–99.
[10] Reynolds M.A.: *The Octagon in Leonardo's Drawings.* Nexus Network Journal 10, 1, 2008, 51-76.
[11] Özdural A.: *Mathematics and Arts: Connections between Theory and Practice in the Medieval Islamic World*. Historia Mathematica 27, 2, 2000, 171–201.
[12] Abdullahi Y. Bin Embi M.R.: Evolution of Islamic Geometric Patterns. *Frontiers of Architectural Research* 2, 2013, 243–51. doi:10.1016/j.foar.2013.03.002.
[13] Critchlow K.: *Islamic Patterns: an analytical and cosmological approach*. Thames and Hudson, London 1989.
[14] Marotta A. De Sanctis A.: *Orienti e Occidenti della rappresentazione come linguaggio complesso. Dalle Selligges nel Khatem al Drakslingor celtico*. In: De Rosa A. (Ed.) Orienti e occidenti della rappresentazione. 139-150, Il Poligrafo, Padova 2005.
[15] Necipoğlu G.: *The Topkapi Scroll: Geometry and Ornament in Islamic Architecture Topkapi Palace Library* MS H.1956. Sketchbooks & Albums. The Getty center for the history of arts and the humanities, Santa Monica, California 1995.
[16] Götze H.: *Friedrich II and the Love of Geometry. Nexus: Architecture and Mathematics*, ed. Williams K. pp. 67-79, Edizioni dell'Erba, Fucecchio (Florence) 1996.
[17] Bruschi A. Frommel C. L. Wolf Metternich F. Thoenes C.: *San Pietro che non c'è: da Bramante a Sangallo il Giovane*. Electa, Milano 1996.
[18] Francia E.: *Storia della costruzione del nuovo San Pietro*. De Luca, Roma 1977.
[19] Streitz R.: *Palladio: la Rotonde et sa géométrie*. Bibliothèque des arts, Paris 1972.
[20] García Salgado T.: *A Perspective Analysis of the Proportions of Palladio's Villa Rotonda*: Making the Invisible Visible. Nexus Network Journal 10, 2, 2008, 269–282.

OŚMIOKĄT W ARCHITEKTURZE NA PRZESTRZENI WIEKÓW

W artykule analizowano znaczenie w architekturze ośmiokąta foremnego jako wielokąta pośredniego między kwadratem a okręgiem. W tym celu porównano zabytki z różnych okresów historycznych, takie jak starożytne rzymskie budowle, chrześcijańskie chrzcielnice, islamskie architektury i dekoracje, renesansowe centralne plany i barokowe fontanny.