Oilfield development and operations planning under geophysical uncertainty

Yury Redutskiy

ABSTRACT

The oil and gas industry nowadays is challenged by dealing with nonconventional reserves and offshore environments. Decision-making associated with projects in the petroleum sector has to handle various technological issues, risks, and uncertainty. The Smart Fields approach was introduced to cope with complicated production conditions and make the production of hydrocarbons economically efficient. A significant part of this approach is proactive planning which implies taking into account the uncertainty, or lack of knowledge of the recoverable reserves, future hydrocarbon prices and various operational issues inherent in the projects. In this study, a multi-stage stochastic programming approach is employed to cover the relevant engineering issues of oilfield development and petroleum production while addressing the geophysical uncertainty related to the developed deposit. The proposed model covers such aspects as well drilling, gathering pipeline infrastructure planning, capacity selection for the infrastructure and the processing units, as well as planning the production operations with consideration of artificial lift efficiency. The model aims to optimise the entire field lifecycle, given the chosen planning criterion, that is an economic criterion of the project's net present value. The contribution of the developed model to the area of planning in the petroleum industry is the detailed consideration of the technology: the flows and pressures in the planned infrastructure, reservoir behaviour, and the artificial lift performance. The goal of including these technological details is to apprehend the economic trade-off between investments, operating costs and the prospective revenues, given the lack of knowledge of the geophysical properties of the developed deposit. The stochastic modelling implemented in this study is relevant to the development projects in nonconventional environments, where several deposits of various sizes are present; however, not each deposit's properties get to be studied in detail.

Corresponding author:

Yury Redutskiy

Molde University College,
Faculty of Logistics, Norway
e-mail: yury.redutskiy@himolde.no

Introduction

Various kinds of energy are vital for the global society to develop. Nowadays, companies producing the energy resources of oil and gas face many challenges. Many of these challenges are associated with the technological complexity of nonconventional and offshore reserves broadly put to use during the past few decades (Mathieson, 2007). To handle the diffi-

culties posed by the nonconventional reserves, the Smart Field or Intelligent Field approach has been developed. The distinctive features of this approach are best described in comparison to the standard, time-tested petroleum production solutions that have been applied for large onshore reserves. The planning capabilities for operations given the standard approach are to a large extent limited to simplified

models and one-at-a-time decisions. The overall life-cycle of the developed reserves is merely monitored, rather than precisely controlled. All this is due to the lack of awareness as to how exactly a certain control decision affects the reservoir draining process, both in the short and the long run. On the contrary, Smart Field solutions employ certain advanced instrumentation and specialised software, all of which aim to improve the knowledge of operations and predict the outcome of each control action (Redutskiy, 2017a). The goal of advanced capabilities of smart solutions is to maintain the economic efficiency of hydrocarbon production in the risky nonconventional environments. An important feature of the smart approach is proactive planning and control which implies coordinating development and production operations while considering the uncertainty about the future, which may include the lack of knowledge of the recoverable reserves, future hydrocarbon prices and many operational issues at various points of a given project.

Making decisions with consideration of uncertainty requires accounting for the phases that the oil and gas industry projects undergo, as well as how the uncertainty manifests during these phases, and how it becomes resolved. Every project begins with the exploration of hydrocarbon occurrences by a geological survey, seismic studies, and drilling of exploratory wells. The next step is the deposit appraisal which mainly implies estimation of oil and gas reserves. Further, the field development activities begin. They include drilling wells, constructing a gathering pipeline and processing facilities. The production of hydrocarbons begins when the infrastructure is ready and ends with the field abandonment after the reserves are depleted. Given the description of the project phases, one may observe that there is a process of gradual uncertainty resolution over the timespan of the project. During earlier stages, there is very little knowledge of the volume of recoverable hydrocarbons in the explored or developed deposit. The quality and the composition of the flow from the reservoir, as well as its change with time, is not entirely apprehended. Also, decision-makers may only anticipate how profitable the whole project will turn out, given the possibilities of sudden changes in the hydrocarbon market price. With time, as the field becomes mature, all these unknowns become revealed, and by the end of the project lifecycle, the full relevant information becomes available to contribute to the knowledge and experience of petroleum engineers for future projects.

A peculiar aspect of uncertainty resolution is attributed to the nonconventional production environments, especially offshore locations. Offshore production sites usually consist of several reservoirs (formations where the petroleum is trapped geologically), some of which are considerably large, while others are small (often referred to as "marginal fields"). All these reservoirs become explored during the geological survey and the seismic studies. However, a more detailed examination of the porous media and the fluid properties through exploratory drilling is often performed only for larger fields for economic reasons. As a result, for these larger deposits, a considerable amount of knowledge is gathered during the appraisal phase. However, for the marginal reservoirs, significant uncertainty prevails, and in many cases, it may lead to these resources not being developed.

To comprehend the relevant uncertainty aspects inherent in the reservoir development and petroleum production processes, these issues, i.e. stochastic events, processes, and characteristics, are accounted below in the three broad categories:

• *functional uncertainty*. A lot of dangerous and expensive machinery guided by a complex automation system is employed in oil and gas projects. This direction addresses the issues of potential technological incidents, hardware failures, repairs and restorations, and systems maintenance in general;

• *geophysical uncertainty*. These issues include the knowledge of formation characteristics, fluid properties, and their behaviour during the deposit draining. More and more information is collected, as the reservoir is being developed and drained, thereby gradually annihilating the lack of knowledge. Among the initially uncertain parameters that might be considered in this group, there are initial flow rates from the production wells (or initial well productivities), initial oil cut (percentage of oil in the produced fluid), production decline rates, total recoverable volume of hydrocarbons, water breakthrough time (production ending time due to water-flooding);

• *economic uncertainty.* Parameters of this group are related to the changing market conditions influencing such parameters as oil price, prices for the machinery/equipment, and so on. Generally speaking, the discount rate may also be an indicator of certain market situations from the investment perspective.

From the very general perspective of uncertainty consideration, an important observation should be made at this point. The nature of petroleum industry projects is complex enough to include both exogenous and endogenous uncertainty types (Jonsbråten, 1998a). The former one may be addressed as an "external" from the decision-making viewpoint on the considered process, i.e. these parameters are revealed independently from the project-related decisions. The economic parameters fall under this category. Further, the functional and the geophysical uncertainty issues make the petroleum industry projects and problems stand out in the area of stochastic programming because these two groups of stochastic factors fall under the category of endogenous, or decision-dependent uncertainty. It means that the decisions made during the earlier stages of the project when only a little information is available, considerably affect the process itself, its efficiency, outcome, profits, etc.

In this paper, the issues of oilfield development and scheduling the production operations under uncertainty will be addressed. In this context, the uncertainty of the functional type appears not quite as relevant as, for instance, for the problems of automation and control system design. See, for example, (Redutskiy, 2017b) that this uncertainty type prompts to focus on choosing the hardware components, making decisions on redundancy (backing-up) the necessary tools, etc. Additionally, incorporating uncertainty of the economic type for the oilfield planning purposes will not be addressed in this study, that is the price of oil will not be regarded as stochastic. This assumption is made due to the primary focus of this research on the technological details. A proper representation of uncertainty in the future market value of hydrocarbons requires a deep analysis and a rather sophisticated representation in the model. Therefore, a stable demand for hydrocarbons from various industries (chemical, textile, pharmaceutical, construction etc.) will be considered further, and thereby, conditions of the hydrocarbons market are presumed as rather favourable.

The main angle of this research is the long-term planning of oilfield development and production operations, that is planning the lifecycle of a given field which is prompted by the state-of-the-art Smart Field approach. From the viewpoint of engineering projects of field development in modern-day conditions, this research addresses a number of strategic decisions like well-drilling and laying out the infrastructure with consideration of the future operations,

that is choosing reasonable production rates, artificial lift running mode, as well as the throughput of the gathering system during the project's planning horizon. An economic indicator of net present value (NPV) is employed for the decision-making model presented further. This indicator allows finding a trade-off between the capital expenditures (CAPEX), i.e. drilling and infrastructure investments, and operational expenditures (OPEX) associated with fluid processing and artificial lift performance.

Thus, the geophysical uncertainty issues are the main stochastic factors taken into consideration in the multi-stage stochastic programming problem presented further. The model covers strategic planning issues relevant to the development of a given oilfield, namely, well drilling, gathering pipeline design, and processing facility choice. All the development activities are planned and hydrocarbon production is scheduled over the given time horizon, corresponding to the deposit lifecycle. The model considers in detail the reservoir description and artificial lift performance. The proposed decision-making framework is especially relevant for the cases of exploring the possibilities including the marginal reservoirs into the development projects, given that the significant level of geophysical uncertainty remains for these marginal deposits by the time the engineering project of the main production site development is commenced. The proposed model is applied to a study example of a marginal offshore field based on a real-life data from several engineering projects available to the author.

1. OVERVIEW OF THE RESEARCH AREA

For almost seven decades, various optimisation models have been developed by many researchers to better comprehend the problems of planning and control over technological processes in the oil and gas industry. Early research in the area of petroleum resource development and production management comprised deterministic models, for example, in research (Aronofsky & Williams, 1962), (Devine & Lesso, 1972), (Frair & Devine, 1975). Later research, for instance (Iyer et al., 1998), (Van den Heever et al., 2001) or (Dawson & Fuller, 1999), included large-scale mixed integer linear and non-linear problem settings which require specialised algorithms to

facilitate their application to real-life problem instances.

Incorporating uncertainty into strategic decision-making appears to be one of the relatively recent directions in research of oil and gas project planning. One of the earlier papers focusing on endogenous resource uncertainties for planning problem sector projects is (Haugen, 1996). Later, another work (Jonsbråten, 1998b) encompassed an optimal field development planning problem under the exogenous uncertainty of the future price for hydrocarbon resources.

Several papers by Grossmann research group at Carnegie Mellon University explore the problems of petroleum field development and production, i.e. in the same research domain as this study. (Goel & Grossmann, 2004) attempt to optimise capital investments and operations of a gas field under uncertainty in reserves. (Tarhan et al., 2009) consider the development and operations planning for an oil and gas field under uncertainty in initial flow rates, recoverable reserves and water breakthrough time. (Gupta & Grossmann, 2014) address the fluid composition in addition to multiple scenarios of geophysical properties of the deposit, while planning the field infrastructure.

Generally, the class of problems addressed in the mentioned papers and also this research is called process network synthesis. A seminal work for this domain of stochastic models is (Tarhan & Grossmann, 2008). The paper incorporates a few theoretical developments concerning the issues of decision-dependent uncertainty.

Another notable direction in the stochastic optimisation of oil and gas field development involves the use of a reservoir simulation models. The paper (Cullick et al., 2004) describes planning the investments into a production system for several deposits with consideration of economic criteria and a certain tolerable level of risk, set up by a field operating company. The authors use finite-difference reservoir model to evaluate the properties of the deposit. Another research incorporating stochastic reservoir properties is presented in the article (Bellout et al., 2007), where the authors contemplate simultaneous decisions on well locations and the anticipated operating modes.

Application of deterministic modelling for planning field infrastructure and operations, for instance, suggested in (Frair & Devine, 1975) or (Iyer et al., 1998), leads to solutions when either all the wells in the plan must be drilled or certain wells or sections of a petroleum deposit are avoided because they are deemed unprofitable in the circumstances of the deterministic data provided for the models. Stochastic modelling solutions also help to choose a certain number of wells for draining of the deposit. However, such solutions set up a reasonable strategy for learning about the deposits with uncertain reserves. That is, there might be an initial or a pilot plan, and the possibilities of expansions if the process is deemed profitable. To sum up, while deterministic models are prone to exclude small deposits from the development plan, the stochastic models might demonstrate certain flexibility and reveal the potential of small deposits due to consideration of optimistic and pessimistic evaluations of various uncertain geophysical parameters.

The model presented in the following section adopts the ideas of incorporating uncertainty in field development and operations planning (Tarhan et al., 2009). However, the necessary adaptations are made to fit the process descriptions into an engineering perspective through incorporating such details as pressure and flow profiles in the gathering system, artificial lift performance, and well interactions in the reservoir.

2. FIELD DEVELOPMENT AND PRODUCTION SCHEDULING MODEL

Further, the problem of oilfield development and production planning is formulated for an offshore field, as demonstrated in Fig. 1. The wellheads located on the seabed are grouped into clusters around manifolds which play the role of gathering stations. The connections between the wells and the manifolds are made with short rigid pipelines called 'jumpers'. The gathered petroleum is delivered from these clusters, first, to a system of risers, and then to a processing facility, a platform called a 'floating production, storage and offloading' (FPSO) unit, where the export-quality petroleum is prepared. Usually, the processed hydrocarbons are offloaded to a shuttle tanker, which delivers the product to an onshore storage base.

An important assumption regarding the nature of a planning problem presented in this research is that the focus is entirely on the structures of the surface facilities (located on the seabed) and the system's
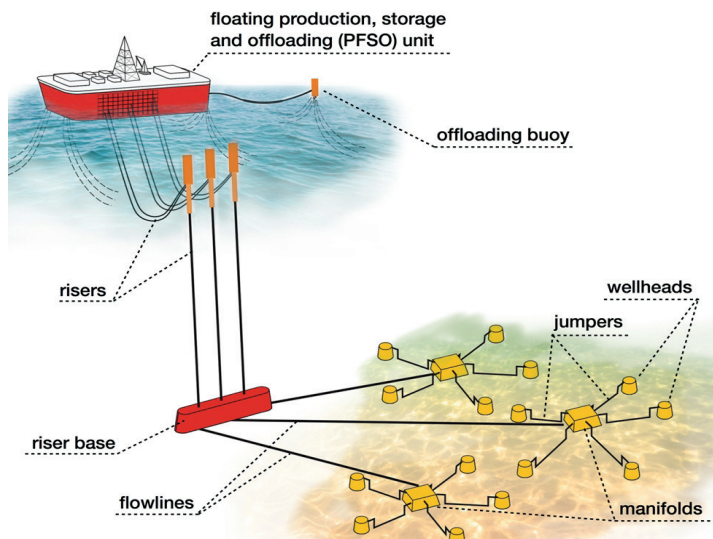
Fig. 1. Standalone offshore production system

Source: author's elaboration based on (Bai & Bai, 2012).

capacities. With this assumption, the layout of the underground structures is disregarded in the model (it is considered as simple as possible, perhaps, ordinary monobore wells).

In this research, the issue of operational efficiency of an artificial lift system is taken under a rather detailed consideration. In some works, e.g. (Tarhan et al., 2009), the operational costs are taken as a fixed amount of money per volume of produced fluid. However, there is a considerable amount of research like (Takács, 2009) or (Wang et al., 2002) pointing to the benefits of considering the operational efficiency of the chosen artificial lift. Given that one of the aims of this research is to capture the economic trade-off between all the capital investments, operational expenditures and profits from producing hydrocarbons, the operational efficiency is addressed here to some extent in details. For our setting, the production of hydrocarbons is assumed to be conducted with electrical submersible pumps (ESP). The decision-making model will drive the machinery's performance to work as closely to the best efficiency point as possible, so that the costs associated with operations can be suitably identified and balanced with other expenditures. The operating mode of the ESP equipment (and by extension its efficiency) is defined by the process values of hydrocarbon production flow rates from the wells and also, the frequency of the alternating current (AC) controlling the pump drive and allowing a certain flexibility in the pump operations.

The flows in the planned structures are considered homogeneous and consisting of oil and water. With this, it is assumed that there are no gas slugs in the flow. This is a reasonable assumption given that ESP technology allows separating the gas from the flow inside the well.

The model below considers in detail the flows and the pressures throughout the entire production system. With this, the pressures in the systems are balanced, and capacity limitations of the chosen gathering structures are considered. Ultimately, this allows avoiding the negative impact of the surface pipeline's backpressure on the wells' operations.

A simple linear model of the reservoir is considered here for long-term planning purposes. The model accounts for the reservoir productivity and interactions between the well during operations. The representation of the resource depletion with time is assumed to take an exponential form.

Given the assumptions above, the goal of the decision-making model is to make the following choices:
- the number of wells to drill and their locations,
- the number, location, and capacities of manifolds,
- the assignment of wells to manifolds (connecting with jumpers), capacities of the jumpers,
- capacities of the flowlines connecting the manifolds to the riser base,
- the processing capacity of the platform, and the need for expanding this capacity,
- the assignment of each development activity (drilling wells, installing manifolds, jumpers, flowlines, and capacity expansions) to periods,
- production operations (production rates, AC frequencies, and so on) during each period,
- the timeline for the end of the production operations after the water breakthrough.

These decisions are made with the consideration of geophysical uncertainty considered in the form of the following stochastic parameters in the provided mathematical formulation:
- *reservoir productivity and well interaction factors* which come in the form of a matrix (coefficients) of a linear model. Note that these parameters

directly correspond to the initial oil flow rates at the start of the production operations;

- *production decline rate*. Note that this parameter in combination with initial production rates leads to an eventual estimation of recoverable reserves of the deposit;

- *water breakthrough time*, the timeline when the hydrocarbon production becomes no longer possible due to the wells starting to produce only water that is being injected into the reservoir to maintain the necessary pressure.

A distinctive trait of the oil and gas industry projects is that knowledge, especially that of the deposit geophysics, is obtained gradually throughout the operations. It means that uncertainty represented by the listed stochastic parameters does not resolve completely at every planning period of the multi-stage problem. The uncertainty is mitigated gradually by discovering certain information based on the decisions made during some earlier periods. The assumptions regarding this learning process for the problem setting considered in this research are listed below. The representation of gradual uncertainty resolution in the mathematical model mostly adopts the ideas of (Tarhan et al., 2009), with necessary adaptations for this paper:

- there is an initial start-up development programme comprising drilling a rather small predefined number of wells N^{init}, install a certain number of manifolds, lay out the pipeline, and eventually start the production process. This initial programme collects the knowledge of the reservoir, i.e. values of the productivity coefficients and well interaction coefficients. By extension, this allows to figure out the initial production rates from the drilled wells, as well as

the intensity of the well interaction through the porous medium;

- there is a possibility to continue the field development by drilling more wells (up to N^{ext} in total), and respectively expand the gathering infrastructure. Fulfilling this extended development programme allows gaining full knowledge of the production decline slopes for the producing wells, and by extension, the deposit's recoverable reserves;

- as an alternative to the expensive and troublesome extended development programme, the knowledge of the recoverable reserves may be unveiled by producing only from the N^{init} wells that were initially drilled. However, a certain time $T^{\dagger}$ of continuous production must elapse so that the values of production decline rates might be evaluated. This is another strategy that the model might suggest estimating the recoverable hydrocarbon reserves;

- the uncertainty in the water breakthrough time is assumed to be independent of the development/drilling decisions. This uncertainty gets resolved when the field becomes mature, i.e. the field must be producing for a certain number of years $T^{\ddagger}$, so that the water breakthrough time becomes known.

Below, the mathematical model is presented. The necessary notations are provided in Tab. 1-3.

The objective function (1) bears the meaning of maximising the expected net present value of the petroleum development and production project. The first term of the NPV for each single scenario represents the revenues from the sales of the produced petroleum. The second term describes the costs of well drilling. The next term corresponds to the manifold installations with the chosen capacities.

Tab. 1. Notations for the lifecycle planning model (sets and indices)

NOTATION	DESCRIPTION	NOTATION	DESCRIPTION
NW	total number of potential well drilling locations	i, j	wells, i, j∈{1…NW}
NM	maximum number of potential manifolds	m	manifolds, m∈{1…NM}
T	total number of time periods	t, τ	time periods, t, τ∈{1…T}
NMC	number of manifold capacity options	k	manifold capacity, k∈{1…NMC}
NJC	number of jumper capacity options	l	jumper capacity, l∈{1…NJC}
NFC	number of flowline capacity options	r	flowline capacity, r∈{1…NFC}
NPC	number of platform processing capacity options	β	platform capacity, β∈{1…NPC}
NEC	number of potential processing capacity expansions	γ	expansion capacity, γ∈{1…NEC}
NS	total number of scenarios	s, s'	scenarios, s, s'∈{1…NS}

Tab. 2. Notations for the lifecycle planning model (parameters)

NOTATION	DESCRIPTION
x^{RB}, y^{RB}	location (horizontal coordinates) of the riser base, [m]
x_i^{WL}, y_i^{WL}	location (horizontal coordinates) of well i, [m]
x_m^{ML}, y_m^{ML}	location (horizontal coordinates) of manifold m, [m]
$L_{i,m}^J$	jumper length, corresponding to the distance between well i and manifold m, [m]
L_m^{MF}	length of the flowline, corresponding to the distance between manifold m and the riser base, [m]
H_i	depth of well i, [m]
H^{sea}	depth of the sea bed, [m]
TD_i	well i production tubing diameter, [m]
S_k	allowed number of connections manifold capacity option k
D_l^J	diameter of option l of the jumper capacity, [m]
D_r^{MF}	diameter of option r of the flowline capacity chosen for the manifold flowlines, [m]
Q_β^{FPSO}	processing capacity option β of initially installed FPSO, [m³/d]
Q_γ^{PCE}	capacity expansion option γ for the gathering system, [m³/d]
Δ_t	duration of time period t, [d]
N^{init}	number of wells to drill to complete the initial development plan
N^{ext}	number of wells to drill to complete the extended development plan, maximum allowed number of well-drillings
$T^\dagger$	number of production years (or time periods) necessary to evaluate the recoverable hydrocarbon reserves
$T^\ddagger$	number of production years (or time periods) necessary to evaluate the water breakthrough time
T_s^{WB}	time interval before the water breakthrough occurs for the entire field and the production stops, scenario s
ε	minimal total field flow rate necessary for the whole system to be viewed as producing, small number, [m³/d]
DC_t	maximum number of wells to be drilled during time period t
MC_t	maximum number of manifolds to be installed during time period t
PC_t	maximum number of pipeline segments to be commissioned during time period t
C	pipeline grade factor for Hazen-Williams formula; assumes the value of 120 for new pipes, 94-100 for old pipes
p^{RBmin}	necessary pressure at the riser base during time period t, [Pa]
p^{BHmin}	the lowest allowed bottomhole pressure, [Pa]
p^R	reservoir pressure, [Pa]
ρ	the density of the produced fluid, [kg/m³]
g	standard acceleration due to gravity, [m/s²]
$\alpha_{i,j,s}$	reservoir model coefficients, whose values are related to the wells' productivity, as well as influence between each pair i and j, scenario s, [Pa·d/m³]
$\sigma_{i,s}$	production decline slope for well i, scenario s, [1/d]
f^{max}	the highest allowed alternating current frequency, [Hz]
B	large number
C_i^W	cost of drilling well i, [USD]
C_k^M	cost of manifold capacity option k, [USD]
C_l^J	cost of jumper capacity option l per unit of length, [USD/m]
C_r^F	cost of flowline capacity option r per unit of length, [USD/m]
C_β^{FPSO}	cost of the initially installed FPSO with the chosen processing capacity option β, [USD]
C_γ^{PCE}	cost of employing a processing capacity expansion option γ for the platform or the gathering system, [USD]
C_t^{proc}	cost of water handling during time period t per volume unit, [USD/m³]
P_t^{oil}	price of oil per volume unit in time period t, [USD/m³]
P_t^{el}	price of electricity during time period t per kWh, [USD/kWh]
δ_t	cash flow discounting coefficient for time period t, fraction
$P_s^{scenario}$	probability of scenario s, fraction $\in [0; 1]$

Tab. 3. Notations for the lifecycle planning model (variables and functions)

Notation	Description
Binary variables (decision variables)	
$z_{i,t,s}^{W}$	equals 1, if well i is drilled and completed at time period t, scenario s; 0, otherwise
$z_{m,k,t,s}^{M}$	equals 1, if manifold m is assigned a capacity option k and installed during time period t, scenario s; 0, otherwise
$z_{i,m,l,t,s}^{J}$	equals 1, if well i is tied back to manifold m via jumper with capacity option l, and the jumper is installed during time period t, scenario s; 0, otherwise
$z_{m,r,t,s}^{MF}$	equals 1, if a flowline from manifold m to the riser base capacity option r is installed during time period t, scenario s; 0, otherwise
$z_{\beta,s}^{FPSO}$	equals 1, if the platform is installed with the initial processing capacity option β, scenario s; 0, otherwise
$z_{\gamma,t,s}^{PCE}$	equals 1, if the processing capacity expansion option γ is engaged starting period t, scenario s; 0, otherwise
Binary variables (state variables)	
$z_{i,t,s}^{compl}$	equals 1, if well i is drilled and completed within the time up to period t, and the chain from the well to the platform is ready for production starting next time period, scenario s; 0, otherwise
$z_{t,s}^{prod}$	equals 1, there the reservoir is producing petroleum, i.e. water breakthrough has not yet happened in time period t, scenario s; 0, otherwise
$z_{t,s}^{flow}$	equals 1, if there is a flow from the drilled and operated wells during time period t, scenario s; 0, otherwise
$\zeta_{t,s}^{init.plan}$	equals 1, if only the initial development plan has been being implemented so far by the end time period t, scenario s; 0, otherwise
$\zeta_{t,s}^{ext.plan}$	equals 1, if an extended development plan is being implemented by the end time period t, scenario s; 0, otherwise
$\zeta_{t,s}^{maturity}$	equals 1, if by the end of time period t, the reservoir is mature enough to resolve the recoverable reserves uncertainty, scenario s; 0, otherwise
$\zeta_{t,s}^{wbt}$	equals 1, if by the end of time period t, the reservoir has been exploited long enough to determine the water breakthrough time, scenario s; 0, otherwise
$z_{s,s',t}$	equals 1, if at the beginning of period t, scenario s and s' are indistinguishable; 0, otherwise
Continuous variables (process values and pipeline segment diameters)	
$q_{i,t,s}$	fluid production rate from well i during period t, scenario s, [m³/d]
$oc_{i,t,s}$	oil cut for the flow from well i during time period t, scenario s, fraction
$q_{i,m,t,s}^{J}$	flow through the jumper between well i and manifold m during time period t, scenario s, [m³/d]
$q_{m,t,s}^{MF}$	flow through manifold m during time period t, scenario s, [m³/d]
$d_{i,m,s}^{J}$	diameter of the jumper between well i and manifold m, scenario s, [m]
$d_{m,s}^{MF}$	diameter of the flowline between manifold m and the riser base, scenario s, [m]
$p_{i,t,s}^{BH}$	bottomhole pressure at well i during time period t, scenario s, [Pa]
$p_{i,t,s}^{WH}$	wellhead pressure at well i, scenario s, [Pa]
$p_{m,t,s}^{M}$	pressure at manifold m during time period t, scenario s, [Pa]
$p_{t,s}^{RB}$	pressure at the riser base during time period t, scenario s, [Pa]
$p_{i,t,s}^{TD}$	total developed pressure of the ESP in well i during time period t, scenario s, [Pa]
$f_{i,t,s}^{ESP}$	AC frequency used in VSD controlling the ESP in well i during time period t, scenario s, [Hz]
$N_{i,t,s}^{HL}$	hydraulic lift power developed by the ESP installation in well i during time period t, scenario s, [kW]
$\eta_{i,t,s}^{ESP}$	total efficiency of the ESP installation in well i during time period t, scenario s, fraction
Functions	
$F_{i}^{TDH}(q,f)$	total developed head of the pump installed in well i
$F_{i}^{eff}(q,f)$	motor efficiency characteristic for the pump drive installed in well i
$p^{fr}(q,d,L)$	friction losses in the pipeline segment
NPV_{s}	net present value of the project, scenario s

$$\max ENPV = \sum_{s=1}^{NS} P_s^{scenario} \cdot NPV_s$$

$$NPV_s = \sum_{t=1}^{T}\delta_t \cdot P^{oil} \cdot \Delta_t \cdot \sum_{i=1}^{NW} oc_{i,t,s} \cdot q_{i,t,s} - \sum_{t=1}^{T}\delta_t \cdot \sum_{i=1}^{NW} C_i^W \cdot z_{i,t,s}^W - \sum_{t=1}^{T}\delta_t \cdot \sum_{m=1}^{NM}\sum_{k=1}^{NMC} C_k^M \cdot z_{m,k,t,s}^M$$

$$- \sum_{t=1}^{T}\delta_t \cdot \sum_{i=1}^{NW}\sum_{m=1}^{NM}\sum_{l=1}^{NMC} C_l^J \cdot L_{i,m}^J \cdot z_{i,m,l,t,s}^J - \sum_{t=1}^{T}\delta_t \cdot \sum_{m=1}^{NM}\sum_{r=1}^{NFC} C_r^F \cdot L_m^{MF} \cdot z_{m,r,t,s}^{MF}$$

$$- \sum_{\beta=1}^{NPC} C_\beta^{FPSO} \cdot z_{\beta,s}^{FPSO} - \sum_{t=1}^{T}\delta_t \cdot \sum_{\gamma=1}^{NEC} C_\gamma^{PCE} \cdot z_{\gamma,t,s}^{PCE}$$

$$- \sum_{t=1}^{T}\delta_t \cdot C_t^{proc} \cdot \Delta_t \cdot \sum_{i=1}^{NW}\left(1-oc_{i,t,s}\right)\cdot q_{i,t,s} - \sum_{t=1}^{T}\delta_t \cdot 24\cdot P_t^{el} \cdot \Delta_t \cdot \sum_{i=1}^{NW} \frac{N_{i,t,s}^{HL}}{\eta_{i,t,s}^{ESP}}$$

(1)

$$\delta_t = (1+i)^{-\left(\sum_{\tau=1}^{t}\Delta_t/365 - 1\right)}$$

(2)

The fourth term corresponds to the costs of the jumpers, given their well-to-manifold allocations. The fifth term goes for the flowline costs. The sixth term is associated with the choice of the initial processing capacity of the platform, while the following term accounts for potential processing capacity expansions in the following years. The eighth term corresponds to the processing of the fluid flow, i.e. separating the petroleum from the water. The ninth term describes the electricity consumption for the chosen ESP operating modes. All the costs and revenues are considered for each period of the planning horizon and the discounting coefficients (2) for the uneven time periods are applied to adjust the cash flows to their present value.

The constraints for the model are provided below, and they are grouped into several broad categories.

2.1. Structure constraints

Constraints (3) state that at any potential well drilling location, no more than one well may be drilled. Constraints (4) state that at any potential manifold placing location, no more than one manifold may be installed, it may be installed only once, and only one capacity option may be chosen for it. Inequalities (5) and (6) say that each drilled well should be included in the gathering network by connecting it to a manifold via a jumper. Equality (7) declares that from any installed manifold, a flowline must be laid to the riser base, and a capacity choice should be made for the pipeline. Expression (8) ensures the installation of a processing platform with a certain chosen capacity.

$$\sum_{t=1}^{T} z_{i,t,s}^W \le 1, \quad \forall i \in \{1..NW\}, \forall s \in \{1..NS\}$$ (3)

$$\sum_{t=1}^{T}\sum_{k=1}^{NMC} z_{m,k,t,s}^M \le 1, \quad \forall m \in \{1..NM\}, \forall s \in \{1..NS\}$$ (4)

$$\sum_{t=1}^{T}\sum_{m=1}^{NM}\sum_{l=1}^{NJC} z_{i,m,l,t,s}^J \ge \sum_{t=1}^{T} z_{i,t,s}^W, \quad \forall i \in \{1..NW\}, \forall s \in \{1..NS\}$$ (5)

$$\sum_{t=1}^{T}\sum_{l=1}^{NJC} z_{i,m,l,t,s}^J \le 1, \quad \forall i \in \{1..NW\}, \forall m \in \{1..NM\}, \forall s \in \{1..NS\}$$ (6)

$$\sum_{t=1}^{T}\sum_{r=1}^{NFC} z_{m,r,t,s}^{MF} = \sum_{t=1}^{T}\sum_{k=1}^{NMC} z_{m,k,t,s}^M, \quad \forall m \in \{1..NM\}, \forall s \in \{1..NS\}$$ (7)

$$\sum_{\beta=1}^{NPC} z_{\beta,s}^{FPSO} = 1, \quad \forall s \in \{1..NS\}$$ (8)

2.2. Gathering system flows and capacities

A capacity choice for each manifold should be made according to the number of jumpers connected to it (9).

The capacity choice for the jumper and flowlines is provided in groups (10)–(12) and (13)–(16), respectively. To choose a suitable jumper diameter, a nonlinear function, also known as the Hazen-Wil-

liams formula (Centrilift, 1997), is used in (12) for determining the pressure drop in a pipeline. In this function, continuous decision variables for jumper lengths, flow rates (10) and diameters (11) are used. The latter two expressions are connected to the corresponding design binaries. The last term on the right-hand side of the expression (12) makes the pressure difference constraints enforced only for those 'well-manifold' pairs that are chosen by the model to be connected with jumpers. Similar logic applies to the description of the flows in the pipelines connecting the manifolds to the riser base (13)–(16).

The constraint (17) declares that the amount of fluid delivered to the processing unit should correspond to its processing capacity, whether it is merely

initially chosen FPSO's capacity or some additional expansions have been employed.

2.3. SCHEDULING CONSTRAINTS

Logical constraints are applied to properly sequence the development activities in time. Specifically, they suggest that the pipeline segments must be installed only after the corresponding structural elements are completed. Expression (18) states that any jumper must be placed only after the corresponding well is drilled and the manifold is commissioned. Similarly, each flowline should be laid after the manifolds are installed (19). The following constraints (20)–(22) represent scheduling limitations, i.e. a maximum number of wells that can be drilled

$$\sum_{t=1}^{T}\sum_{i=1}^{NW}\sum_{l=1}^{NJC} z_{i,m,l,t,s}^{J} \leq \sum_{t=1}^{T}\sum_{k=1}^{NMC} S_k \cdot z_{m,k,t,s}^{M}, \quad \forall m \in \{1..NM\}, \forall s \in \{1..NS\} \tag{9}$$

$$q_{i,m,t,s}^{J} = \sum_{\tau=1}^{t}\sum_{l=1}^{NJC} q_{i,t,s} \cdot z_{i,m,l,\tau,s}^{J}, \quad \forall i \in \{1..NW\}, \forall m \in \{1..NM\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{10}$$

$$d_{i,m,s}^{J} = \sum_{t=1}^{T}\sum_{l=1}^{NJC} D_l^{J} \cdot z_{i,m,l,\tau,s}^{J}, \quad \forall i \in \{1..NW\}, \forall m \in \{1..NM\}, \forall s \in \{1..NS\} \tag{11}$$

$$p_{i,t,s}^{WH} - p_{m,t,s}^{M} \geq p^{fr}\left(q_{i,m,t,s}^{J}, d_{i,m,s}^{J}, L_{i,m}^{J}\right) - B \cdot \left(1 - \sum_{\tau=1}^{t}\sum_{l=1}^{NJC} z_{i,m,l,\tau,s}^{J}\right), \forall i \in \{1..NW\}, \forall m \in \{1..NM\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\};$$

$$\text{where } p^{fr}(q,d,L) = 7.68 \cdot 10^{-9} \cdot \rho \cdot g \left(\frac{q}{C}\right)^{1.85} \cdot \frac{L}{D^{4.89}}, \text{and lengths } L_{i,m}^{J} = \sqrt{\left(x_i^{WL} - x_m^{ML}\right)^2 + \left(y_i^{WL} - y_m^{ML}\right)^2} \tag{12}$$

$$q_{m,t,s}^{MF} = \sum_{i=1}^{NW} q_{i,m,t,s}^{J}, \quad \forall m \in \{1..NM\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{13}$$

$$d_{m,s}^{MF} = \sum_{t=1}^{T}\sum_{r=1}^{NFC} D_r^{MF} \cdot z_{m,r,t,s}^{MF}, \quad \forall m \in \{1..NM\}, \forall s \in \{1..NS\} \tag{14}$$

$$p_{m,t,s}^{M} - p_{t,s}^{RB} \geq p^{fr}\left(q_{m,t,s}^{MF}, d_{m,s}^{MF}, L_m^{MF}\right) - B \cdot \left(1 - \sum_{\tau=1}^{t}\sum_{k=1}^{NMC} z_{m,k,\tau,s}^{M}\right), \forall m \in \{1..NM\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\};$$

$$\text{and lengths } L_m^{MF} = \sqrt{\left(x^{RB} - x_m^{ML}\right)^2 + \left(y^{RB} - y_m^{ML}\right)^2} \tag{15}$$

$$p_{t,s}^{RB} \geq p^{RB\min}, \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{16}$$

$$\sum_{i=1}^{NW} q_{i,t,s} \leq \sum_{\beta=1}^{NPC} Q_{\beta}^{FPSO} \cdot z_{\beta,s}^{FPSO} + \sum_{\gamma=1}^{NEC}\sum_{\tau=1}^{t} C_{\gamma}^{PCE} \cdot z_{\gamma,\tau,s}^{PCE}, \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{17}$$

$$\sum_{\tau=1}^{t}\sum_{l=1}^{NJC}z_{i,m,l,t,s}^{J}\leq\sum_{\tau=1}^{t}z_{i,\tau,s}^{W}\cdot\sum_{\tau=1}^{t}\sum_{k=1}^{NMC}z_{m,k,\tau,s}^{M},\quad\forall i\in\{1..NW\},\forall m\in\{1..NM\},\forall t\in\{1..T\},\forall s\in\{1..NS\}\tag{18}$$

$$\sum_{\tau=1}^{t}\sum_{r=1}^{NFC}z_{m,r,t,s}^{MF}\leq\sum_{\tau=1}^{t}\sum_{k=1}^{NMC}z_{m,k,\tau,s}^{M},\quad\forall m\in\{1..NM\},\forall t\in\{1..T\},\forall s\in\{1..NS\}\tag{19}$$

$$\sum_{i=1}^{NW}z_{i,t,s}^{W}\leq DC_{t},\quad\forall t\in\{1..T\},\forall s\in\{1..NS\}\tag{20}$$

$$\sum_{m=1}^{NM}\sum_{k=1}^{NMC}z_{m,k,t,s}^{M}\leq MC_{t},\quad\forall t\in\{1..T\},\forall s\in\{1..NS\}\tag{21}$$

$$\sum_{i=1}^{NW}\sum_{m=1}^{NM}\sum_{l=1}^{NMC}z_{i,m,l,t,s}^{J}+\sum_{m=1}^{NM}\sum_{r=1}^{NFC}z_{m,r,t,s}^{MF}\leq PC_{t},\quad\forall t\in\{1..T\},\forall s\in\{1..NS\}\tag{22}$$

and completed, and a maximum number of manifolds and pipeline segment that can be installed in each period.

2.4. Production scheduling

Logical constraints connecting the development and production phases require certain intricacy for proper decision-making in the setting that includes uncertainty considerations. Firstly, the intermediate (state) binary variable is introduced in (23) declaring that production from a well may only start no earlier than the entire path of infrastructure from the well to the riser base is finished. The processing platform is assumed to be installed and ready by the end of the very first period. Secondly, another intermediate variable is described in (24), ensuring the production from the reservoir is possible, i.e. that the water breakthrough has not happened. Finally, the logical constraints (25) allow the production to take place when the necessary conditions are met.

The constraint (26) describes yet another intermediate (state) binary, accounting for the periods when production is not only allowed but actually happens, i.e. if the production from the entire deposit is bigger than a pre-defined small value ε.

Technological constraints (27) represent the reservoir performance. The coefficients $\alpha_{i,j,s}$ correspond to the productivity indices for the wells and well interaction factors. The constraint (28) intends to provide an advisable range of the reservoir draining pressures. The next set of constraints (29) demonstrates how the operational conditions change from

period to period, i.e. how the oil content in the produced fluid declines over time.

Constraints (30) through (32) characterise the performance of the artificial lift for every well in every period. Equation (30) describes the pressure developed by the ESP unit given the well inflow and the alternating current (AC) frequency controlling the motor. The function employs a polynomial approximation of a characteristic provided in the chosen pump documentation. The corrections are also applied to the curve given the variable AC frequency. The efficiency function (30) describes the total ESP system efficiency, a characteristic partly obtained from the curves from the equipment documentation and partly from the engineering reference sources like (Takács, 2009) and (Centrilift, 1997). Again, a polynomial approximation of the efficiency function is employed, with consideration of the variable AC frequency. Constraint (32) sets a technological limitation for the variable frequency drive used with the pumps. To estimate the energy used to lift the fluid from the formation to the processing unit, a formula of hydraulic lift power (33) adopted from (Takács, 2009) is used, and this concludes the evaluation of the artificial lift efficiency.

Finally, the pressure difference between the bottomhole and wellhead of each well is obtained in (34), given the chosen production operating mode for each period.

2.5. Uncertainty resolution

The following set of constraints (35)–(39) compute the values of the state binary variables that cor-

$$z_{i,t,s}^{compl} = \sum_{\tau=1}^{t} z_{i,\tau,s}^{W} \cdot \sum_{m=1}^{NM} \left[\sum_{\tau=1}^{t} \sum_{k=1}^{NMC} z_{m,k,\tau,s}^{M} \cdot \sum_{\tau=1}^{t} \sum_{l=1}^{NJC} z_{i,m,l,\tau,s}^{J} \cdot \sum_{\tau=1}^{t} \sum_{r=1}^{NFC} z_{m,r,\tau,s}^{MF} \right],$$
$$\forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{23}$$

$$T_s^{WB} \cdot \left(1 - z_{t,s}^{flow}\right) \leq \sum_{\tau=1}^{t} \Delta_t \cdot z_{t,s}^{prod} \leq T_s^{WB} - 1 + B \cdot \left(1 - z_{t,s}^{flow}\right), \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{24}$$

$$q_{i,t,s} \leq B \cdot z_{i,t-1,s}^{compl} \cdot z_{t,s}^{flow}, \quad \forall i \in \{1..NW\}, \forall t \in \{2..T\}, \forall s \in \{1..NS\} \tag{25}$$

$$\varepsilon \cdot z_{t,s}^{prod} \leq \sum_{i=1}^{NW} q_{i,t,s} \leq B \cdot z_{t,s}^{prod}, \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{26}$$

$$p^R - p_{i,t,s}^{BH} = \sum_{j=1}^{NW} \alpha_{i,j,s} \cdot q_{j,t,s}, \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{27}$$

$$p_{i,t,s}^{BH} \geq p^{BHmin}, \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{28}$$

$$oc_{i,t,s} \cdot q_{i,t,s} \leq oc_{i,t-1,s} \cdot q_{i,t-1,s} - \sigma_{i,s} \cdot oc_{i,t-1,s} \cdot q_{i,t-1,s}, \quad \forall i \in \{1..NW\}, \forall t \in \{2..T\}, \forall s \in \{1..NS\} \tag{29}$$

$$p_{i,t,s}^{TD} = \rho \cdot g \cdot F_i^{TDH}\left(q_{i,t,s}, f_{i,t,s}^{ESP}\right), \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\};$$
$$F_i^{TDH}(q,f) = \left(a_2^{TDH} \cdot \left(q \cdot \frac{f_0}{f}\right)^2 + a_1^{TDH} \cdot q \cdot \frac{f_0}{f} + a_0^{TDH}\right) \cdot \left(\frac{f}{f_0}\right)^2 \tag{30}$$

$$\eta_{i,t,s}^{ESP} = F_i^{eff}\left(q_{i,t,s}, f_{i,t,s}^{ESP}\right), \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\};$$
$$F_i^{eff}(q,f) = -\eta^{max}\left(\frac{2 \cdot f_0}{q^{max} \cdot f}\right)^2 \cdot q \cdot \left(q - q^{max} \cdot \frac{f}{f_0}\right) \tag{31}$$

$$f_{i,t,s}^{ESP} \leq f^{max}, \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{32}$$

$$N_{i,t,s}^{HL} = 10^{-3} \cdot \frac{q_{i,t,s}}{24 \cdot 60 \cdot 60} \cdot \left[\rho \cdot g \cdot \left(H_i + H^{sea}\right) - p_{i,t,s}^{BH}\right] \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{33}$$

$$p_{i,t,s}^{BH} - p_{i,t,s}^{WH} \geq \rho \cdot g \cdot H_i + p^{fr}\left(q_{i,t,s}, TD_i, H_i\right) - p_{i,t,s}^{TD}, \quad \forall i \in \{1..NW\}, \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{34}$$

$$N_{t,s}^{wells} = \sum_{i=1}^{NW} \sum_{\tau=1}^{t} z_{i,\tau,s}^{compl}, \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{35}$$

$$N^{init} \cdot \left(1 - \xi_{t,s}^{init.plan}\right) \le N_{t+1,s}^{wells} \le N^{init} - 1 + B \cdot \left(1 - \xi_{t,s}^{init.plan}\right), \quad \forall t \in \{1..(T-1)\}, \forall s \in \{1..NS\} \tag{36}$$

$$N^{ext} \cdot \left(1 - \xi_{t,s}^{ext.plan}\right) \le N_{t+1,s}^{wells} \le N^{ext} - 1 + B \cdot \left(1 - \xi_{t,s}^{ext.plan}\right), \quad \forall t \in \{1..(T-1)\}, \forall s \in \{1..NS\} \tag{37}$$

$$T^{\dagger} \cdot \left(1 - \xi_{t,s}^{maturity}\right) \le \sum_{\tau=1}^{t} \Delta_t \cdot z_{t,s}^{prod} \le T^{\dagger} - 1 + B \cdot \left(1 - \xi_{t,s}^{maturity}\right), \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{38}$$

$$T^{\ddagger} \cdot \left(1 - \xi_{t,s}^{wbt}\right) \le \sum_{\tau=1}^{t} \Delta_t \cdot z_{t,s}^{prod} \le T^{\ddagger} - 1 + B \cdot \left(1 - \xi_{t,s}^{wbt}\right), \quad \forall t \in \{1..T\}, \forall s \in \{1..NS\} \tag{39}$$

respond to the learning process. These variables will further allow identifying the pairs of scenarios indistinguishable from the point of view of the accumulated knowledge by the end of each period, which in its turn influences the decisions that are to be made. The constraint (35) counts the number of wells drilled and completed up to the end of each period. The constraints (36) attains the proper value for the variable indicating whether the project is in the initial development phase or past it. The constraints (37) calculate a value for the variable indicating whether the project has moved to the phase of extended field development, i.e. more than N^{init} wells are being drilled. The constraints (38) account for the binary state variable indicating whether the field is mature enough to estimate its recoverable reserves. The con-

straints (39) account for the binary state variable indicating whether the field is mature enough to evaluate the end of production due to the water breakthrough.

Tab. 4 describes the scenario subsets *M1...M7* which differ only in the given uncertainty kinds specified with checkmarks in the table. Further, the constraints (40)–(46) reducing the decision space due to the scenario pairs (*s, s'*) being indistinguishable at certain time periods, are provided.

Decision "reduction". Conditional non-anticipativity constraints (40)–(46) reveal if any scenario pair is indistinguishable during the multi-stage decision-making process, based on the scenario subsets in Tab. 4.

Tab. 4. Scenario sets that differ only in the specified parameters

PARAMETER	M_1	M_2	M_3	M_4	M_5	M_6	M_7
Productivity and interactions	✓			✓	✓		✓
Decline slopes (recoverable reserves)		✓		✓		✓	✓
Water breakthrough time			✓		✓	✓	✓

$$\left(1 - \xi_{t,s}^{init.plan}\right) + z_{s,s',t} \ge 1, \ \xi_{t,s}^{init.plan} \ge z_{s,s',t}, \quad \forall t \in \{2..T\}, (s,s') \in M_1 \tag{40}$$

$$\left(1 - \xi_{t,s}^{init.plan}\right) + \left(1 - \xi_{t,s}^{maturity}\right) + z_{s,s',t} \ge 1, \ \xi_{t,s}^{init.plan} \ge z_{s,s',t}, \ \xi_{t,s}^{maturity} \ge z_{s,s',t}, \ \forall t \in \{2..T\}, (s,s') \in M_2 \tag{41}$$

$$\left(1-\xi_{t,s}^{wbt}\right)+z_{s,s',t}\geq1,\ \xi_{t,s}^{wbt}\geq z_{s,s',t},\quad \forall t\in\{2..T\},(s,s')\in M_3 \tag{42}$$

$$\left(1-\xi_{t,s}^{init.plan}\right)+\left(1-\xi_{t,s}^{ext.plan}\right)+\left(1-\xi_{t,s}^{maturity}\right)+z_{s,s',t}\geq1,$$
$$\xi_{t,s}^{init.plan}\geq z_{s,s',t},\ \xi_{t,s}^{ext.plan}\geq z_{s,s',t},\ \xi_{t,s}^{maturity}\geq z_{s,s',t},\ \forall t\in\left\{2..T\right\},\left(s,s'\right)\in M_4 \tag{43}$$

$$\left(1-\xi_{t,s}^{init.plan}\right)+\left(1-\xi_{t,s}^{wbt}\right)+z_{s,s',t}\geq1,\xi_{t,s}^{init.plan}\geq z_{s,s',t},\xi_{t,s}^{wbt}\geq z_{s,s',t},\forall t\in\{2..T\},(s,s')\in M_5 \tag{44}$$

$$\left(1-\xi_{t,s}^{ext.plan}\right)+\left(1-\xi_{t,s}^{maturity}\right)+\left(1-\xi_{t,s}^{wbt}\right)+z_{s,s',t}\geq1,$$
$$\xi_{t,s}^{ext.plan}\geq z_{s,s',t},\ \xi_{t,s}^{maturity}\geq z_{s,s',t},\ \xi_{t,s}^{wbt}\geq z_{s,s',t},\ \forall t\in\left\{2..T\right\},\left(s,s'\right)\in M_6 \tag{45}$$

$$\left(1-\xi_{t,s}^{init.plan}\right)+\left(1-\xi_{t,s}^{ext.plan}\right)+\left(1-\xi_{t,s}^{maturity}\right)+\left(1-\xi_{t,s}^{wbt}\right)+z_{s,s',t}\geq1,$$
$$\xi_{t,s}^{init.plan}\geq z_{s,s',t},\ \xi_{t,s}^{ext.plan}\geq z_{s,s',t},\ \xi_{t,s}^{maturity}\geq z_{s,s',t},\ \xi_{t,s}^{wbt}\geq z_{s,s',t},\ \forall t\in\left\{2..T\right\},\left(s,s'\right)\in M_7 \tag{46}$$

Now that the conditions for the decision space reduction (variable $z_{s,s',t}$ values) are provided, the similarities in the decisions across the indistinguishable scenario sets may be determined in (47).

Finally, the initial non-anticipativity constraints (48) are presented. They represent the lack of knowledge of the deposit's geophysics at the very beginning of the decision-making process. For small examples, these constraints are usually needed only for the period $t=1$. However, for larger examples, it might take more periods until the very first uncertainty kind gets resolved (for our model, it is the uncertainty in the reservoir productivity). That is why below, the constraints are written down for the general case, where *INAC* is a set of scenarios and periods for whom these initial non-anticipativity constraints are required.

$$z_{i,t,s'}^{W}-B\cdot(1-z_{s,s',t})\leq z_{i,t,s}^{W}\leq z_{i,t,s'}^{W}+B\cdot(1-z_{s,s',t}),\qquad \forall i\in\left\{1..NW\right\},\forall t\in\left\{2..T\right\},(s,s')\in\left\{1..NS\right\}$$
$$z_{m,k,t,s'}^{M}-B\cdot(1-z_{s,s',t})\leq z_{m,k,t,s}^{M}\leq z_{m,k,t,s'}^{M}+B\cdot(1-z_{s,s',t}),\qquad \forall m\in\left\{1..NM\right\},\forall k\in\left\{1..NMC\right\},\forall t\in\left\{2..T\right\},(s,s')\in\left\{1..NS\right\}$$
$$z_{i,m,l,t,s'}^{J}-B\cdot(1-z_{s,s',t})\leq z_{i,m,l,t,s}^{J}\leq z_{i,m,l,t,s'}^{J}+B\cdot(1-z_{s,s',t}),\quad \forall i\in\left\{1..NW\right\},\forall m\in\left\{1..NM\right\},\forall l\in\left\{1..NJC\right\},\forall t\in\left\{2..T\right\},(s,s')\in\left\{1..NS\right\} \tag{47}$$
$$z_{m,r,t,s'}^{MF}-B\cdot(1-z_{s,s',t})\leq z_{m,r,t,s}^{MF}\leq z_{m,r,t,s'}^{MF}+B\cdot(1-z_{s,s',t}),\qquad \forall m\in\left\{1..NM\right\},\forall r\in\left\{1..NFC\right\},\forall t\in\left\{2..T\right\},(s,s')\in\left\{1..NS\right\}$$
$$z_{\gamma,t,s'}^{PCE}-B\cdot(1-z_{s,s',t})\leq z_{\gamma,t,s}^{PCE}\leq z_{\gamma,t,s'}^{PCE}+B\cdot(1-z_{s,s',t}),\qquad \forall\gamma\in\left\{1..NEC\right\},\forall t\in\left\{2..T\right\},(s,s')\in\left\{1..NS\right\}$$

$$z_{i,t,s}^{W}=z_{i,t,s'}^{W}\qquad \forall i\in\left\{1..NW\right\},\left(s,s',t\right)\in INAC$$
$$z_{m,k,t,s}^{M}=z_{m,k,t,s'}^{M},\qquad \forall m\in\left\{1..NM\right\},\forall k\in\left\{1..NMC\right\},\left(s,s',t\right)\in INAC$$
$$z_{i,m,l,t,s}^{J}=z_{i,m,l,t,s'}^{J},\qquad \forall i\in\left\{1..NW\right\},\forall m\in\left\{1..NM\right\},\forall l\in\left\{1..NJC\right\},\left(s,s',t\right)\in INAC$$
$$z_{m,r,t,s}^{MF}=z_{m,r,t,s'}^{MF},\qquad \forall m\in\left\{1..NM\right\},\forall r\in\left\{1..NFC\right\},\left(s,s',t\right)\in INAC \tag{48}$$
$$z_{\beta,s}^{FPSO}=z_{\beta,s'}^{FPSO},\qquad \forall\beta\in\left\{1..NPC\right\},\left(s,s',t\right)\in INAC$$
$$z_{\gamma,t,s}^{PCE}=z_{\gamma,t,s'}^{PCE},\qquad \forall\gamma\in\left\{1..NEC\right\},\left(s,s',t\right)\in INAC$$

3. Computational experiment. Problem setting and optimisation algorithm

For the computational experiment, a marginal oilfield of five potential wells is considered. The planning horizon is ten years, divided into five problem stages (two years each). Eight scenarios representing geophysical uncertainty are provided in Tab. 5. All the scenarios are deemed equally probable. The matrix A_1 represents rather high reservoir productivity and at the same time high influence between the wells. These conditions allow flow rates from the five wells to be up to 500m³/d. The matrix A_2 stands for a relatively low reservoir productivity and at the same time low influence between the wells. These conditions allow flow rates from the five wells to be up to 200m3/d. Vectors S_1 represents relatively fast production decline: the decline rates are ranging from $4·10^{-4}$ to $9·10^{-4}$1/d. Vectors S_2, on the contrary, represents slow decline: the rates are ranging from $2·10^{-4}$ to $4.5·10^{-4}$1/d. Water breakthrough time is estimated to be either T_1^{wbt}=8 years or T_2^{wbt}=6 years.

According to Tab. 4 and 5, the subsets $M1...M7$ include the following scenario pairs:

- M1: (1,2); (3,4); (5,6); (7,8)
- M2: (1,3); (2,4); (5,7); (6,8)
- M3: (1,5); (2,6); (3,7); (4,8)
- M4: (1,4); (2,3); (5,8); (6,7)
- M5: (1,6); (2,5); (3,8); (4,7)
- M6: (1,7); (2,8); (3,5); (4,6)
- M7: (1,8); (2,7); (3,6); (4,5)

The initial development plan includes drilling two wells, while the extended plan implied drilling all five wells. The production decline slopes are considered to be revealed after two years from the start of the production. The same timeline applies to the revealing of water breakthrough time. Initial non-anticipativity constraints for such problem setting are required during the first period (years one and two) only. Conditional non-anticipativity constraints work during the second period (years three and four). From t=3 and further (years five to ten) all

the relevant geophysical characteristics are considered known.

The model was run with the MIDACO solver in the Matlab environment. This solver implements a black-box optimisation algorithm for large-scale mixed-integer nonlinear problems (MINLP). The employed algorithm combines a meta-heuristic ant-colony optimisation with an Oracle penalty method of constraint handling. The model was run 30 times, and each time the result of the previous run was used as a starting point. The techniques suggested in the user manual of the solver were employed during the runs to avoid getting stuck in local optima and to refine the obtained solution. The results of the modelling computations are presented in Tab. 6.

4. Discussion of the modelling results

As seen in the results, the problem setting for the marginal field taken as an example is viewed as generally favourable by the algorithm both in the deterministic and the stochastic settings.

For the stochastic problem setting, the initial production plan (drilling two wells) is initiated for all the scenarios. Moreover, the algorithm finds it profitable to drill at least three wells in every single scenario, even for the cases of small production rates and fast production decline (s=2 and s=6).

For the processing platform, the algorithm chooses the largest available processing capacity (1000 m³/d), which turns out suitable for the initial production plan as well as for the scenarios with low reservoir productivity ($s \in \{2,4,6,8\}$). For the scenarios with higher reservoir productivity ($s \in \{1,3,5,7\}$) and thereby higher production rates, the algorithm suggests a capacity expansion of 2000m³/d to be installed during the second period, after the uncertainty in productivity has become resolved. Also, for the scenarios with high production rates, the algorithm suggests drilling a second cluster of wells: two more wells connected to a newly installed manifold.

Tab. 5. Scenarios for the computational example

Parameter	s=1	s=2	s=3	s=4	s=5	s=6	s=7	s=8
Productivity and interactions matrix	A_1	A_2	A_1	A_2	A_1	A_2	A_1	A_2
Decline slopes vector	S_1	S_1	S_2	S_2	S_1	S_1	S_2	S_2
Water breakthrough time	T_1^{wbt}	T_1^{wbt}	T_1^{wbt}	T_1^{wbt}	T_2^{wbt}	T_2^{wbt}	T_2^{wbt}	T_2^{wbt}

Tab. 6. Modelling results

SCENARIO	DESCRIPTION	CAPEX	OPEX	NPV
s=1	During t=1, drill wells W1, W3, install manifold M1 with 4 connections, install corresponding jumpers J11, J31 with diameter 10'', and flowline MF1 with diameter 20''. Install FPSO with processing capacity 1000m³/d. Begin production. During t=2, drill W2, install J21. During t=3, drill W4 and W5, install M2 with 2 connections, install J42, J52 (10''), and MF2 (20''). Expand processing capacity by 2000m³/d	141.709	37.766	883.906
s=2	During t=1, drill W1, W3, install M1 (4 conn.), install J11 and J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000 m³/d. Begin production. During t=2, drill W2, install J21 (4'')	90.209	0.75	154.491
s=3	During t=1, drill W1, W3, install M1 (4 conn.), install J11, J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000 m³/d. Begin production. During t=2, drill W2, install J21 (10''). During t=3, drill W4, W5, install M2 (2 conn.), install J42 and J52 (10''), and MF2 (20''). Expand processing capacity by 2000m³/d	139.151	41.237	889.131
s=4	During t=1, drill W1, W3, install M1 (4 conn.), install J11, J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000m³/d. Begin production. During t=2, drill W2, install J21 (4''). During t=3, drill W4, W5, install M2 (2 conn.), install J42, J52 (4''), and MF2 (8'')	135.038	0.58	198.753
s=5	During t=1, drill W1, W3, install M1 (4 conn.), install J11, J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000m³/d. Begin production. During t=2, drill W2, install J21 (10''). During t=3, drill W4, W5, install M2 (2 conn.), install J42, J52 (10''), and MF2 (20''). Expand processing capacity by 2000m³/d. For t=5, no production; water breakthrough.	139.151	34.470	613.795
s=6	During t=1, drill W1, W3, install M1 (4 conn.), install J11 and J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000m³/d. Begin production. During t=2 drill W2, install J21 (4''). For t=5, no production; water breakthrough.	90.209	0.48	113.985
s=7	During t=1, drill W1, W3, install M1 (4 conn.), install J11 and J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000m³/d. Begin production. During t=2, drill W2, install J21 (10''). During t=3, drill W4, W5, install M2 (2 conn.), install J42, J52 (10''), and MF2 (20''). Expand processing capacity by 2000m³/d. For t=5, no production; water breakthrough.	141.709	19.617	584.600
s=8	During t=1, drill W1, W3, install M1 (4 conn.), install J11 and J31 (10''), and MF1 (20''). Install FPSO with processing capacity 1000m³/d. Begin production. During t=2, drill W2, install J21 (4''). During t=3, drill W4, W5, install M2 (2 conn.), install J42, J52 (4''), and MF2 (8''). For t=5, no production; water breakthrough.	135.038	0.31	143.049
				ENPV: 447.714
Average values solution	During t=1, drill W1, W3, install M1 (4 conn.), install J11 and J31 (4''), and MF1 (8''). Install FPSO with processing capacity 500m³/d. Begin production. During t=2, drill W2, W4, W5, install M2 (2 conn.), install J21, J42, J52 (4''), and MF2 (8''). Expand processing capacity by 1000m³/d	136.083	32.537	274.863

These things get done in the third period, after the uncertainty in production decline, and water breakthrough becomes resolved.

When it comes to the capacity choices for the pipeline segments, that is the corresponding diameters, the initial choice (for t=1) is larger diameters. This is attributed mainly to the non-anticipativity constraints. When the very first decisions are made, there is no information on the flow rates from the wells draining the reservoir. The set of constraints (11)–(16) monitors the pressure drops along the gathering system to ensure the necessary pressure at the riser base. Pipeline segment diameters influ-

ence these pressure drops dramatically: refer to constraint (12) where the value of the diameter in the pressure loss function is included with the power of −4.89. Thereby, the algorithm chooses the larger diameters to get the pressure drops to be rather small. When the productivity is revealed, the diameter choice is made with respect to the reservoir draining rates. This is how the choice is made for the jumper diameter from Well 2 to Manifold 1 in period t=2, and also for the second cluster of wells (Well 4, Well 5, Manifold 2, Flowline 2) in period t=3. For low production rates (s∈{2,4,6,8}), the lower pipeline diameters are chosen for both jumpers and flowlines,

and for the higher rates ($s \in \{1,3,5,7\}$), the higher capacity options are selected.

The last line in Tab. 6 describes the solution for the model, where instead of the stochastic parameters their average evaluations were assumed. One may observe that unlike the stochastic solution, this average values solution does not wait for the 3rd period to finish the field development. However, given that the values of the productivities and well influence factors are considered as the average values, the resulting production rates are rather low, only up to 270m³/d. This explains the fact the NPV of average values solution is lower than the expected NPV (ENPV) of the stochastic one. In the stochastic solution, there four outcomes that are far better than the outcome of the deterministic solution, there are two outcomes that are a little worse than the deterministic one and only two that are far worse than in the average values case. Therefore, generally, the stochastic formulation of the development and production planning problem views the problem setting for this example as more favourable than the deterministic formulation.

Conclusions

The strategic planning problem of optimising oilfield development and production operations has been addressed in this research with consideration of geophysical uncertainty. It has been demonstrated that the approach to planning under uncertainty and the approach of planning based on deterministic modelling with average values of stochastic parameters reveal a different attitude to the project lifecycle. The results of the stochastic modelling approach prove to be more nuanced and considerate to learning about the uncertain behaviour of the planned process and applying the gained knowledge to achieve good results. These characteristic attributes of the stochastic modelling make it beneficial for engineering departments working with planning the oilfield development activities in nonconventional environments. It is especially relevant for development projects where marginal fields and reservoirs are present. These small deposits are usually not studied in detail; however, the decision whether to develop them or not needs to be made.

With regards to the model composed in this study and the example, this research demonstrates certain limitations and drawbacks regarding the sto-

chastic parameter representation. First of all, the 2-point distributions for the values of the stochastic parameters given in the example may not be good enough for some cases. Second, the stochastic representation of the reservoir may prove to benefit from the use of the specialised reservoir-simulating software that may work in conjunction with the black-box optimisation algorithm.

Generally speaking, in the modern context of energy resources and the pursuit of renewable energy, it may be of interest for planning projects in the petroleum sector to consider the energy market situation by introducing the hydrocarbon market price as a stochastic parameter. It may prove to have some bearing on whether to develop certain reserves or not, or whether it is best to produce the petroleum with rather low rates despite the possibility of producing faster.

From the modelling perspective, any of the mentioned ideas for further research in this domain will definitely increase the complexity of the model. The model as presented in this study is already a large nonlinear non-convex combinatorial problem, and even for a small example, it has thousands of variables and constraints. Thus, adding more complexity to the model would require a specialised and highly efficient heuristic algorithm that should perhaps include some decomposition procedures.

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