Advances in Science and Technology
Research Journal

Advances in Science and Technology Research Journal 2023, 17(3), 257–268
https://doi.org/10.12913/22998624/166567
ISSN 2299-8624, License CC-BY 4.0

Received: 2023.03.27
Accepted: 2023.05.18
Published: 2023.06.01

On a Possibility of a Side Impact Collision Analysis Based on the Coefficients of Restitution

Jerzy Kisilowski[1], Jarosław Zalewski[2], Rafał Kowalik[3*]

[1] Kazimierz Pulaski University of Technology and Humanities, Malczewskiego 29, 26-660 Radom, Poland
[2] Warsaw University of Technology, Pl. Politechniki 1, 00-661 Warsaw, Poland
[3] Military University of Aviation, Dywizjonu 303 No. 35, 08-521 Dęblin, Poland
* Corresponding author's e-mail: r.kowalik@law.mil.pl

ABSTRACT

In this paper a potential use of the tangential coefficient of restitution has been analyzed on the basis of a computer simulation regarding a typical side oblique collision of two motor vehicles. The simulation was performed in PC-Crash 8.0 software. The results obtained for a side impact have been analyzed along with the tangential phenomena occurring between the colliding vehicles. This simulation was repeated for the three adopted values of the coefficient of restitution. Next the adopted coefficient of restitution was divided into two components, one normal and one tangential, which has been described in further parts of the paper. The obtained results were then compared with the certain analytical calculations basing on the vehicle crash model for a vehicles performing a planar motion but with the phenomena between their bodies included. Also a rough surface collision theory was used. The aim of this analysis was to examine whether the coefficient of restitution divided into two perpendicular components will alter the results obtained from the analytical calculations and if it is worthy considering such division in analyzing the real accidents in road traffic. Also it has been considered whether such complication of a vehicle crash model is useful and necessary.

Keywords: impact collision, car accidents, coefficients of restitution

INTRODUCTION

Modeling of road vehicle collisions developed over the years. One of the problems mostly considered was a determination of the coefficient of restitution which is an important parameter when a vehicle crash is considered, either in planar or in resultant motion. Several papers were devoted to determine or at least specify a magnitude of this coefficient either for the frontal or for the side impact collision [2, 4] where the authors attempted to review the current trends in collision modeling, or even for the high speed collisions [15] where the author assumed the dependence of the restitution coefficient on the mutual location of both vehicles and the speed. Some of them based on the simplified tools, such as the spring – mass – damper systems as in [25].

In [5] and [6] the stiffness and the mass based coefficient of restitution was considered as a basis to accident reconstruction, along with the basic Newton's law of motion. Of course, the coefficient of restitution is not the only parameter of a road accident as it was presented in [3] or [11] where other necessary parameters of a collision were identified. In order to understand the collision process via the collision modeling research was conducted in various aspects beginning with a mathematical model based on a planar motion of the vehicles involved [7, 9] and ending with some more complex models, containing analyzes on three dimensional motion during the collision [1, 14].

Some specific problems related to the collision modeling or reconstruction also were the subject of multiple works. One of them is the impact analysis presented in [8] where the problem of interlocking of the colliding parts of vehicles was considered, or in [10] where the equations of impulse and momentum were used.

Other problems can be related to the loss of the kinetic energy of a vehicle during a collision [12] and the uncertainty of the collected results related to the velocity change and the loss of the kinetic energy [13].

An important aspect of the collision theory seems to be determination of the point of application of the impact force impulse at the beginning of a collision [19] as well as the restitution of the force impulses during a collision [24].

Also several works undertaken some various aspects of the use and determination of a restitution coefficient [30], such as dividing it into two components: one parallel (tangential) and one perpendicular (normal) to the common plane of the mutual contact of vehicles (e.g. [16]) as well as dependence of the coefficient of restitution on the initial velocity of a collision [36].

One of the important additions to the collision theory is performing the computer simulations used to verify the analytical calculations during accident reconstruction. The use of a specific PC-Crash software to create a basis to verification of the simulation results was presented in [40] whereas in [23] the same software was used to analyze the results of pedestrian impact collisions.

The coefficient of restitution was also determined on the basis of real crash tests as in [26] and plays an important role in the process of collision reconstruction, which can be observed in some selected works, such as [32] and [33].

In this paper a specific simulation of a side oblique vehicles collision was carried out (Fig. 1) in order to analyze the influence of the adopted coefficient of restitution in two directions (the normal and the tangential) versus the mutual plane of a crash (the common vertical plane of the contact between the colliding vehicles) on the results of calculations with the use of a mathematical model based on [37]. Preliminary analysis has been presented in [38] but in this paper some certain results were obtained and compared with the results of the crash simulations prepared in PC Crash 8.0 software.

From the above presented literature analysis it seems that only a minor part of works on vehicle crash modeling includes the tangential coefficient of restitution [20, 21] or the use of a finite element method [35], and even less takes the resultant motion during the crash into account [38].

There were other approaches towards collision modeling, such as [17] where a throw concept was used in a frontal collision analysis or predicting the vehicle motion after the so called 'light collisions' meaning the low speed collisions. Of course there is a group of works considering the collision theory in a general scope of applications to motor vehicles, such as [27] and [28]. A broad area of collision mechanics related to motor vehicles and the reconstruction of road accidents was considered in [18, 34].

Some papers show attempts to determine the coefficient of restitution [22, 31]. Therefore analysis in this paper focuses on answering the question whether it is useful to consider both the normal and the tangential coefficient of restitution in a crash model with planar motion adopted.

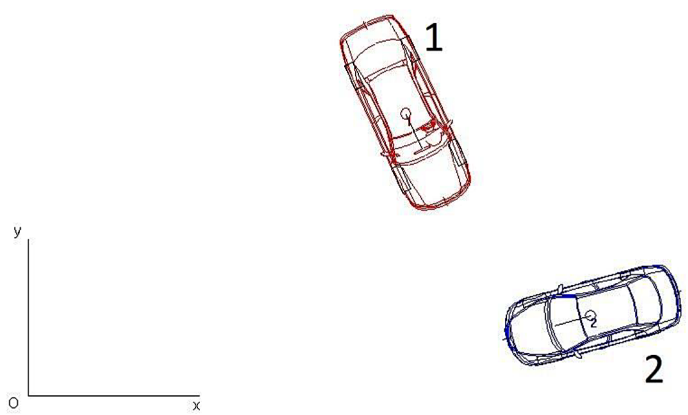

Figure. 1. Mutual position of both vehicles before the collision in PC-Crash (Source: PC-Crash)

Such approach can also enable verification if the adoption of the specific coefficients facilitates obtaining the real values of the parameters of a collision. The approach is up to date in face of the modern software enabling both the complexion of the collision models and the use of computing power to run the simulations of various road accidents. New elements of research in this paper contain the analysis on a potential implementation of the tangential coefficient of restitution into a collision analysis in a simplified way as well as the attempt to specify whether the impulse given in the results of the simulations can be decomposed int two components – one normal and one tangential.

ASSUMPTIONS

Before performing the simulation of a collision presented in Figure 2 where the initial location of both vehicle at the beginning of a collision is shown, some essential assumptions need to be made in relation to the mathematical model of collision which can simplify the discussed problem and enable comparison of the simulation results with the analytical calculations at the same time. These considerations were partially based on [16] and [38] because these works contain the mechanics of collisions in theoretical approach. The adopted assumptions are as follows:

- the bodies of the vehicles involved in the collision are considered as rectangular cuboids having constant mass (before and after the collision) and stiffness. There is also an initial point of contact in the middle of a contact plane between the vehicles (a vertical plane which contains the area of contact of the colliding vehicles). This point is an origin of a local coordinate system *Ont*, where *O* is a geometric center of the collision. Of course this point is only needed to apply the *Ont* system, because in further analysis some parameters will be determined in relation to it;
- vehicles used in the simulations had the linear suspension and their mass-inertia parameters were adopted from the program database containing the real values. Hence, the vehicles No. 1 and 2 reflect the real ones;
- the vehicles moved on a road plane during the collision which occurred on a dry surface where the coefficient of adhesion was $\mu = 0.8$;
- before the collision the vehicle No. 1 was doing 60 km/h, and the vehicle No. 2 – 50 km/h;
- the initial mass of the unladen vehicle No. 1 was 2020 kg, and was enhanced to 2300 kg by the mass of a driver and three passengers;
- the initial mass of vehicle No. 2 was increased from 1500 kg to 1750 kg. Both vehicles did not carry any baggage;
- in order to simulate a collision in which both vehicles are represented by solids rather than plane figures, it was necessary to specify the center of mass in each of the laden vehicles which was adopted in accordance to [29]. As a result the center of mass of the vehicle No. 1 was 0.56 m and No. 2 – 0.57 m above the road plane in a vertical direction;
- the initial adopted values of the coefficients of restitution were $R = 0.1$, $R = 0.05$ and $R = 0.01$ in three separate simulations in PC-Crash. Of course it seems that the values 0.1 and 0.05 are not real due to the fact that a typical collision lasts very short and the typical damages to the vehicles involved are usually rather plastic than elastic. Nevertheless, these values of R were adopted also in order to compare the simulation results for a side impact collision;
- two coefficients of restitution in the mathematical model describing the collision have been

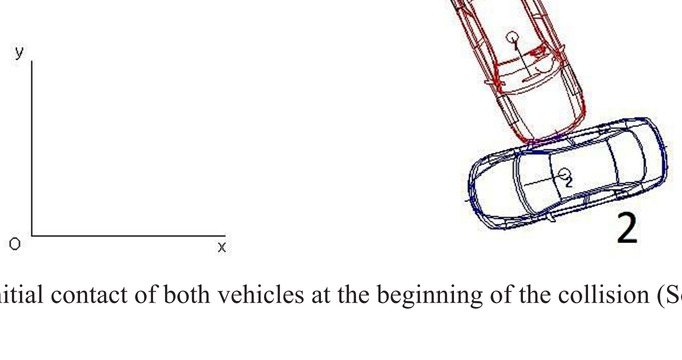

Figure 2. The initial contact of both vehicles at the beginning of the collision (Source: PC-Crash)

adopted as in [16]: R_n which is the coefficient in the normal direction and R_t in the tangential direction in relation to the plane of impact (a mutual plane of contact of the vehicles which is vertical and contains the area in which the vehicles collide).

It should be stressed that in PC-Crash the coefficient of restitution is only considered in the direction normal to the plane of impact.

The presented example of a collision is side and oblique, which means that impulses of the collision force has to be taken into account in the tangential direction as well. The mutual plane of impact is perpendicular to the road surface and at the same time tangent to the colliding vehicles at the point of an initial contact. The mathematical model describing such collisions can be adopted from, e.g. [16] and [38].

SIMULATION DESCRIPTION

To simplify a vehicle model used in PC-Crash in order to make it easy to use in a mathematical model of a collision, let us regard the vehicle's body as a set of quasi-rigid but deformable cuboids having a predefined mass, stiffness and the moments of inertia. The loss of mass of each cuboid during the collision is omitted.

In a PC-Crash manual available in the Internet it was stated that the stiffness of the vehicle models is different for various parts of each vehicle [37], which could be contrary to what has been assumed earlier. The wheels are half as stiff as a vehicle's body, while the roof and the side pillars are 75% as stiff as the lower vehicle's body [37]. Of course, in order to simplify the considerations presented here, the vehicles, especially in case of the analytical calculations were assumed as equally stiff, because the main problem was to determine the most crucial parameters of the adopted collision in the aspect of side impact and the coefficients of restitution. The main aim was to perform the simulation of a collision with the predetermined selected parameters, and to verify the obtained results with the use of the analytical calculations based on mathematical modeling and the already adopted values of the restitution coefficient.

The simulation of a side impact collision for the three adopted values of the restitution coefficient (R) has been conducted in accordance with, e.g. in [30]. For each case of R (0.01, 0.05 and 0.1) the simulation time was 2 s and the duration of the collision was about 0.25 s.

SIMULATION RESULTS

Results of the simulations for each value of R have been presented in Table 1. They contain both the translational and the angular velocities determined before and after the collision, with the post-collision values marked with an apostrophe. Also the mass and the moments of inertia for both vehicles along with the resultant impact force impulses have been presented, because they were the key parameters needed to complete the analysis of vehicles collisions in planar motion. Basing on these results a further analysis with the use of certain calculations will be presented.

From the presented results some preliminary conclusions can be made. At first it should be

Table 1. Results of a side oblique collision simulations for three different values of the coefficient of restitution

Vehicle No.	1			2		
	initial parameters					
Mass [kg]	m_1=2300			m_2=1750		
Moment of inertia relative to the vertical axis passing through the center of mass of a vehicle [kg/m²]	I_1=4277			I_2=2729		
Duration of the collision	0.23s					
Coefficient of restitution	R=0.1		R=0.05		R=0.01	
Vehicle No.	1	2	1	2	1	2
Translational speed before the crash [km/h]	V_1=53.2	V_2=46.5	V_1=57.8	V_2=47.5	V_1=60	V_2=50
Angular velocity before the crash [1/s]	ω_1=-0.88	ω_2=-0.8	ω_1=-0.57	ω_2=-0.42	ω_1=0	ω_2=0
Translational speed after the crash [km/h]	V_1'=35.2	V_2'=45.3	V_1'=36.7	V_2'=44.3	V_1'=38	V_2'=43.6
Angular velocity after the crash [1/s]	ω_1'=-3.31	ω_2'=-4.73	ω_1'=-3.12	ω_2'=-5.07	ω_1'=-2.96	ω_2'=-5.09
Impulse of the impact force [Ns]	12850		14864		15689	

stressed that the speed before the collision appearing in the first two cases (R=0.1 and R=0.5) was smaller than predefined. It seems that PC Crash, with the use of a feature called 'collision detection' brakes the vehicles right before the collision as if a driver of each vehicle managed to brake slightly before crashing the opposite vehicle. However, a decrease in speed was largest in case of the greatest value of R. Strangely though, in the third case (R=0.01) no preliminary speed decrease occurred. This could induce a hypothesis that the greater value of the restitution coefficient the greater decrease in a preliminary speed before a collision.

Another conclusion may be related to the impulse of an impact force which, as presented in Table 1, had the greater value, the smaller coefficient of restitution was. This agrees with the crash mechanics and the Newton hypothesis, according to which an impulse from the phase of restitution divided by an impulse from the deformation phase produce the coefficient of restitution [16]. Moreover, the coefficient of restitution can in an indirect manner provide some elementary information about vehicles' damages due to a collision. Hence, the lower value of the impulse of a collision force may result from the greater value of a the coefficient of restitution. This in turn may cause less damage to a vehicle taking part in a collision even if the same preliminary parameters are applied but with different values of the given coefficient.

To prove the above considerations the maximum depth of damages to the vehicles taking part in a simulated collision are presented in Table 2 for each adopted value of a coefficient of restitution.

The results presented in Table 2 correspond with the crash mechanics theory and the above considerations. The greater value of the coefficient of restitution provided the lower depth of damages to the vehicles involved in the simulated collision which shows that the coefficient of restitution may partially influence reducing the part of an impulse of an impact force lost due to a deformation of the vehicle's body.

ANALYTICAL VERIFICATION OF THE SIMULATION RESULTS

In order to verify if the analytical calculations are suitable for checking the correctness of the simulation software it is necessary to determine some geometric and trigonometric dimensions. Let us now return to the initial position of both vehicles at the beginning of the collision (Fig. 3).

In Figure 4a the input angles specifying the initial positions of both vehicles at the first moment (the initial contact) of the collision have been presented. If two additional axes, attached at the point of the first contact, would be applied as in Figure 4, both perpendicular, then the angles between them and the axes of symmetry of both vehicle will be as shown in Figure 4. This in turn allows determination of the more necessary angles α and β which will be useful to determine the components of the velocity of both vehicles.

Basing on the simple calculations the sought after angles can be determined:

$\alpha = 90^o - 65^o = 25^o$, and $\beta = 195^o - 180^o = 15^o$

It is important to notice that the axes of symmetry of both vehicles in Figure 4a are not perpendicular, so the α and β angles are only to help determine the further, more necessary collision parameters.

Let us then introduce a local coordinate system Ont at the point of an initial point of a

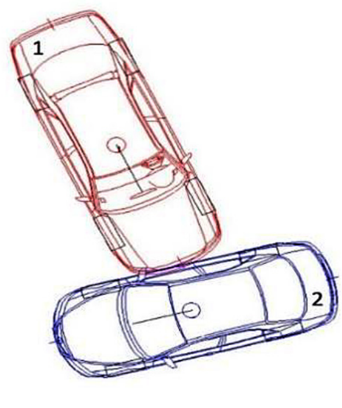

Figure. 3. The initial location of both vehicles at the beginning of a collision – preparation for determining the necessary angles (Source: PC-Crash)

Table 2. Depth of deformation in a vehicle's body at the adopted coefficient of restitution

Depth of the deformation in a vehicle	R = 0.1	R = 0.05	R = 0.01
No. 1 [m]	0.36	0.39	0.39
No. 2 [m]	0.37	0.4	0.42

b)

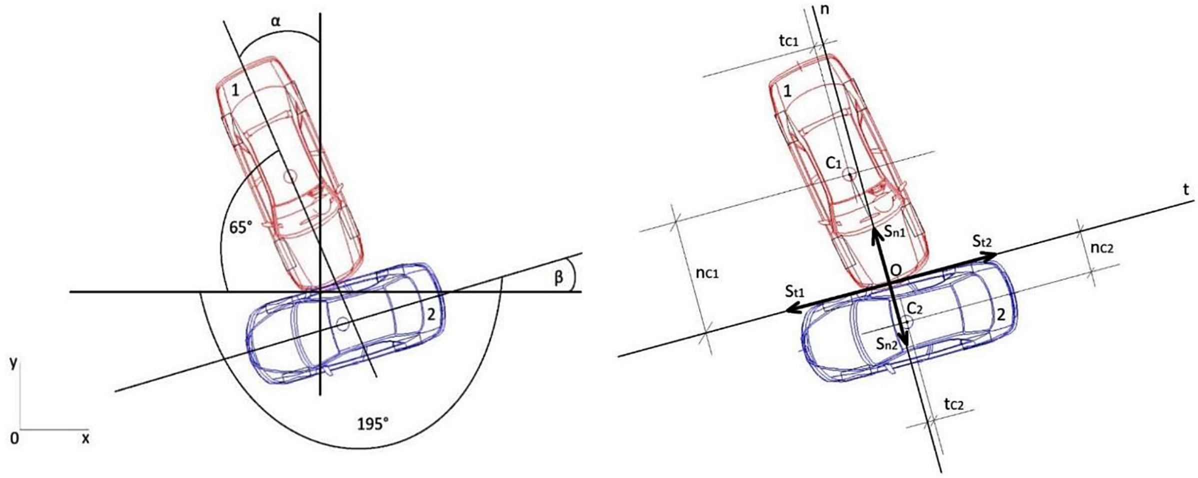

Figure. 4. a) Mutual location of the vehicle 1 and 2 at the initial moment of the collision, b) The coordinate system *Ont* for the side impact collision Source: PC-Crash.

mutual contact between the vehicles, where *O* is the origin of this system (Fig. 4b). This will enable marking the distance between the center of mass of each vehicle and the center of the collision located at the origin of the *Ont* system. These distances at the beginning of the collision, marked with n_{C1}, t_{C1}, n_{C2} and t_{C2} for the vehicle No. 1 and 2 respectively, will be necessary for the equations of motion used to describe this collision. From this figure it is clear that the only angle which will be furtherly used is α, because the vehicle No. 2 is parallel to the *Ot* axis.

In Figure 4b also the impulses of the collision forces respective for each vehicle (S_{n1}, S_{t1}, S_{n2}, S_{t2}) have been presented. They can be presented as scalars because they are only the components of the resultant impulses of a collision force acting on each vehicle. It is worth remembering here that the motion during the collision was assumed as planar.

The distances between the geometric center of a collision (O) and the center of mass of each vehicle has been determined with the use of PC-Crash, and have been presented in Figure 6, this time without the impulses so that the figure can be clearer. The measured distances are:

$$n_{C1} = 2.39 \text{ m}, \; t_{C1} = 0.19 \text{ m},$$
$$n_{C2} = 0.89 \text{ m}, \; t_{C2} = 0.14 \text{ m},$$

with the length of the vehicle No. 1 equaling 5.06 m and No. 2 – 4.7 m.

a) b)

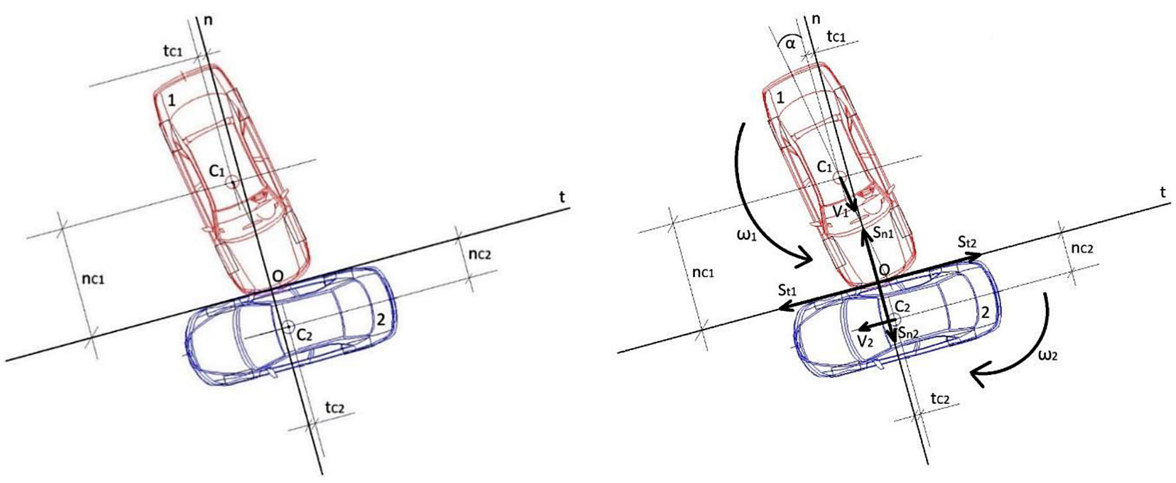

Figure 5. a) Location of the center of mass of each vehicle from O at the beginning of a collision, b) The directions of rotation on a road plane during the collision Source: PC-Crash

In order to complete the scheme of the collision prepared in PC-Crash it is necessary to mark all the parameters needed to create a mathematical model describing the kinematic state of both vehicles during the crash. This has been presented in Figure 5a where the following parameters are marked:
- S_{n1}, S_{t1}, S_{n2}, S_{t2} – components of both impulses of the collision force for the vehicle No. 1 and 2 respectively;
- V_1, V_2 – the speed of the vehicle No. 1 and 2 respectively (at the start the crash);
- ω_1, ω_2 – angular velocity of the vehicle No. 1 and 2 respectively (at the start of the crash);
- n_{C1}, t_{C1}, n_{C2}, t_{C2} – coordinates specifying the center of mass of each vehicle relatively to the geometric center of the analyzed collision;
- α – the angle between the vector of the speed of the vehicle No. 1 and the *On* axis.

In Figure 5b the positive rotation of each vehicle on the road plane has also been marked. It was assumed that during the collision the vehicle No. 1 rotates counterclockwise and the vehicle No. 2 spins clockwise.

To determine the components of the velocity in the normal and the tangential direction versus the plane of the collision (to remind it is a vertical plane of a contact between both vehicles) the α angle was used, i.e. the angle between the velocity vector of the vehicle No. 1 and its longitudinal axis of symmetry which is parallel to the On axis (Fig. 5b). This angle will then be used to distribute of the velocity of the vehicle No. 1 into two components: the normal and the tangential one (Fig. 6). Although the components in Figure 8 are marked with the arrow which does not match the scale of the vectors in Fig. 5b the main purpose of presenting them in such a way was to highlight them as the necessary parameters used in a mathematical model of the presented example of road collision.

If the measured distances along with α are taken into consideration both velocity components of the vehicle No. 1 (Fig. 6) will be:

$$v_{1n} = V_1 cos\alpha,\ v_{1t} = V_1 sin\alpha.$$

There is one simplification regarding the velocity of the vehicle No. 2. Since it is parallel to the Ot axis (Fig. 5b), it can be assumed that its lateral component (v_{2n}) is zero. So the longitudinal component $v_{2t} = V_2$.

In Figure 8 most of the necessary collision parameters have been presented, except for the coefficients of restitution (R_n and R_t) where R_n is the restitution coefficient in the normal and R_t in the tangential direction. It should be mentioned that both symbols of these coefficients have been adopted in accordance with [16].

Before completing the mathematical model of this collision it is necessary to mention that the variables after the collision were marked with an apostrophe. The indexes '1' used in the equations of motion indicate the parameters of the vehicle No. 1, and '2' indicate the parameters of the vehicle No. 2. The indexes 'n' and 't' respectively denote the normal and the tangential direction (along the On and Ot axes respectively).

Using the symbols and the above assumptions the model of the analyzed collision can be represented by a set of the following equations based, among others, on [16]:

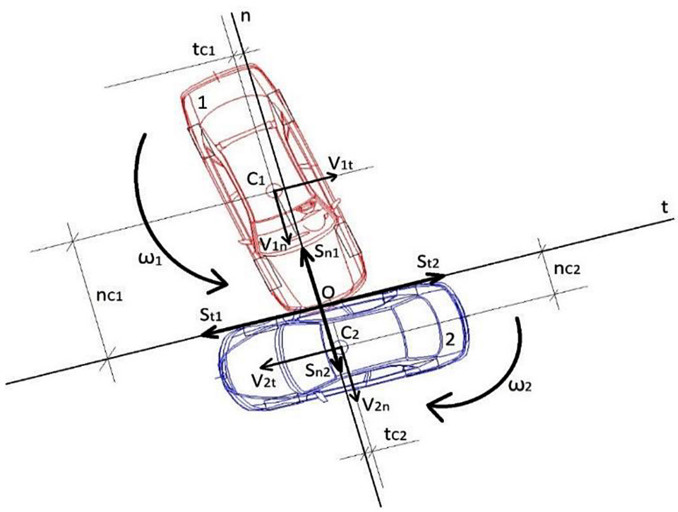

Figure 6. Velocity components and the angular velocities at the start of the collision

a) normally to the impact plane (mutual contact plane):

$$m_1(v_{1n} - v'_{1n}) = S_{n1}, m_2(v'_{2n} - v_{2n}) = -S_{n2}, \quad (1)$$

b) tangentially to the impact plane:

$$m_1(v'_{1t} - v_{1t}) = -S_{t1}, m_2(v_{2t} - v'_{2t}) = S_{t2}, \quad (2)$$

c) in the rotation direction on a road plane, according to the marked angular velocities as in Figure 8. Let us assume that the counterclockwise motion is positive and the clockwise – negative:

$$I_1(-\omega'_1 + \omega_1) = -S_{n1}t_{C1} + S_{t1}n_{C1},$$
$$I_2(\omega'_2 - \omega_2) = -S_{n2}t_{C2} + S_{t2}n_{C2}, \quad (3)$$

Such a set of six equations contains ten unknowns, of which six are the post-collision velocities (v'_{1n}, v'_{1t}, v'_{2n}, v'_{2t}, ω'_1, ω'_2) and four are the components of the impulse of a collision force (S_{n1}, S_{t1}, S_{n2}, S_{t2}), two for each vehicle, as in Figure 6. These components can be determined in a simplified way as follows:

$$S_1 = \sqrt{S_{n1}^2 + S_{t1}^2}, S_2 = \sqrt{S_{n2}^2 + S_{t2}^2} \quad (4)$$

$$S_{n1} = S_1 \cdot cos\alpha, S_{t1} = S_1 \cdot sin\alpha \quad (5)$$

$$S_{n2} = S_2 \cdot cos\alpha, S_{t2} = S_2 \cdot sin\alpha \quad (6)$$

where α is the angle specifying the direction of motion (speed) of the vehicle No. 1 and the *On* axis (Fig. 5b). In this simplified method one can observe that the impulses S_1 and S_2 are equal as the resultant of the normal and the tangential components and the product of a force acting on both vehicles mutually. Hence another simplification resulting from the data obtained in simulations:

$$S_1 = S_2 \quad (7)$$

which can be made on the basis of only one value of the impulse given in the results (Table 1) for the two colliding vehicles.

The impulse of a collision force is given in Table 1 for each configuration of the coefficient of restitution. Since it is the side impact collision, it seems advisable to consider the adoption of the impulse in the direction of motion (speed) of the vehicle No. 1 (Fig. 7a) as a simplifying assumption. Hence, it will be easier to determine the normal (S_n) and the tangential (S_t) component of the impulse of a collision force for each vehicle, as the α angle is also known.

The kinematic state of both vehicles at the end of the collision (post-collision parameters) can then be described by such formulas:

$$v'_{1n} = v_{1n} - \frac{S_{n1}}{m_1}, v'_{2n} = v_{2n} - \frac{S_{n2}}{m_2}, \quad (8)$$

$$v'_{1t} = v_{1t} - \frac{S_{t1}}{m_1}, v'_{2t} = v_{2t} - \frac{S_{t2}}{m_2}, \quad (9)$$

$$\omega'_1 = \omega_1 + \frac{S_{n1}t_{C1} - S_{t1}n_{C1}}{I_1},$$
$$\omega'_2 = \omega_2 + \frac{S_{t2}n_{C2} - S_{n2}t_{C2}}{I_2} \quad (10)$$

DISCUSSION

In Table 3 the results of the analytical calculations based, e.g. on [16] have been presented along with the results obtained in the simulations of the oblique side impact collision and the three adopted values of the restitution coefficient. Also the components of the impulse of a collision force have been

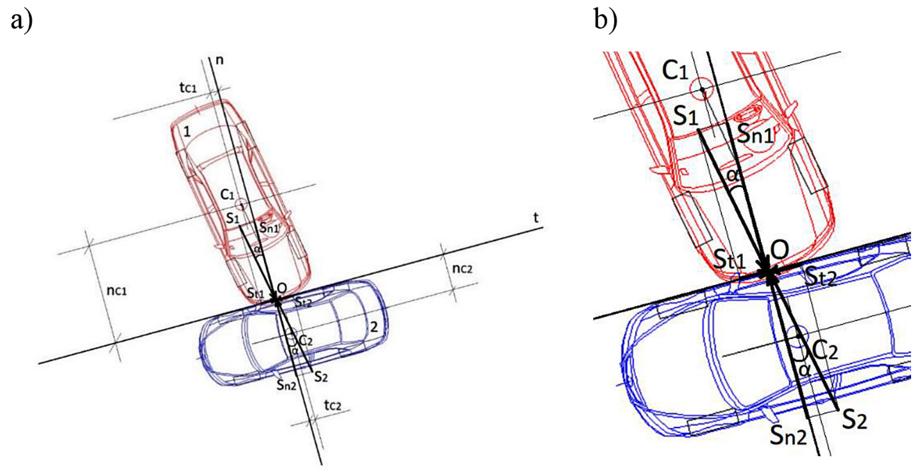

Figure 7. a) The components of the impulses of collision force, b) Magnification of the selected area of Fig. 7a

Table 3. Results of the analytical calculations

Vehicle	1		2	
Coefficient of restitution	R=0.1			
	calculations	simulation	calculations	simulation
Translational speed after the crash [km/h]	V_1'= 33	V_1'= 35.2	V_2'= 42.6	V_2'= 45.3
Angular velocity after the crash [1/s]	ω_1'= -3.39	ω_1'= -3.31	ω_2'= 0.37	ω_2'= -4.73
Impulse of a collision force [Ns]	S_{n1} = 11646 S_{t1} = 5430		S_{n2} = 11646 S_{t2} = 5430	
Coefficient of restitution	R=0.05			
Translational speed after the crash [km/h]	V_1'= 34.5	V_1'= 36.7	V_2'= 44.3	V_2'= 44.3
Angular velocity after the crash [1/s]	ω_1'= -3.48	ω_1'= -3.12	ω_2'= 0.93	ω_2'= -5.07
Impulse of a collision force [Ns]	S_{n1} = 13471 S_{t1} = 6282		S_{n2} = 13471 S_{t2} = 6282	
Coefficient of restitution	R=0.01			
Translational speed after the crash [km/h]	V_1'= 35.4	V_1'= 38	V_2'= 46.6	V_2'= 43.6
Angular velocity after the crash [1/s]	ω_1'= -3.07	ω_1'= -2.96	ω_2'= 1.43	ω_2'= -5.09
Impulse of a collision force [Ns]	S_{n1} = 14219 S_{t1} = 6630		S_{n2} = 14219 S_{t2} = 6630	

presented. As previously mentioned, this impulse was adopted equal for both vehicles.Now the question is whether the coefficient of restitution could be useful if all of the unknowns have to be determined without the knowledge of the impulses of a collision force. If no such information is provided then it is necessary to consider another attempt towards the solution of determination the post-collision velocities. Let us introduce two coefficients of restitution, one related to the normal, other to the tangential velocities:

$$R_n = -\frac{w_n'}{w_n} = -\frac{(v_{2n}^{'}-v_{1n}^{'})}{(v_{2n}-v_{1n})},$$

$$R_t = \frac{w_t'}{w_t} = \frac{(v_{2t}^{'}-v_{1t}^{'})}{(v_{2t}-v_{1t})}. \tag{11}$$

Such description of the coefficients still does not give any useful information as how to use them in determination of the unknowns from the formulas (8) – (10). Therefore some additional assumptions about the nature of the collision can be made as, e.g. in [16]. Let us assume that this is a model of a non-slip collision where both vehicles remain in contact during the entire period of the collision and their surfaces do not slide over each other. In case of such an example deformations occur both in the shape and the volume of the vehicles and the following transformations can be made [16].

Let us introduce three indicators combined as in [16], for the non-slip collision with the restitution of the tangential velocities:

$$A = \frac{1}{m_1} + \frac{1}{m_2} + \frac{n_{C1}^2}{I_1} + \frac{n_{C2}^2}{I_2},$$

$$B = \frac{1}{m_1} + \frac{1}{m_2} + \frac{t_{C1}^2}{I_1} + \frac{t_{C2}^2}{I_2},$$

$$. \quad C = \frac{n_{C1}t_{C1}}{I_1} - \frac{n_{C2}t_{C2}}{I_2}. \tag{12}$$

The above formula has previously been presented in [38] as well but only in theoretical approach. Further in [16] it was assumed that the relative tangential velocity in a non-slip collision can be described as:

$$w_{1t} = -AS_t - CS_n = \theta w_t, \tag{13}$$

whereas the relative normal velocity as [16]:

$$w_n' = w_n - CS_t - BS_n = -Rw_n, \tag{14}$$

where: w_n – is the relative velocity of both vehicles in the normal direction (*On*) before the collision;
w_t – is the relative velocity of both vehicles in the tangential direction (*Ot*) before the collision;
w'_n – is the relative velocity of both vehicles in the normal direction (*On*) after the collision;
w'_t – is the relative velocity of both vehicles in the tangential direction (*On*) after the collision.

After solving the equations (13) and (14) both the tangential and the normal components of the impulse of a collision force can be nominated as in [16] but with the assumption that the relative velocities are taken from the initial moment of the collision:

$$S_t = \frac{-(1+R)Cw_n + (1-\theta)Bw_t}{BA - C^2},$$

$$S_n = \frac{(1+R)Aw_n - (1-\theta)Cw_t}{BA - C^2}. \quad (15)$$

Equations (15) allow determination of the components of the impulse of a collision force for the given example. Of course, one simplification has been made, i.e. the impulses were assumed equal for both vehicles, so the equations (15) provide results for both vehicles.

The main difficulty is to specify the value of both coefficients of restitution in order to receive similar results.

Here a hypothesis can be made, that because it is the non-slip collision, i.e. both vehicle do not slide on each other while in contact, the tangential coefficient of restitution can be close to 1, although it was discovered, e.g. in [20], that it could have different values, even more than 1 or less than -1.

- Let us assume that R_t = 0.99 and R_t = -0.99.
- Let us also assume that the normal coefficient of restitution will remain at the adopted values, i.e. R_n = 0.1, R_n = 0.05 and R_n = 0.01.

This will allow to compare the values of the resultant impulse of a collision force with those obtained in simulation (Table 1). In Table 4 the results of analytical calculations with the use of the values of both normal and tangential coefficients of restitution have been presented in such a way that the resultant impulse of the collision force is closest to those obtained in the simulations by changing the values of both coefficients of restitution. The adoption of the tangential restitution coefficient results from the previous research, for example in [20], showing that it can change from -1 to 1 because the vehicles perform both translational and rotational motion during a collision remaining in contact at the same time. In this paper the author assume that the vehicles remain in contact during the whole period of the collision.

The obtained analytical results show that the determined impulses are of the same order of magnitude as those specified by PC-Crash, although in general they are greater than those obtained in the simulations because both the normal and the tangential coefficient of restitution has been used. Also it can be noticed that the closest pair of coefficients enabling almost the same impulse as in the simulation are either R_n = 0.05, R_t = 0.99 or R_n = 0.01, R_t = 0.99. Of course this do not give the whole spectrum of possible solutions as the normal coefficient of restitution can be between 0 and 1, whereas the tangential has more possible values, even the negative ones because it specifies the phenomena between the vehicles while in contact.

Having the specified impulse of the collision force it is easy to use the formulas from (7) to (9)

Table 4. Calculation results based on a model discussed, e.g. in [38]

Coefficients of restitution	Calculation of the impulse of a collision force [ns]	Impulse of a collision force obtained in simulation (without r_t) [ns]
R_n = 0.1, R_t = 0.99	14433	12850
	15691	14864
	16281	15689
R_n = 0.05, R_t = 0.99	13768	12850
	14978	14864
	15548	15689
R_n = 0.01, R_t = 0.99	13261	12850
	14408	14864
	14956	15689
R_n = 0.1, R_t = -0.99	15688	12850
	16802	14864
	17495	15689
R_n = 0.05, R_t = -0.99	15068	12850
	16120	14864
	16789	15689
R_n = 0.01, R_t = -0.99	14574	12850
	15577	14864
	16227	15689

to calculate the post-collision velocities and the resultant speeds.

Despite the differences in the results obtained in the impulses (Table 4), some regularity can be noticed, because the impulse of a collision force tend to increase along with the decrease of the coefficient of restitution which shows how important this parameter is. Secondly, the PC-Crash software has an initial value of R = 0.1 which seems too much for such phenomena as a vehicle collision, lasting e.g. about 0.05 s.

The differences in the obtained results may be discussed basing on some facts. The impulse of the collision force in the simulation was determined by the algorithm used in the Kudlich-Slibar method and the analytical calculations were implemented on the basis of a model proposed in a book regarding the crash mechanics in the discrete structures. What is more is that the PC-Crash duration of the collision seems too long if the momentary phenomena in crash mechanics would be taken into account.

CONCLUSIONS

After adoption of the collision model for the vehicles regarded as the bodies with rough surfaces and the lack of sliding between them the restitution coefficient can influence mainly the impulses of the collision force and indirectly the post-crash parameters. In practice, such a collision model of the side impact is rather difficult to analyze thoroughly due to short period and some simplifications, thanks to which analytical calculations can be prepared. As a tool for the preliminary impact assessment and testimony, as well as accident reconstruction, this obviously simplified procedure may be used if the crash mechanics would be implemented in the forensic work.

The results obtained here are only an attempt to stress the importance of the restitution coefficients for the normal velocities and a possibility of the use of a coefficient of the tangential velocities in modeling of the road traffic collisions.

Further research may provide analyzes on the influence of the discussed coefficients of restitution in case of a collision with the 3D motion (resultant motion) included as has already been highlighted by the authors in the previous selected papers.

REFERENCES

1. Abe M., Morisawa M., Arai K. Sato T. B., Three-dimensional Behavior of Vehicle at Oblique Side Collision - Computer Simulation Using Dynamical Model, JSAE Review, 16(3), 1995.
2. Ahmad M., Ismail K. A., Mat F., Impact models and coefficient of restitution: a review, ARPN Journal of Engineering and Applied Sciences, 11(10), 2016.
3. Aleksandrowicz P., Modeling Head-On Collisions: The Problem of Identifying Collision Parameters, Applied Sciences, 10(18), 2020, DOI: https://doi.org/10.3390/app10186212.
4. Antonetti V.W., Estimating the coefficient of restitution of vehicle-to-vehicle bumper impacts, SAE technical paper 980552, 1998, DOI: https://doi.org/10.4271/980552.
5. Batista M., On the Mutual Coefficient of Restitution in Two Car Collinear Collisions, Physics and Society 2006, arXiv, 2006. https://doi.org/10.48550/arXiv.physics/0601168,
6. Batista M., Zovak G., A restitution model of two-car collinear collisions, Promet – Traffic & Transportation, Vol. 19, No. 1, 2007.
7. Brach R., Brach R., A Review of Impact Models for Vehicle Collision, SAE Technical Paper 870048, 1987, DOI: https://doi.org/10.4271/870048.
8. Brach R.M., An impact moment coefficient for vehicle collision analysis. SAE paper 770014, SAE Transactions, Vol. 86, Section 1: 770010–770187, 1977, DOI: https://doi.org/10.4271/770014.
9. Brach R., Brach M., Analysis of Collisions, Point Mass Mechanics and Planar Impact Mechanics, Collision Magazine, 2(1), 2007.
10. Brach R., Impact Analysis of Two-Vehicle Collisions, SAE Technical Paper 830468, 1983, DOI: https://doi.org/10.4271/830468.
11. Brach R. M., Identification of Vehicle and Collision Impact Parameters from Crash Tests, Journal of Vibration Accoustics 106(2), 1984, DOI: https://doi.org/10.1115/1.3269180.
12. Brach R. M., Momentum and Energy Analysis of Automobile Collisions, J. Morton, Ed, Structural Impact and Crashworthiness, Vol. 2, Imperial College, London, England, 1984.
13. Brach R., Brach R., Louderback A., Uncertainty of CRASH3 ΔV and Energy Loss for Frontal Collisions, SAE Technical Paper 2012-01-0608, 2012, DOI: https://doi.org/10.4271/2012-01-0608.
14. Brach R., Brach R., Pongetti K., Analysis of High-Speed Sideswipe Collisions Using Data from Small Overlap Tests, SAE Int. J. Trans. Safety 2(1):86-99, 2014, DOI: https://doi.org/10.4271/2014-01-0469.
15. Cannon, J. W., Dependence of a Coefficient of Restitution on Geometry for High Speed Vehicle

Collisions, SAE Transactions, 110: 1063-076, 2001, DOI: 10.4271/2001-01-0892.

16. Gryboś R., Teoria uderzenia w dyskretnych układach mechanicznych, PWN, Warsaw, Poland, 1969 (in Polish).
17. Han I., Brach R., Throw Model for Frontal Pedestrian Collisions, SAE Technical Paper 2001-01-0898, Journal of Passenger Car: Mechanical Systems, 110(6), 2001.
18. Huang, M., Vehicle Crash Mechanics, Applied Mechanics Reviews, 56(5), 2003, DOI: https://doi.org/10.1115/1.1584416.
19. Ishikawa H., Impact center and restitution coefficients for accident reconstruction, SAE technical paper 940564, Journal of Passenger Cars, 103(6), 1994, DOI: https://doi.org/10.4271/940564.
20. Ishikawa H., Impact model for accident reconstruction - normal and tangential restitution coefficients, SAE technical paper 930654, 1993, DOI: https://doi.org/10.4271/930654.
21. Ishikawa H., Restitution Coefficients in Car-to-Car Collisions, JSAE 1995
22. Kisilowski J. Zalewski J., Analysis of the Selected Aspects of a Crash between Two Vehicles, Proceedings of the 2nd International Conference on Advances in Computer Science and Engineering (CSE 2013), Atlantis Press, 2013, DOI: https://doi.org/10.2991/cse.2013.23.
23. Linder A., Douglas C., Clark A., Fildes B., Yang J., Otte D., Mathematical simulations of real-world pedestrian-vehicle collisions, NHTSA Paper no. 05-285, 2005.
24. Michalczyk J., Phenomenon of force impulse restitution in collision modelling, Journal of Theoretical and Applied Mechanics, 46(4), 2008.
25. Mnyazikwiye B.B., Karimi H.R., Robbersmyr K.G., Mathematical Modeling of Vehicle Frontal Crash by a Double Spring-Mass-Damper Model, IEEE, 2013, DOI: 10.1109/ICAT.2013.6684071.
26. Monson, K., Germane, G., Determination and Mechanisms of Motor Vehicle Structural Restitution from Crash Test Data, SAE Technical Paper 1999-01-0097, 1999, DOI: https://doi.org/10.4271/1999-01-0097.
27. Neades J., Equivalence of impact phase models in two vehicle planar collisions, Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 227(9), 2013, DOI: 10.1177/0954407013491905.
28. Pawlus W., Karimi H. R., Robbersmyr K.G., Investigation of vehicle crash modeling techniques: theory and application, The International Journal of Advanced Manufacturing Technology 2014, 70(5), DOI: https://doi.org/10.1007/s00170-013-5320-3.
29. Reimpell J., Stoll H., Betzler J., The Automotive Chassis: Engineering Principles, 2nd Edition, Butterworth-Heinemann, 2001.
30. Rose N. A., Fenton S., J., Beauchamp G., Restitution Modeling for Crush Analysis: Theory and Validation, SAE paper 2006-01-0908, 2006, DOI: https://doi.org/10.4271/2006-01-0908.
31. Savova-Mratsenkova M., Djonev G, Determining the values of the Coefficient of Restitution in the meanwhile of a crash between two vehicles, IOP Conference Series: Materials Science and Engineering 618:012058, 618, 2019, DOI: 10.1088/1757-899X/618/1/012058.
32. Smit S., Tomasch E., Kolk H., Plank M., Gugler J., Glaser H, Evaluation of a momentum based impact model in frontal car collisions for the prospective assessment of ADAS, European Transport Research Review, 11(2), 2019, DOI: 10.1186/s12544-018-0343-3.
33. Vangi D., Collision stage reconstruction in traffic accidents: Sensitivity analysis, Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 219(3), 2005, DOI: https://doi.org/10.1243/095440705X6613.
34. Vangi D., Vehicle Collision Dynamics. Analysis and Reconstruction, Butterworth – Heinemann, 2020, DOI: https://doi.org/10.1016/C2016-0-03879-2.
35. Yuta Y., Yuqing Z., Koji M., Analysis of normal and tangential restitution coefficients in car collisions based on finite element method, International Journal of Crashworthiness, 2021, DOI: 10.1080/13588265.2021.1926825.
36. Zaikin O., Korablin A., Dyulger N., Barnenkov N., Model of the Relationship between the Velocity Restitution Coefficient and the Initial Car Velocity during Collision, Transportation Research Procedia, 20, 2017, DOI: https://doi.org/10.1016/j.trpro.2017.01.116.
37. Zalewski J., Analysis of a road accident in the aspect of mechanics of a frontal crash between two vehicles, Diagnostyka, 16(4), 2015.
38. Zalewski J. Selected problems of motor vehicle maintenance after side impact collision, MATEC Web of Conferences, 182, 2018, DOI: 10.1051/matecconf/201818201019.
39. Zhoua J., Pengb H., Luc J., Collision model for vehicle motion prediction after light impacts, Vehicle System Dynamics Supplement, 46, 2008, DOI: https://doi.org/10.1080/00423110701882256.
40. Zioła A., Verification of road accident simulation created with the use of PC-Crash software, Scientific Journal of Silesian University of Technology, Series Transport, 98, 2018, DOI: https://doi.org/10.20858/sjsutst.2018.98.20.