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Feasibility of a Nonlinear Acoustic Method for Monitoring of Bubbles and Fishes

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A problem of remote counting of bubbles and fishes in shallow waters is complicated due to strong reverberation. Traditional way to solve this problem is the use of narrow acoustic beams and short pulse signals, but it is not applicable for all the situations, in particular, in rivers. Reverberation can be suppressed by methods of nonlinear acoustics, since signals scattered by nonlinear targets contain frequencies which are different from primary emitted ones. Bubble-like objects (free bubbles or swimbladder fishes) have strong nonlinear responses. In the present paper a difference frequency method of counting of bubble-like objects is theoretically considered. Since such a nonlinear scattering is resonant bubble and fish sizing becomes possible. For moving targets nonlinear acoustic response exhibits a specific frequency shift, which can be used for velocity estimation.

1. Introduction

A problem of remote counting of bubbles and fishes in shallow waters is complicated due to strong reverberation. Reverberation produced by sound scattering from rough water surface and bottom masks small-amplitude signals from targets [1,2]. Traditional linear acoustic way to solve this problem is the use of narrow acoustic beams and short pulse signals, but it is not applicable for all cases, in particular, in rivers.

Nonlinear acoustic methods can be powerful tools to suppress reverberation because they are based on receiving scattered signal at frequencies which are different from primary emitted ones. It is known that some inclusions may drastically increase nonlinearity of water. Among different underwater objects bubbles exhibit the strongest nonlinear acoustic properties [2,3]. Similar nonlinear responses can be expected for swimbladder fishes. The nonlinearity of bubble-like objects can be several orders higher than that of pure water or other scatterers. Hence, this may be used for acoustical detection of such objects. Bubble-like targets are resonant and the resonance frequency is related to bubble radius [1,2]. Consequently, nonlinear responses also develop resonance properties, thus, acoustic bubble sizing becomes possible.

Different nonlinear acoustic methods have been suggested for bubble density measurements and sizing. They include the second harmonics method [4,5], the combination frequency method, [6,7], the modulation method [8], the subharmonics method [9], the subharmonic-modulation method [10,11].

Among all of them the difference frequency method is the most promising for application for remote diagnostics because it is very selective and does not need so high levels of primary acoustic beams, as for example, the subharmonic method. In this method a bubble is insonified by two primary acoustic waves of different frequencies, and the scattered field at the difference frequency is registered [3,6,7]. The difference frequency signal arises if the difference frequency is equal to the bubble eigenfrequency [6,7]. In the above mentioned experimental works the nonlinear scattering from a single bubble was mainly studied. If there are many bubbles in the interaction zone of two acoustic waves the total field scattered by bubbles is related to the bubble size density distribution. Such scattering, generally, consists of coherent and incoherent parts. For stochastic bubble distribution signal is incoherent. The difference frequency method for incoherent bubble density measurements was considered in [12].

The methods of nonlinear acoustic bubble sizing have been developed previously for the nonmoving bubbles. If a bubble moves a specific Doppler frequency shift arises in scattered field at harmonics or combination frequencies [13-15]. Such a frequency shift at the difference frequency can be abnormally high that makes possible measurement of target velocities. This effect opens new feasibility for size and velocity spectroscopy of bubble-like objects.

In the present paper the difference frequency method is theoretically considered for remote nonlinear acoustic diagnostics of bubble-like objects in shallow waters. Estimations are done to prove applicability of the method for counting of swimbladder fishes in rivers.

2. Difference Frequency Acoustic Scattering by a Moving Bubble

Let us suppose that monochromatic plane waves of frequencies ω_1 and ω_2 from the transducers 1 and 2, respectively, are incident on a moving bubble. The angles between the wave vectors of two incident waves and the velocity vector of the bubble are ϑ_1 and ϑ_2, respectively. Scattered signals are considered to be received by the transducer 3, whose beam axis is at the angle ϑ_3 to the bubble velocity vector. The primary waves from the transducers 1 and 2 can be written as

$$P_n = A_n \exp(i\omega_n t - \vec{k}_n \vec{r}) \quad (1)$$

where $k_n = \omega_n / c_0$, $n = 1,2$, and $\vec{r}$ is radius-vector. Subscripts "1" and "2" denote each component of primary waves, respectively.

The Doppler effect for sound scattering by a moving target can be considered as a two-stage process - signal reaching to a moving scatterer and after that signal emitting into the medium [2]. By analogy with linear scattering if the bubble moves with a constant velocity $\vec{v}$, then, by substituting $\vec{r} = \vec{r}_0 + \vec{v}t$ into Eq.(1), the pressure fields at the bubble are given:

$$P_n^{(b)} = A_n \exp(i\omega_n^{(b)} t) , \quad (2)$$

where

$$\omega_n^{(b)} = \omega_n - \vec{k}_n \vec{v} = \omega_n (1 - \frac{v}{c_0} \cos \vartheta_n) . \quad (3)$$

For a nonlinear scatterer such as a bubble an intermediate stage - transformation of fields $P_1^{(b)}$ and $P_2^{(b)}$ into the combination fields due to nonlinearity of the target must be introduced. The equation of small oscillations of bubble volume V ($V << V_0$, where V_0 is an equilibrium volume) up to the second order terms under the acoustic pressure $p_a = P_1^{(b)} + P_2^{(b)}$ is [3]:

$$\ddot{V} + \omega_0^2 V + \mu \dot{V} - \frac{1}{8\pi R_0^3}[3(\gamma+1)\omega_0^2 V^2 + 2V\ddot{V} + \left(\dot{V}\right)^2] = \frac{-4\pi R_0}{\rho_0} p_a \quad (4)$$

where R_0 is the equilibrium bubble radius and ω_0 is the bubble resonance frequency given by the Minnart formula as [2]

$$\omega_0 = \frac{1}{R_0}\sqrt{\frac{3\gamma P_0}{\rho_0}} , \quad (5)$$

and $\mu = \omega / Q = \delta\omega$ is the loss factor, where Q is the quality factor and δ is the bubble damping constant. Assuming that there is no bubble with radius which has a resonance at the primary frequencies one can get from Eq.(4) an amplitude of the bubble volume variation at the difference frequency $\Omega_b = \omega_1^{(b)} - \omega_2^{(b)}$ ($\Omega_b << \omega_{1,2}$):

$$V_\Omega = \frac{2\pi A_1 A_2}{\rho_0^2 R_0 \omega_1^2 [\omega_0^2 - \Omega_b^2 + i\delta\Omega_b^2]} . \quad (6)$$

Let us consider now the second stage of the process of nonlinear scattering by a moving bubble that is the emitting of the signal produced by the oscillating moving bubble into a medium. This stage is the same as for the linear scattering case. Therefore, a pressure field P_Ω of the difference frequency, emitted by the bubble into a medium can be written in the form

$$P_\Omega = \frac{\rho_0 \Omega_b^2 V_\Omega}{4\pi r} \exp(i\Omega_s t - \vec{k}_3 \vec{r}) \ , \quad (7)$$

where Ω_s is a frequency of field scattered into a medium and received by the transducer 3 [13-15]:

$$\Omega_s = \frac{\Omega_b}{1-(v/c_0)\cos\vartheta_3} \approx \Omega - \frac{v}{c_0}[\omega_1 \cos\vartheta_1 - \omega_2 \cos\vartheta_2 - \Omega\cos\vartheta_3] \quad (8)$$

and $\Omega = \omega_1 - \omega_2$, $k_3 = \Omega_s / c_0$.

The nonlinear scattering cross section σ_Ω is introduced for the nonlinear scattering as [12]

$$P_\Omega^2 = \frac{\sigma_\Omega \left|P_1^{(b)}\right| \left|P_2^{(b)}\right|}{4\pi r^2} = \frac{\sigma_\Omega A_1 A_2}{4\pi r^2} \ . \quad (9)$$

From equations (7), (6) and (9) σ_Ω can be expressed in the next form:

$$\sigma_\Omega = \frac{\pi \Omega_b^4 A_1 A_2}{\rho_0^2 R_0^2 \omega_1^4 [(\omega_0^2 - \Omega_b^2)^2 + \delta^2 \Omega_b^4]} \ . \quad (10)$$

Finally one can get from Eqs.(9), (10) and (7)

$$P_\Omega = \frac{\sqrt{\pi \sigma_\Omega A_1 A_2}}{2\pi r} \exp(i\Omega_s t - \vec{k}_3 \vec{r}\} \ . \quad (11)$$

Equations (11), (10) and (8) give the difference frequency field produced by a single moving bubble.

One can see from Eq.(8) that the frequency of the scattered field depends essentially on angles ϑ_1 and ϑ_2 between axes of primary beams and the velocity vector. In the particular case $\vartheta_1 = \vartheta_2$ (the same transducer emits both primary waves)

$$\Omega_s = \Omega[1 - \frac{v}{c_0}(\cos\vartheta_1 - \cos\vartheta_2)] \ . \quad (12)$$

It gives the same result as if the transducer 1 directly emits the wave of the difference frequency Ω.

The most interesting case takes place if the angle between primary beams is not small and satisfies the following condition

$$\pi/2 \le \vartheta_1 - \vartheta_2 \le 3\pi/2 \ .$$

In this case it can be seen that Doppler frequency shifts produced by two primary waves are not subtracted each other but added in the receiving wave frequency. Further, the Doppler frequency shift arising due to the scatterer motion relative to the transducer 3 is small enough to be neglected if $|\omega_1 - \omega_2| << \omega_{1,2}$. The result is that the Doppler frequency shift for the difference frequency can be as large as twice of the Doppler frequency shift for the high frequency primary waves. For example, when $\vartheta_1 = \vartheta_2 + \pi$ one can get

$$\Omega_s \cong \Omega - 2\omega_{1,2}\left(\frac{v}{c_0}\right)\cos\vartheta_1 \ . \quad (13)$$

It is seen from Eq.(13) that if two primary beams are directed opposite to each other the Doppler frequency shift at the difference frequency can be very large compared to the linear Doppler frequency shift for frequency Ω. One can easily obtain from Eq.(13) a ratio between nonlinear and linear Doppler frequency shifts as follows

$$\frac{\Delta\Omega_{nonlinear}}{\Delta\Omega_{linear}} \sim \frac{\omega_{1,2}}{\Omega} >> 1 \ .$$

This feature of the nonlinear Doppler effect for difference frequency scattering is very intriguing for various applications, because opens a possibility to detect slow motion of nonlinear acoustic targets - bubble-like objects. It is also worthy to point the resonance character of sound scattering by bubbles. This fact results in that the amplitude of scattered signal is not vanishing only if the difference frequency is closed to the bubble resonance frequency $\Omega \approx \omega_0$.

3. Nonlinear Doppler effect for ensemble of moving bubbles

Consider the nonlinear difference frequency scattering by an ensemble of moving bubbles. Let us suppose that the scattering volume formed by crossing primary beams contains many bubbles of different sizes moving with different velocities. Such may occur, for example, in a turbulent flow. Fishes may also have similar behaviour. We consider that the velocity v_j $(j = 1,2,...)$ of any bubble can be represented as a sum of regular velocity v_0 (progressive motion of the scattering volume) and a stochastic velocity component $\tilde{v}$. Another supposition for simplicity is that velocities $\tilde{v}$ are statistically independent of bubble sizes and evenly distributed in the entire space angle 4π and in the range of absolute values $(0, v')$. Further, the scattering signal consists of incoherent and coherent parts. For the first one the mean intensity can be found as a sum of intensities of signal scattered by different bubbles. The coherent part of scattered filed can be observed only for some special conditions (scattering at definite angles related with the geometry of the scattering volume and spatial distributions of bubbles) and for the general case of stochastic bubble distribution vanishes. So, the total intensity of scattered signal can be presented as incoherent sum of partial intensities produced by individual bubble scattering. In the far field zone from the interaction volume Δ_0, whose size is much smaller than the distance |r| from the scattering volume to the receiving transducer 3, the nonlinear scattering field is given as follows

$$P_\Omega^2 = \frac{S_\Omega A_1 A_2 \Delta_0}{4\pi r^2}, \quad (14)$$

where S_Ω is the nonlinear volume scattering coefficient, which is equal to the effective nonlinear scattering cross section per unit volume of a medium. It can be calculated supposing that the main contribution into the scattered field is due to the resonant bubbles, so the integration may be done only over the resonance curve [12]. It yields

$$S_\Omega = \int \sigma_\Omega(R) n(R) dR = \frac{\pi^2 n(R_\Omega) A_1 A_2}{2\rho_0^2 \delta \omega_1^4 R_\Omega}, \quad (15)$$

where R_Ω is the bubble radius having the resonance frequency of Ω.

It is evident that the spectrum of acoustic field scattered by the stochastic bubble ensemble is a broad line having the mean Doppler frequency shift and width. Statistical properties of the spectral line can be derived from Eq.(8) with assumptions made in the beginning of this chapter. One may easily find the mean frequency of scattered field as follows

$$< \Omega_s >= \Omega - \omega_1 \left(\frac{v_0}{c_0} \right) [\cos \vartheta_1 - \cos \vartheta_2] - \Omega \left(\frac{v}{c_0} \right) \cos \vartheta_3 . \quad (16)$$

Correspondingly, the r.m.s. width of the spectral line is derived as

$$< (\Omega_s)^2 >= \frac{1}{6} \left(\frac{v'}{c_0} \right)^2 \left[4\omega_1^2 \sin(\vartheta/2) + \Omega^2 \right], \quad (17)$$

where $\vartheta = \vartheta_1 - \vartheta_2$ is the angle between two primary beams. For the particular case of opposite directed primary beams $\vartheta_1 = \vartheta_2 + \pi$ one may have from Eqs.(16) and (17)

$$< \Omega_s > \cong \Omega - 2\omega_1 \left(\frac{v_0}{c_0} \right) \cos \vartheta_1 , \quad (18)$$

$$< (\Omega_s)^2 >= \frac{2}{3} \omega_1^2 \left(\frac{v'}{c_0} \right)^2 . \quad (19)$$

The last expressions show that parameters of bubble motion can be obtained from measurements of the mean frequency and the frequency dispersion of the difference frequency spectral line of the scattered signal. The projection of the mean, *i.e.* progressive velocity of bubbles on beams axes one can obtain from the measurement of the mean Doppler frequency shift, and the turbulent motion velocity dispersion is related to the width of the spectral line.

4. Application of the Method to Fish Monitoring in Rivers

It follows from the consideration in the previous part that the scattered field of difference frequency contains the information on the bubble concentration and bubble velocities. Eq.(14) relates the intensity of the difference frequency field with the concentration of bubbles in the scattering volume which are resonant at the difference frequency. This equation allows to measure the bubble size distribution density if the difference frequency is swept. A spectral analysis of the scattered field allows one to determine the mean velocity of the scattered volume (Eq.(18)) and the turbulent velocities of bubbles (Eq.(19)). Such a method can be applied for observation of bubbles and bubble-like objects in shallow waters.

Let us estimate possibility of monitoring swimbladder fishes in rivers with the nonlinear acoustic method. This problem is important for evaluation of fish resources. A block-scheme of such a measurement includes two high frequency acoustic transducers deployed underwater near both river banks and whose beam axes are directed to each other. Transducers must have relatively narrow beam pattern in horizontal plane across the river. Distribution of intensities of primary acoustic fields through the depth of water layer would be approximately homogeneous. Thus, an "acoustical sheet" across the river is formed. Further, the angle ϑ_1 between beam axes and river water flow must not be 90 degrees, one may suggest for this angle a range of values of 20-60 degrees. A receiving low frequency transducer (hydrophone) must be deployed near one of primary ones. In the simplest case the receiving transducer may be a hydrophone, but to increase gain it would have directivity pattern similar to that of primary transducers. When a swimbladder fish having the resonance frequency closed to the difference frequency passes through the "acoustical sheet" the difference frequency signal is generated and received by the hydrophone.

Typical values of fish velocities can be of order 1-3 m/s. For seasonal fish motion they go mainly up- or downstream. Let us set $\omega_{1,2}/2\pi \sim 100$ kHz, $v_0 = 1.5$ m/s, and $\vartheta_1 = 60^\circ$. Consequently, from Eq.(18) we obtain $< \Omega_s > - \Omega \cong 100$ Hz. This frequency shift which arises at the difference frequency can be easily detected with spectrum analyser. The resonance frequency of fish swimbladder depends on its size, which is in turn determined by a fish size [2]. For fishes of medium sizes (about 20 cm) swimbladder resonance frequencies in shallow waters may be of order 1 kHz and $\delta \approx 0.3$ [2]. The equivalent sphere radius for such a swimbladder can be roughly estimated from Eq.(5) as $R_0 \approx 3$ mm. Now one can evaluate the possibility of detection of difference frequency signals. Let us suppose that a single fish crosses the "acoustical sheet". Then, the minimum detectable signal level is the ambient noise level, which is of order 30 dB at 1 kHz [1,2]. For this case Eqs.(9) and (10) must be applied. Estimation shows that P_Ω exceeds noise level at distances up to several tens meters for $A_{1,2} \sim 10^4 - 10^5$ Pa. Therefore, this method can be useful for small rivers having width of about several tens meters.

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References

1. R. J. Urick, *Principles of Underwater Sound*, NY, McGraw-Hill, 1983.

2. H. Medwin, C. S. Clay, *Fundamentals of Acoustical Oceanography*, Boston, Academic Press, 1998, pp. 384-391.

3. E. A. Zabolotskaya, S. I. Soluyan, Emission of harmonic and combination frequency waves by air bubbles,. Sov. Phys. Acoust., 18, 396-398, (1972).

4. F. N. Fenlon, J. W. Wohn, On the Amplification of Modulated Acoustic Waves in Gas-Liquid Mixtures, *Cavitation and Inhomogeneities in Underwater Acoustics, ed. W. Lauterborn,* Heidelberg, Springer-Verlag, 1980, pp. 141-150.

5. L. A. Ostrovsky, A. M. Sutin, Nonlinear acoustic diagnostics of discrete inhomogeneities in liquids and solids, *1st Int. Congr. Acoust.*, Paris, 1983, v.2, pp. 137-140.

6. V. A. Zverev, Y. A. Kobelev, D. A. Selivanovsky, A. Y. Sokolov, On a method of detection of gas bubbles in a liquid, Sov. Tech. Phys., 50, 1544-1545, (1980).

7. B. M. Sandler, D. A. Selivanovsky, A. Y. Sokolov, The measurements of gas bubble concentration in a subsurface sea layer,. Doklady Akademii Nauk SSSR, 260, 1474-1476, (1981).

8. V. L. Newhouse, P. M. Shankar, Bubble size measurement using the nonlinear mixing of two frequencies, J. Acoust. Soc. Am., 75, 1473-1477, (1984).

9. A. Eller, H. G. Flynn, Generation of subharmonics of order one-half by bubbles in a sound field, J. Acoust. Soc. Am., 46, 722-727, (1969).

10. T. G. Leighton, A. J. Walton, J. E. Field, Acoustic bubble sizing by combination of subharmonic emission with imaging frequency, Ultrasonics, 29, 319-323, (1991).

11. A. D. Phelps, T. G. Leighton, Investigations into the use of two frequency excitation to accurately determine bubble sizes, *Bubble Dynamics and Interface Phenomena, ed. J. R. Blake,* Dordrecht, Kluwer Academic Publishers, 1994, pp. 475-484.

12. A. M. Sutin, S. W. Yoon, E. J. Kim, I. N. Didenkulov, Nonlinear acoustic method for bubble density measurements in water, J. AcoUst. Soc. Am., 103, 2377-2384, (1998).

13. I. N. Didenkulov, E. J. Kim, S. W. Yoon, A. M. Sutin, Nonlinear acoustic Doppler effect in a bubble flow. J. Acoust. Soc. Am., 99, 2514, (1996).

14. I. N. Didenkulov, Nonlinear acoustic spectroscopy of gas bubbles and their motion in a subsurface ocean layer, *Natural Physical Processes Associated with Sea Surface Sound, Ed. T. G. Leighton,* Institute of Sound and Vibration Research, University of Southampton, UK, 1997, pp. 189-197.

15. I. N. Didenkulov, S. W. Yoon, A. M. Sutin, E. J. Kim, Nonlinear acoustic Doppler effect and its use for bubble flow velocity measurement, J. Acoust. Soc. Am., to be published, (1999).