THE DOPPLER EFFECT IN A BISTATIC SYSTEM FOR DETERMINING THE POSITION OF MOVING TARGETS

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The article presents the theoretical analysis and the results of numerical calculations of the Doppler effect it occurs in a system designed to determine the position and speed of a moving target. The transmitter is the source of the signal and it emits a sinusoidal, acoustic and continuous wave. Signal reflected off a moving target is received by four hydrophones. Based on the signals, four Doppler shifts are determined and inserted into a set of equations. The solution gives instantaneous target coordinates and the vector of target speed. Analytical formulas are given which describe the spectra of the Doppler shift of echo signals in a bistatic system, as well as numerically determined examples. Spectral lines are assigned to instantaneous target positions. This provides the basis for solving the above set of equations and the functioning of the system.

INTRODUCTION

Terrorist attacks are an increasing threat to bodies of water which are an economic, tourist and defence significance. Facilities which can be accessed on shore and off shore (such as power plants, dams, quays) should have a system to detect and prevent access if they have to be secure. For example, sending a diver or unmanned underwater vehicle to the protected body of water with the intent of obtaining information and/or cause of damage. The basic objective of such a security system is to detect the moving target and then determine its position and speed.

The position and speed of moving underwater targets can be determined through analysis of the Doppler shifts of echo signals deriving from the moving underwater target. This is enabled by a multi-static system consisting of a transmitter and four hydrophones in known locations, as explained in the paper [2, 3]. The authors demonstrated that recording and analysing four Doppler shifts in a system of transmitter-target-hydrophones can help to determine the speed and direction of a moving target. The solution, however, was based on a mono-frequency Doppler shift, which works only if the target is far away from both the transmitter and hydrophones. This assumption is no longer used in this system but target can move near the transmitting transducer and hydrophones. In this case the spectrum of the Doppler shift lasts for a short observation period. In addition, it was assumed that the transmitting transducer may be installed anywhere in the observed or monitored area, unlike the previous solution if it is was put near one of the hydrophones. While it makes signal processing algorithms more complicated, it is no longer have to reduce the crosstalk between the transmitter and receiver, which is technically difficult to achieve.

In the system in question, a single Doppler shift must be determined from the echo signal received by each hydrophone which is clearly related to the target's instantaneous position. This is the objective of this analysis and these numerical calculations.

1. THE DOPPLER EFFECT BETWEEN THE TRANSMITTER AND TARGET

Fig. 1 shows the coordinates of hydrophones marked with indices from 1 to 4 and the coordinates of the transmitting transducer, which is marked with the index *n*. It is assumed that only one moving target with instantaneous coordinates x,y is under observation and when observation begins the coordinates are x_o,y_o. It would be left out the effects of draught of the transmitter, hydrophones and target under observation. This is justified by size of the body of water and its small depth. Hence, it would be assumed that the geometric structure of the system is two dimensional.

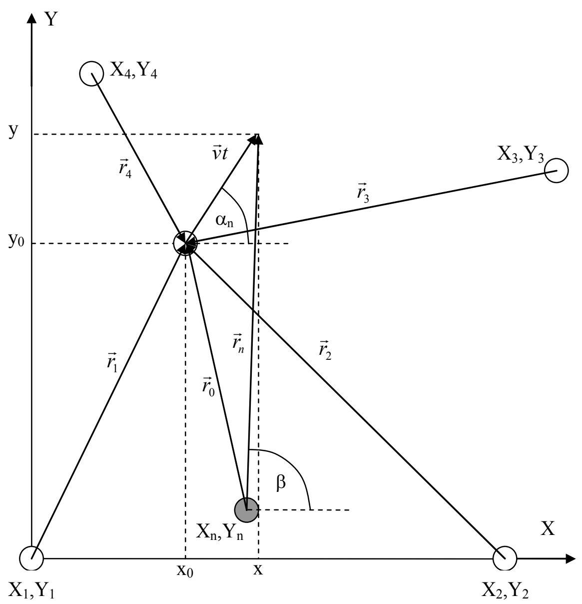

Fig. 1. Position of the transmitter (grey), hydrophones and the target under observation in a body of water.

The system is bistatic because the transmitting transducer and each hydrophone are in different places as it is shown in Fig.1. This is why the Doppler effect will be analysed in two stages: between the transmitter and target, and then between the target and hydrophone.

In the system the acoustic signal is generated by the transmitting transducer with coordinates X_n, Y_n describing its position in the reservoir, as shown in Fig. 1. The transducer emits a sinusoidal spherical acoustic wave whose pressure can be written as follows:

$$p(r,t) = \frac{A}{r_n}\sin\{2\pi f_0[t - r_n(t)/c]\}, \tag{1}$$

where t is time, f_0 – the frequency of acoustic wave emitted by the transmitter, c – velocity of acoustic wave in water, r_n – distance between target and sound source, and A - constant dependent on sound source efficiency.

As the target moves, distance r_n changes over time and hence the frequency of the wave incident on the target is equal to:

$$f_n(t) = \frac{1}{2\pi}\frac{d}{dt}\{2\pi f_0[t - r_n(t)/c]\} = f_0 - \frac{f_0}{c}\frac{dr_n(t)}{dt}. \tag{2}$$

It can be assumed that when observation begins, the target's position is x_0, y_0 and that it moves at constant speed $\vec{v}$ whose vector is shown in Fig.1. The distance between the target and source (after time t) is:

$$r_n(t) = |\vec{r}_n + \vec{v}t| = \sqrt{(x_0 - X_n + v_x t)^2 + (y_0 - Y_n + v_y t)^2}\ , \tag{3}$$

where v_x and v_y are the components of the target's speed vector.

It is calculated the derivative of the expression and insert it into formula (2) and obtain as follows:

$$f_n(t) = f_0 - \frac{f_0}{c}\frac{v_x(x_0 - X_n + v_x t) + v_y(y_0 - Y_n + v_y t)}{\sqrt{(x_0 - X_n + v_x t)^2 + (y_0 - Y_n + v_y t)^2}}. \tag{4}$$

It would be called the difference $F_n(t)=f_n(t)-f_0$ the Doppler shift of the frequency of the acoustic wave incident on the moving target. The shift is generally a function of time as it is shown in formula (4). The observation time is short and the speed of the moving object is low if there is a long distance between the target and transmitting transducer. The shift's formula can be presented in reduced form:

$$F_n(0) = -\frac{f_0}{c}\frac{v_x(x_0 - X_n) + v_y(y_0 - Y_n)}{\sqrt{(x_0 - X_n)^2 + (y_0 - Y_n)^2}}. \tag{5}$$

Fig. 2 shows a module of the spectrum $|P(F_n)|$ of signal $p(r,t)$ numerically determined from formula (1) because the assumptions regarding formula (5) are achieved. Frequency F_n obtained from the spectrum is -94 Hz, and the frequency calculated from formula (5) is equal to -94.3 Hz. The error which results from the reduction is smaller than spectrum resolution, which in this case is $\Delta f=1/T=1$ Hz, where $T=1$ s is the signal observation time.

The expression in the numerator of formula (5) is the scalar product of vectors $\vec{r}_n$ and $\vec{v}$, and the expression in the denominator describes length r_n of vector $\vec{r}_n$, hence:

$$F_n \cong -\frac{f_0}{c}\frac{\vec{v}\vec{r}_n}{r_n}, \tag{6}$$

If vector $\vec{r}_n$ is tilted relative to axis OX at angle β, and vector $\vec{v}$ at angle α_n, then $(x_0-X_n)/r_n=\cos\beta$, $(y_0-Y_n)/r_n=\sin\beta$, $v_x/v=\cos\alpha_n$, $v_y/v=\sin\alpha_n$ and formula (5) can be written as:

$$F_n \cong -\frac{f_0 v}{c}(\cos\beta\cos\alpha_n + \sin\beta\sin\alpha_n) = -\frac{f_0 v}{c}\cos(\beta - \alpha_n). \tag{7}$$

The conditions under which the above formulas were derived occur in the majority of radar and sonar systems. As a result, the basic literature describes the Doppler effect using these formulas only. In the system in question the distance between the target and transmitting transducer may be relatively short, which does not justify the use of above approximations even for a low target speed. The spectrum of the signal cannot be described with a single frequency because it contains several components as illustrated in Fig. 3.

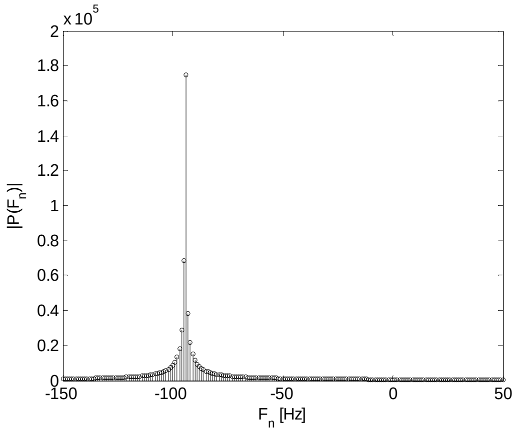

Fig. 2. Spectrum of pressure of an acoustic wave incident on a distant moving target (f_0=100kHz, x_0-X_n=100m, y_0-Y_n=100m, v_x=1m/s, v_y=1m/s, t_o=1 s, c=1500m/s).

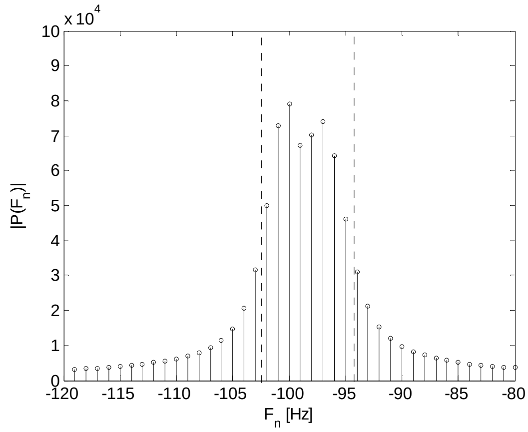

Fig. 3. Spectrum of pressure of an acoustic wave incident on a close moving target (f_0=100kHz, x_0-X_n=10m, y_0-Y_n=10m, v_x=2m/s, v_y=0m/s).

The cut-off frequencies of the Doppler shift $F_n(t)$ can be determined from formula (4). Results are as follows:

$$F_n(0) = -\frac{f_0}{c}\frac{v_x(x_0 - X_n) + v_y(y_0 - Y_n)}{\sqrt{(x_0 - X_n)^2 + (y_0 - Y_n)^2}}, \tag{8}$$

$$F_n(t_o) = -\frac{f_0}{c}\frac{v_x(x_0 - X_n + v_x t_o) + v_y(y_0 - Y_n + v_y t_o)}{\sqrt{(x_0 - X_n + v_x t_o)^2 + (y_0 - Y_n v_y t_o)^2}}. \tag{9}$$

As it is shown from the formulas above, the shift $F_n(0)$ refers to the target with coordinates x_0, y_0, and the shift $F_n(t_o)$ refers to the target whose coordinates are $x_0+v_x t_o$, $y_0+v_y t_o$. The shifts marked in Fig. 3 between dotted lines are derived from formulas (8) and (9). The difference in the shifts depends on the distance covered by the target in time t_0 and the angle between vector $\vec{r}_n$ and vector $\vec{v}$. This is illustrated in Fig. 4 and Fig. 5, which show the difference in Doppler shifts $\Delta F_n = F_n(t_o) - F_n(0)$ in the function of angular differences between the vectors. The difference is smallest when the vectors are parallel and the biggest when they are perpendicular. The spectrum of the Doppler shift fits in between one or two lines with resolution $\Delta f = 1/t_0$= 1 Hz for a low speed (Fig. 4). It means that the target can be treated as a distant target even for a distance r_n=10 m. When the speed increases four times (Fig. 5) the spectrum of the Doppler shift becomes considerably wider and its width is not negligible for distances r_n shorter than about 100 m.

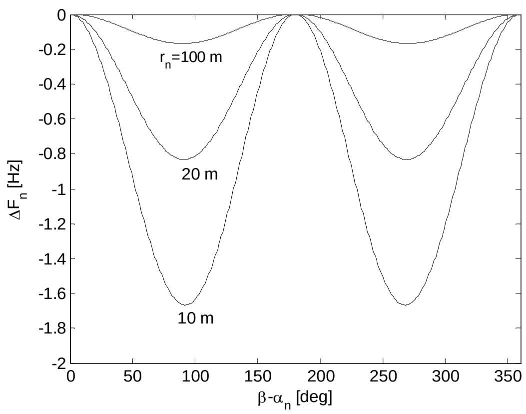

Fig. 4. Width of Doppler shift spectrum (f_0=100kHz, v=0.5m/s, t_o=1s).

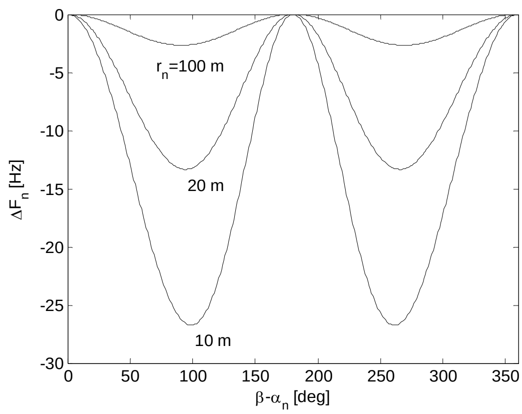

Fig. 5. Width of Doppler shift spectrum (f_0=100kHz, v=2m/s, t_o=1s).

By rearranging formula (9), we can write the spectrum of the Doppler shift $F_n(t)$ approximately as:

$$F_n(t) \cong -\frac{f_0 v}{c}\cos(\beta-\alpha_n) - \frac{f_0 v}{c}\frac{vt}{r_n}\sin^2(\beta-\alpha_n)\,. \tag{10}$$

Doppler effect causes linear frequency modulation what is shown in above relation. As it is known, the spectrum of a regular sinusoidal signal with linear frequency modulation is almost rectangular with its mid-frequency equal to carrier frequency [1, 4, 5, 6]. The Doppler effect causes an extra shift of the edge of the spectrum by shift $F_n(0)$. The width of the spectrum is described with the second term of formula (10). Fig. 6 shows examples of changes in frequency $F_n(t)$, derived from formula (9) and Fig. 7 shows the errors caused by formula approximation (10). As it is shown, approximation errors have no technical significance.

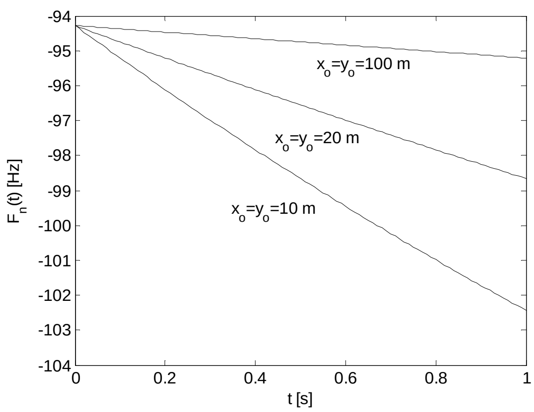

Fig. 6. Doppler shifts in the function of time (f_0=100kHz, v=2m/s, β=0°, α_n=45°).

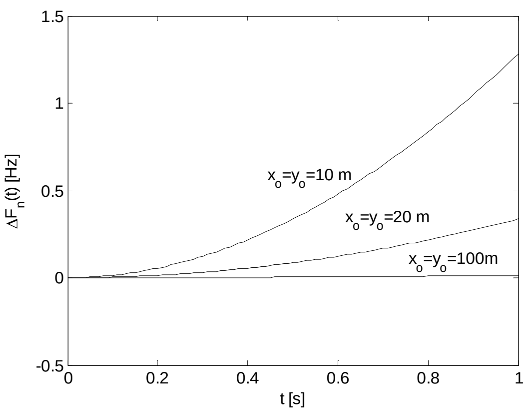

Fig. 7. Error of the Doppler shift in the function of time (f_0=100kHz, v=2m/s, β=0°, α_n=45°).

2. DOPPLER EFFECT BETWEEN TARGET AND HYDROPHONE

Both an incident and a reflected acoustic wave, if observed on the surface of a moving target, have a frequency that is different from the frequency of the wave emitted by the transmitter by the above Doppler shift. The surface of the target becomes the source of spherical waves with frequencies $f_n(t)$. When the target moves at speed $\vec{v}$, then the frequencies of the wave received by the hydrophones change again. The physical cause is a change in the length of the wave emitted in the space surrounding the moving target. This is illustrated in Fig. 8, which shows the geometric places of the front of acoustic wave emitted by the target moving at speed $\vec{v}$ with components v_x, v_y. The front of that wave emitted in time t is marked with a solid line. The dotted line shows the front of a wave emitted by a stationary target in time $t+T$, and the broken line represents the target moving at speed $\vec{v}\,(v_x, v_y)$.

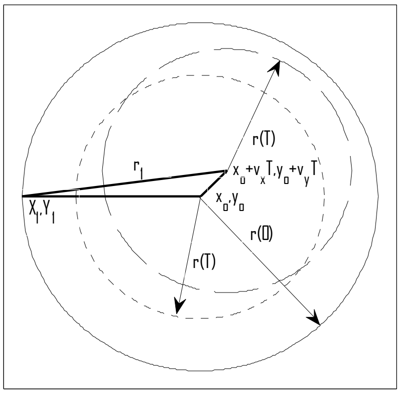

Fig. 8. Front of the sinusoidal wave emitted by a moving target.

If the target is stationary, and T is the period of the emitted wave, the difference between radius, length $r(0)$ and $r(T)$ is equal to the length of the wave $\lambda=cT$. If the target is moving, the length of the wave will change and is equal to $\lambda_1=cT_1=r_1-r(T)$. The change in length is caused by the Doppler effect and amounts to $\Delta\lambda=\lambda_1-\lambda=r_1-r(0)$. To determine the difference it would be first calculated the difference in radius square:

$$r_1^2-r(0)^2=(x_0-X_1+v_xT)^2+(y_0-Y_1+v_yT)^2-(x_0-X_1)^2(y_0-Y_1)^2. \tag{11}$$

After reductions, the formula takes this shape:

$$r_1^2-r(0)^2=2v_xT(x_0-X_1)+2v_yT(y_0-Y_1)+v^2T^2, \tag{12}$$

where $v^2=v_x^2+v_y^2$.

If the distance between the target and hydrophone is large, it can be left out component v^2T^2. In addition, expression $r_1^2-r(0)^2$ can be reduced to $[r_1-r(0)]\,[r_1+r(0)]\cong 2[r_1-r(0)]\cdot r(0)$. Using this relation, we obtain:

$$\Delta\lambda=r_1-r(0)\cong\frac{v_xT(x_0-X_1)+v_yT(y_0-Y_1)}{\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}}. \tag{13}$$

Because $\Delta\lambda=\lambda_1-\lambda=c(T_1-T)$, the period of the wave from the moving target is:

$$T_1\cong T[1+\frac{v_x(x_0-X_1)+v_y(y_0-Y_1)}{c\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}}]. \tag{14}$$

In the bistatic situation in question, period $T=T_n=1/f_n$. The frequency of the acoustic wave received by the hydrophone is equal to $f_1=1/T_1$, hence:

$$f_1 \cong f_n \cdot [1+\frac{v_x(x_0-X_1)+v_y(y_0-Y_1)}{c\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}}]^{-1}, \tag{15a}$$

and after expanding it into Taylor series and leaving out insignificant terms:

$$f_1 \cong f_n \cdot [1-\frac{v_x(x_0-X_1)+v_y(y_0-Y_1)}{c\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}}]. \tag{15b}$$

It would be defined the Doppler shift between the target and hydrophone as:

$$F_1(0) \cong -f_0\frac{v_x(x_0-X_1)+v_y(y_0-Y_1)}{c\sqrt{(x_0-X_1)^2+(y_0-Y_1)^2}}. \tag{16}$$

Using this definition, formulas (14) can be written as:

$$f_1 \cong [f_0+F_n(0)]\cdot[1+F_1(0)/f_0] = f_0+F_n(0)+F_1(0)+F_n(0)\cdot F_1(0)/f_0. \tag{17}$$

The last term of the above relation is always significantly lower than the others, which means that it can be presented in a reduced form as:

$$f_1 \cong f_0+F_n(0)+F_1(0). \tag{18}$$

where $F_n(0)$ and $F_1(0)$ are described with formulas (8) and (16) respectively.

The above relation was derived using reducing assumptions under which, technically speaking, the target is a long distance away from the transmitter and hydrophone, and moves at a slow speed. What these general assumptions mean the distance covered by the target during observation time is much smaller than the distance between the target, transmitter and hydrophone. Failure to meet this criterion increases the width of the Doppler shift spectrum between the transmitter and target, as shown in the previous section. If the target is close to the transmitter, then $F_n(0)$ should be replaced with spectrum $F_n(t)$, described with formula (9), after replacing constant delay t_o with time t. As a consequence, frequency f_1 of a specific numerical value will be replaced with a spectrum with frequencies $f_1(t)$ that depend on the observation time. The spectrum will be shifted on the frequency axis by a constant value $F_1(0)$ if the target is far away from the hydrophone. Otherwise, frequency shift $F_1(0)$ is replaced with spectrum $F_1(t)$, because the above reductions have not been applied. This spectrum will be determined in the next section.

3. SIGNAL APPROACH TO THE DOPPLER EFFECT IN A BISTATIC SYSTEM

Consider the situation shown in Fig. 1, where the acoustic sinusoidal signal is emitted by the transmitting transducer with coordinates X_n,Y_n. It reaches a moving point target, reflected off the target and reaches the hydrophone with coordinates X_1,Y_1. The acoustic pressure of the wave incident on the target is described in formula (1). After it has covered the distance $r_1(t)$, the reflected wave has an acoustic pressure on the surface of the hydrophone which can be described with this formula:

$$p_1(r,t)=\frac{B}{r_n r_1}\sin\{2\pi f_0\{t-[r_n(t)+r_1(t)]/c\}\}, \tag{19}$$

where B is the constant; dependent on the efficiency of the sound source, the conditions under which the wave reflected off the target and the absorption coefficient in water.

The instantaneous frequency of pressure, and by the same token, of the signal at hydrophone output, is calculated as a derivative of the phase, which - as shown in formula (2) - is equal to:

$$f_1 = f_0 - \frac{f_0}{c}\left(\frac{dr_n(t)}{dt} + \frac{dr_1(t)}{dt}\right). \tag{20}$$

Distance $r_n(t)$is described with formula (3), and distance $r_1(t)$ is:

$$r_1(t) = \sqrt{(x_0 - X_1 + v_x t)^2 + (y_0 - Y_1 + v_y t)^2}\ . \tag{21}$$

After calculating the derivatives in formula (19) is obtained:

$$f_1(t) = f_0 - \frac{f_0}{c}\left[\frac{v_x(x_0 - X_n + v_x t) + v_y(y_0 - Y_n + v_y t)}{\sqrt{(x_0 - X_n + v_x t)^2 + (y_0 - Y_n + v_y t)^2}} + \frac{v_x(x_0 - X_1 + v_x t) + v_y(y_0 - Y_1 + v_y t)}{\sqrt{(x_0 - X_1 + v_x t)^2 + (y_0 - Y_1 + v_y t)^2}}\right]. \tag{22}$$

The first fraction in the above expression is proportional to the Doppler shift between the transmitter and target (formula 4). The second one is proportional to the Doppler shift between the target and hydrophone. Both Doppler shifts are described with the same relations. If the target is far away from the hydrophone and moving at a slow speed, in the second fraction it is possible to disregard $v_x t$ and $v_y t$, which gives the Doppler shift F_n a constant numerical value - as shown in formula (16).

Formulas (22) describe the frequencies of signals received by the other three hydrophones, if their coordinates are put in place of X_1, Y_1.

By rearranging formula (9) we can write the total Doppler shift in a reduced form as:

$$F_n(t) + F_1(t) \cong -\frac{f_0 v}{c}\left\{\cos(\beta - \alpha_n) + \cos(\beta - \alpha_1) + vt\left[\frac{\sin^2(\beta - \alpha_n)}{r_n} + \frac{\sin^2(\beta - \alpha_1)}{r_1}\right]\right\}. \tag{23}$$

The first two components of the above relation describe the total Doppler shift when $vt<<r_n$ and $vt<<r_1$. The shift can be presented as:

$$F_n(0) + F_1(0) \cong -2\frac{f_0 v}{c}\cos\left(\beta - \frac{\alpha_n + \alpha_1}{2}\right)\cdot\cos\left(\frac{\alpha_n - \alpha_1}{2}\right). \tag{24}$$

If $\alpha_1 = \alpha_n = \alpha$, the bistatic situation transitions into a monostatic situation then the above formula is reduced to a form commonly used in literature [5]:

$$F_n(0) + F_1(0) \cong -2\frac{f_0 v}{c}\cos(\beta - \alpha). \tag{25}$$

If $\beta=\alpha$, or the sum of Doppler shifts takes on minimal value, and when $\beta-\alpha=90^o$, the sum is zero. The sum of the shifts takes on the maximum value when $\beta-\alpha=180^o$. In the bistatic situation the sum of the Doppler shifts is zero, if $\alpha_n-\alpha_1=90^o$.

Fig. 9 shows the spectra of Doppler shifts when the target under observation is approaching the transmitter (positive Doppler shift) and moving away from the hydrophone (negative Doppler shift). The spectra in Fig. 10 occur when the target moves away from the transmitter and hydrophone. In both figures the target is close to the transmitter and hydrophone in relation to distance vt, which makes the spectra relatively wide. As the spectrum width increases, its lines become shorter making detection more difficult.

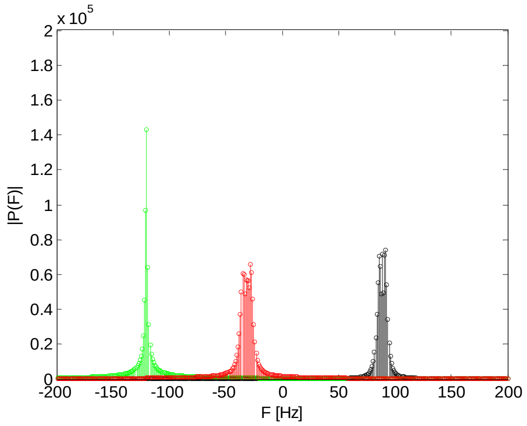

Fig. 9. Doppler shifts between transmitter and target (black), target and hydrophone (green) and transmitter, target and hydrophone (red) (f_0=100kHz, X_n, Y_n=(0,0), X_1, Y_1=(20,30), (x_0, y_0)=(10,10) [m], v_x=0m/s, v_y=-2m/s, t_o=1s).

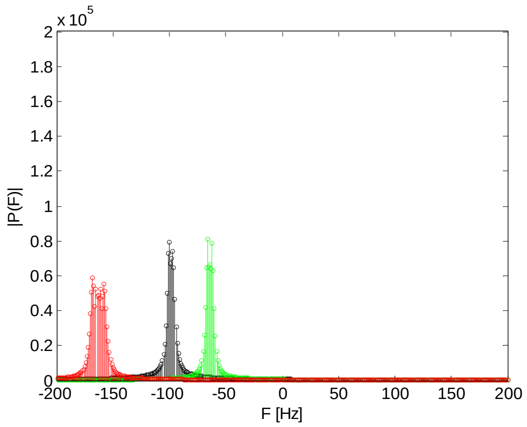

Fig. 10. Doppler shifts between transmitter and target (black), target and hydrophone (green) and transmitter, target and hydrophone (red) (f_0=100kHz, X_n, Y_n=(0,0), X_1, Y_1=(30,0), (x_0, y_0)=(10,10) [m], v_x=0m/s, v_y=2m/s, t_o=1s).

The particular lines of the Doppler shift spectrum correspond to the instantaneous position of the target which results from formula (23). Another conclusion from the formula is that the initial target position (x_0, y_0) always corresponds to the extreme right spectral line. This fact will be used in the procedure for determining target coordinates and speed. This is further confirmed in the figures below which show the changes in Doppler shift spectrum corresponding to changes in observation time, as a result of a larger distance covered by the target.

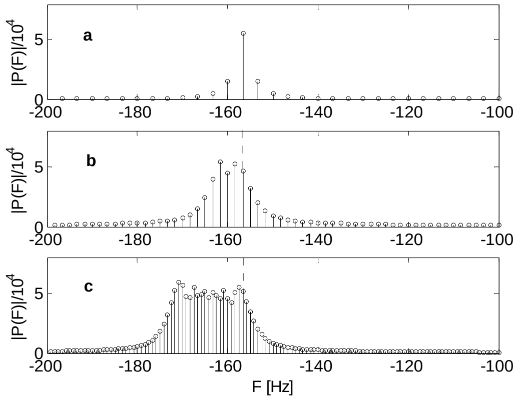

Fig. 11. Change in Doppler shift spectrum
for different observation times (a: t=0.3 s, b: t=0.6 s, c: t=1.2 s).

The basis for solving the set of equations with the unknowns representing the coordinates of a moving target, and components of its speed vector, is the Doppler shift, which corresponds to the initial target position. This can be determined by analysing the recorded Doppler shift spectrum.

4. CONCLUSION

The analysis above shows that the particular lines of the Doppler shift spectrum can be clearly assigned to the target's instantaneous position, as well as its position in the initial instance of subsequent observation periods. As a consequence, it is possible to solve the set of equations [2, 3] with the solution by providing the target's instantaneous position and speed.

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