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Overview of the Development of Nonlinear Acoustics

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ABSTRACT

Developments in nonlinear acoustics are followed from gases to liquids to solids. For all materials the nonlinearity parameter can be defined as the negative ratio of the coefficient of the nonlinear term to the coefficient of the linear term in the nonlinear wave equation. With this definition we can compare the nonlinear behaviors of gases, liquids, and solids. In gases $\beta = \gamma + 1 \approx 2$. In liquids $6 \leq \beta = \frac{B}{A} + 2 \leq 14$. In crystalline solids $3 \leq \beta = \frac{K_3}{K_2} + 3 \leq 15$. The pure number β gives an impression of the nonlinearity to be expected in each instance. However, recent measurements in solids such as rocks and PZT give a much larger nonlinearity parameter. In this case, and similar ones, one must exercise care. Very large nonlinearity parameters often can be explained by including previously ignored effects in the analysis. As an introduction, the nonlinear behavior of PZT is discussed.

INTRODUCTION

Early theoretical work on the propagation of finite amplitude waves in an isentropic gas was done by Stokes [1], Earnshaw [2], and Riemann [3]. Various special approximations and simplifications are intended to obtain a solution in spite of the nonlinearity. In 1935 Thuras, Jenkins and O'Neill [4] demonstrated unequivocally that waveform distortion occurs in an air-filled tube because of nonlinear effects. Soon thereafter it became necessary to investigate the propagation of finite amplitude waves in liquids. In water, nonlinear effects could be studied using light diffraction. A plane ultrasonic wave serves as a grating for the light. As waveform distortion occurs the ultrasonic wave becomes a blaze grating and the diffraction patterns become asymmetrical. Diffraction patterns produced by a 1.7 MHz ultrasonic wave [5] are shown in Fig. 1. The progressive waveform distortion can be followed as one increases the amplitude or the propagation distance. Such waveform distortion also occurs in solids, but its effect is not as easy to demonstrate.

NONLINEAR DISTORTION IN SOLIDS

We were able to demonstrate the presence of finite amplitude effects in aluminum [6] by measuring the presence of second harmonics of a 30 MHz fundamental ultrasonic wave. The fact that the amplitude of the second harmonic was proportional to the square of the fundamental amplitude was evidence that nonlinear effects were the cause of the second harmonic generation.

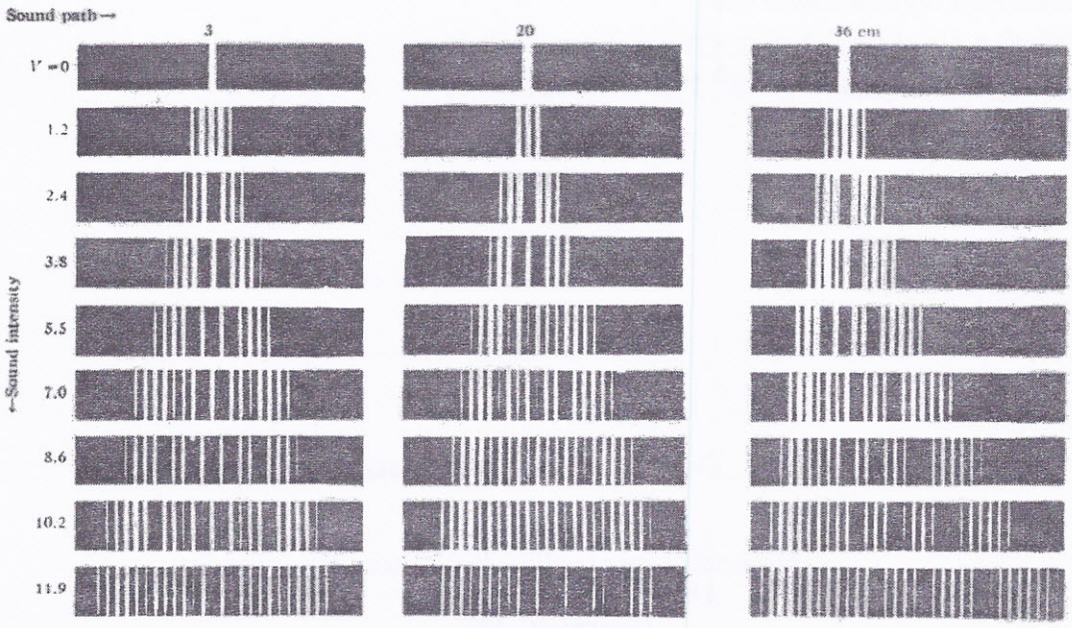

Figure 1. Light diffraction by ultrasonic waves in water. f = 1.76 MHz.

The theoretical analysis for single crystal solids could follow that for fluids, provided the nonlinear equations describing ultrasonic wave propagation in solids were the same as those for fluids. This was true for longitudinal wave propagation. (Shear waves in solids do not couple to their second harmonics in the first nonlinear terms, so they can be ignored.) Our objective was to evaluate coefficients of the nonlinear terms in solids because they were a measure of the deviation from the predictions of Hooke's law.

THEORETICAL ANALYSIS FOR FLUIDS AND CRYSTALLINE SOLIDS

The non-dissipative equation describing the propagation of finite amplitude sound in an ideal gas is [7]

$$\frac{\partial^2 U}{\partial t^2} = \frac{c_o^2 \dfrac{\partial^2 U}{\partial x^2}}{\left(1+\dfrac{\partial U}{\partial x}\right)^{\gamma+1}} \quad , \tag{1}$$

where U is the particle displacement, and c_o is the velocity of propagation of a small amplitude wave. This equation can be generalized by using an equation of state in the form of a Taylor's series:

$$p = p_o + A\frac{\rho-\rho_o}{\rho_o} + \frac{B}{2}\left(\frac{\rho-\rho_o}{\rho_o}\right)^2 + \ldots \tag{2}$$

This leads to an equation of the form

$$\frac{\partial^2 U}{\partial t^2} = \frac{c_o^2 \dfrac{\partial^2 U}{\partial x^2}}{\left(1+\dfrac{\partial U}{\partial x}\right)^{\frac{B}{A}+2}} \tag{3}$$

This equation is to be compared with an equation resulting from the application of elasticity to crystalline solids. If one uses elasticity and keeps higher order terms one obtains an equation of the form

$$\rho_o \frac{\partial^2 U}{\partial t^2} = K_2\left(\frac{\partial^2 U}{\partial a^2} + 3\frac{\partial U}{\partial a}\frac{\partial^2 U}{\partial a^2}\right) + K_3\frac{\partial U}{\partial a}\frac{\partial^2 U}{\partial a^2}$$

$$= K_2\frac{\partial^2 U}{\partial a^2}\left[1+\left(\frac{K_3}{K_2}+3\right)\frac{\partial U}{\partial a}\right] + \cdots \tag{4}$$

where a is the distance in the propagation direction and the expressions for K_2 and K_3 are given in Table I. If one applies a binomial expansion to the denominator in Eq. 3 and recognizes that $c_o^2 = \frac{A}{\rho_o}$, then Eq. 3 becomes

$$\rho_o \frac{\partial^2 U}{\partial t^2} = A \frac{\partial^2 U}{\partial x^2}\left[1 - \left(\frac{B}{A} + 2\right)\frac{\partial U}{\partial x} + \cdots\right] \qquad (5)$$

These two equations have the same mathematical form. This means that a finite amplitude ultrasonic wave in a solid behaves in the same way as a finite amplitude wave in a fluid. The scaling factor is the ratio of $-\left(\frac{K_3}{K_2} + 3\right)$ to $\left(\frac{B}{A} + 2\right)$. As long as other processes do not enter in either medium, we can use the appropriate equation to define the nonlinearity parameter as the negative ratio of the coefficient of the nonlinear term to the coeficient of the linear term in the nonlinear wave equation. The appropriate expressions for the nonlinearity parameter in gases, liquids, and solids are found in Table II.

EXPERIMENTAL RESULTS

We have used the harmonic generation technique to evaluate the nonlinearity parameters of a number of crystalline solids. Our results indicate that the nonlinearity parameter in crystalline solids ranges from 3 to 15 (According to recent calculations it may be slightly larger than this in certain solids.) Typical behavior of the nonlinearity parameter as a function of temperature in the [100] direction in crystalline solids is shown in Fig. 2. One notices immediately that the results for fused silica are different from those for the other solids. Upon examination, one finds that fused silica is not truly a crystalline solid.

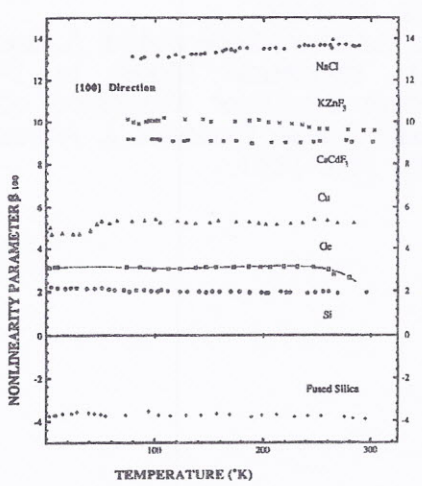

Figure 2. Nonlinearity parameter as a function of temperature.

Direction	K_2	K_3
[100]	C_{11}	C_{111}
[110]	$\frac{C_{11}+C_{12}+2C_{44}}{2}$	$\frac{C_{111}+3C_{112}+12C_{164}}{4}$
[111]	$\frac{C_{11}+2C_{12}+4C_{44}}{3}$	$(1/9)(C_{111}+6C_{112}+12C_{144}+24C_{164}+2C_{124}+16C_{456})$

Table I. Expressions for K_2 and K_3 for longitudinal waves in cubic crystals.

Material	c_0^2	β
Gas	$\frac{\gamma P}{\rho_0}$	$\gamma + 1$
Liquid	$\frac{A}{\rho_0}$	$\frac{B}{A} + 2$
Cubic Solid	$\frac{K_2}{\rho_0}$	$\frac{K_3}{K_2} + 3$

Table II. Small amplitude sound velocity and nonlinearity parameter in various media.

The exact reason for the behavior of fused silica may be elusive, but I can confirm the fact that the nonlinearity parameter is negative. We have examined harmonic generation in fused silica with a phase sensitive detector [7] and found that our results are consistent with a negative nonlinearity parameter. The point really is that fused silica is not a single crystal.

We have examined the behavior of another noncrystalline solid PZT and found some equally interesting behavior [8]. In Fig. 3 we show that the measured nonlinearity parameter in one sample of PZT goes to approsimately 1500 at the Curie Temperature. Such behavior is far from that of a single crystal. For this reason we examined the frequency dependence of the nonlinearity parameter of two samples of PZT. The results are shown in Fig. 4. The nonpolarized data were taken after the temperature had been raised above the Curie Temperature, then lowered without appplication of a polarizing voltage.

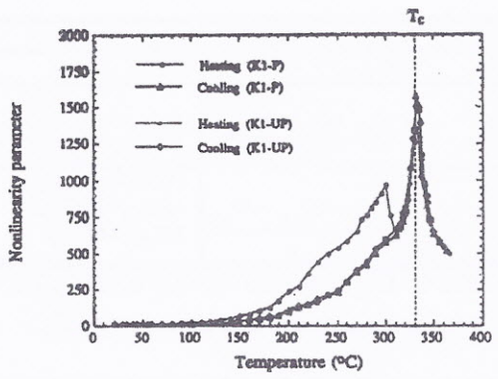

Figure 3. Change of nonlinearity parameter as a function of temperature in K1-Polarized and -Unpolarized PZT ceramic samples at frequency 10 MHz.

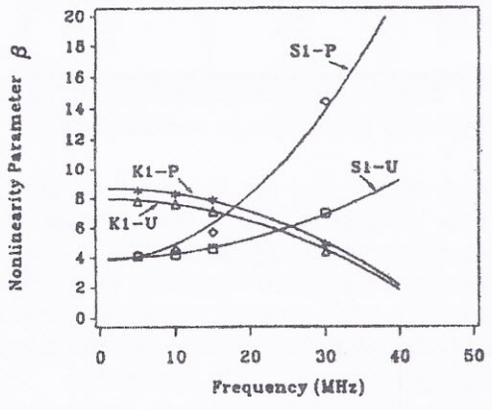

Figure 4. Frequency dependence of the nonlinearity parameter β for different PZT samples. Data points represent experimental measurements. The lines are theoretical prediction using the perturbation solution of the dispersive nonlinear differential equation.

These results suggested that at least one source of nonlinearity had been ignored. The variation of nonlinearity with frequency could be traced by an equation [9] of the form

$$\rho_o \frac{\partial^2 U}{\partial t^2} = K_2 \frac{\partial^2 U}{\partial a^2} + (3K_2 + K_3)\frac{\partial U}{\partial a}\frac{\partial^2 U}{\partial a^2} + \Gamma_2 \frac{\partial^4 U}{\partial a^4} + \Gamma_3 \frac{\partial U}{\partial a}\frac{\partial^4 U}{\partial a^4} \qquad (6)$$

in which variation with frequency has been included with the last two terms. The curves in Fig. 4 actually are theoretical curves drawn from Eq. 6.

SUMMARY

A large nonlinearity parameter may result from nonlinear effects not considered in the original equation. This is especially true of non-crystalline solids. When this happens it is important to trace down the origin of the nonlinearity and to attribute the nonlinearity to the proper physical origin.

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