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INDEPENDENCE OF THE CONTROLLABILITY AND OBSERVABILITY OF RESISTANCES IN LINEAR ELECTRICAL CIRCUITS

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Abstract – Sufficient conditions are given under which the controllability and observability of linear electrical circuits is independent of their resistances. In some particular cases the observability depends only on the capacitances or inductances of the electrical circuits.

Key words – independence, controllability, observability, resistance, linear, electrical circuit.

INTRODUCTION

The notion of controllability and observability and the decomposition of linear systems have been introduced by Kalman [9, 10]. These notions are the basic concepts of the modern control theory [1, 3, 8, 11-13]. They have been also extended to positive linear systems [2, 4, 6, 7]. It is well-known that the controllability and observability of linear systems are generic properties of the systems [11].

A dynamical system is called positive if its trajectory starting from any nonnegative initial condition state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive system theory is given in the monographs [2, 4, 6, 7] and in the paper [5]. Models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc.

In this paper the independence of the controllability and observability of linear electrical circuits of their resistances will be addressed.

The paper is organized as follows. In Section 1 the basic definitions and theorems concerning the controllability and observability are recalled. Sufficient conditions for the independence of the controllability and observability of linear electrical circuits of their resistances are presented in Section 2 and illustrating example of electrical circuit is given in Section 3. Concluding remarks are given in Section 4.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ real matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n\times n$ identity matrix.

I. CONTROLLABILITY AND OBSERVABILITY OF LINEAR ELECTRICAL CIRCUITS

Consider the linear continuous-time electrical circuit described by the state equations

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1a}$$

$$y(t) = Cx(t) + Du(t), \tag{1b}$$

where $x(t)\in\Re^n$, $u(t)\in\Re^m$, $y(t)\in\Re^p$ are the state, input and output vectors and $A\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{p\times n}$, $D\in\Re^{p\times m}$.

It is well-known [7] that any linear electrical circuit composed of resistors, coils, capacitors and voltage (current) sources can be described by the state equations (1). Usually as the state variables $x_1(t),\ldots,x_n(t)$ (the components of the state vector $x(t)$) the currents in the coils and voltages on the capacitors are chosen.

Definition 1. [7] The electrical circuit (1) is called (internally) positive if $x(t)\in\Re_+^n$ and $y(t)\in\Re_+^p$, $t\in[0,+\infty)$ for any $x_0 = x(0)\in\Re_+^n$ and every $u(t)\in\Re_+^m$, $[0,+\infty)$.

Theorem 1. [7] The electrical circuit (2) is positive if and only if

$$A\in M_n,\quad B\in\Re_+^{n\times m},\quad C\in\Re_+^{p\times n},\quad D\in\Re_+^{p\times m}. \tag{2}$$

Theorem 2. [7] The linear electrical circuit composed of resistors, coils and voltage sources is positive for any values of the resistances, inductances and source voltages if the number of coils is less or equal to the number of its linearly independent meshes and the direction of the mesh currents are consistent with the directions of the mesh source voltages.

Theorem 3. [7] The linear electrical circuit composed of resistors, capacitors and voltage sources is not positive for all values of its resistances, capacitances and source voltages if each its branch contains resistor, capacitor and voltage source.

Theorem 4. [7] The *R*, *L*, *C*, *e* electrical circuits are not positive for any values of its resistances, inductances, capacitances and source voltages if at least one its branch contains coil and capacitor.

Definition 2. [7] The electrical circuit (1) (or the pair (A,B)) is called controllable (in the time $[0, t_f]$, $t_f > 0$) if there exists an input $u(t) \in \Re^m$, $t \in [0, t_f]$ which steers the state of the system from initial state $x_0 \in \Re^n$ to any given final state $x_f \in \Re^n$, i.e. $x(t_f) = x_f$.

Theorem 5. [7] The electrical circuit (1) is controllable in time $[0, t_f]$ if and only if

$$\operatorname{rank}[I_n s - A \quad B] = n \quad for \quad s \in \mathbf{C}. \qquad (3)$$

Definition 3. [7] The electrical circuit (2) (or the pair (A,C)) is called observable (in the time $[0, t_f]$, $t_f > 0$) if it is possible to find unique initial state $x_0 \in \Re^n$ of the system knowing its input $u(t) \in \Re^m$ and its output $y(t) \in \Re^p$, $t \in [0, t_f]$.

Theorem 6. [7] The electrical circuit (1) is controllable in time $[0, t_f]$ if and only if

$$\operatorname{rank}\begin{bmatrix} C \\ I_n s - A \end{bmatrix} = n \quad for \quad s \in \mathbf{C}. \qquad (4)$$

II. Independence of the Controllability and Observability

In this section sufficient conditions for the independence of the controllability and observability of the linear electrical circuits of their resistances will be presented.

Consider the linear electrical circuit shown in Figure 1 with given resistances R_k, $k = 1, \dots, 13$, inductances L_i, $i = 1,2,3,4$, capacitances C_j, $j = 1,2,3,4$ and source voltages e_j, $j = 1,2,3,4$.

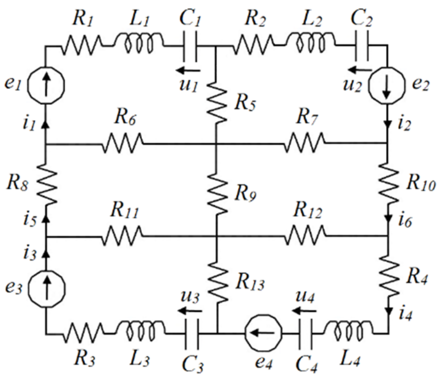

Fig. 1. **Electrical circuit**

It is well-known that the number of linearly independent meshes n_m of the linear electrical circuit is given by the formula

$$n_m = n_b - n_o + 1, \qquad (5)$$

where n_b is the number of branches and n_o is the number of the nodes of the electrical circuit.

The linear independent meshes of the electrical circuit will be divided into following two classes: external and internal meshes.

Definition 4. Linearly independent mesh is called external if it contains only one branch with resistance, inductance, capacitance and source voltage and branches with only resistances. Linearly independent mesh is called internal if it contains only resistances.

Using the Kirchhoff's laws we may write for the external meshes the equations

$$\begin{aligned} e_1 &= L_1 \frac{di_1}{dt} + u_1 + \overline{R}_1 i_1 - R_5 i_2 - R_6 i_5, \\ e_2 &= L_2 \frac{di_2}{dt} + u_2 + \overline{R}_2 i_2 - R_5 i_1 - R_7 i_6, \\ e_3 &= L_3 \frac{di_3}{dt} + u_3 + \overline{R}_3 i_3 - R_{11} i_5 - R_{13} i_4, \\ e_4 &= L_4 \frac{di_4}{dt} + u_4 + \overline{R}_4 i_4 - R_{13} i_3 - R_{12} i_6 \end{aligned} \qquad (6a)$$

and for external meshes

$$\begin{aligned} \overline{R}_5 i_5 - R_9 i_6 - R_6 i_1 - R_{11} i_3 &= 0, \\ \overline{R}_6 i_6 - R_9 i_5 - R_7 i_2 - R_{12} i_4 &= 0, \end{aligned} \qquad (6b)$$

where

$$\begin{aligned} &\overline{R}_1 = R_1 + R_5 + R_6, \quad \overline{R}_2 = R_2 + R_5 + R_7, \\ &\overline{R}_3 = R_3 + R_{11} + R_{13} \quad \overline{R}_4 = R_4 + R_{12} + R_{13}, \\ &\overline{R}_5 = R_6 + R_8 + R_9 + R_{11}, \quad \overline{R}_6 = R_7 + R_9 + R_{10} + R_{12}. \end{aligned} \qquad (6c)$$

The currents in capacitors and their voltages are related by

$$i_k = C_k \frac{du_k}{dt}, \quad k = 1,2,3,4. \tag{7}$$

From the equations (6a) we have

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -\frac{\overline{R}_1}{L_1} & \frac{R_5}{L_1} & 0 & 0 \\ \frac{R_5}{L_2} & -\frac{\overline{R}_2}{L_2} & 0 & 0 \\ 0 & 0 & -\frac{\overline{R}_3}{L_3} & \frac{R_{13}}{L_3} \\ 0 & 0 & \frac{R_{13}}{L_4} & -\frac{\overline{R}_4}{L_4} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} + \begin{bmatrix} \frac{R_6}{L_1} & 0 \\ 0 & \frac{R_7}{L_2} \\ \frac{R_{11}}{L_3} & 0 \\ 0 & \frac{R_{12}}{L_4} \end{bmatrix}\begin{bmatrix} i_5 \\ i_6 \end{bmatrix} - \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 & 0 \\ 0 & 0 & \frac{1}{L_3} & 0 \\ 0 & 0 & 0 & \frac{1}{L_4} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 & 0 \\ 0 & 0 & \frac{1}{L_3} & 0 \\ 0 & 0 & 0 & \frac{1}{L_4} \end{bmatrix}\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{8}$$

and from (6b)

$$\begin{bmatrix} i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} \overline{R}_5 & -R_9 \\ -R_9 & \overline{R}_6 \end{bmatrix}^{-1}\begin{bmatrix} R_6 & 0 & R_{11} & 0 \\ 0 & R_7 & 0 & R_{12} \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}. \tag{9}$$

Substitution of (9) into (8) yields

$$\frac{d}{dt}\begin{bmatrix} u_{14} \\ i_{14} \end{bmatrix} = A\begin{bmatrix} u_{14} \\ i_{14} \end{bmatrix} + Be_{14}, \tag{10a}$$

where

$$u_{14} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}, \quad i_{14} = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \quad e_{14} = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}, \tag{10b}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{C_4} \\ -\frac{1}{L_1} & 0 & 0 & 0 & \frac{\overline{R}_{11}}{L_1} & \frac{\overline{R}_{12}}{L_1} & \frac{\overline{R}_{13}}{L_1} & \frac{\overline{R}_{14}}{L_1} \\ 0 & -\frac{1}{L_2} & 0 & 0 & \frac{\overline{R}_{21}}{L_2} & \frac{\overline{R}_{22}}{L_2} & \frac{\overline{R}_{23}}{L_2} & \frac{\overline{R}_{24}}{L_2} \\ 0 & 0 & -\frac{1}{L_3} & 0 & \frac{\overline{R}_{31}}{L_3} & \frac{\overline{R}_{32}}{L_3} & \frac{\overline{R}_{33}}{L_3} & \frac{\overline{R}_{34}}{L_3} \\ 0 & 0 & 0 & -\frac{1}{L_4} & \frac{\overline{R}_{41}}{L_4} & \frac{\overline{R}_{42}}{L_4} & \frac{\overline{R}_{43}}{L_4} & \frac{\overline{R}_{44}}{L_4} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 & 0 \\ 0 & 0 & \frac{1}{L_3} & 0 \\ 0 & 0 & 0 & \frac{1}{L_4} \end{bmatrix} \tag{10c}$$

and

$$\overline{R} = \begin{bmatrix} \overline{R}_{11} & \overline{R}_{12} & \overline{R}_{13} & \overline{R}_{14} \\ \overline{R}_{21} & \overline{R}_{22} & \overline{R}_{23} & \overline{R}_{24} \\ \overline{R}_{31} & \overline{R}_{32} & \overline{R}_{33} & \overline{R}_{34} \\ \overline{R}_{41} & \overline{R}_{42} & \overline{R}_{43} & \overline{R}_{44} \end{bmatrix} = \begin{bmatrix} -\overline{R}_1 & R_5 & 0 & 0 \\ R_5 & -\overline{R}_2 & 0 & 0 \\ 0 & 0 & -\overline{R}_3 & R_{13} \\ 0 & 0 & R_{13} & -\overline{R}_4 \end{bmatrix} + \begin{bmatrix} R_6 & 0 \\ 0 & R_2 \\ R_{11} & 0 \\ 0 & R_{12} \end{bmatrix}\begin{bmatrix} \overline{R}_5 & -R_9 \\ -R_9 & \overline{R}_6 \end{bmatrix}^{-1} \times \begin{bmatrix} R_6 & 0 & R_{11} & 0 \\ 0 & R_7 & 0 & R_{12} \end{bmatrix} \tag{10d}$$

The electrical circuit described by (10a) is not positive since its matrix A defined by (10c) is not a Metzler matrix.

Theorem 7. The controllability of linear electrical circuits is independent of their resistances if the number n_e of their external meshes satisfies the condition

$$n_e \le n_m, \tag{11}$$

where n_m is defined by (5).

Proof. To simplify the notation the proof will be given for the electrical circuit shown in Figure 1. In this case we have $n_b = 13$, $n_0 = 8$ and $n_e = 4$. Therefore, the condition (11) is satisfied since $n_m > n_e$. Using the condition (3) of Theorem 5 to the matrices (10c) we obtain (12) for $s \in \mathbf{C}$. The condition (12) is satisfied for all values of the resistances of the electrical circuit. Therefore, the controllability of the linear electrical circuit is independent of its resistances.

Theorem 8. The controllability of linear electrical circuits depends only on their inductances and capacitances if each branch with inductance contains also source voltage.

Proof. Note that if each branch with inductance contains also source voltage then to each row in the matrix A with resistances we have a nonzero entry in corresponding row in the matrix B. In this case using elementary column operations it is possible to eliminate in the matrix $[I_n s - A \quad B]$ entries with the resistances.

Now the observability of the linear electrical circuit shown in Figure 1 will be analyzed for the following three cases:

Case 1. $C = [C_1 \quad 0] \in \Re^{4\times 8}$, $\det C_1 \neq 0$.

Case 2. $C = [0 \quad C_2] \in \Re^{4\times 8}$, $\det C_2 \neq 0$.

Case 3. $C = [C_1 \quad C_2] \in \Re^{4\times 8}$, $\operatorname{rank} C = 4$.

In Case 1 we have the following theorem.

Theorem 9. In Case 1 the observability of linear electrical circuits is independent of their resistances and inductances if the condition (11) is satisfied.

Proof. Using the condition (4) of Theorem 6 to the matrix *A* defined by (15) and $C = [C_1 \quad 0]$, $\det C_1 \neq 0$ we obtain

$$\operatorname{rank}\begin{bmatrix} C \\ I_8 s - A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} C_1 & 0 \\ I_4 s & -F \\ G & I_4 s - \overline{R} \end{bmatrix} = 8 \quad \text{for} \quad \det C_1 \neq 0, \quad \det F \neq 0, \tag{12}$$

where

$$F = \begin{bmatrix} \frac{1}{C_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & 0 \\ 0 & 0 & \frac{1}{C_3} & 0 \\ 0 & 0 & 0 & \frac{1}{C_4} \end{bmatrix}, \quad G = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 & 0 \\ 0 & 0 & \frac{1}{L_3} & 0 \\ 0 & 0 & 0 & \frac{1}{L_4} \end{bmatrix}, \tag{13}$$

and the matrix $\overline{R}$ is defined by (10d).

The condition (12) is satisfied for all values of the resistances and inductances of the electrical circuit. Therefore, the observability of the linear electrical circuits in Case 1 is independent of their resistances and inductances if the condition (11) is satisfied.

Theorem 10. In Case 2 the observability of linear electrical circuits is independent of their resistances and capacitances if the condition (11) is satisfied.

Proof. Using the condition (4) of Theorem 6 to the matrix A defined by (10c) and $C = [0 \quad C_2]$, $\det C_2 \neq 0$ we obtain

$$\operatorname{rank}\begin{bmatrix} C \\ I_8 s - A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 0 & C_2 \\ I_4 s & -F \\ G & I_4 s - \overline{R} \end{bmatrix} = 8 \quad \text{for} \quad \det C_2 \neq 0, \quad \det G \neq 0. \tag{14}$$

The condition (14) is satisfied for all values of the resistances and capacitances of the electrical circuit. Therefore, the observability of the linear electrical circuits in Case 2 is independent of their resistances and capacitances if the condition (11) is satisfied.

Theorem 11. In Case 3 the observability of linear electrical circuits is independent of their resistances and capacitances if the condition (11) is satisfied.

Proof. The proof is similar to the proofs of Theorems 9 and 10.

III. Example

Consider the linear electrical circuit shown in Figure 2 with given resistances R_k, $k=1,\dots,6$, inductances L_i, $i=1,2,3$, capacitances C_j, $j=1,2,3$ and source voltages e_j, $j=1,2,3$.

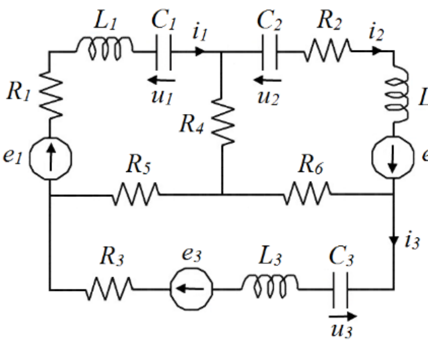

Fig. 2. **Electrical circuit**

Using the Kirchhoff's laws we may write the equations

$$\begin{aligned} e_1 &= L_1\frac{di_1}{dt}+R_{11}i_1-R_4i_2-R_5i_3+u_1,\\ e_2 &= L_2\frac{di_2}{dt}+R_{22}i_2-R_4i_1-R_6i_3+u_2,\\ e_3 &= L_3\frac{di_3}{dt}+R_{33}i_3-R_5i_1-R_6i_2+u_3, \end{aligned} \tag{15a}$$

where

$$R_{11}=R_1+R_4+R_5,\quad R_{22}=R_2+R_4+R_6,\quad R_{33}=R_3+R_5+R_6. \tag{15b}$$

The currents in capacitors and their voltages are related by

$$i_k=C_k\frac{du_k}{dt},\quad k=1,2,3. \tag{16}$$

The equations (15) and (16) can be written in the form

$$\frac{d}{dt}\begin{bmatrix}u_{13}\\ i_{13}\end{bmatrix}+A\begin{bmatrix}u_{13}\\ i_{13}\end{bmatrix}+Be_{13}, \tag{17a}$$

where

$$u_{13}=\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix},\quad i_{13}=\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix},\quad e_{13}=\begin{bmatrix}e_1\\ e_2\\ e_3\end{bmatrix}, \tag{17b}$$

$$A=\begin{bmatrix} 0 & 0 & 0 & \frac{1}{C_1} & 0 & 0\\ 0 & 0 & 0 & 0 & \frac{1}{C_2} & 0\\ 0 & 0 & 0 & 0 & 0 & \frac{1}{C_3}\\ -\frac{1}{L_1} & 0 & 0 & -\frac{R_{11}}{L_1} & \frac{R_4}{L_1} & \frac{R_5}{L_1}\\ 0 & -\frac{1}{L_2} & 0 & \frac{R_4}{L_2} & -\frac{R_{22}}{L_2} & \frac{R_6}{L_2}\\ 0 & 0 & -\frac{1}{L_3} & \frac{R_5}{L_3} & \frac{R_6}{L_3} & -\frac{R_{33}}{L_3} \end{bmatrix}, \quad B=\begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ \frac{1}{L_1} & 0 & 0\\ 0 & \frac{1}{L_2} & 0\\ 0 & 0 & \frac{1}{L_3} \end{bmatrix}. \tag{17c}$$

Note that in this case we have $n_b=6$, $n_o=4$, $n_e=3$ and $n_e=n_m$. Therefore, the electrical circuit has only external meshes. The electrical circuit is not positive since its matrix A is not a Metzler matrix.

The electrical circuit is controllable since we have (18) and the controllability is independent of its resistances R_k, $k=1,\dots,6$.

The following three cases of the choice of the output matrix C of the electrical circuit will be considered:

$$\operatorname{rank}[I_8s-A\quad B]=\operatorname{rank}\left[\begin{array}{cccccccc|cccc} s & 0 & 0 & 0 & -\frac{1}{C_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & s & 0 & 0 & 0 & -\frac{1}{C_2} & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & s & 0 & 0 & 0 & -\frac{1}{C_3} & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & s & 0 & 0 & 0 & -\frac{1}{C_4} & 0 & 0 & 0 & 0\\ \frac{1}{L_1} & 0 & 0 & 0 & s-\frac{\bar R_{11}}{L_1} & -\frac{\bar R_{12}}{L_1} & -\frac{\bar R_{13}}{L_1} & -\frac{\bar R_{14}}{L_1} & \frac{1}{L_1} & 0 & 0 & 0\\ 0 & \frac{1}{L_2} & 0 & 0 & -\frac{\bar R_{21}}{L_2} & s-\frac{\bar R_{22}}{L_2} & -\frac{\bar R_{23}}{L_2} & -\frac{\bar R_{24}}{L_2} & 0 & \frac{1}{L_2} & 0 & 0\\ 0 & 0 & \frac{1}{L_3} & 0 & -\frac{\bar R_{31}}{L_3} & -\frac{\bar R_{32}}{L_3} & s-\frac{\bar R_{33}}{L_3} & -\frac{\bar R_{34}}{L_3} & 0 & 0 & \frac{1}{L_3} & 0\\ 0 & 0 & 0 & \frac{1}{L_4} & -\frac{\bar R_{41}}{L_4} & -\frac{\bar R_{42}}{L_4} & -\frac{\bar R_{43}}{L_4} & s-\frac{\bar R_{44}}{L_4} & 0 & 0 & 0 & \frac{1}{L_4} \end{array}\right]=8 \tag{12}$$

Case 1. $C = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$

Case 2. $C = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$

Case 3. $C = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$

In Case 1 using the condition (4) we obtain

$$\operatorname{rank}\begin{bmatrix} C \\ I_6 s - A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline s & 0 & 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & s & 0 & 0 & -\frac{1}{C_2} & 0 \\ 0 & 0 & s & 0 & 0 & -\frac{1}{C_3} \\ \frac{1}{L_1} & 0 & 0 & s+\frac{R_{11}}{L_1} & -\frac{R_4}{L_1} & -\frac{R_5}{L_1} \\ 0 & \frac{1}{L_2} & 0 & -\frac{R_4}{L_2} & s+\frac{R_{22}}{L_2} & -\frac{R_6}{L_2} \\ 0 & 0 & \frac{1}{L_3} & -\frac{R_5}{L_3} & -\frac{R_6}{L_3} & s+\frac{R_{33}}{L_3} \end{bmatrix} = 6 \quad \text{for } C_k \neq 0,\ k = 1,2,3. \tag{19}$$

Therefore, the electrical circuit is observable and its observability is independent of its resistances and inductances.

In Case 2 using the condition (4) we obtain

$$\operatorname{rank}\begin{bmatrix} C \\ I_6 s - A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline s & 0 & 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & s & 0 & 0 & -\frac{1}{C_2} & 0 \\ 0 & 0 & s & 0 & 0 & -\frac{1}{C_3} \\ \frac{1}{L_1} & 0 & 0 & s+\frac{R_{11}}{L_1} & -\frac{R_4}{L_1} & -\frac{R_5}{L_1} \\ 0 & \frac{1}{L_2} & 0 & -\frac{R_4}{L_2} & s+\frac{R_{22}}{L_2} & -\frac{R_6}{L_2} \\ 0 & 0 & \frac{1}{L_3} & -\frac{R_5}{L_3} & -\frac{R_6}{L_3} & s+\frac{R_{33}}{L_3} \end{bmatrix} = 6 \quad \text{for } L_k \neq 0,\ k = 1,2,3. \tag{20}$$

Therefore, the electrical circuit is observable and its observability is independent of its resistances and capacitances. In Case 3 using the condition (4) we obtain

$$\operatorname{rank}\begin{bmatrix} C \\ I_6 s - A \end{bmatrix} = \operatorname{rank}\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \hline s & 0 & 0 & -\frac{1}{C_1} & 0 & 0 \\ 0 & s & 0 & 0 & -\frac{1}{C_2} & 0 \\ 0 & 0 & s & 0 & 0 & -\frac{1}{C_3} \\ \frac{1}{L_1} & 0 & 0 & s+\frac{R_{11}}{L_1} & -\frac{R_4}{L_1} & -\frac{R_5}{L_1} \\ 0 & \frac{1}{L_2} & 0 & -\frac{R_4}{L_2} & s+\frac{R_{22}}{L_2} & -\frac{R_6}{L_2} \\ 0 & 0 & \frac{1}{L_3} & -\frac{R_5}{L_3} & -\frac{R_6}{L_3} & s+\frac{R_{33}}{L_3} \end{bmatrix} = 6 \quad \text{for } L_k \neq 0,\ C_k \neq 0,\ k = 1,2,3. \tag{21}$$

$$\operatorname{rank}\begin{bmatrix} I_6 s - A & B \end{bmatrix} = \operatorname{rank}\left[\begin{array}{cccccc|ccc} s & 0 & 0 & -\frac{1}{C_1} & 0 & 0 & 0 & 0 & 0 \\ 0 & s & 0 & 0 & -\frac{1}{C_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & s & 0 & 0 & -\frac{1}{C_3} & 0 & 0 & 0 \\ \frac{1}{L_1} & 0 & 0 & s+\frac{R_{11}}{L_1} & -\frac{R_4}{L_1} & -\frac{R_5}{L_1} & \frac{1}{L_1} & 0 & 0 \\ 0 & \frac{1}{L_2} & 0 & -\frac{R_4}{L_2} & s+\frac{R_{22}}{L_2} & -\frac{R_6}{L_2} & 0 & \frac{1}{L_2} & 0 \\ 0 & 0 & \frac{1}{L_3} & -\frac{R_5}{L_3} & -\frac{R_6}{L_3} & s+\frac{R_{33}}{L_3} & 0 & 0 & \frac{1}{L_3} \end{array}\right] = 3 \tag{18}$$

Therefore, the electrical circuit is observable and its observability is independent only of its resistances.

It is well-known [7] that the linear electrical circuits are not positive if at least one of their branch contains resistance, inductance and capacitance. The linear electrical circuits analyzed in this paper belong to this class. Therefore, we have the following conclusion.

Conclusion 1. The linear electrical circuits analyzed in this paper are not positive.

IV. CONCLUDING REMARKS

The independence of the controllability and observability of resistances, inductances and capacitances of linear electrical circuits has been investigated. Sufficient conditions have been given:

for the independence of the controllability of linear electrical circuits of their resistances (Theorems 7 and 8);

for the independence of the observability of linear electrical circuits of their resistances and inductances (Theorem 9), of their resistances and capacitances (Theorem 10) and of their resistances (Theorem 11).

It is shown that the electrical circuits satisfying the sufficient conditions are not positive electrical circuits (Conclusion 1).

The considerations have been illustrated by examples of linear electrical circuits.

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