Applications of the combinatorial configurations for optimization of technological systems

V. Riznyk

Lviv Polytechnic National University, e-mail: rvv@polynet.lviv.ua

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Abstract: This paper involves techniques for improving the quality indices of engineering devices or systems with non-uniform structure (e.g. arrays of sonar antenna arrays) with respect to performance reliability, transmission speed, resolving ability, and error protection, using novel designs based on combinatorial configurations such as classic cyclic difference sets and novel vector combinatorial configurations. These design techniques makes it possible to configure systems with fewer elements than at present, while maintaining or improving on the other operating characteristics of the system. Several factors are responsible for distinguish of the objects depending an implicit function of symmetry and non-symmetry interaction subject to the real space dimensionality. Considering the significance of circular symmetric field, while an asymmetric subfields of the field, further a better understanding of the role of geometric structure in the behaviour of system objects is developed. This study, therefore, aims to use the appropriate algebraic results and techniques for improving such quality indices as combinatorial varieties, precision, and resolving ability, using remarkable properties of circular symmetry and non-symmetry mutual penetration as an interconnection cyclic relationships, and interconvertible dimensionality models of optimal distributed systems. Paper contains some examples for the optimization relating to the optimal placement of structural elements in spatially or temporally distributed technological systems, to which these techniques can be applied, including applications to coded design of signals for communications and radar, positioning of elements in an antenna array, and development vector data coding design.

Key words: Ideal Ring Bundle, circular sequence, circular symmetry, model, optimal proportion, vector data coding, self-correcting, resolving ability.

INTRODUCTION

The latest advances in the modern theory of systems of certifying the existence of a direct link of circular symmetry and non-symmetry relationships with delivering the totality of philosophical, methodological, specifically-scientific and applied problems, which allowed her to gain the status of theoretical foundation of system engineering in modern science. Particularly relevant emerging study of the physical laws of nature, what in their treatises paid attention even the ancient philosophers. Studies include the use of modern mathematical methods of optimization of systems that exist in the structural analysis, the theory of combinatorial configurations, analysis of finite groups and fields, algebraic number theory and coding. Two aspects of the matter the issue are examined useful in applications of symmetrical and non-symmetrical models: optimization of technology, and hypothetic unified “universal informative field of harmony” [1].

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

In [2] developed Verilog Analog Mixed-Signal simulation (Verilog-AMS) model of the comp-drive sensing element of integrated capacitive micro-accelerometer. This model allows simulate the reaction of the sensing elements effected by the applied force of acceleration, changes of its comb-drive capacities, output voltages and currents for determining its constructive parameters and for analysis of mechanical module the integrated device, but precision are of very important indices for these models. Proposed in [3] method of adaptive data transmission in telecommunication access networks with a combine modulation types ensures the lowest possible bit error rate during data transmission at some ratio of signal power to noise power. General problem of systems optimization relates to finding the best placement of its structural elements and events. Research into underlying mathematical area involves the appropriate algebraic structures and modern combinatorial analysis [4], finite projective geometry [5], difference sets and finite groups in extensions of the Galois fields [4]. We’re seeing a remarkable progress in developing of innovative techniques in systems optimization and design combinatorial Sequencing Theory, namely the concept of Ideal Ring Bundles (IRB)s [6-16]. The concept of the IRBs can be used for finding optimal solutions for wide classes of technological problems. A new vision of the concept with point of view of the role of geometric circular symmetry and non-symmetry mutual relation laws allows better understand the idea of “perfect” combinatorial constructions, and to apply this concept for multidimensional systems optimization [17-20].

OBJECTIVES

The objectives of the underlying concept are as research into the underlying mathematical principles relating to the optimal placement of structural elements in spatially or temporally distributed systems, including appropriate algebraic constructions based on cyclic groups. Development of the scientific basis for technologically optimum systems theory, and the generalization of these methods and results to the improvement and optimization of technological systems.

THE MAIN RESULTS OF THE RESEARCH

IRBs are cyclic sequences of positive integers which form perfect partitions of a finite interval $[1,N]$ of integers. The sums of connected sub-sequences of an IRB enumerate the set of integers $[1,N\text{-}1]$ exactly R-times[8]. Example: The IRB {1, 2, 6, 4} containing four elements allows an enumeration of all numbers from 1 to 12= 1+2+6+4 exactly once (R=1). The chain ordered approach to the study of sequences and events is known to be of widespread applicability, and has been extremely effective when applied to the problem of finding the optimum ordered arrangement of structural elements in a sequence. Let us regard n-fold symmetric sequence as to an ability to reproduce the maximum number of combinatorial varieties in the sequence using two-part distribution. Clearly, the maximum number N_n of such variants in n-stage sequence is taken two connected sub-sequences of the sequence:

$$N_n = n - 1 \qquad (1)$$

The maximum number of variants N in ring ordered (closed loop) sequence divided of two connected subsequence is a number of ordered combinations of n elements taken 2 at a time as below

$$N = n(n-1) \qquad (2)$$

Comparing the equations (1) and (2), we see that the number N of ordered combinations for binary consecutive sub-sequences of close-loop topology is n as many the number N_n of combination in the non-closed topology, for the same sequence of n elements.

To extract meaningful information from the underlying comparison let us apply to circular S-fold symmetry as a quantized planar field of two complementary completions of the symmetric field.

Example: The 3-fold (S=3) circular symmetry combined with two complementary completions (Fig.1).

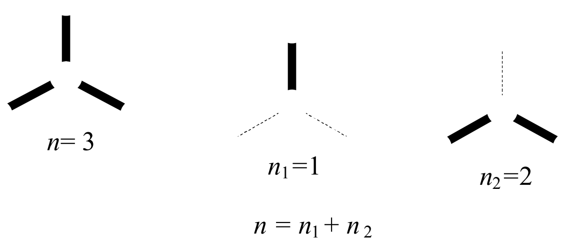

Fig.1. The 3-fold circular symmetry (left) combined with two complementary completions.

In general the order S of the circular symmetry may be chosen arbitrarily. The objectives of the proposed research and applications of circular symmetric and non-symmetric relationships are improving such quality of information technology as precision, resolving ability, and a better understanding of the role of geometric structure in the behaviour of natural objects. Underlying models help to understand the evolutionary aspects of the role of geometric structure in the behaviour of natural and man-made objects. In this reasoning, we precede from a visible geometric circular symmetry.

A plane circular S-fold symmetry, is known can be depicted graphically as a set of S lines diverged from a central point O uniformly (to be equal in spacing angles). Regarding n_1 and n_2 of S lines as being complementary sets ($S = n_1 + n_2$), we require all angular distances between n_1 lines enumerate the set of spacing angles $[\alpha_{min}, 360°\text{-}\alpha_{min}]$ exactly R_1 times, while between n_2 - exactly R_2 times, we call this a Perfect Circular Relation (PCR). Our reasoning proceeds from the fact, that minimal and maximal angular distances relation initiated by S-fold circular symmetry to be of prime importance for finding the PCR origin (Fig.2)

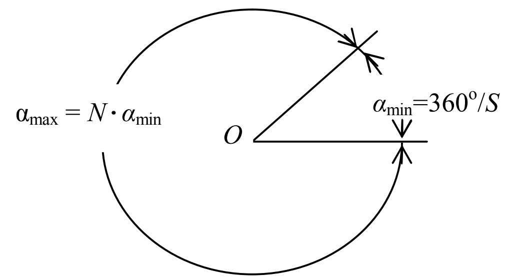

Fig. 2. The minimal- maximal relationship of spacing angles of a plane S –fold natural PCR origin

Clearly the set of all $N=n(n\text{-}1)$ angular distances $[\alpha_{min}, N\cdot\alpha_{min}]$ of S-fold PCR plane quantized field allows an enumeration of all integers $[1, S–1]$ exactly R-times (Fig. 2):

$$N = (S-1)R \qquad (3)$$

From equations (2) and (3) follows the integer relation, the PCR for $7 \le S \le 19$, $n_{1,2} \ge R_{1,2}+2$ are tabulated (Table 1):

$$S = \frac{n(n-1)}{R} + 1 \qquad (4)$$

Table 1. PCR for $7 \le S \le 19$, $n_{1,2} \ge R_{1,2}+2$

№	S	n_1	R_1	n_2	R_2
1	7	3	1	4	2
2	11	5	2	6	3
3	13	9	6	4	1
4	15	7	3	8	4
5	19	9	4	10	5

Here are some examples of the simplest PCR. The elementary is 3-fold (S=3) circular symmetry that splits into 1-fold (n_1=1, R_1=1), and 2-fold (n_2=2, R_2=1) complementary asymmetries. The first of them enumerates the set {1} by quantization level $\alpha_{min}=360°$, while the second {1, 2} by $\alpha_{min}=120°$ exactly once. The 7-fold rotational symmetry (S=7) splits into 3-fold (n_1=3) asymmetry, which allows an enumeration the set of all angular intervals [360°/7, 6×360°/7] of n_1=3, R_1=1, and n_2 =4, R_2 =2 by quantization level α_{min}=360°/7, etc.[13].

From Table 1 we can see that PCR is two complementary asymmetries of even (n_1), and odd (n_2) orders, each of them allows an enumeration the set of all angular distances the precise numbers of fixed times.

Optimal numerical circular proportion is cyclic relationship of positive integers, which form perfect partitions of a finite interval [1, N] of integers. The sums of connected numbers of a relationship enumerate the set of integer [1, N] exactly R-times.

Let us regard a graphic vision of an optimal circular proportion as image segmentation, for example numerical circular proportion {1:3:2:7} based on 13-fold PCR, n=4, R=1 (Fig.3)

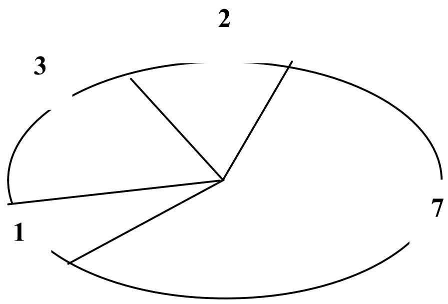

Fig 3. The {1:3:2:7} cyclic ratio segmentation

Observing the {1:3:2:7} image segmentation (Fig.3), we can form complete set of integer harmonious two-body cyclic proportions from 1:12 to 12:1 as follows: 1:12, 2:11, 3:10, 4:9, 5:8, 6:7, 7:6, 8:5. 9:4, 10:3, 11:2, 12:1. The numerical circular proportion {1:3:2:7} based on 13-fold PCR, n=4, R=1 provides the maximum number of harmonious two-body relationships with four (n=4) cross-sections.

Next we regard a graphic vision of the circular proportion {1:1:2:3} based on 7-fold PCR, n=4, R=2 (Fig.4)

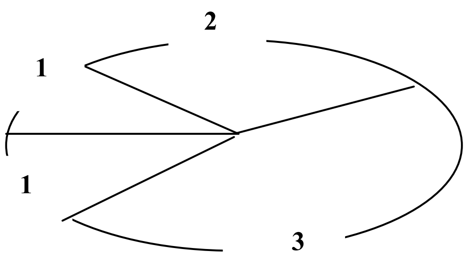

Fig. 4. The {1:1:2:3} cyclic ratio segmentation

Here is an example of complete set of harmonious two-body relationships from 1:6 to 6:1 obtained exactly twice (R=2) each of them over the {1:2:3:1} optimal cyclic segmentation. The cyclic relationship {1:2:3:1^*} containing four (n=4) elements allows an enumeration of all numbers from 1 to 6 exactly twice (R=2): **1**=1=1^*, **2**=2=1^*+1, **3**=3=1+2, **4**=3+1^*=1^*+1+2, **5**=2+3=3+1^*+1, **6**=1+2+3=2+3+1^*. This property makes optimal cyclic relationships useful in application which need to partition sets with the smallest possible number of intersections exactly twice. Here we can see that underlying segmentation provide an ability to reproduce the maximum number of harmonious combinatorial varieties in the system using sequential parting technology.

To discuss concept of Torus Cyclic Groups (TCG)s let us regard structural model of t-dimensional vector ring as ring n- sequence $C_{nt}=\{K_1,K_2,..,K_i,...,K_n\}$ of t-stage sub-sequences (terms) $K_i=(k_{i1},k_{i2},...,k_{it})$ each of them to be completed with nonnegative integers (Fig.5).

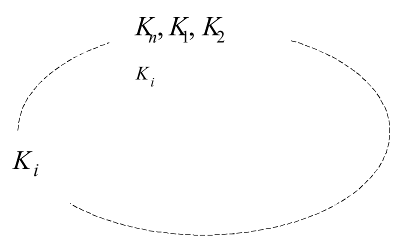

Fig.5. Schematic model of t-dimensional n-stage ring sequence.

Optimal 2D vector circular proportion is a cyclic relationship of 2-tiple (t=2) integers (2D vectors) based on S-fold PCR, which form consecutive 2D partitions over 2D torus surface exactly R times of fixed 2D quantization level. Here is an example of cyclic 2D vector relationship based on 7-fold PRC {(0,1):(0,2):(1,1)} containing three (n=3) 2D vectors, which form complete set of six (S-1=6) 2D vectors as lattice 2×3 covered torus surface exactly once (R=1), and all 2D vector-sums are taken modulo 2,3:

(0,0)≡(0,1)+(0,2)(mod2,3),
(0,1)=(0,1),
(0,2)=(0,2),
(1,0)≡(0,2)+(1,1)(mod2,3),
(1,1)=(1,1),
(1,2)≡(0,1)+(1,1)(mod2,3).

Next we see cyclic 2D vector circular proportion {(0,2):(1,1):(1,1):(1,0)} containing four (n=4) 2D vectors, which form complete set of 2D vectors covered torus surface 2×3 exactly twice (R=2).

The 7-fold PCR is common to optimal circular proportions {1:2:4}, {1:2:3:1},{(0,1):(0,2):(1,1)}, and {(0,2):(1,1):(1,1):(1,0)}. Clearly the set of these optimal proportions corresponds to set of IRBs as follows: {1:2:4}↔{1,2,4}, {1:2:3:1}↔{1,2,3,1}, {(0,1):(0,2):(1,1)}↔{(0,1),(0,2),(1,1)}, {(0,2):(1,1):(1,1):(1,0)} ↔{(0,2),(1,1),(1,1),(1,0)}.

In this we have 1D IRB {1,2,4}, and {1,2,3,1} as well as 2D IRB {(0,1),(0,2),(1,1)}, and {(0,2),(1,1),(1,1),(1,0)}. Both pairs of the IRBs come from the 7-fold circular symmetry. Moreover, the underlying combinatorial configurations are interconvertible dimensionality models of optimal distributed systems.

Regarding formula (4) with n = m_1, and n -1 = m_2 we have equation:

$$S = \frac{m_1 m_2}{R} + 1 \qquad (5)$$

Space coordinate grid $m_1 \times m_2$ forms a frame of two modular (close-loop) axes modulo m_1 and modulo m_2, respectively, over a surface of torus as an orthogonal two modulo cyclic axes of the system being the product of two (t=2) circles. We call this two-dimensional Ideal Ring Bundle (2D IRB).

Here is an example of set completed from the 2D IRBs with m_1=2, m_2 =3, R=1, and takes four variants as follows:

{(0,1),(0,2),(1,1)}, {(1,0),(1,1),(1,2)},
{(0,1),(0,2),(1,0)}, {(0,1),(0,2),(1,2)} (6)

To observe ring sequence {(1,0), (1,1), (1,2)} we can see the next circular vector sums to be consecutive terms in this sequence:

So long as the terms (1,0), (1,1), (1,2) of the cyclic sequence themselves are two-dimensional vector sums also, the set of the modular vector sums (m_1=2,m_2=3) forms a set of nodal points of annular reference grid over 2×3 exactly once (R=1).

To observe ring sequence {(0,1), (0,2), (1,0)} we can see the next circular vector sums to be consecutive terms in this sequence:

$$\left.\begin{array}{l}(0,1)+(0,2)=(0,0)\\(1,0)+(0,1)=(1,1)\\(0,2)+(1,0)=(1,2)\end{array}\right\}(\bmod 2, \bmod 3)$$

So long as the terms (0,1), (0,2), (1,0) of the cyclic sequence themselves are two-dimensional vector sums also, the set of the modular vector sums (m_1=2,m_2=3) forms a set of nodal points of annular reference grid over 2×3 exactly once (R=1).

In much the same way can be formed sets of nodal points of reference grids for the rest 2D IRBs with m_1=2, m_2 =3, R=1. We call this torus cyclic group of 7-fold circular symmetry [18].

Here is an example of 3D IRB with n= 6, m_1 =2, m_2 =3, m_3 =5, and R=1 which contains circular 6-stage sequence of 3-stage (t=3) sub-sequences $\{K_1,...,K_6\}$:
$K_1 \Rightarrow (k_{11}, k_{21}, k_{31})$= (0,2,3), $K_2 \Rightarrow (k_{12}, k_{22}, k_{32})$ = (1,1,2), $K_3 \Rightarrow (k_{13}, k_{23}, k_{33})$ = (0,2,2), $K_4 \Rightarrow (k_{14}, k_{24}, k_{34})$ = (1,0,3), $K_5 \Rightarrow (k_{15}, k_{25}, k_{35})$ = (1,1,1), $K_6 \Rightarrow (k_{16}, k_{26}, k_{36})$ = (0,1,0). The set of all circular sums over the 6-stage sequence, taking 3-tuple (t=3) modulo (2, 3, 5) gives the next result:

(0,0,1) ⇒ ((0,2,2) + (1,0,3) + (1,1,1)),
(0,0,2) ⇒ ((1,1,2) + (0,2,2) + (1,0,3)),
(0,0,3) ⇒ ((0,2,3) + (0,1,0)),
(0,0,4)⇒((0,2,2) + (1,0,3) + (1,1,1) + (0,1,0) + (0,2,3)),
(1,2,4) ⇒ ((0,2,3)+(1,1,2)+(1,1,1)+(1,0,3)+(0,1,0)).

So, the set of all circular vector-sums of six (n=6) consecutive 3D vectors of this ring sequence covers surface of torus 2 × 3 × 5 exactly once (R=1).

Next, we regard the n-stage ring sequence K_{tD} ={(k_{11}, $k_{12,...,}k_{1t}$), ($k_{21},k_{22},\ldots k_{2t}$),... ($k_{i1},k_{i2},\ldots,k_{it}$),... ($k_{n1},k_{n2},\ldots,k_{nt}$)}, where all terms in each modular vector-sum to be consecutive t-stage sub-sequences as elements of the sequence. A modular vector-sum of consecutive terms in the ring sequence can have any of the n terms as its starting point, and can be of any length from 1 to n-1 exactly R-times.

Easy to see this verify of the next conditions:

$$\prod_1^t m_i = \frac{n(n-1)}{R} \ , \ \text{or} \ \prod_1^t m_i = \frac{n(n-1)}{R} + 1$$

$$(m_1, m_2,\ldots, m_t) = 1 \ , \qquad (7)$$

where: n, R , and m_1, m_2,…, m_t are numerical parameters of a t-dimensional Ideal Ring Bundle (tD IRB) [8,10].

Remarkable combinatorial properties of 2D and 3D IRB can be used for improving the quality indices of optic or acoustic systems with non-uniform structure (e.g. overlapping masks utilizing the entire ultra-acoustic aperture) with respect to resolving ability due to avoid the interference of signal components of the same spatial frequency [11-13].

Characteristics of optimum cyclic IRB-code for some parameters of $7 \le S_n \le 103$ diapason is presented in Tabl.2.

Table 2. Characteristic of optimum cyclic IRB code for $7 \le S_n \le 103$

Optimum cyclic IRB code							
Parameters				Autocorrelation function			
n	S_n	t_2	C_n	+1	-1	Δ	$\|\Delta/S_n\|$
4	7	1	14	3	4	-1	0,143
5	11	2	22	5	6	-1	0,091
6	11	2	22	5	6	-1	0,091
7	15	3	30	7	8	-1	0,066
8	15	3	30	7	8	-1	0,066
9	19	4	28	9	10	-1	0,053
10	19	4	28	9	10	-1	0,053
… …	… …	… …	… …	… …	… …	… …	… …
49	99	24	198	49	50	-1	0,010
50	99	24	198	49	50	-1	0,010
51	103	25	206	51	52	-1	0,010

Here n- number of terms in an IRB, S_n – code combinations length, t_2 – number of corrected errors, C_n- code size, Δ- evaluated expression of autocorrelation function. The function calculates taking summation with respect to all items +1 and -1 after a full cycle set of step-by-step shifts an IRB –sequence. Clearly, the correcting ability of optimum cyclic IRB-code increase as length S_n of the code, and number t_2 of corrected errors tends to 25% in increasing this length non linearly, and sidelobe level ratio is better than Barker code [21].

IRBs are useful for high performance coded design of optimum discrete signals such as correcting cyclic codes [22], code of the better autocorrelation function than Barker code, and self-correcting monolithic codes [8].

Here is a set of two 2D IRBs -{(0,1),(0,2),(1,1)} and {(1,1),(1,0),(0,2),(1,1)}, which form complete set of cyclic 2D IRB code combinations (0,0), (0,1), (0,2), (1,0), (1,1), (1,2) over 2D ignorable array 2×3 covered torus surface exactly R-times. A developed view of the torus array 2×3 appears below:

(1,0) (1,1) (1,2)
(0,0) (0,1) (0,2)

Note, each of these code combinations forms massive symbols "1" or "0". We call this code a "Monolithic Ideal Ring Bundles" (MIRB) codes [8,10,19]. An example of 2D Monolithic IRB code with five (n=5) cyclic binary digits is tabulated (Table 3)

Table 3. 2D-IRB Monolithic Code {(1,3), (1,1), (2,3), (0,3), (3,3)}

Vector	Cyclic binary digits				
	(1,3)	(1,1)	(2,3)	(0,3)	(3,3)
(0,0)	1	1	1	1	0
(0,1)	1	0	0	0	1
(0,2)	1	1	1	0	0
(0,3)	0	0	0	1	0
(0,4)	1	0	0	1	1
…….. ……..	…… …….	…… …….	…… …….	…… …….	…… …….
(3,4)	0	1	1	0	0

This code forms complete set of cyclic 2D IRB code combinations (0,0), (0,1),…,(3,4) over 2D ignorable array 4×5 covered torus surface exactly once (R=1).

Code size of 2D MIRB coding system of five (n=5) binary digits coincides in number of cells in the array 4×5 that is $n(n$-1) =20, and m_1 = 4, m_2 =5.

To see table 3, we observe all code combinations of the 2D IRB Monolithic code exhaust a set of collected similar signals in the combinations.In the same way can be formed 3D Monolithic IRB code {(1,1,2),(0,2,2), (1,0,3),(1,1,1), (0,1,0), (0,2,3)} (Table 4)

Table 4. 3D-IRB Monolithic Code {(1,1,2),(0,2,2), (1,0,3),(1,1,1), (0,1,0), (0,2,3)}

Binary digits					
(1,1,2)	(0,2,2)	(1,0,3)	(1,1,1)	(0,1,0)	(0,2,3)
0	1	1	1	0	0
1	1	1	0	0	0
1	0	0	0	0	1
0	1	1	1	1	1
…….. ……..	…….. ……..	…….. ……..	…….. ……..	…….. ……..	…….. ……..
1	1	1	1	0	1

To see Table 4, we observe:

3D vector (0,0,1) represented in cyclic binary digits as 011100, vector (0,0,2) as 111000, (0,0,3) as 100001, (0,0,4) as (01111) ……, finally vector (1,2,4) as 111101.

Hence the 3D-IRB Monolithic Code {(1,1,2),(0,2,2), (1,0,3),(1,1,1), (0,1,0), (0,2,3)}forms complete set of cyclic 3D IRB code combinations over 3D ignorable array 2 ×3×5 covered torus surface exactly once (R=1).

Code size of the coding system of six (n=6) binary digits coincides in number of cells in the array 2 ×3×5 that is $n(n$-1) =30, and m_1 = 2, m_2 =3, m_3=5.

Underlying property makes MIRBs useful in applications to high performance coded design of signals for communications with respect to self-correcting, transmission speed, vector data information technology, and fetch protection [23].

The S-fold PCR is common to one- , 2D and 3D models for technologically optimum distributed processes on sequentially ordered of structural elements and operations.

The purpose of the study is to improve the quality indices of technology for accuracy, resolution and functionality by distributing of the minimal number of structural elements and interconnections of technological system in spatially or/and temporally distributed coordinates. The challenge is to find a method of the optimal placement of structural elements in technical systems with a limited number of elements and bonds, while maintaining or improving on resolving ability and the other significant operating characteristics of the system.

Application profiting from the PCR concept are for example optimum vector (two- and three-dimensional) technology, vector data computing systems, and development high speed vector information technology**.**

The Ideal Ring Bundles (IRBs) provide a new conceptual model of radio- and information technologies or systems based on symmetry laws [1].

It is known that cardinality set of IRBs increases out many times in increasing order n of circular symmetry, e.g., tabled 1D IRBs of order 168 have 4676 distinct its variants [8]. The cardinality set of two- and three-dimensional IRBs outnumber in part analogous to cyclic difference sets heavily, e.g., there are 360 distinct variants of 2D IRBs of order 7, and 180 3D ones [18], while the existence of the perfect difference sets [4] for n =7 is unknown yet.

Remarkable geometrical properties of real space-time based on perfect circular symmetry and non-symmetry reflected in the underlying models. These properties make useful in applications to high performance coded design of signals for communications and radar, positioning of elements in an antenna array and visual coding systems with respect to redundancy, signal reconstruction and low side lobe antenna design [11-13].

CONCLUSIONS

1. Applications of the symmetrical and non-symmetrical combinatorial configurations for optimization of technology, namely the concept of Perfect Circular Relation (PCR), can be used for finding optimal solutions of technological problems in systems engineering.
2. The behavior of a self-locked quantum PRC system with parameters n, R is in step with S-fold circular symmetry.
3. Proposed code design based on the combinatorial configurations make it possible for reading a signal in signal/noise ratio under 1.
4. Application profiting from concept of PCR are for example vector data coding, self -correcting codes and problems of high-resolution interferometry for radar, data communications, and signal design.

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