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Monitoring of water distribution system effectiveness using fractal geometry

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Abstract. The paper discusses issues related to monitoring quality and pressure of water transmitted using water supply networks. Special attention was paid to methods of determining location of measuring points, which to a large extent influence effectiveness of the monitoring system. The purpose of the paper is to present authors' own method of determining location of points of measuring quality and pressure of transmitted water. The basis for considerations was a real water supply network in a city of about 10.000 residents. The presented method is based on existence of self-similarity properties of the set of fractals formed by the geometrical structure of the water supply network. It is a rank-ordered method involving 3 basic stages – reduction of the number of potential measuring points, providing more details of a target location and checking usefulness of selected points for monitoring purposes. At the preparatory stage, existence of fractal properties of the examined network structure is required to be demonstrated as well as the construction of its numerical model. The ranking is based on two indicators referring by analogy to human circulatory system monitoring and elements of the risk theory. This theory was also used to evaluate usefulness of selected measuring points for monitoring purposes.

Key words: water supply networks, monitoring, effectiveness.

1. Introduction

The main purpose of functioning of water supply companies is provision of water to consumers [1]. Besides economic criteria, it is highly important to meet consumer safety criteria. The key to meeting the aforesaid requirements is production process monitoring – of water intake, treatment and distribution [2]. Due to public-like character of water distribution systems, their managers are obliged to meet a number of requirements [3], [4], and [5]. All mentioned legislative acts include a provision that the monitoring system should be *effective*, and sampling points should be *representative* for the examined system. All mentioned documents fail to specify both of these terms however, which requires designers of the monitoring system to define them independently [6]. One of the defining criteria is effectiveness.

Monitoring system effectiveness may be defined by means of: accuracy of measurements and range of parameters recorded by the sensors, representativeness of obtained measuring results for the entire monitored area, speed of response to emerging interference and notifying the dispatch room about it, as well as a whole range of risk factors, such as probability of interference detection or size of defective batch to be transmitted to consumers [7–9]. In case of water distribution systems, apart from technical characteristics of measuring equipment and systems for transmitting collected data, the largest impact on monitoring system effectiveness defined in this manner is exerted by location of the measuring points. Until today, no universal, rationalized method of determining their location has been developed [10]. One reason for this is a specific, quasi chaotic character of water supply system geometrical structures. Descriptions of these structures used to date are not universal enough. In the paper, it was assumed that this problem may be solved by using the language of fractal geometry in describing geometrical structures of water supply systems.

As mentioned above, monitoring system effectiveness is also determined by representativeness of obtained measurement data. This problem has also remained unsolved. Various definitions of representativeness have led to development of various methods of determining location of measuring points [11–18]. In case of complex distribution systems, a possible solution to the above problem may be reference to nature. Human circulatory system may serve here as analogy. Optimized control of this system has naturally evolved over millions of years [19, 20].

Using the analogy between human circulatory system and a water distribution system, as well as based on fractal description of structures of this network, the authors have developed their own method for determining location of points monitoring quality and pressure of transmitted water. The main purpose of this paper is to present the method, together with a model application in a water supply network of a medium-sized city.

2. Description of the examined object

The basis for considerations presented in the paper was a real water supply network in a city of about 10,000 residents. Excluding connections, the water supply network is about 55 km long and pipe diameters range from 80 to 200 mm. The network is supplied with water by two pumping stations. Geo-

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metrical structure of the network is presented in Fig. 1. It is a mixed network, containing 14 rings and multiple branches. It is composed of 356 nodes and 639 pipes.

Fig. 1. Geometrical structure of the examined water supply network. Arrows indicate locations of water pumping stations

Geometrical network model presented in Fig. 1 was developed and calibrated using EPANET 2.0 [21] software, and subsequently used to determine locations of water pressure and quality monitoring points in the examined network. Performed simulations have shown that just as in the case of majority of such facilities in Poland, the examined network operates in considerably oversized conditions. In addition, it was verified whether the examined geometrical network structure meets formal conditions required of fractal sets [22]. As part of the verification works, it was confirmed that changing the scale (definition) of observation leads to changes in the image of the observed network (Fig. 2). Based on the observation method presented in Fig. 2, topological dimension of the examined set was calculated as well. To that end, the box-counting method [23, 24] developed on the basis of works by Kołmogorow and Minkowski was used. The unknown dimension was calculated using the formula:

$$\dim_B F = \lim_{\delta \to 0} \frac{\log N_\delta(F)}{\log(1/\delta)}, \tag{1}$$

where $\dim_B F$ – the unknown box-counting dimension of F geometrical set, $N_d\ (F)$ – smallest number of squares with d side able to cover the F set.

In the examined case, the unknown value was 1.439. As expected, it was not expressed as an integer.

Subsequently, it was confirmed that the examined structure may be mapped in 19 steps using the so called tree-like structure. To that end, author's own formula [25, 26] was used based on papers by Lindenmayer [27] and Prusinkiewicz [28]:

$$\text{axiom: } L_o\text{– beginning segment,}$$
$$\text{formula of transformations: } L_{i+1} \to \begin{cases} a \cdot L_i, \alpha' \\ b \cdot L_i, \alpha'' \\ c \cdot L_i, \alpha''' \end{cases} . \tag{2}$$

The formula corresponds to iterative transformations of a previous segment L_i to a consecutive one L_{i+1}, where a, b, c are the length parameters of the newly created segments, and $\alpha', \alpha'', \alpha$”’ – angles describing the localisation of the newly created segments in accordance with the previous segment.

Thus, a recursive principle of constructing the examined network was confirmed, as well as existence of self-similarity properties – network fragments are similar to the wholeness.

The above results show that according to papers by Mandelbrot [29] and Falconer [24], geometrical structure of the analysed water supply network meets requirements to be met by fractal sets with statistical self-similarity.

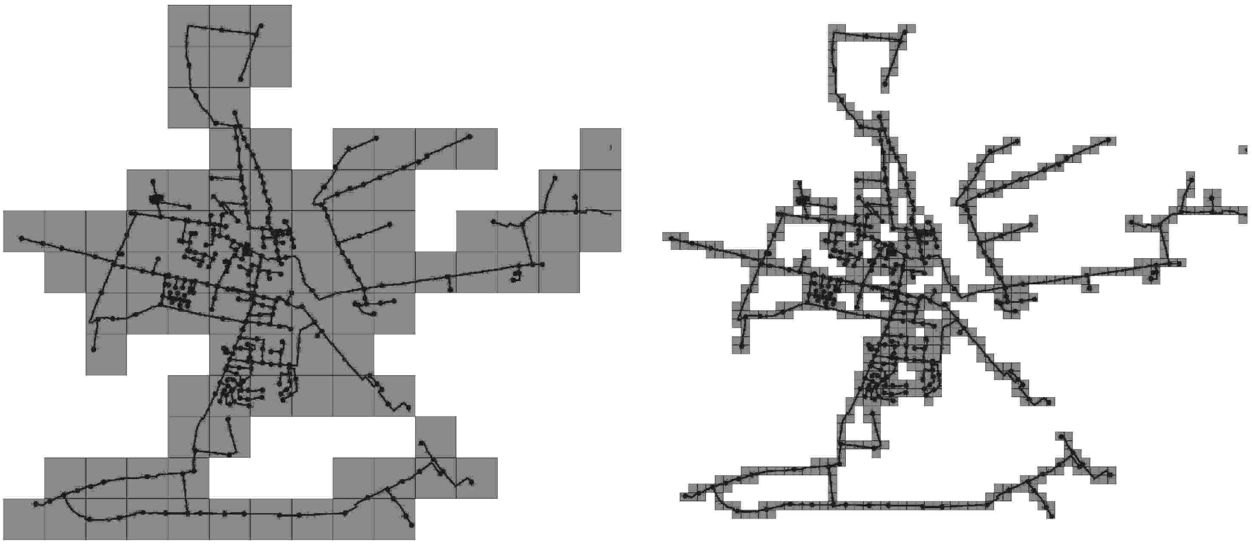

Fig. 2. Image of the examined network depending on the length of covering square $\delta = 1/15$ (on the left) and $\delta = 1/65$ (on the right) of the longest horizontal dimension of the examined network

3. Study method

Detailed description of the presented method for determining location of measuring points in the quality and pressure monitoring system for transmitted water was included in studies by [30, 31]. This method is based on the following premises: dominance of the heuristic factor, use of fractal imaging of geometrical structures of water supply networks, reference to physiology of the human circulatory system and universal character – possible application to the broadest possible class of water supply networks. The basis for calculations in the method is a calibrated numerical model of the examined network and good knowledge of major water consumers.

Suggested method was based on the following assumptions:

- separate consideration of location of quality and pressure measuring points for transmitted water, in the nodes of the examined network,
- controlling sensors should be located in key network points, definition of 'significance' of these points is based on heuristic criteria,
- sensors measuring the same parameter should not be located near each other,
- location of the measuring points is determined according to three stages presented below.

Stage 1

It is a preparatory stage involving model-based mapping of hydraulic working conditions of the examined network and verification whether the analysed geometrical structure of the network bears self-similarity properties. Hydraulic conditions should include directions and distribution of flow velocities and pressure over twenty-four hours in all pipes and nodes of the examined network. The used numerical model must also provide for evaluation of water age. Demonstration of existence of self-similarity properties is necessary for commencement of the next stage of works. According to the authors, method presented in item 2 is relatively easy to use.

Stage 2

At this stage, approximated location of measuring points is determined. To that end, areas in which monitoring sensors are expected to be mounted, are selected. Authors suggest to use here the network observation method presented in item 2. Existence of self-similarity properties demonstrated above enables observation of the covering squares at this stage instead of performance of a detailed analysis of all network nodes. This allows to significantly reduce the number of potential measuring point locations. Selection of a proper location comes down to preparation of the rank-ordered usefulness of individual covering squares for monitoring purposes. The actual problem however, is determination of their side length. This problem was solved by making variant-based calculations and by comparing effectiveness of obtained results.

Stage 3

At this stage, location indications for specific network nodes are specified in greater detail. This is done by increasing definition of observation, which allows to analyse specific network nodes, localised within covering squares, with the highest score in the usefulness ranking prepared at the previous stage. The number of analysed covering areas depends on the number of measuring devices required by economic factors. In the proposed method, it was assumed that only one node from a given selected area may be chosen. The node is also selected based on the usefulness rank for monitoring purposes.

3.1. The problem of usefulness of measuring points. Reference to solutions originated by nature.

At the second and third stage of works, there emerges the problem of determining usefulness of measuring point location for monitoring purposes. The authors decided to refer here to solutions elaborated by nature during evolution, assuming that naturally existing solutions, due to self-organisation, strive to reach optimum status quo under given conditions – homeostasis [32]. The sought-after analogous system should be found on a relatively high level of evolution, due to the time needed to conduct optimum adjustments. It should also operate under conditions typical for a complex system incorporating a considerable number of components. Human circulatory system appears to serve as a good analogy. The system of blood vessels with the heart acting as the major pumping station compares very well with the water supply network. Using this example, it is worth asking a question about management techniques applied by a human body. From the point of view of water supply network monitoring, the component related to maintenance of blood parameters, in particular: pressure, oxygen and carbon dioxide concentration, and pH, appears to be the most interesting. The system of sensors monitoring the above parameters includes the following elements [33]: arterial *baroreceptors* controlling blood pressure and *chemosensors* controlling the remaining parameters. From the point of view of location of the above sensors, it should be stated that their task primarily involves protection and control of facilities and areas of the highest importance for the body (in this case, the central nervous system). Other organs are treated as secondary, especially those that do not demand continuous blood supply. When looking for analogies to the water supply network, it could be stated that the monitoring system should include the water supply station (main pumping station) and network fragments located near the largest consumers. Such a system was suggested by Ghimire and Barkdoll [34]. They recommend placement of pressure and quality sensors on pipe crossings, in the nodes and in the largest mass of released contaminations. Similar procedure is applied during project-related water calculations for water supply networks. Demanded height to which the pumps are lifted is calculated for roads with the most pressure-adverse conditions [35].

Authors' own suggestion of criteria for ranking of usefulness. It appears, however, that the above rules of monitoring areas with the largest consumers cannot be directly and fully applied to water supply networks. Quality of supplied water must be relatively high within the entire network area. Therefore, what elements should be copied from the human circulatory system to the water supply network? According to

Table 1
Values of coefficients included in formulas (3) and (4)

Category/Value	Q	d	e	f	g	h
1	0–20% of max between all nodes	Residential buildings	Low-rise buildings	0–20% of max between all nodes	Above 20 mH$_2$O	up to 20% of maximum differences displayed by the model
2	21–40% max	Schools, boarding schools, services other than water-consuming services	Average-rise buildings	21–40% max	16 ± 20 mH$_2$O	21–40% max.
3	41–60% max	Water-consuming services, shopping and administration centres, small shops	High-rise buildings	41–60% max	11 ± 15 mH$_2$O	41–60% max.
4	61–80% max	Outpatient clinics, entertainment and sports facilities, industrial and storage premises, gastronomic facilities	Administration centres, industrial areas	61–80% max	6 ± 10 mH $_2$O	61–80% max
5	80–100% max	Water-consuming areas, fire brigade stations, hospitals	Life-saving facilities	80–100% max	0 ± 5 mH$_2$O	Above 80%

the authors, these should include: measurements of pressure and quality of water introduced to the network, protection of the key and largest consumers, relatively scattered network of sensors beyond the area of key consumers. In case of water supply networks, it is necessary however to define the term “key consumers”. The suggested definition is provided below.:

$$W1 = Q \cdot d \cdot e \cdot f, \tag{3}$$

$$W2 = Q \cdot d \cdot e \cdot g \cdot h, \tag{4}$$

where Q – volume of water demand over twenty-four hours, d – coefficient describing the required certainty of water supply of demanded quality, e – coefficient describing consequences of the lack of such supply, f – coefficient describing water age, g – coefficient depending on the difference between pressure on a given node and the required pressure, h – coefficient depending on node sensitivity (pressure variability over a calculation day).

Values of specific coefficients are provided in Table 1. They were determined based on a 5-score risk scale used by Rak [36] and Tchórzewska-Cieślak, [37]. Category number is also the value of the unknown indicator describing water demand.

3.2. Evaluation of correctness – effectiveness of selected points. The last problem faced by the authors was evaluation of correctness – effectiveness of indicated locations. To that end, authors’ own method [30, 31] was used, based on elements of the risk theory using the following coefficients:

- water quality monitoring: V – volume of water accumulated in network conduits, not covered by the monitoring system, T – maximum time of detecting the pollutant by the monitoring system, T/V_{pom} – ratio of time of detecting the pollutant to volume of water accumulated in network conduits, covered by the monitoring system,
- pressure monitoring: V_2 – volume of water in the network in which the detector (detectors) will not show outflow equal to full hydrant opening $d = 80$ mm (detection threshold for pressure drop caused by water outflow from the network was adopted as 2 mH$_2$O), V_3 – volume of water in the network in which the detector (detectors) will show pressure drop caused by assumed outflow of water from the network by at least 4 mH$_2$O.

To facilitate the analyses, risk matrix was used [36, 37] according to ranks based on indicators discussed above. Values of specific matrix elements were made dependent by the authors in every location indications variant on the place taken by a given solution in the ranking based on examined risk indicators. The lowest place in the ranking corresponds to 1 point, whereas the highest place – to the number of all examined variants. In case when values of indicators used to evaluate the risk repeated, the number of possibly available points was reduced. Total number of points obtained in this manner may be treated as the major indicator enabling comparative evaluation of correctness of water quality monitoring points location in specific variants.

4. Results and discussion

In line with assumptions, location of quality and pressure measuring points was determined according to specific variants, using methods presented above. Calculations were focused on determination of 3 points of measuring quality of transmitted water and 3 points for measuring its pressure. This was done according to the following variants:

- variant 1, based on water demand, according to Ghimire and Barkdoll method [34],
- variant 2 – based on formula (3) and (4). Length of side of the covering areas at the second stage was assumed as corresponding to the distance travelled by water in the network within 2, 4 and 6 hours, with average flow rate determined on the basis of all results reported by the numerical model in an hour with average water consumption.

Calculation results are provided in Table 2. An hour with average water consumption was assumed at 10:00 a.m. Weighted average (water flow velocity and section length) was 0.045 m/s.

Evaluation of usefulness (effectiveness) of determined monitoring points was performed according to risk categories, using methods presented in item 3. Obtained risk matrix was included in Table 3. The highest number of points identified with the highest usefulness of determined measuring points for monitoring purposes, both in terms of quality and pressure of transmitted water, was obtained in variant 2b. This variant also proved to be slightly better than the variant suggested by Ghimre and Barkdol. Locations determined therein are presented in Fig. 3. What is interesting, is the fact that only in one case selected location indications for quality and pressure measuring points differed from each other.

Table 2
Summary results of calculations of measuring point location indications

Variant	Covering square side length	Number of covering squares	Range of ranking indicator values	
			W1	W2
1	–	–	$Q_{\max} = 68.0$ m^3/d	$Q_{\min} = 0.1$ m^3/d
2a	2 hours	49	160	2
2b	4 hours	17	200	2
2c	6 hours	6	250	2

Table 3
The risk matrix of lack or to late failure detection of investigated network

No. of measuring points	Considered calculation variant							
	Quality monitoring				Pressure monitoring			
	1	2a	2b	2c	1	2a	2b	2c
	Volume of water not covered by the monitoring system (V)				V_2 – not covered by detection with 2 mH$_2$O threshold			
1	1	2	1	1	1	1	1	1
2	1	2	1	1	1	1	1	1
3	1	3	2	2	2	1	1	1
	Maximum pollutant detection time (T)				V_3 – not covered by detection with 4 mH$_2$O threshold			
1	2	1	2	1	1	1	2	1
2	2	1	2	1	2	1	2	1
3	3	2	3	2	2	2	3	2
	Relative maximum pollutant detection time (T/Vpom)							
1	1	1	2	1				
2	1	1	2	2				
3	2	1	3	2				
TOTAL	14	14	**18**	13	9	7	**10**	7

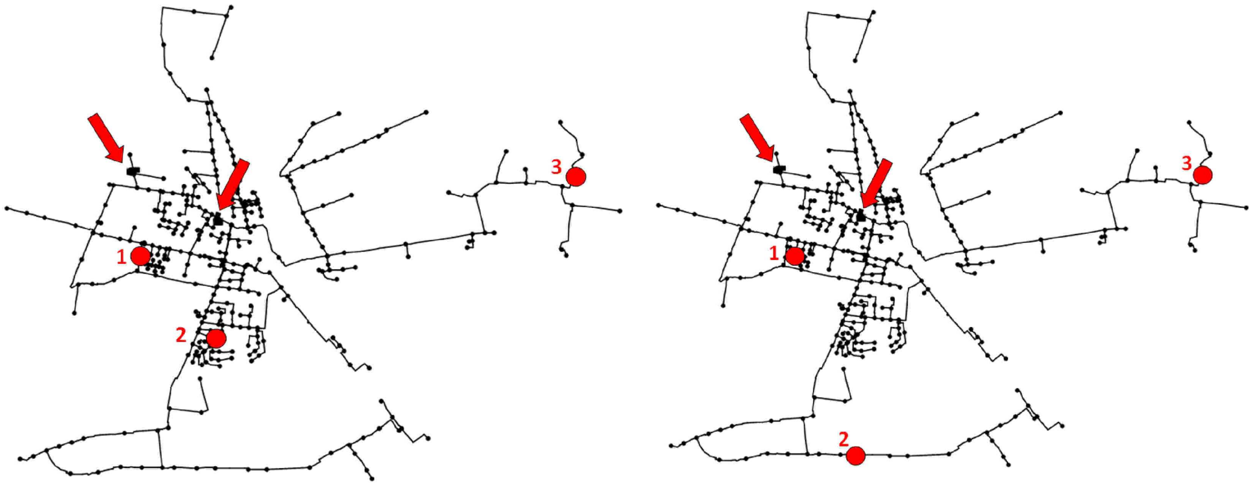

Fig. 3. Geometrical structure of the examined network and indicated locations of the measuring points. On the left – quality, on the right– pressure of transmitted water. Arrows indicate locations of network water supply sources

5. Conclusions

- The key issue, from the point of view of effectiveness of water quality and pressure monitoring systems in water supply networks, is location of the measuring points. This task still not been solved in a satisfactorily manner. One of the reasons for absence of such solution may be the lack of a universal method of describing geometrical structures of water supply networks. As a solution, the authors suggested the use of elements of fractal geometry.
- The use of fractal properties of water supply network geometrical structures, in combination with analogy to human circulatory system, allowed the authors to develop a concept of a their own method of determining location of points of measuring quality and pressure of transmitted water. Its major advantages include: universal character, consideration of the network structure and characteristics of water consumers, avoidance of problems related with measuring points too much approximating each other, and simplicity.
- The above-described method was successfully implemented in real-life conditions corresponding to a water supply network in a city of 10,000 residents. Conducted studies have shown that the quality of obtained location indications depends on the length of sides of covering squares in the examined distribution network used at the initial stage of works. In the analysed conditions it should correspond to four-hour flow of network water, determined on the basis of all values reported by the numerical model in an hour with average water consumption.
- Comparison of effectiveness of thus designed monitoring system, performed according to risk categories, in case of absence of or too late detection of an adverse event, with the system proposed by Ghimire and Barkdoll showed slight advantage of the solution proposed by authors of this paper.

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