Matlab, Simulink, air spring, machine

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OPTIMIZATION OF MOUNTING OF A MACHINE AND ITS PARTS

Frequency dependence values of input or mutual mechanical impedances of vibrating machines and their parts can be measured using an impedance hammer and a two-channel frequency analyzer. The knowledge of the frequency response of mechanical impedances provides for the determination of properties of vibration energy transmission in the structure of the machine based on which it is possible to adjust the mounting arrangement of the machine and its parts in order to attenuate vibrations. The obtained data concerning the dynamic behaviour of the machine then serve as a basis for a proper design and implementation of elastic mounting of the machine and its parts on damping elements which attenuate unwanted vibrations to the minimum. The design consists in assembling a model of the machine assuming that elastic mounting has been provided for, describing the model using equations of motion and elaborating their numerical solution. An efficient damping arrangement is achieved by mounting the vibrating parts of the machine using air springs. The air pressure in the air springs can be regulated according to deflections of the individual machine parts measured by accelerometers using a control element. The numeric solution provides for determination of values of stiffness coefficients *k* and damping coefficients *c* which would be optimal for the specified machine parameters in order to minimise the deflections of the individual machine parts.

1. INTRODUCTION

Machine vibrations are caused by unbalanced rotating machine parts, or operating impact load which results in the occurrence of mounting clearance, which may subsequently – by effect of increasing centrifugal forces – cause even a breakdown of the machine. Therefore it is recommended to place the individual machine parts on damping elements which would minimize these undesirable vibrations. The article features a numerical solution of an appropriate mounting of a vibrating machine using the Matlab-Simulink program [4]. Vibration-isolating elements were designed in the form of a vibration-isolating mount (installed between the frame and the base) and air springs which allow for the calculation of the air pressure in order to minimize the deflections of the vibrating parts of the machine. According to harmonic load as well as stiffness and damping constants specific sizes of air springs have been designed. The vibrating machine parts can be effectively damped by installing these machine parts on air springs 2 and 3 (Fig. 1) in the

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case of which it is possible to regulate the air pressure $p_i(t)$, $(i = 2, 3)$ in the air springs according to deflections of the individual machine parts x_1, x_2, x_3 , measured by accelerometers a_1, a_2, a_3 using a control element.

2. MODEL OF A VIBRATING MACHINE

The machine model displayed in Fig. 1 consists of:

1) Machine frame 1 of a weight of m_l installed on 4 special vibration-isolating mounts of type ISOLOC-UMS5 [1], comprising vibration-isolating material of IPL30 of a rubber basis and a metal screwed connection allowing for attachment to the machine frame. The maximum distributed force which this vibration-isolating mount is able to transfer amounts to 50 kN at a vertical frequency of 38.5 Hz. The measurement determined the vibrationisolating material stiffness constant at IPL30 k_1 =960016 Nm⁻¹.

2) Plate 2 of a weight of m_2 which – together with four air springs 2 of a stiffness constant of k_2 and four air springs 3 [3] of stiffness constants k_3 – forms a vibration absorber.

3) Working part of the machine of a weight m_3 affected by harmonic load of $f_3(t)$, which is mounted on four steel springs of stiffness constants of *k4*.

Fig. 1 Model of a machine with all its individual parts mounted on vibration-isolating mounts 1, air springs 2, 3 and springs 4

The use of air springs proves appropriate for the purposes of air pressure regulation in the case of variable load affecting the individual parts of the machine.

The air spring consists of a rubber-textile bellows, two clamping rings, an upper lid with an air inflow, lower lid and a ring between the waves. The lids allow for an attachment to the vibrating parts of the machine. The air source is linked to a valve which provides for air regulation.

The mechanical system displayed on Fig. 1 features three degrees of looseness x_1, x_2, x_3 and it can be described using the following equations of motion

$$
m_1\ddot{x}_1 + 4(c_1 + c_2 + c_4)\dot{x}_1 - 4c_2\dot{x}_2 - 4c_4\dot{x}_3 + 4(k_1 + k_2 + k_4)x_1 - 4k_2x_2 - 4k_4x_3 = 0,
$$

\n
$$
m_2\ddot{x}_2 - 4c_2\dot{x}_1 + 4(c_2 + c_3)\dot{x}_2 - 4c_3\dot{x}_3 + 4k_2x_1 - 4(k_2 + k_3)x_2 - 4k_4x_3 = 0,
$$

\n
$$
m_3\ddot{x}_3 - 4c_4\dot{x}_1 - 4c_3\dot{x}_2 + 4(c_3 + c_4)\dot{x}_3 - 4k_3x_1 - 4k_3x_2 - 4(k_3 + k_4)x_3 = f_3(t),
$$
\n(1)

where c_i , $(i = 1 \div 4)$ are the damping constants in the isolating mount 1, air springs 2 and 3, spring 4 and $f_3(t)$ represents the harmonic load affecting the working part of machine 3.

3. DESIGN OF VIBRATION-ISOLATING FITTINGS

Provided the vibrations in the working part of machine 3 of a weight of m_3 are to be minimal the kinetic energy of this material is to be taken over by a vibrating absorber. This case occurs when provided $\omega_3 = \omega_2$ and provided the absorber 2 material oscillates in the phase opposite to the vibrations of the working part of the machine 3 [2].

A numerical solution using Matlab-Simulink program was performed for the following machine parameters (Fig. 1):

The dampening degree of material IPL30 of vibration isolating mounts 1 and air springs 2 and 3 is indicated by the producers as very low [1]. Damping provided by spring 4 is also considered negligible.

The numerical solution [4],[7],[9] and optimization [5,6], [8] of the system of equations (1) makes it possible to determine the stiffness constant *k* in air springs 2 and 3 depending on the deflection of absorber 2 (Fig. 2).

Fig. 2. Course of displacement x_2 of the absorber 2 on stiffness coefficient k_2 of air springs 2 and 3

Deflections x_1, x_2 of frame 1 and absorber 2 for stiffness constants $k=30 \div 100 \text{ kNm}^{-1}$ determined based on a numerical system of equations (1) are displayed in Fig. 3 *÷* Fig. 10.

The air pressure *p* in the individual air springs can be determined based on the relation of

$$
p = \frac{k \cdot S_i}{\kappa \cdot V_i},\tag{2}
$$

where S_i , $i = (2,3)$ stands for the section of the spring inner active surface, κ stands for the adiabatic exponent of the incoming air and V_i , $(i = 2, 3)$ represents the active inner volume of the air spring.

The individual parts are mounted using air springs type Dunlop 2, 3/4x2 [3] specifically designed for this purpose. These Dunlop air springs are fit for mounting machines producing impacts and vibrations. The bellows of the Dunlop air spring 2 of 3/4x2

can be used up to a max. working overpressure of $p_{\text{max}} = 0.7MPa$, which applies to a static deflection of H_{stat} . The dimensions of this spring are $S_i = 25.5 \cdot 10^{-4}$ m² $i = (2,3)$, $H_{max} = 0.115$ m, $H_{min} = 0.065$ m, $H_{stat} = 0.092$ m (maximum, minimum and static height of the air spring rubber-textile bellows). This relation (2) then serves for the calculation of the maximum stiffness of air springs 2 and 3 of $k=90.16 \text{ k Nm}^{-1}$ (Fig. 2) for max. working overpressure $p_{\text{max}} = 0.7MPa$ using stiffness constants *k* and the above-mentioned Dunlop 2, 3/4x2 air spring parameters.

The deflection of the absorber 2 is then directly proportional to the air pressure *p* in the air spring and it can be determined based on Fig. 2. The air pressure p in air spring, corresponding stiffness constants $k=30 \div 100$ kNm⁻¹ and maximum deflection x_1 and x_2 frame 1 and absorber 2 are mentioned in Tab. 1.

Fig. 7. Deflection x_l of the frame 1 ($k=70$ kNm⁻¹)

Fig. 9. Deflection x_l of the frame 1 ($k=100$ kNm⁻¹)

) Fig. 8. Deflection x_2 of the absorber 2 ($k=70$ kNm⁻¹)

) Fig. 10. Deflection x_2 of the absorber 2 ($k=100 \text{ kNm}^{-1}$)

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k [kNm ⁻¹]	p [MPa]	x_l [mm]	x_2 [mm]
30	0.233	3.6	120
50	0.388	4.3	97
70	0.543	5.1	59
100	0.776	79	

Table 1. Stiffness constant *k*, air pressure *p* and max. deflections $x₁$ and $x₂$ frame 1 and absorber 2

4. RESULT AND CONCLUSION

Based on a numerical solution of a model of a vibrating machine (Fig. 1) designed using the Simulink program for specified machine parameters and stiffness constants of *k¹* and k_4 , while neglecting stiffness constants of c_i , $(i=1 \div 4)$, deflections of x_1, x_2 of the individual machine parts 1 and 2 have been determined (Fig. $3 \div$ Fig. 10) for different stiffness constant $k=30 \div 100 \text{ kNm}^{-1}$ (Tab. 1).

The air pressure p in air springs 2 and 3 (Fig. 1), corresponding stiffness constants $k=30 \div 100$ kNm⁻¹ is also mentioned in the Tab. 1. In order to attenuate vibrations of the working part of machine 3 the use of absorber 2 absorbing vibrations by means of air springs 2 and 3 was proposed. The working part of machine 3 features a zero deflection provided $\omega_3 = \omega_2$ and provided the material of absorber 2 oscillates in a phase opposite to the vibrations of the working part of the machine 3 [2].

The numerical solution of these equations (1) derived by means of Matlab-Simulink incorporating a requirement of a minimum deflection of x_3 can be used in order to determine the optimization of the stiffness constant k of air springs 2 and 3 (Fig.2). The graph in Fig. 2 then indicates that for an absorber deflection of $x_2 = 51$ mm the stiffness constant of the air spring equals to $k=90.16$ kNm⁻¹ which, based on relation (2), corresponds with the maximum permitted pressure in the air spring of *p=*0.7 MPa.

The numerical solution and the designed dimensions of the airs springs will be verified using a real-scale model of a vibrating material placed on air springs and considering various load types.

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