$\theta_{12}^{1} = 120, \ \theta_{12}^{2} = 60, \ \theta_{12}^{3} = 60, \ \theta_{12}^{4} = 215, \ \theta_{12}^{5} = 109, \ \theta_{12}^{6} = 122,$ 

- the variable  $\theta_{13}$ , the number of realizations  $n_{13} = 11$ , realizations:

 $\theta_{13}^{1} = 60, \ \theta_{13}^{2} = 105, \ \theta_{13}^{3} = 100, \ \theta_{13}^{4} = 60,$  $\theta_{13}^{5} = 275, \ \theta_{13}^{6} = 70, \ \theta_{13}^{7} = 80, \ \theta_{13}^{8} = 75, \ \theta_{13}^{9} = 298,$  $\theta_{13}^{10} = 182, \ \theta_{13}^{11} = 75,$ 

- the variable  $\theta_{14}$ , the number of realizations  $n_{14} = 2$ , realizations:

$$\theta_{14}^1 = 235, \ \theta_{14}^2 = 70$$

- the variable  $\theta_{15}$ , the number of realizations  $n_{15} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{21}$ , the number of realizations  $n_{21} = 6$ , realizations:

 $\theta_{21}^{1} = 180, \ \theta_{21}^{2} = 120, \ \theta_{21}^{3} = 120, \ \theta_{21}^{4} = 200, \ \theta_{21}^{5} = 137, \ \theta_{21}^{6} = 595,$ 

- the variable  $\theta_{22}$ , the number of realizations  $n_{22} = 0$ , realizations:

 $\theta_{22}^{k}$  - these realizations are not possible, - the variable  $\theta_{23}$ , the number of realizations  $n_{23} = 8$ , realizations:

$$\theta_{23}^{1} = 360, \ \theta_{23}^{2} = 290, \ \theta_{23}^{3} = 60, \ \theta_{23}^{4} = 180, \ \theta_{23}^{5} = 140, \ \theta_{23}^{6} = 60, \ \theta_{23}^{7} = 60, \ \theta_{23}^{8} = 220,$$

- the variable  $\theta_{24}$ , the number of realizations  $n_{24} = 3$ , realizations:

$$\theta_{24}^{1} = 120, \ \theta_{24}^{2} = 330, \ \theta_{24}^{3} = 240,$$

- the variable  $\theta_{25}$ , the number of realizations  $n_{25} = 1$ , realizations:

 $\theta_{25}^{1} = 240,$ 

- the variable  $\theta_{_{31}}$ , the number of realizations  $n_{_{31}} = 8$ , realizations:

 $\theta_{31}^{1} = 60, \ \theta_{31}^{2} = 120, \ \theta_{31}^{3} = 190, \ \theta_{31}^{4} = 200,$ 

$$\theta_{31}^{5} = 410, \ \theta_{31}^{6} = 440, \ \theta_{31}^{7} = 250, \ \theta_{31}^{8} = 280$$

- the variable  $\theta_{32}$ , the number of realizations  $n_{32} = 10$ ,

realizations:

$$\theta_{32}^1 = 180, \ \theta_{32}^2 = 165, \ \theta_{32}^3 = 180, \ \theta_{32}^4 = 120, \ \theta_{32}^5 = 180, \ \theta_{32}^6 = 410, \ \theta_{32}^7 = 255, \ \theta_{32}^8 = 150, \ \theta_{32}^9 = 495, \ \theta_{32}^{10} = 1105,$$

- the variable  $\theta_{33}$ , the number of realizations  $n_{33} = 0$ , realizations:

 $\boldsymbol{\theta}_{\scriptscriptstyle 33}^{\scriptscriptstyle k}\,$  - these realizations are not possible,

- the variable  $\theta_{_{34}}$ , the number of realizations  $n_{_{34}} = 2$ , realizations:

$$\theta_{34}^1 = 310, \ \theta_{34}^2 = 270,$$

- the variable  $\theta_{35}$ , the number of realizations  $n_{35} = 2$ , realizations:

$$\theta_{35}^{1} = 180, \ \theta_{35}^{2} = 360,$$

- the variable  $\theta_{41}$ , the number of realizations  $n_{41} = 2$ , realizations:

$$\theta_{41}^1 = 175, \ \theta_{41}^2 = 140,$$

- the variable  $\theta_{42}$ , the number of realizations  $n_{42} = 2$ ,

realizations:

$$\theta_{42}^{1} = 180, \ \theta_{42}^{2} = 180,$$

- the variable  $\theta_{_{43}}$ , the number of realizations  $n_{_{43}} = 2$ , realizations:

$$\theta_{43}^1 = 135, \ \theta_{43}^2 = 180,$$

- the variable  $\theta_{_{44}}$  , the number of realizations  $n_{_{44}} = 0$  , realizations:

 $\theta_{44}^{k}$  - these realizations are not possible,

- the variable  $\theta_{45}$ , the number of realizations  $n_{45} = 1$ , realizations:

$$\theta_{45}^{1} = 150$$

- the variable  $\theta_{s_1}$ , the number of realizations  $n_{s_1} = 2$ , realizations:

$$\theta_{51}^{1} = 240, \ \theta_{51}^{2} = 360,$$

- the variable  $\theta_{52}$ , the number of realizations  $n_{52} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{53}}$ , the number of realizations  $n_{_{53}} = 2$ , realizations:

$$\theta_{53}^{1} = 180, \ \theta_{53}^{2} = 720,$$

- the variable  $\theta_{54}$ , the number of realizations  $n_{54} = 0$ , realizations:

there are no realizations.

- the variable  $\theta_{55}$ , the number of realizations  $n_{55} = 0$ , realizations:

 $\theta_{55}^{k}$  - these realizations are not possible.

# **5.2.4. Evaluating the unknown parameters of the shipyard ship-rope elevator operation process**

On the basis of the statistical data from Section 4.2.3, using the formulae given in Section 3.3, it is possible to evaluate

- the vector of realizations

[p(0)] = [0.4211, 0.3684, 0.2105, 0, 0]

of the initial probabilities  $p_b(0)$ , b = 1,2,3,4,5, of the ship-rope elevator operation process transients in the particular states  $z_b$  at the moment t = 0,

- the matrix of realizations

$[p_{bl}] =$				
0	0,3158	0,5789	0,1053	0
0,3333	0	0,4444	0,1667	0,0556
0,3636	0,4546	0	0,0909	0,0909
0,2857	0,2857	0,2857	0	0,1429
0,5	0	0,5	0	0

of the transition probabilities  $p_{bl}$ , b, l = 1, 2, 3, 4, 5, of the ship-rope elevator operation process from the operation state  $z_b$  into the operation state  $z_l$  during the experiment time  $\Theta = 616$  days.

Until now there are too small numbers of realizations to determine, presented in Section 3.4 empirical characteristics of the realizations of the conditional sojourn time of the system operation process in the particular operation states.

# **5.2.5. Identifying the distributions of the conditional sojourn times in the operation states of the shipyard ship-rope elevator**

The realizations of the system operation process sojourn times  $\theta_{bl}$ , in the state  $z_b$  while the next transition is to the state  $z_l$  may allow us to formulate and to verify the hypotheses about their conditional distribution functions  $H_{bl}(t)$ . Unfortunately until now there are too small numbers of realizations to formulate any hypotheses about the distributions of the conditional sojourn times in the particular operation states.

# **5.2.6. Identifying the mean values of the system conditional sojourn times in operation states of the shipyard ship-rope elevator**

Due to the fact pointed in Section 5.2.5, the conditional empirical mean values  $M_{bl} = E[\theta_{bl}]$ ,  $b, l = 1, 2, ..., 5, b \neq l$ , of the sojourn times in the particular operation states on the basis of the realizations of the ship-rope elevator operation process conditional sojourn times  $\theta_{bl}$  and after applying formula the (7) are estimated and they amount:

 $M_{12} = 114.33, M_{13} = 125.45, M_{14} = 152.50,$   $M_{21} = 225.33, M_{23} = 171.25, M_{24} = 230.00,$   $M_{25} = 240.00, M_{31} = 243.75, M_{32} = 324.00,$   $M_{34} = 290.00, M_{35} = 270.00, M_{41} = 157.50,$   $M_{42} = 180.00, M_{43} = 157.50, M_{45} = 150.00,$  $M_{51} = 300.00, M_{53} = 450.00.$ 

As there are no realizations of conditional sojourn times  $\theta_{15}$ ,  $\theta_{52}$  and  $\theta_{54}$  it is impossible to estimate their conditional mean values  $M_{15}$ ,  $M_{52}$  and  $M_{54}$ .

# **5.3.** Statistical identification of the shipyard ground ship-rope transporter operation process

## **5.3.1.** The shipyard ground ship-rope transporter description

The ground ship-rope transporter in the Naval Shipyard in Gdynia is composed of three broaching machines working independently equipped in the steel ropes "Drumet" with the diameter 30 mm. This system is used to transfer ships coming to the shipyard for repairs from platform to the repair post and back from repair post to the platform. The load of steel ropes in the broaching machines is measured as a power consumption of amperage. The maximum of power consumption of broaching machines is 100 Ampere. First, during ship docking the ship settled in special supporting carriages on the platform is raised to the wharf level and then the ship is transferred from the platform with the rope broaching machine on a traverse. Next the ship with the traverse, on which the ship is settled, is shifted in the repair post direction. Then after stretching the ropes from the ship to the broaching machine through some blocs, the ship is transferred from the traverse to the repair post. After some repair measures, the ship is transferred back to the traverse and then on the platform. Finally, during undocking the ship on the platform is moved down to the water. There are nine repair posts, denoted by symbols R1-R9. The first repair post R1 can be lengthening to the post R1/B1 for long ships. There are also available two repair depots denoted by symbols B and D. Generally all kind of repairs can be carried out in any repair post. The repair posts R1 and R2 are equipped in crane. The submarines are repaired in the depot. Additionally large vessels are transferred to the repair post R1/B1. The scheme of the plan of repair post placing is given in Figure 22.



*Figure 22.* The scheme of the plan of repair post placing

The ground ship-rope transporter reliability depends strongly on the tonnage of transferred ships and the place where the ship should be transferred. The broaching machines in the transportation system are numbered 1, 2, 3. There is used one or there are used two or possibly three broaching machines depending on weight and length of the ship and on which repair post the ship should be transferred. All three broaching machines are working in the extreme situation when large vessel over 1800 tones is transferred.

# **5.3.2.** Defining the parameters of the shipyard ground ship-rope transporter operation process

Taking into account the expert opinion on the operation process of the considered ground ship-rope transporter we fix:

- the number of the ground ship-rope transporter operation process states v = 7

and we distinguish the following as its seven operation states:

- an operation state  $z_1$  the ship with a tonnage up to 1300 tones is transferred from the platform to the traverse, from the traverse to the repair posts R1-R5 and from the repair posts R6-R9 to the traverse (the broaching machine number 1 is used ( $S_1$ )),
- an operation state  $z_2$  the ship with a tonnage up to 1300 tones is transferred from the traverse to the repair posts R6-R9, from the repair posts R1-R5 to the traverse and from the traverse to the platform (the broaching machine number 3 is used ( $S_3$ )),
- an operation state  $z_3$  the ship with a tonnage up to 1300 tones is transferred from the repair posts R1-R5 to the traverse and the access to the broaching machine number 3 is difficult (the broaching machine number 2 is used ( $S_2$ )),
- an operation state  $z_4$  the ship with a tonnage over 1300 up to 1800 tones (or the ship with a tonnage up to 1300 tones after long period of renovation or after taking some special kind of measures) is transferred from the platform to the traverse, from the traverse to the repair posts R1-R5 or from the repair posts R6-R9 to the traverse (the broaching machines 1 and 3 are used ( $S_1$ ,  $S_3$ )),
- an operation state  $z_s$  the ship with a tonnage over 1300 up to 1800 tones (or the ship with a tonnage up to 1300 tones after long period of renovation or after taking

some special kind of measures) is transferred from the platform to the traverse, from the traverse to the repair posts R1-R5 or from the repair posts R6-R9 to the traverse and the access to the broaching machine number 3 is difficult (the broaching machines 1 and 2 are used ( $S_1$ ,  $S_2$ )),

- an operation state  $z_6$  the ship with a tonnage over 1300 up to 1800 tones (or the ship with a tonnage up to 1300 tones after long period of renovation or after taking some special kind of measures) is transferred from the traverse to the repair posts R6-R9, from the repair posts R1-R5 to the traverse or from the traverse to the platform (the broaching machines 2 and 3 are used ( $S_2$ ,  $S_3$ )),
- an operation state z<sub>7</sub> the ship with a tonnage over 1800 tones is transferred (all broaching machines 1, 2 and 3 are used (S<sub>1</sub>, S<sub>2</sub>, S<sub>3</sub>)).

Moreover, we fix that there are possible the transitions between all system operation states. Thus, the unknown parameters of the system operation process semi-markov model are:

- the initial probabilities  $p_b(0)$ , b = 1, 2, ..., 7,  $b \neq l$ , of the ground ship-rope transporter operation process transients in the particular states  $z_b$  at the moment t = 0,
- the transition probabilities  $p_{bl}$ , b = 1, 2, ..., 7,  $b \neq l$ , of the ground ship-rope transporter operation process from the operation state  $z_b$  into the operation state  $z_l$ ,
- the distributions of the conditional sojourn times  $\theta_{bl}$ , b = 1, 2, ..., 7,  $b \neq l$ , in the particular operation states and their mean values.

To identify all these parameters of the ground shiprope transporter operation process the statistical data about this process is needed. The statistical data that has been collected is given in the Appendix 3A in *Tables 2-21* [4]. From data given in these Tables, on the basis of methods and procedures given in the previous sections, in further sections the ground shiprope transporter operation process statistical data are fixed and its unknown parameters are estimated.

## **5.3.3.** The shipyard ground ship-rope transporter operation process data collection

The collected statistical data necessary to evaluating the initial transient probabilities of the ground shiprope transporter operation process in the particular states are:

- the ground ship-rope transporter operation process observation/experiment time  $\Theta = 616$  days,

- the number of the ground ship-rope transporter operation process realizations n(0) = 19,

- the realization  $n_b(0)$  of the number of the ground ship-rope transporter operation process transients in the particular operation states  $z_b$  at the initial moment t = 0

$$n_1(0) = 12, n_2(0) = 2, n_3(0) = 3, n_4(0) = 0,$$

$$n_5(0) = 0, n_6(0) = 0, n_7(0) = 2,$$

- the vector of realizations of the numbers of the ground ship-rope transporter operation process transitions in the particular operation states  $z_{b}$  at the initial moment t = 0

$$[n_b(0)] = [n_1(0), n_2(0), n_3(0), n_4(0), n_5(0), n_6(0), n_7(0)]$$
$$= [12, 2, 3, 0, 0, 0, 2].$$

The collected statistical data necessary to evaluating the transition probabilities of the ground ship-rope transporter operation process between the operation states are:

- the realization  $n_{bl}$  of the numbers of ground shiprope transporter operation process transitions from the state  $z_b$  into the state  $z_l$  during the experiment time  $\Theta = 616$  days

$$n_{12} = 27, n_{13} = 6, n_{14} = 1, n_{16} = 2,$$
  

$$n_{21} = 26, n_{23} = 7, n_{25} = 1, n_{26} = 5, n_{27} = 3,$$
  

$$n_{32} = 14,$$
  

$$n_{41} = 1, n_{46} = 1,$$
  

$$n_{51} = 1, n_{56} = 1,$$
  

$$n_{61} = 6, n_{63} = 1, n_{64} = 1, n_{67} = 1,$$
  

$$n_{71} = 1, n_{72} = 2, n_{75} = 1,$$

- the matrix of realizations  $n_{\scriptscriptstyle bl}$  of the numbers of the ground ship-rope transporter operation process transitions from the state  $z_{\scriptscriptstyle b}$  into the state  $z_{\scriptscriptstyle l}$  during the experiment time  $\Theta = 616$  days

	0	27	6	1	0	2	0
	26	0	7	0	1	5	3
	0	14	0	0	0	0	0
$[n_{bl}] =$	1	0	0	0	0	1	0
	1	0	0	0	0	1	0
	6	0	1	1	0	0	1
	1	2	0	0	1	0	0

- the realization  $n_b$  of the total numbers of the ground ship-rope transporter operation process transitions from the operation state  $z_b$  during the experiment time  $\Theta = 616$  days (the sums of the numbers of the matrix  $[n_{bl}]$ )

$$n_1 = 36, n_2 = 42, n_3 = 14, n_4 = 2, n_5 = 2,$$
  
 $n_6 = 9, n_7 = 4,$ 

- the matrix of realizations of the total numbers of the ground ship-rope transporter operation process transitions from the operation state  $z_b$  during the experiment time  $\Theta = 616$  days

$$[n_{b}] = [n_{1}, n_{2}, n_{3}, n_{4}, n_{5}, n_{6}, n_{7}] = [36, 42, 14, 2, 2, 9, 4].$$

The collected statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the ground ship-rope transporter operation process in the particular operation states are as follows:

- the realizations  $\theta_{bl}^{k}$ ,  $k = 1, 2, ..., n_{bl}$ , of the conditional sojourn times  $\theta_{bl}$  of the ground ship-rope transporter operation process at the operation state  $z_{b}$  when the next transition is to the operation state  $z_{l}$  during the observation time:

- the variable  $\theta_{11}$ , the number of realizations  $n_{11} = 0$ , realizations:

 $\theta_{11}^{k}$  - these realizations are not possible,

- the variable  $\theta_{12}$ , the number of realizations  $n_{12} = 27$ , realizations:

 $\begin{aligned} \theta_{12}^{1} &= 40, \ \theta_{12}^{2} = 45, \ \theta_{12}^{3} = 240, \ \theta_{12}^{4} = 80, \\ \theta_{12}^{5} &= 205, \ \theta_{12}^{6} = 45, \ \theta_{12}^{7} = 50, \ \theta_{12}^{8} = 50, \\ \theta_{12}^{9} &= 80, \ \theta_{12}^{10} = 115, \ \theta_{12}^{11} = 60, \ \theta_{12}^{12} = 70, \\ \theta_{12}^{13} &= 155, \ \theta_{12}^{14} = 125, \ \theta_{12}^{15} = 40, \ \theta_{12}^{16} = 265, \end{aligned}$ 

$$\theta_{12}^{17} = 145, \ \theta_{12}^{18} = 40, \ \theta_{12}^{19} = 60, \ \theta_{12}^{20} = 40,$$
  
$$\theta_{12}^{21} = 140, \ \theta_{12}^{22} = 192, \ \theta_{12}^{23} = 33, \ \theta_{12}^{24} = 15,$$
  
$$\theta_{12}^{25} = 185, \ \theta_{12}^{26} = 65, \ \theta_{12}^{27} = 170,$$

- the variable  $\theta_{13}$ , the number of realizations  $n_{13} = 6$ , realizations:

$$\theta_{13}^1 = 135, \ \theta_{13}^2 = 125, \ \theta_{13}^3 = 100, \ \theta_{13}^4 = 390, \ \theta_{13}^5 = 115, \ \theta_{13}^6 = 210,$$

- the variable  $\theta_{14}$ , the number of realizations  $n_{14} = 1$ , realizations:

$$\theta_{14}^{1} = 145,$$

- the variable  $\theta_{15}$ , the number of realizations  $n_{15} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{16}$ , the number of realizations  $n_{16} = 2$ , realizations:

$$\theta_{16}^1 = 250, \ \theta_{16}^2 = 180,$$

- the variable  $\theta_{17}$ , the number of realizations  $n_{17} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{21}$ , the number of realizations  $n_{21} = 26$ , realizations:

realizations:

$$\begin{aligned} \theta_{21}^{1} &= 45, \ \theta_{21}^{2} &= 55, \ \theta_{21}^{3} &= 35, \ \theta_{21}^{4} &= 40, \\ \theta_{21}^{5} &= 75, \ \theta_{21}^{6} &= 270, \ \theta_{21}^{7} &= 55, \ \theta_{21}^{8} &= 50, \\ \theta_{21}^{9} &= 115, \ \theta_{21}^{10} &= 70, \ \theta_{21}^{11} &= 35, \ \theta_{21}^{12} &= 205, \\ \theta_{21}^{13} &= 130, \ \theta_{21}^{14} &= 240, \ \theta_{21}^{15} &= 30, \ \theta_{21}^{16} &= 190, \\ \theta_{21}^{17} &= 60, \ \theta_{21}^{18} &= 15, \ \theta_{21}^{9} &= 20, \ \theta_{21}^{20} &= 65, \\ \theta_{21}^{21} &= 60, \ \theta_{21}^{22} &= 75, \ \theta_{21}^{23} &= 45, \ \theta_{21}^{24} &= 60, \\ \theta_{21}^{25} &= 40, \ \theta_{21}^{26} &= 40, \end{aligned}$$

- the variable  $\theta_{22}$ , the number of realizations  $n_{22} = 0$ , realizations:

 $\theta_{22}^{k}$  - these realizations are not possible,

- the variable  $\theta_{23}$ , the number of realizations  $n_{23} = 7$ , realizations:

 $\theta_{23}^{_1} = 27, \ \theta_{23}^{_2} = 20, \ \theta_{23}^{_3} = 15, \ \theta_{23}^{_4} = 75, \ \theta_{23}^{_5} = 40, \ \theta_{23}^{_6} = 60, \ \theta_{23}^{_7} = 50,$ 

- the variable  $\theta_{24}$ , the number of realizations  $n_{24} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{25}$ , the number of realizations  $n_{25} = 1$ , realizations:

$$\theta_{25}^{1}=32,$$

- the variable  $\theta_{26}$ , the number of realizations  $n_{26} = 5$ , realizations:

$$\theta_{26}^{1} = 50, \ \theta_{26}^{2} = 70, \ \theta_{26}^{3} = 45, \ \theta_{26}^{4} = 45, \ \theta_{26}^{5} = 40,$$

- the variable  $\theta_{27}$ , the number of realizations  $n_{27} = 3$ , realizations:

$$\theta_{27}^1 = 90, \ \theta_{27}^2 = 165, \ \theta_{27}^3 = 70,$$

- the variable  $\theta_{_{31}}$ , the number of realizations  $n_{_{31}} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{32}$ , the number of realizations  $n_{32} = 14$ ,

realizations:

 $\theta_{32}^{1} = 95, \ \theta_{32}^{2} = 25, \ \theta_{32}^{3} = 20, \ \theta_{32}^{4} = 25, \ \theta_{32}^{5} = 62,$  $\theta_{32}^{6} = 60, \ \theta_{32}^{7} = 70, \ \theta_{32}^{8} = 45, \ \theta_{32}^{9} = 70, \ \theta_{32}^{10} = 50,$  $\theta_{31}^{11} = 90, \ \theta_{32}^{12} = 60, \ \theta_{32}^{13} = 55, \ \theta_{32}^{14} = 60,$ 

- the variable  $\theta_{_{33}}$ , the number of realizations  $n_{_{33}} = 0$ , realizations:

 $\theta_{33}^{k}$  - these realizations are not possible,

- the variable  $\theta_{34}$ , the number of realizations  $n_{34} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{35}$ , the number of realizations  $n_{35} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{36}$ , the number of realizations  $n_{36} = 0$ ,

realizations:

there are no realizations,

- the variable  $\theta_{37}$ , the number of realizations  $n_{37} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{41}}$ , the number of realizations  $n_{_{41}} = 1$ , realizations:

 $\theta_{41}^{1} = 150,$ 

- the variable  $\theta_{_{42}}$ , the number of realizations  $n_{_{42}} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{43}}$ , the number of realizations  $n_{_{43}} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{44}}$ , the number of realizations  $n_{_{44}} = 0$ , realizations:

 $\theta_{_{44}}^{_k}$  - these realizations are not possible, - the variable  $\theta_{_{45}}$ , the number of realizations  $n_{_{45}} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{46}$ , the number of realizations  $n_{46} = 1$ , realizations:

 $\theta_{46}^{1} = 70,$ 

- the variable  $\theta_{_{47}}$ , the number of realizations  $n_{_{47}} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{s_1}$ , the number of realizations  $n_{s_1} = 1$ , realizations:

$$\theta_{51}^{1} = 240,$$

- the variable  $\theta_{s_2}$ , the number of realizations  $n_{s_2} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{53}}$ , the number of realizations  $n_{_{53}} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{54}$ , the number of realizations  $n_{54} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{ss}$ , the number of realizations  $n_{ss} = 0$ , realizations:

 $\theta_{\scriptscriptstyle 55}^{\scriptscriptstyle k}\,$  - these realizations are not present,

- the variable  $\theta_{56}$ , the number of realizations  $n_{56} = 1$ , realizations:

 $\theta_{56}^{1} = 180,$ 

- the variable  $\theta_{57}$ , the number of realizations  $n_{57} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{61}$ , the number of realizations  $n_{61} = 6$ , realizations:

 $\theta_{61}^1 = 210, \ \theta_{61}^2 = 210, \ \theta_{61}^3 = 80, \ \theta_{61}^4 = 180, \ \theta_{61}^5 = 180, \ \theta_{61}^6 = 190,$ 

- the variable  $\theta_{62}$ , the number of realizations  $n_{62} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{63}}$ , the number of realizations  $n_{_{63}} = 1$ , realizations:

 $\theta_{63}^{1} = 190,$ 

- the variable  $\theta_{64}$ , the number of realizations  $n_{64} = 1$ , realizations:

$$\theta_{64}^{1} = 190,$$

- the variable  $\theta_{65}$ , the number of realizations  $n_{65} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{_{66}}$ , the number of realizations  $n_{_{66}} = 0$ , realizations:

 $\boldsymbol{\theta}^{\scriptscriptstyle k}_{\scriptscriptstyle 66}\;$  - these realizations are not possible,

- the variable  $\theta_{\gamma_1}$ , the number of realizations  $n_{\gamma_1} = 1$ , realizations:

$$\theta_{71}^{1} = 245,$$

- the variable  $\theta_{_{72}}$ , the number of realizations  $n_{_{72}} = 2$ , realizations:

$$\theta_{72}^1 = 230, \ \theta_{72}^2 = 240,$$

- the variable  $\theta_{73}$ , the number of realizations  $n_{73} = 0$ ,

realizations:

there are no realizations,

- the variable  $\theta_{74}$ , the number of realizations  $n_{74} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{75}$ , the number of realizations  $n_{75} = 1$ , realizations:

 $\theta_{75}^{1} = 60,$ 

- the variable  $\theta_{76}$ , the number of realizations  $n_{76} = 0$ , realizations:

there are no realizations,

- the variable  $\theta_{\gamma\gamma}$ , the number of realizations  $n_{\gamma\gamma} = 0$ , realizations:

 $\theta_{\gamma\gamma}^{k}$  - these realizations are not possible.

# **5.3.4.** Evaluating the unknown parameters of the shipyard ground ship-rope transporter operation process

On the basis of the statistical data from Section 5.3.3, using the formulae given in Section 3.4, it is possible to evaluate

- the vector of realizations

[p(0)] = [0.6315, 0.1053, 0.1579, 0, 0, 0, 0.1053]

of the initial probabilities  $p_b(0)$ , b = 1, 2, ..., 7, of the ground ship-rope transporter operation process

transients in the particular states  $z_b$  at the moment t = 0,

#### - the matrix of realizations

$[p_{bl}] =$						
0	0.7500	0.1667	0.0278	0	0.0555	0 ]
0.6190	0	0.1667	0	0.0238	0.1191	0.0714
0	1	0	0	0	0	0
0.5	0	0	0	0	0.5	0
0.5	0	0	0	0	0.5	0
0.6667	0	0.1111	0.1111	0	0	0.1111
0.25	0.5	0	0	0.25	0	0

of the transition probabilities  $p_{bl}$ , b, l = 1, 2, ..., 7, of the ground ship-rope transporter operation process from the operation state  $z_b$  into the operation state  $z_l$ during the experiment time  $\Theta = 616$  days.

On the basis of the statistical data from Section 5.3.3, using the formulae given in Section 3.4, it is possible to determine the following empirical characteristics of the realizations of the conditional sojourn time of the ground ship-rope transporter operation process in the particular operation states:

- the realizations of the mean values  $\overline{\theta}_{12}$  of the conditional sojourn times  $\theta_{12}$  of the ground ship-rope transporter operation process at the operation state  $z_1$  when the next transition is to the operation state  $z_2$ 

$$\overline{\theta}_{12} = \frac{1}{27} \sum_{k=1}^{27} \theta_{12}^{k} \cong 101.9 \quad b, l = 12, ..., 7 \quad b \neq l,$$

- the number  $\bar{r}_{12}$  of the disjoint intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, ..., \bar{r}_{12}$ , that include the realizations  $\theta_{12}^k$ , k = 1, 2, ..., 27, of the conditional sojourn times  $\theta_{12}$  at the operation state  $z_1$  when the next transition is to the operation state  $z_2$ 

$$\overline{r}_{12} \cong \sqrt{27} \cong 5$$
,

- the length  $d_{12}$  of the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots, 5$ ,

$$d_{12} = \frac{\overline{R}_{12}}{\overline{r}_{12} - 1} = \frac{250}{4} \cong 63$$

where

$$\overline{R}_{12} = \max_{1 \le k \le 27} \theta_{12}^{k} - \min_{1 \le k \le 27} \theta_{12}^{k} = 265 - 15 = 250,$$

- the ends  $a_{bl}^{j}$ ,  $b_{bl}^{j}$ , of the intervals  $I_{j} = \langle a_{bl}^{j}, b_{bl}^{j} \rangle$ , j = 1, 2, ..., 5, according to the formulae

$$\min_{1 \le k \le 27} \theta_{12}^{k} - \frac{d_{12}}{2} = 15 - \frac{63}{2} = -16.5,$$

$$a_{12}^{1} = \max\{-16.5, 0\} = 0,$$

$$b_{12}^{1} = a_{12}^{1} + 63 = 0 + 63 = 63,$$

$$a_{12}^{2} = b_{12}^{1} = 63, \quad b_{12}^{2} = a_{12}^{1} + 2 \cdot 63 = 0 + 126 = 126,$$

$$a_{12}^{3} = b_{12}^{2} = 126, \quad b_{12}^{3} = a_{12}^{1} + 3 \cdot 63 = 0 + 189 = 189,$$

$$a_{12}^{4} = b_{12}^{3} = 189, \quad b_{12}^{4} = a_{12}^{1} + 4 \cdot 63 = 0 + 252 = 252,$$

$$a_{12}^{5} = b_{12}^{4} = 252, \quad b_{12}^{5} = a_{12}^{1} + 5 \cdot 63 = 0 + 315 = 315,$$

- the numbers  $n_{12}^{j}$  of the realizations  $\theta_{12}^{k}$  in particular intervals  $I_{j}$ , j = 1, 2, ..., 5,

$$n_{12}^1 = 12, n_{12}^2 = 6, n_{12}^3 = 5, n_{12}^4 = 3, n_{12}^5 = 1$$

Histogram of the conditional sojourn time $\theta_{12}$								
$I_j = < a_{bl}^j, b_{bl}^j) \qquad 0 - 63 \qquad 63 - 126 \qquad 126 - 189 \qquad 189 - 252 \qquad 252 - 315$								
$n_{_{12}}^{_{j}}$	12 6 5 3 1							
$\overline{h_{12}}(t) = n_{12}^{j} / n_{12} \qquad 12/27 \qquad 6/27 \qquad 5/27 \qquad 3/27 \qquad 1/27$								



*Figure 23.* Histogram of the conditional sojourn time  $\theta_{12}$ 

- the realizations of the mean values  $\overline{\theta}_{21}$  of the conditional sojourn times  $\theta_{21}$  of the ground shiprope transporter operation process at the operation state  $z_2$  when the next transition is to the operation state  $z_1$ 

$$\overline{\theta}_{21} = \frac{1}{26} \sum_{k=1}^{26} \theta_{21}^{k} \cong 81.5, \ b, l = 12, ..., 7 \ b \neq l,$$

- the number  $\bar{r}_{21}$  of the disjoint intervals  $I_{i} = \langle a_{bl}^{j}, b_{bl}^{j} \rangle, \quad j = 1, 2, ..., \overline{r}_{21}, \text{ that include the}$ realizations  $\theta_{21}^{k}$ , k = 1, 2, ..., 26, of the conditional sojourn times  $\theta_{21}$  at the operation state  $z_2$  when the next transition is to the operation state  $z_1$ 

$$\bar{r}_{21} = \sqrt{26} \cong 5 \,,$$

- the length  $d_{21}$  of the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots, 5$ ,

$$d_{21} = \frac{\overline{R}_{12}}{\overline{r}_{21} - 1} = \frac{255}{4} \cong 64$$

where

$$\overline{R}_{21} = \max_{1 \le k \le 26} \theta_{21}^{k} - \min_{1 \le k \le 26} \theta_{21}^{k} = 270 - 15 = 255 ,$$

- the ends  $a_{bl}^{j}$ ,  $b_{bl}^{j}$ , of the intervals  $I_{j} = \langle a_{bl}^{j}, b_{bl}^{j} \rangle$ ,  $j = 1, 2, \dots, 5$ , according to the formulae

$$\min_{1 \le k \le 26} \theta_{21}^{k} - \frac{d_{21}}{2} = 15 - \frac{64}{2} = -17,$$

$$a_{21}^{1} = \max\{-17, 0\} = 0, \ b_{21}^{1} = a_{21}^{1} + 64 = 0 + 64 = 64,$$

$$a_{21}^{2} = b_{21}^{1} = 64, \ b_{21}^{2} = a_{21}^{1} + 2 \cdot 64 = 0 + 128 = 128,$$

$$a_{21}^{3} = b_{21}^{2} = 128, \ b_{21}^{3} = a_{21}^{1} + 3 \cdot 64 = 0 + 192 = 192,$$

$$a_{21}^{4} = b_{21}^{3} = 192, \ b_{21}^{4} = a_{21}^{1} + 4 \cdot 64 = 0 + 256 = 256,$$

$$a_{21}^{5} = b_{21}^{4} = 256, \ b_{21}^{5} = a_{21}^{1} + 5 \cdot 64 = 0 + 320 = 320,$$

- the numbers  $n_{21}^{j}$  of the realizations  $\theta_{21}^{k}$  in particular intervals  $I_{j}$ , j = 1, 2, ..., 5,

$$n_{21}^1 = 16, n_{21}^2 = 5, n_{21}^3 = 2, n_{21}^4 = 2, n_{21}^5 = 1.$$

Histogram of the conditional sojourn time $\theta_{21}$									
$I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ $0 - 64$ $64 - 128$ $128 - 192$ $192 - 256$ $256 - 320$									
$n_{21}^{j}$	16	5	2	2	1				
$\overline{h}_{21}(t) = n_{21}^{j} / n_{21}$	$\frac{1}{\overline{h}_{21}(t) = n_{21}^{j} / n_{21}} = \frac{16/26}{16/26} = \frac{5/26}{5/26} = \frac{2}{26} = \frac{1}{26}$								



*Figure 24.* Histogram of the conditional sojourn time  $\theta_{21}$ 

# **5.3.5. Identifying the distributions of the conditional sojourn times in the operation states of the shipyard ground ship-rope transporter**

Using the procedure given in Section 3.5 we may verify hypotheses on the distributions of the conditional sojourn times  $\theta_{bl}$ ,  $b, l = 1, 2, ..., 7, b \neq l$ , in the particular operation states. At the moment, because of the lack of statistical data coming from experiment it was possible to verify only two hypotheses on the distributions of the sojourn times. Namely, we have the following results:

- the conditional sojourn time  $\theta_{12}$  has an exponential distribution with the density function

$$h_{12}(t) = \begin{cases} 0, & t < 0, \\ 0.0098 \exp[-0.0098t], & t \ge 0, \end{cases}$$

- the conditional sojourn time  $\theta_{21}$  has an exponential distribution with the density function

$$h_{21}(t) = \begin{cases} 0, & t < 0, \\ 0.0123 \exp[-0.0123t], & t \ge 0. \end{cases}$$

# **5.3.6. Identifying the mean values of the system conditional sojourn times in operation states of the shipyard ground ship-rope transporter**

For the above distributions identified in Section 5.3.5, using either the formulae (8) or (13), we can find the following mean values of the conditional sojourn times in the particular operation states:

 $M_{12} = 102.04, M_{21} = 81.30.$ 

In the remaining cases not identified in Section 5.3.5 distributions, using formula (7), it is possible to find only the approximate values of the empirical mean values  $M_{bl} = E[\theta_{bl}]$  of the conditional sojourn times in the particular operation states that are as follows:

$$M_{13} = 179.17, M_{14} = 145, M_{16} = 215,$$
  
 $M_{23} = 41, M_{25} = 32, M_{26} = 50, M_{27} = 108.33,$   
 $M_{32} = 56.21,$   
 $M_{41} = 150, M_{46} = 70,$   
 $M_{51} = 240, M_{56} = 180,$   
 $M_{61} = 175, M_{63} = 190, M_{64} = 190, M_{67} = 150,$   
 $M_{71} = 245, M_{72} = 235, M_{75} = 60.$ 

As there are no realizations of conditional sojourn times  $\theta_{15}$ ,  $\theta_{17}$ ,  $\theta_{24}$ ,  $\theta_{34}$ ,  $\theta_{35}$ ,  $\theta_{36}$ ,  $\theta_{37}$ ,  $\theta_{42}$ ,  $\theta_{43}$ ,  $\theta_{45}$ ,  $\theta_{47}$ ,  $\theta_{52}$ ,  $\theta_{53}$ ,  $\theta_{54}$ ,  $\theta_{57}$ ,  $\theta_{62}$ ,  $\theta_{65}$ ,  $\theta_{73}$ ,  $\theta_{74}$  and  $\theta_{76}$  it is impossible to estimate their conditional mean values  $M_{15}$ ,  $M_{17}$ ,  $M_{24}$ ,  $M_{34}$ ,  $M_{35}$ ,  $M_{36}$ ,  $M_{37}$ ,  $M_{42}$ ,  $M_{43}$ ,  $M_{45}$ ,  $M_{47}$ ,  $M_{52}$ ,  $M_{53}$ ,  $M_{54}$ ,  $M_{57}$ ,  $M_{62}$ ,  $M_{65}$ ,  $M_{73}$ ,  $M_{74}$  and  $M_{76}$ .

#### 5.4. Statistical identification of the Stena Baltica ferry operation process

#### 5.4.1. The Stena Baltica ferry description

The m/v Stena Baltica is a passenger Ro-Ro ship operating in Baltic Sea between Gdynia and Karlskrona ports on regular everyday line. Her owner is Stena Line Scandinavia AB. She was build in Gdańsk Shipyard in 2005. She is characterized by the following parameters: the length of 164.41m, the breadth moulded of 27.60 m, the summer load draft of 6.313 m, DWT of 4456, the displacement of 16618 tons, the cargo capacity of 466 cars, the total numbers of passengers and crew capacity of 1200 + 96 = 1296. The number of cabins is 379 with the number of beds 949 and total number of seats on a board is 981. The main engines are 4 of the kind MAN 4840 kW, the propellers are 2 of the kind Ka Me Wa with diameter 4800 mm, the BOW thrusters are 2 of the kind 1275 kW and 735 kW and the aft thruster is 1 of the kind 735 kW. The navigation and communication equipments are according to SOLAS Convention. The ferry speed is 19.5 knots (calm water) (RPM - 178). The service restriction are: maximum of 350 NM from land and wave height of 3.1 m, according to the Stockholm Agreement.

We assume that the ferry is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in *Figure 25*. On the scheme of the ferry presented in *Figure 25*, there are distinguished its following subsystems:

- $S_1$  a navigational subsystem,
- $S_2$  a propulsion and controlling subsystem,
- $S_{3}$  a loading and unloading subsystem,
- $S_4$  a hull subsystem,
- $S_{\scriptscriptstyle 5}\,$  an anchoring and mooring subsystem,
- $S_{6}$  a protection and rescue subsystem,
- $S_{\gamma}$  a social subsystem.



*Figure 25.* Subsystems having an essential influence on ferry safety

### **5.4.2. Defining the parameters of the Stena Baltica ferry operation process**

Taking into account the expert opinion on the operation process of the considered Stena Baltica ferry we fix:

- the number of the ferry operation process states  $\nu = 18$ 

and we distinguish the following as its eighteen operation states:

• an operation state  $z_1$  – loading at Gdynia Port,

- an operation state z<sub>2</sub> unmooring operations at Gdynia Port,
- an operation state z<sub>3</sub> leaving Gdynia Port and navigation to "GD" buoy,
- an operation state z<sub>4</sub> navigation at restricted waters from "GD" buoy to the end of Traffic Separation Scheme,
- an operation state z<sub>s</sub> navigation at open waters from the end of Traffic Separation Scheme to "Angoring" buoy,
- an operation state z<sub>6</sub> navigation at restricted waters from "Angoring" buoy to "Verko" Berth at Karlskrona,
- an operation state z<sub>7</sub> mooring operations at Karlskrona Port,
- an operation state z<sub>8</sub> unloading at Karlskrona Port,
- an operation state  $z_9$  loading at Karlskrona Port,
- an operation state z<sub>10</sub> unmooring operations at Karlskrona Port,
- an operation state  $z_{11}$  ship turning at Karlskrona Port,
- an operation state  $z_{12}$  leaving Karlskrona Port and navigation at restricted waters to "Angoring" buoy,
- an operation state  $z_{13}$  navigation at open waters from "Angoring" buoy to the entering Traffic Separation Scheme,
- an operation state  $z_{14}$  navigation at restricted waters from the entering Traffic Separation Scheme to "GD" buoy,
- an operation state  $z_{15}$  navigation from "GD" buoy to turning area,
- an operation state  $z_{16}$  ship turning at Gdynia Port,
- an operation state z<sub>17</sub> mooring operations at Gdynia Port,
- an operation state  $z_{18}$  unloading at Gdynia Port.

Moreover, we fix that there are possible only the transitions between the neighboring system operation states, i.e., from the operation states  $z_b$  to the operation states  $z_{b+1}$ , b = 1, 2, ..., 17, and from the operation state  $z_{18}$  to the operation state  $z_1$ .

Thus, the unknown parameters of the system operation process semi-markov model are:

- the initial probabilities  $p_b(0)$ , b = 1,2,...,18, of the Stena Baltica ferry operation process transients in the particular states  $z_b$  at the moment t = 0,

- the transition probabilities  $p_{bb+1}$ , b = 1, 2, ..., 17, and  $p_{181}$  of the Stena Baltica ferry operation process from the operation state  $z_b$  into the operation state  $z_{b+1}$  and from the operation state  $z_{18}$  into the operation state  $z_1$ ,
- the distributions of the conditional sojourn times  $\theta_{bb+1}$ , b = 1, 2, ..., 17, and  $\theta_{181}$  in the particular operation states and their mean values.

To identify all these parameters of the Stena Baltica ferry operation process the statistical data about this process is needed. The statistical data that has been collected during spring is given in the Appendix 4A in *Tables 1-7* [7].

From data given in these Tables, on the basis of methods and procedures given in the previous section, in further sections the Stena Baltica ferry operation process statistical data is fixed and its unknown parameters are estimated.

## **5.4.3.** The Stena Baltica ferry operation process data collection

The collected statistical data necessary to evaluating the initial transient probabilities of the Stena Baltica ferry operation process in the particular states are:

- the ship operation process observation/experiment time  $\Theta = 42$  days,

- the number of the ship operation process realizations n(0) = 42,

- the numbers  $n_b(0)$  of the system operation process transients in the particular operation states  $z_b$  at the initial moment t = 0

$$n_1(0) = 42, n_2(0) = 0, \dots, n_{18}(0) = 0,$$

where

$$n_1(0) + n_2(0) + \ldots + n_{\nu}(0) = 42$$
,

- the vector of realizations of the numbers of the system operation process transients in the particular operation states  $z_b$  at the initial moment t = 0

$$[n_{b}(0)] = [n_{1}(0), n_{2}(0), \dots, n_{v}(0)] = [42, 0, \dots, 0].$$

The collected statistical data necessary to evaluating the transition probabilities of the Stena Baltica ferry operation process between the operation states are:

- the realization  $n_{bl}$  of the numbers of the ship operation process transitions from the state  $z_{b}$  into the state  $z_{l}$  during the experiment time  $\Theta = 42$  days

$$n_{11} = 0, n_{12} = 42, n_{13} = 0, \dots, n_{117} = 0, n_{118} = 0,$$
  

$$n_{21} = 0, n_{22} = 0, n_{23} = 42, \dots, n_{217} = 0, n_{218} = 0,$$
  

$$\dots$$
  

$$n_{171} = 0, n_{172} = 0, n_{173} = 0, \dots, n_{1717} = 0, n_{1718} = 42,$$
  

$$n_{181} = 42, n_{182} = 0, n_{183} = 0, \dots, n_{1817} = 0, n_{1818} = 0,$$

- the matrix of realizations  $n_{\scriptscriptstyle bl}$  of the numbers of the ferry operation process transitions from the state  $z_{\scriptscriptstyle b}$  into the state  $z_{\scriptscriptstyle l}$  during the experiment time  $\Theta = 42$  days

$$[n_{bl}] = \begin{bmatrix} 0.42 & 0...0 & 0 \\ 0 & 0.42 & ...0 & 0 \\ ... & & \\ 0 & 0 & 0...0 & 42 \\ 42 & 0 & 0...0 & 0 \end{bmatrix},$$

- the realization  $n_b$  of the total numbers of the ship operation process transitions from the operation state  $z_b$  during the experiment time  $\Theta = 42$  days (the sums of the numbers of the matrix  $[n_{bl}]$ )

$$n_{1} = n_{11} + n_{12} + \dots + n_{118} = 42,$$
  

$$n_{2} = n_{21} + n_{22} + \dots + n_{218} = 42,$$
  

$$\dots$$
  

$$n_{17} = n_{171} + n_{172} + \dots + n_{1718} = 42,$$
  

$$n_{18} = n_{181} + n_{182} + \dots + n_{1818} = 42$$

- the matrix of realizations of the total numbers of the ship operation process transitions from the operation state  $z_{b}$  during the experiment time  $\Theta = 42$  days

$$[n_{b}] = [n_{1}, n_{2}, ..., n_{v}] = [42, 42, ..., 42].$$

The collected statistical data necessary to evaluating the unknown parameters of the distributions of the conditional sojourn times of the Stena Baltica ferry operation process in the particular operation states are as follows: - the realizations  $\theta_{bl}^{k}$ ,  $k = 1, 2, ..., n_{bl}$ , of the conditional sojourn times  $\theta_{bl}$  of the Stena Baltica ferry operation process at the operation state  $z_{i}$ when the next transition is to the operation state  $z_i$ during the observation time are given in the Tables 1-7. In the Tables 1-7 there are given realizations of the conditional sojourn times in particular operation states on the basis of a sample composed of n = 42realizations of the Stena Baltica ferry operation process. It is assumed that one voyage from Gdynia to Karlskrone and back to Gdynia of the ferry is a single realization of its operation process. The conditional sojourn times in particular operation states of each single realization of the ferry operation process are given in minutes in separate columns. The operation process is very regular in the sense that the operation state changes are from the particular state  $z_{b}$ , b = 1, 2, ..., 17, to the neighbouring state  $z_{b+1}$ , b = 1, 2, ..., 17, only and from  $z_{18}$  to  $z_1$ . Therefore, the realizations of the conditional sojourn times  $\theta_{bb+1}^{j}$ , b = 1, 2, ..., 17, j = 1, 2, ..., 42, are given in the Tables b-th row and the realizations of the conditional sojourn time  $\theta_{181}^{j}$ , b = 1, 2, ..., 17, are given in the Tables 18-th row.

Table 1. Realization of conditional sojourn times  $\theta_{\scriptscriptstyle bl}$ at operations states

Oper	$\theta^{\scriptscriptstyle 1}_{\scriptscriptstyle hh+1}$	$\theta_{\rm ab+1}^2$	$\theta^{3}_{_{bb+1}}$	$ heta_{_{hh+1}}^{_4}$	$\theta^{\scriptscriptstyle 5}_{\scriptscriptstyle bb+1}$	$ heta_{_{bb+1}}^{_{6}}$
ation	00+1	00+1	00+1	00+1	00+1	00+1
state						
$Z_{b}$						
$Z_1$	55	52	47	75	60	60
Z 2	4	3	3	2	2	2
Z 3	28	31	32	35	37	48
Z 4	52	46	48	65	53	47
Z 5	598	635	539	572	499	507
Z 6	35	42	42	44	35	37
Z 7	7	9	8	7	7	5
$Z_8$	25	20	23	27	20	31
Ζ,	75	59	56	40	66	47
$Z_{10}$	5	3	2	3	2	3
Z 11	6	5	4	5	4	5
Z 12	25	22	25	25	23	25
Z <sub>13</sub>	574	427	461	501	498	490
$Z_{14}$	61	43	43	46	49	52
Z 15	33	32	33	36	35	33
$Z_{16}$	4	4	5	4	4	4

$Z_{17}$	8	10	6	5	5	6
$Z_{18}$	26	26	30	20	16	17

*Table 2.* Realization of conditional sojourn times  $\theta_{_{bl}}$  at operations states

Oper	$\theta^{7}_{_{bb+1}}$	$ heta_{\scriptscriptstyle hb+1}^{\scriptscriptstyle 8}$	$\theta^{9}_{_{bb+1}}$	$ heta_{_{hh+1}}^{_{10}}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 11}$	$ heta_{_{bb+1}}^{_{12}}$
ation	00+1	00+1	00+1	00+1	00+1	00+1
state						
$\mathcal{Z}_{b}$						
$Z_1$	62	43	50	61	65	63
$Z_2$	2	3	3	4	3	2
Z 3	33	38	39	43	40	42
$Z_4$	49	62	45	46	51	47
$Z_5$	621	580	507	511	497	496
$Z_{6}$	34	40	36	33	38	38
$Z_7$	5	5	5	5	8	7
$Z_8$	15	17	16	21	33	34
$Z_9$	26	60	65	25	55	40
$Z_{10}$	5	6	3	4	4	2
$Z_{11}$	4	4	4	6	4	5
$Z_{12}$	20	33	24	24	22	22
$Z_{13}$	438	561	491	513	496	500
$Z_{14}$	42	63	46	60	50	50
$Z_{15}$	35	34	31	33	34	36
$Z_{16}$	3	4	4	4	4	4
Z <sub>17</sub>	4	5	8	7	6	7
$Z_{18}$	16	22	17	8	17	17

*Table 3.* Realization of conditional sojourn times  $\theta_{\scriptscriptstyle bl}$  at operations states

Oper	$\theta^{\scriptscriptstyle 13}_{\scriptscriptstyle bb+1}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 14}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 15}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 16}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 17}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 18}$
ation						
state						
$\mathcal{Z}_{b}$						
$Z_1$	45	45	40	20	33	50
$Z_2$	2	2	2	2	2	3
$Z_3$	35	36	36	36	37	35
Z 4	51	51	51	49	53	44
$Z_5$	595	495	504	507	498	483
Z 6	34	39	38	39	38	35
Z 7	7	8	7	10	8	8
$Z_8$	18	16	13	3	15	6
$Z_9$	75	77	60	73	82	118
$Z_{10}$	5	2	2	2	3	4

Soszyńska Joanna, Kołowrocki Krzysztof, Blokus-Roszkowska Agnieszka, Guze Sambor Identification of complex technical systems operation processes

<i>z</i> <sub>11</sub>	4	4	4	4	4	4
Z <sub>12</sub>	24	24	25	24	23	22
Z <sub>13</sub>	582	491	499	488	464	484
$Z_{14}$	72	50	48	50	48	52
Z 15	34	35	35	34	35	34
$Z_{16}$	5	5	5	4	4	4
Z 17	7	7	6	4	4	7
$Z_{18}$	26	40	21	34	40	35

Table 4. Realization of conditional sojourn times  $\theta_{M}$ at operations states

Oper	$oldsymbol{ heta}^{^{19}}_{^{bb+1}}$	$oldsymbol{ heta}_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 20}$	$\boldsymbol{ heta}^{\scriptscriptstyle 21}_{\scriptscriptstyle bb+1}$	$oldsymbol{ heta}_{^{22}}^{^{22}}$	$\theta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 23}$	$\boldsymbol{ heta}^{^{24}}_{^{bb+1}}$
ation						
state						
$Z_{b}$						
$Z_1$	43	15	45	57	97	68
Z 2	2	2	3	2	2	3
$Z_3$	34	34	36	36	39	36
Z 4	51	52	50	53	53	54
Z 5	497	504	507	503	500	492
Z 6	37	36	37	34	38	40
Z 7	7	8	8	8	7	9
$Z_8$	9	25	19	31	30	35
Ζ,	71	55	30	24	34	41
$Z_{10}$	2	2	3	3	2	5
Z 11	4	4	4	4	4	4
Z 12	23	22	22	22	26	22
Z 13	498	496	505	595	493	499
Z 14	47	53	51	61	61	48
Z 15	31	32	33	46	34	34
$Z_{16}$	5	5	3	4	6	6
Z 17	5	5	7	5	4	5
Z 18	28	22	8	2	12	13

Table 5. Realization of conditional sojourn times  $\theta_{\scriptscriptstyle bl}$ at operations states

	-					
Oper	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 25}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 26}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 27}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 28}$	$ heta^{29}_{_{bb+1}}$	$ heta^{\scriptscriptstyle 30}_{\scriptscriptstyle bb+1}$
ation						
state						
$Z_{b}$						
$Z_1$	58	35	45	75	72	62
Z 2	3	4	3	3	2	3
$Z_3$	37	36	35	39	37	36
$Z_4$	67	51	50	62	49	48

$Z_5$	573	498	506	576	494	505
$Z_6$	36	37	35	38	38	36
$Z_{7}$	8	7	5	7	10	9
$Z_8$	25	11	17	31	23	25
Z 9	55	55	43	45	52	48
$Z_{10}$	3	3	3	3	2	3
$Z_{11}$	4	4	5	5	4	5
Z <sub>12</sub>	23	22	23	26	23	23
Z <sub>13</sub>	573	497	531	500	492	496
$Z_{14}$	58	51	54	47	40	51
$Z_{15}$	34	35	33	35	35	34
$Z_{16}$	5	5	6	5	4	6
Z <sub>17</sub>	4	5	5	5	7	6
$Z_{18}$	18	20	11	10	16	18

Table 6. Realization of conditional sojourn times  $\theta_{\scriptscriptstyle bl}$ at operations states

Oper	$\theta^{\scriptscriptstyle 31}_{\scriptscriptstyle bb+1}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 32}$	$\theta^{\scriptscriptstyle 33}_{\scriptscriptstyle bb+1}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 54}$	$\boldsymbol{ heta}^{\scriptscriptstyle 35}_{\scriptscriptstyle bb+1}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 36}$
ation						
state						
$Z_{b}$						
$Z_1$	37	44	46	78	59	65
$Z_2$	6	3	2	2	2	2
$Z_3$	37	36	36	37	36	36
$Z_4$	64	51	53	63	55	53
$Z_5$	576	495	502	574	492	497
$Z_6$	35	39	37	36	38	37
$Z_7$	10	6	7	7	6	6
$Z_8$	23	15	18	19	18	24
$Z_9$	50	58	53	30	30	45
$Z_{10}$	2	2	3	3	2	2
$Z_{11}$	4	5	4	5	4	4
Z <sub>12</sub>	24	23	24	23	28	24
Z <sub>13</sub>	590	508	520	502	508	508
$Z_{14}$	47	47	56	47	46	42
Z <sub>15</sub>	33	34	35	36	35	35
$Z_{16}$	5	5	4	4	5	4
Z 17	5	6	6	10	5	4
Z.,	25	18	12	12	17	14

Oper	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 37}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 38}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 39}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 40}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 41}$	$ heta_{\scriptscriptstyle bb+1}^{\scriptscriptstyle 42}$
ation						
state						
$Z_{b}$						
$Z_1$	53	25	55	84	71	67
$Z_2$	2	2	3	2	2	2
$Z_3$	38	37	40	36	37	34
$Z_4$	60	49	46	57	53	51
$Z_5$	584	504	505	573	494	495
$Z_{6}$	38	35	36	39	36	36
$Z_7$	5	7	5	5	6	6
$Z_8$	15	6	40	28	32	28
$Z_9$	70	35	35	47	40	50
$Z_{10}$	2	2	3	3	3	2
$Z_{11}$	5	4	5	5	4	4
$Z_{12}$	25	25	24	23	26	24
$Z_{13}$	595	506	535	506	503	503
$Z_{14}$	42	45	47	46	51	43
$Z_{15}$	34	35	34	34	33	33
$Z_{16}$	6	4	4	5	5	4
Z 17	5	3	4	5	3	5
$Z_{18}$	20	11	11	10	13	18

Table 7. Realization of conditional sojourn times	$ heta_{\scriptscriptstyle bl}$
at operations states	

### **5.4.4. Evaluating the unknown parameters of the Stena Baltica ferry operation process**

On the basis of the statistical data from Section 5.4.3, using the formulae given in Section 3.4, it is possible to evaluate

- the vector of realizations

[p(0)] = [1,0,0,...,0,0]

of the initial probabilities  $p_b(0)$ , b = 1, 2, ..., 18, of the ship operation process transients in the particular operation states  $z_b$  at the moment t = 0,

- the matrix of realizations

$$[p_{bl}] = \begin{bmatrix} 0 \ 1 \ 0 \ \dots \ 0 \ 0 \\ 0 \ 0 \ 1 \ \dots \ 0 \ 0 \\ \dots \\ 0 \ 0 \ 0 \ \dots \ 0 \ 1 \\ 1 \ 0 \ 0 \ \dots \ 0 \ 0 \end{bmatrix},$$

of the transition probabilities  $p_{bl}$ , b, l = 1, 2, ..., 18, of the system operation process from the operation state  $z_b$  into the operation state  $z_l$  during the experiment time  $\Theta = 42$  days.

On the basis of the statistical data from Section 5.4.3, using the formulae given in Section 3.4, it is possible to determine the following empirical characteristics of the realizations of the conditional sojourn time of the Stena Baltica ferry operation process in the particular operation states:

- the realizations of the mean values  $\overline{\theta}_{12}$  of the conditional sojourn times  $\theta_{12}$  of the Stena Baltia ferry operation process at the operation state  $z_1$  when the next transition is to the operation state  $z_2$ 

$$\overline{\theta}_{12} = \frac{1}{42} \sum_{k=1}^{42} \theta_{12}^{k} = 54.3, \ b, l = 12, ..., 18 \ b \neq l,$$

- the number  $\bar{r}_{12}$  of the disjoint intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, ..., \bar{r}_{12}$ , that include the realizations  $\theta_{12}^k$ , k = 1, 2, ..., 42, of the conditional sojourn times  $\theta_{12}$  at the operation state  $z_1$  when the next transition is to the operation state  $z_2$ 

$$\overline{r}_{12} = \sqrt{n_{bl}} = \sqrt{42} \cong 6$$

- the length  $d_{12}$  of the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ , j = 1, 2, ... 6,

$$d_{12} = \frac{\overline{R}}{\overline{r}_{12} - 1} = \frac{82}{5} \cong 16,$$

where

$$\overline{R}_{12} = \max_{1 \le k \le 42} \theta_{12}^k - \min_{1 \le k \le 42} \theta_{12}^k = 97 - 15 = 82,$$

- the ends  $a_{bl}^{j}$ ,  $b_{bl}^{j}$ , of the intervals  $I_{j} = \langle a_{bl}^{j}, b_{bl}^{j} \rangle$ ,  $j = 1, 2, \dots 6$ , according to the formulae

$$\min_{1 \le k \le 42} \theta_{12}^{k} - \frac{d_{12}}{2} = 15 - \frac{16}{2} = 15 - 8 = 7,$$
  
$$a_{12}^{1} = \max\{7, 0\} = 7,$$
  
$$b_{12}^{1} = a_{12}^{1} + d = 7 + 16 = 23,$$
  
$$a_{12}^{2} = b_{12}^{1} = 23, \ b_{12}^{2} = a_{12}^{1} + 2 \cdot 16 = 7 + 32 = 39,$$

 $a_{12}^3 = b_{12}^2 = 39, \ b_{12}^3 = a_{12}^1 + 3 \cdot 16 = 7 + 48 = 55,$ 

$$a_{12}^4 = b_{12}^3 = 55, \ b_{12}^4 = a_{12}^1 + 4 \cdot 16 = 7 + 64 = 71,$$

 $a_{12}^5 = b_{12}^4 = 71, \ b_{12}^5 = a_{12}^1 + 5 \cdot 16 = 7 + 80 = 87,$ 

- the numbers  $n_{12}^{j}$  of the realizations  $\theta_{12}^{42}$  in particular intervals  $I_{j}$ , j = 1, 2, ...6,

$$n_{12}^1 = 2, n_{12}^2 = 4, n_{13}^3 = 15, n_{12}^4 = 14, n_{12}^5 = 6, n_{12}^6 = 1,$$

 $a_{12}^6 = b_{12}^5 = 87, \ b_{12}^6 = a_{12}^1 + 6 \cdot 16 = 7 + 96 = 103,$ 

Histogram of the conditional sojourn time $\theta_{12}$								
$I_j = < a_{bl}^j, b_{bl}^j)$	7 – 23	23 – 39	39 – 55	55 - 71	71 - 87	87 – 103		
$n_{12}^{j}$	2	4	15	14	6	1		
$\overline{h}_{12}(t) = n_{12}^{j} / n_{12}$	2/42	4/42	15/42	14/42	6/42	1/42		



*Figure 26.* Histogram of the conditional sojourn time  $\theta_{12}$ 

- the realizations of the mean values  $\overline{\theta}_{23}$  of the conditional sojourn times  $\theta_{23}$  of the Stena Baltia ferry operation process at the operation state  $z_2$  when the next transition is to the operation state  $z_3$ 

$$\overline{\theta}_{23} = \frac{1}{42} \sum_{k=1}^{42} \theta_{23}^{k} = 2.6, \ b, l = 1, 2, ..., 18, \ b \neq l,$$

- the number  $\overline{r}_{23}$  of the disjoint intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, ..., \overline{r}_{23}$ , that include the realizations  $\theta_{23}^k$ , k = 1, 2, ..., 42, of the conditional sojourn times  $\theta_{23}$  at the operation state  $z_2$  when the next transition is to the operation state  $z_3$ 

 $\overline{r}_{12} = \sqrt{n_{bl}} = \sqrt{42} \cong 6,$ 

- the length  $d_{23}$  of the intervals  $I_j = \langle a_{bl}^j, b_{bl}^j \rangle$ ,  $j = 1, 2, \dots 6$ ,

$$d_{23} = \frac{\overline{R}_{23}}{\overline{r}_{23} - 1} = \frac{82}{5} \cong 16$$

where

$$\overline{R}_{12} = \max_{1 \le k \le 42} \theta_{12}^{k} - \min_{1 \le k \le 42} \theta_{12}^{k} = 97 - 15 = 82,$$

- the ends  $a_{bl}^{j}$ ,  $b_{bl}^{j}$ , of the intervals  $I_{j} = \langle a_{bl}^{j}, b_{bl}^{j} \rangle$ ,  $I_{j} = \langle a_{bl}^{j}, b_{bl}^{j} \rangle$ , according to the formulae

$$\min_{1 \le k \le 42} \theta_{23}^{k} - \frac{d_{23}}{2} = 2 - \frac{0.8}{2} = 2 - 0.4 = 1.6,$$

$$a_{23}^{1} = \max\{1.6,0\} = 1.6,$$
  

$$b_{23}^{1} = a_{23}^{1} + 0.8 = 1.6 + 0.8 = 2.4,$$
  

$$a_{23}^{2} = b_{23}^{1} = 2.4, \quad b_{23}^{2} = a_{23}^{1} + 2 \cdot 0.8 = 1.6 + 1.6 = 3.2,$$
  

$$a_{23}^{3} = b_{23}^{2} = 3.2, \quad b_{23}^{3} = a_{23}^{1} + 3 \cdot 0.8 = 1.6 + 2.4 = 4.0,$$
  

$$a_{23}^{4} = b_{23}^{3} = 4.0, \quad b_{23}^{4} = a_{23}^{1} + 4 \cdot 0.8 = 1.6 + 3.2 = 4.8,$$

 $a_{23}^5 = b_{23}^4 = 4.8, \ b_{23}^5 = a_{23}^5 + 5 \cdot 0.8 = 1.6 + 4.0 = 5.6,$  $a_{23}^6 = b_{23}^5 = 5.6, \ b_{23}^6 = a_{23}^6 + 6 \cdot 0.8 = 1.6 + 4.8 = 6.4,$ 

- the numbers  $n_{23}^{j}$  of the realizations  $\theta_{23}^{42}$  in particular intervals  $I_{j}$ , j = 1, 2, ... 6,

$$n_{12}^1 = 24, n_{12}^2 = 14, n_{13}^3 = 0, n_{12}^4 = 3, n_{12}^5 = 0, n_{12}^6 = 1.$$

Histogram of the conditional sojourn time $\theta_{23}$							
$I_j = < a_{bl}^j, b_{bl}^j)$	1.6 – 2.4	2.4 - 3.2	3.2 - 4.0	4.0 - 4.8	4.8 - 5.6	5.6 - 6.4	
n <sup>j</sup> <sub>23</sub>	24	14	0	3	0	1	
$\overline{h}_{23}(t) = n_{23}^{j} / n_{23}$	24/42	14/42	0/42	3/42	0/42	1/42	



*Figure 27.* Histogram of the conditional sojourn time  $\theta_{23}$ 

At the remaining cases we are proceeding afterwards in an analogous way.

## **5.4.5. Identifying the distributions of the conditional sojourn times in the operation states of the Stena Baltica**

Using the procedure given in Section 3.5 we may verify the hypotheses on the distributions of the conditional sojourn times  $\theta_{bl}$ , b, l = 1, 2, ..., 18,  $b \neq l$ , in the particular operation states and we have the following results:

- the conditional sojourn time  $\theta_{12}$  has a triangular distribution with the density function

$$h_{12}(t) = \begin{cases} 0, & t < 7, \\ \frac{2}{103 - 7} \frac{t - 7}{54 - 7} = \frac{1}{48} \frac{t - 7}{47}, & 7 \le t < 54, \\ \frac{2}{103 - 7} \frac{103 - t}{103 - 54} = \frac{1}{48} \frac{103 - t}{49}, & 54 \le t < 103, \\ 0, & t \ge 103, \end{cases}$$

- the conditional sojourn time  $\theta_{23}$  has an exponential distribution with the density function

$$h_{23}(t) = \begin{cases} 0, & t < 1.6, \\ 1.03 \exp[-1.03(t-1.6)], & t \ge 1.6, \end{cases}$$

- the conditional sojourn time  $\theta_{\rm 34}$  has a chimney distribution with the density function

$$h_{34}(t) = \begin{cases} 0, & t < 29, \\ 0.0278, & 29 \le t < 35, \\ 0.1984, & 35 \le t < 38, \\ 0.0266, & 38 \le t < 47, \\ 0, & t \ge 47; \end{cases}$$

- the conditional sojourn time  $\theta_{45}$  has a chimney distribution with the density function

$$h_{45}(t) = \begin{cases} 0, & t < 41, \\ 0.0095, & 41 \le t < 46, \\ 0.0762, & 46 \le t < 56, \\ 0.0127, & 56 \le t < 71, \\ 0, & t \ge 71; \end{cases}$$

- the conditional sojourn time  $\theta_{56}$  has a double trapezium distribution with the density function

$$h_{56}(t) = \begin{cases} 0, & t < 467.8, \\ -0.00004t + 0.0277, & 467.8 \le t \le 525.95, \\ -0.00006t + 0.0397, & 525.95 \le t \le 650.2, \\ 0, & t < 650.2; \end{cases}$$

- the conditional sojourn time  $\theta_{\rm 67}$  has a double trapezium distribution with the density function

$$h_{67}(t) = \begin{cases} 0, & t < 31.9, \\ 0.0067t - 0.1747, & 31.9 \le t \le 37.17, \\ 0.0031t - 0.0395, & 37.17 \le t \le 45.1, \\ 0, & t > 45.1; \end{cases}$$

- the conditional sojourn time  $\theta_{78}$  has a double trapezium distribution with the density function

$$h_{78}(t) = \begin{cases} 0, & t < 4.5, \\ -0.0183t + 0.2922, & 4.5 \le t \le 7.02, \\ -0.0069t + 0.2122, & 7.02 \le t \le 10.5, \\ 0, & t > 10.5; \end{cases}$$

- the conditional sojourn time  $\theta_{89}$  has a triangular distribution with the density function

$$h_{89}(t) = \begin{cases} 0, & t < 0, \\ 0.0021t, & 0 \le t \le 21.4 \\ -0.002t + 0.087, & 21.4 \le t \le 44.4, \\ 0, & t > 44.4; \end{cases}$$

- the conditional sojourn time  $\theta_{_{910}}$  has a double trapezium distribution with the density function

$$h_{910}(t) = \begin{cases} 0, & t < 14.6, \\ 0.0001t + 0.0109, & 14.6 \le t \le 52.26, \\ 0.0062, & 52.2 \le t \le 127.4, \\ 0, & t > 127.4; \end{cases}$$

- the conditional sojourn time  $\,\theta_{_{1011}}\,$  has a chimney distribution with the density function

$$h_{1011}(t) = \begin{cases} 0, & t < 1.6, \\ 0.506, & 1.6 \le t \le 3.2, \\ 0.0595, & 3.2 \le t \le 6.4, \\ 0, & t > 6.4; \end{cases}$$

- the conditional sojourn time  $\theta_{1112}$  has a chimney distribution with the density function

$$h_{1112}(t) = \begin{cases} 0, & t < 3.8, \\ 1.6667, & 3.8 \le t \le 4.2, \\ 0.1667, & 4.2 \le t \le 6.2, \\ 0, & t > 6.2; \end{cases}$$

- the conditional sojourn time  $\theta_{_{1213}}$  has a chimney distribution with the density function

$$h_{1213}(t) = \begin{cases} 0, & t < 18.7, \\ 0.0092, & 18.7 \le t \le 21.3, \\ 0.1786, & 21.3 \le t < 26.5, \\ 0.0061, & 26.5 \le t < 34.3, \\ 0, & t > 34.3; \end{cases}$$

- the conditional sojourn time  $\,\theta_{_{1314}}\,$  has a chimney distribution with the density function

$$h_{1314}(t) = \begin{cases} 0, & t < 410.2, \\ 0.0014, & 410.2 \le t < 477.4, \\ 0.0184, & 477.4 \le t < 511, \\ 0.0028, & 511 \le t < 611.4, \\ 0, & t \ge 611.4; \end{cases}$$

- the conditional sojourn time  $\theta_{_{1415}}$  has a Weibull's distribution with the density function

$$h_{1415}(t) = \begin{cases} 0, & t < 36.8, \\ 0.1415(t - 36.8)^{1.1403} \exp[-0.0661(t - 36.8)^{2.1403}], & t > 36.8 \end{cases}$$

- the conditional sojourn time  $\theta_{\rm 1516}$  has a chimney distribution with the density function

$$h_{1516}(t) = \begin{cases} 0, & t < 30, \\ 0.0317, & 30 \le t < 33, \\ 0.2698, & 33 \le t < 36, \\ 0.0084, & 36 \le t < 48, \\ 0, & t \ge 48; \end{cases}$$

- the conditional sojourn time  $\theta_{\rm \scriptscriptstyle 1617}$  has a triangular distribution with the density function

$$h_{1617}(t) = \begin{cases} 0, & t < 2.7, \\ 0.305t - 0.823, & 2.7 \le t \le 4.52, \\ -0.313t + 1.9719, & 4.52 \le t \le 6.3, \\ 0, & t > 6.3; \end{cases}$$

- the conditional sojourn time  $\theta_{_{1718}}$  has a double trapezium distribution with the density function

$$h_{1718}(t) = \begin{cases} 0, & t < 2.3, \\ -0.1134t + 0.6707, & 2.3 \le t \le 5.62, \\ 0.0071t - 0.0063, & 5.62 \le t \le 10.7, \\ 0, & t > 10.7; \end{cases}$$

- the conditional sojourn time  $\theta_{_{181}}$  has a Weibull's distribution with the density function

$$h_{181}(t) = \begin{cases} 0, & t < 0, \\ 0.1113t^{1.3519} \exp[-0.0473t^{2.3519}], & t \ge 0. \end{cases}$$

# **5.4.6. Identifying the mean values of the system conditional sojourn times in operation states of the Stena Baltica ferry**

For the distributions identified in Section 5.4.5, according to either the formulae (8) or (9)-(16), the mean values  $M_{bl} = E[\theta_{bl}]$ , b, l = 1, 2, ..., 18,  $b \neq l$ , of the system operation process Z(t) conditional sojourn times in particular operation states ware determined and they amount:

$$\begin{split} M_{12} &= 54.33, \ M_{23} = 2.57, \ M_{34} = 36.57, \\ M_{45} &= 52.5, \ M_{56} = 525.95, \ M_{67} = 37.16, \\ M_{78} &= 7.02, \ M_{89} = 21.43, \ M_{910} = 53.69, \\ M_{1011} &= 2.93, \ M_{1112} = 4.38, \ M_{1213} = 23.86, \\ M_{1314} &= 509.69, \ M_{1415} = 50.14, \ M_{1516} = 34.28, \\ M_{1617} &= 4.52, \ M_{1718} = 5.62, \ M_{181} = 18.74. \end{split}$$

# 6. Identification of the operation processes of real complex technical systems – using computer program

The computer program allows to identify of the unknown basic parameters of the system operation processes and to verify the hypotheses concerning the unknown forms of the distribution functions of the conditional sojourn times in the particular operation states on the basis of empirical data coming from the operation processes of complex technical systems. The program is based on the methods and algorithms for the estimating unknown parameters of the operations process included in [1]. Particularly, the computer program is useful in determining the probabilities of the initial operation states of the system operation process and the probabilities of the system operation process transitions between the operation states. The computer program allows to estimate the unknown parameters of the distributions of the conditional sojourn times of the system operation process in the particular operation states distinguished in [5] as the suitable for these variables uniform, triangular, double trapezium, quasi-trapezium, exponential, Weibull, normal and chimney distributions. Next, the program allows for testifying the hypotheses about fitting empirical distributions with the the distinguished distributions. Moreover, in the cases of the hypotheses acceptance, the computer program allows to determine the theoretical mean values of the sojourn times of the system operation process in the particular operation states. If the hypotheses are rejected, the program allows to find the empirical values of the mean values of these variables.

The computer program may be used for unknown parameters identification of the operation processes of real technical systems, particularly, the operation processes of port, shipyard and maritime transportation systems [3]-[4], [6]-[7]. It may also be used to construct the integrated safety and reliability decision support systems for various maritime and coastal transport sectors. This program together with the description may also be included into these training courses addressed to industry.

#### Acknowledgements

The paper describes the part of the work in the Poland-Singapore Joint Research Project titled "Safety and Reliability of Complex Industrial Systems and Processes" supported by grants from the Poland's Ministry of Science and Higher Education (MSHE grant No. 63/N-Singapore/2007/0) and the Agency for Science, Technology and Research of Singapore (A\*STAR SERC grant No. 072 1340050).

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