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A method for estimating the probability distribution of the lifetime for new technical equipment based on expert judgement

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Highlights

- A new method for estimating the probability distribution of the lifetime based on expert assessments is developed.
- The expert lifetime elicitation procedure is developed and applied to the Weibull lifetime.
- The quantile function is used to develop the expert method.
- The subjective Bayesian approach with models of classical probability theory is integrated.
- The objectification of the evaluation of experts to assign weights to their opinions is proposed.

Abstract

Managing the exploitation of technical equipment under conditions of uncertainty requires the use of probabilistic prediction models in the form of probability distributions of the lifetime of these objects. The parameters of these distributions are estimated with the use of statistical methods based on historical data about actual realizations of the lifetime of examined objects. However, when completely new solutions are introduced into service, such data are not available and the only possible method for the initial assessment of the expected lifetime of technical objects is expert methods. The aim of the study is to present a method for estimating the probability distribution of the lifetime for new technical facilities based on expert assessments of three parameters characterizing the expected lifetime of these objects. The method is based on a subjective Bayesian approach to the problem of randomness and integrated with models of classical probability theory. Due to its wide application in the field of maintenance of machinery and technical equipment, a Weibull model is proposed, and its possible practical applications are shown. A new method of expert elicitation of probabilities for any continuous random variable is developed. A general procedure for the application of this method is proposed and the individual steps of its implementation are discussed, as well as the mathematical models necessary for the estimation of the parameters of the probability distribution are presented. A practical example of the application of the developed method on specific numerical values is also presented.

Keywords

uncertainty, expert elicitation of lifetime, quantile function, Weibull distribution.

Acronyms

CDF	Cumulative Distribution Function
ED	Expert Data
EEL	Expert Elicitation of Lifetime
ELV	Expanded Lover Value
ETD	Expanded Triangle Distribution
EUV	Expanded Upper Value
IRF	Invers Reliability Function
LF	Lifetime Family
PDF	Probability Density Function
REE	Reliability Engineer Expert
RF	Reliability Function
TD	Technical Device

Notation

β	shape parameter
$\tilde{\beta}$	shape parameter in the EEL procedure
$\breve{\beta}$	aggregated shape parameter
η	scale parameter
$\tilde{\eta}$	scale parameter in the EEL procedure
$\breve{\eta}$	aggregated scale parameter
$\lambda(t)$	failure rate function
$\Gamma(\cdot)$	gamma function
$f(t)$	PDF
$F(t)$	CDF
k	number of experts

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$M_k(r_1;r_2)$	matrix of the theoretical values of the location parameters
$\tilde{M}_k(r_1;r_2)$	matrix of the expert location parameters
p	unreliability level
$\Pr(.)$	probability function
r	reliability level
$R(t)$	RF
$R^{-1}(p)$	IRF
t	exposure variable (e.g., time)
t_p	potential lifetime at the unreliability level p
$wbl(\beta)$	one-parameter family of Weibull distributions
$wbl(\beta;\eta)$	two-parameter family of Weibull distributions
$wbl((t_1,r_1),(t_2,r_2))$	two-parameter Weibull distribution in the EEL parametrization

1. Introduction

In today's increasingly competitive environment, designing and manufacturing reliable products is essential to the company's survival. An innovative reliability program for a manufacturing company can significantly improve the quality, performance and durability of a product, and ultimately the company's profitability and customer satisfaction. Reliability analysis of industrial equipment is one of the most dynamic branches of research and continues to be a challenge for many applications. For decades, statistical methods have been developed and used in reliability research, see, e.g., [1, 15, 24, 29, 31]. Software tools to support more and more complex reliability analyses are being developed, see, e.g., [16, 17, 18].

Nowadays, empirical statistical methods are supported by other methods. The Bayesian modelling framework is based on incorporation of different sources of quantitative and qualitative data in the model [4, 22, 37]. The article [8] concerns the estimation of low probabilities of failure in terms of structural reliability. Analytic models for predicting system lifetime are based on reliability block diagrams [22], fault trees [25], Markov chains, semi-Markov processes [14], stochastic Petri nets [10] or hierarchical models. Typically, such models capture uncertainty that is natural in the system being modelled. This includes random times to failure of components, random times for various recovery actions and randomness in the ability to detect a failure. The methodology of examining uncertainty in various aspects is presented in the articles [20, 33, 38]. Such uncertainty, known as aleatory uncertainty, is usually captured by beta, gamma, exponential, triangular, Weibull, lognormal, Bernoulli and other distributions. Computations and results obtained from such models thus account for the aleatory uncertainty in the system. Results of the model will depend upon the validity of the assumed distribution forms as well as the parameter values attached to these distributions. Assuming that the distribution forms are valid, parametric uncertainty is the subject of this paper.

The main challenge of fitting distribution to reliability data is finding the family of distribution and the values of the parameters that give the highest probability of producing the observed data. One of the most common probability density functions used in industry is the Weibull distribution [1]. The paper [2] gives an extensive review of some discrete and continuous versions of the modifications of the Weibull distribution.

Other concepts of uncertainty description are based on the notion of imperfect knowledge [9] and use methods beyond classical probability theory. Such concepts include methods of so-called generalized uncertainty [5], which also allow the use of expert knowledge based on data and information of an incomplete and sometimes ambiguous nature. These methods provide opportunities for quantitative uncertainty assessment considering three main criteria, which can sometimes conflict with each other, namely:

- inclusion in the analysis and calculation of all verified data and information at the disposal of the expert,
- the abandonment of assumptions in the model which cannot be clearly and reliably justified,
- the orientation of the modelling process towards achieving the main objective, which is to develop an effective tool to support decision-making under uncertainty.

As the predominant type of uncertainty within this concept is epistemic uncertainty, the most used methods for its description are subjective probabilities (e.g., in the Bayesian approach) and the so-called imprecise probabilities (e.g., in the approach of fuzzy set theory).

In many industrial applications the basic criterion for the usability of a technical device is the quality of the product, which is a function of the technical condition of this device. However, in the case of other types of technical devices, such as e.g., infrastructural facilities, and especially of unique character, this methodology is not applicable. Our proposal concerns exactly such devices, for which it is not possible to obtain either direct – historical data, or indirect – data concerning the influence of the degradation of the examined device on the quality of the product.

The aim of this article is to present a method of estimating the lifetime probability distribution of new technical devices based on expert assessments of only a few parameters characterizing the expected lifetime of these objects. The method is based on a subjective Bayesian approach to the problem of randomness and integrated with models of classical probability theory. Due to its widespread use in maintenance of machinery and technical equipment, a Weibull model is proposed, and possible practical applications are shown for it.

This article is organized as follows. Section 2 presents a literature survey on the determination of subjective probability distributions based on expert opinion data. Special emphasis is placed on discussing methods that have been positively validated in so-called critical infrastructure (e.g., in risk analysis of dams). On this basis, and in particular the analysis of the strengths and weaknesses of these methods, a modified procedure for expert elicitation of probabilities for any continuous random variable, consisting of eight main steps is proposed in Section 3. A general procedure for applying this method is developed and the various steps in its implementation are discussed. Section 4 proposes a formal construction of the expert lifetime elicitation procedure and presents the mathematical models necessary to estimate the parameters of its distribution. Application of the Expert Elicitation of Lifetime (EEL) procedure to the Weibull lifetime distribution is the subject of Section 5. The next section presents a practical example of using the developed method on concrete numerical values. The article ends with a summary, conclusions and plans for further work within the ongoing research project.

2. Determination of subjective probability distribution based on expert judgement – literature review

The subjective probability should reflect a starting point of knowledge of an object of interest (so-called prior probability distribution), based on which a rational person would use Bayes' methodology, by means of new available information, to determine the modified probability distribution (so-called posterior probability distribution). Thus, this methodology is implemented in multiple steps; first the prior probability is elicited and then it is modified based on further available information.

The stimulus for the dynamic development of methods based on Bayesian inference has been the challenge of managing the risk of unitary systems with high levels of reliability and potentially high safety risks, such as reactors in the nuclear power industry. An example of an attempt to solve this problem can be found in the safety study of nuclear reactors, concluded with a guide recommending the use of appropriate elicitation methods [36]. This type of methodology has also been used to assess environmental risks and their impact on the safety and health of whole populations as well as individual people [27].

As interest in this issue grew, more and more papers appeared in the field of psychology on human decision-making under uncertainty. The experiments generally consisted of asking questions to which the subjects did not know the answers, and then respondents were asked to quantify the degree of uncertainty in these responses. Mostly the psychologists who compiled the results of these studies assigned corresponding probabilities to the different degrees of uncertainty. As a result of this research, it was found that assessing the uncertainty of one's own knowledge tends to be subject to systematic errors, which were called biases. Galwey's publication [12] defines the most important of these biases, namely:

- accessibility - overestimating the chance of events that have happened recently and that we have easy access to in our memory,
- representativeness – assessing the chance of events based on irrelevant data, often incidentally linked to those events,
- anchoring – ignoring new data and information about events about which we have already formed an opinion, particularly in terms of the likelihood of their occurrence, and
- overconfidence – overestimating our knowledge and therefore underestimating the uncertainty of our assessment.

Until the early-1990s, assessments of these errors were descriptive based on widely accepted concepts presented in the work by Kahneman, Slovic, and Tversky [21]. In contrast, Morgan and Henrion's book [27] proposed a general procedure that could be used as a basis for developing a guide for performing rational elicitation. Summarizing the literature in this area, it can be stated that (based on [12]):

- the selection of experts should consider their technical, technological, managerial, and economic competence in the subject matter of the expert opinion, and ensure their independence from the owner of the object under assessment,
- elicitation should take place under the minimum constraints of both time and money, and should provide the experts with full access to all information on the object of the evaluation,
- the elicitation methodology should be carefully prepared before the experts start their work, and the experts should know and accept it,
- the entire elicitation process should be carefully and explicitly documented so that it can be reproduced in the future and its correctness and effectiveness critically analysed.

Current Best Practices by determination of subjective probability distribution based on expert judgement can be synthesized to the following procedure, which is based on several sources (e.g., [11, 12, 26, 27]):

a) Using multiple experts, if possible, the more the better. It is particularly important to ensure that independent experts with in-depth knowledge and engineering experience participate in the elicitation.
b) Asking experts not only about the expected or most likely value, but also about the smallest and largest possible values of the parameter being evaluated. It is recommended that the order of the questions should force the experts to first ask for the dispersion of the values of the parameter and only then for the expected value.
c) Use of triangular decomposition for graphical description of elicitation results. In works [6] and [13] it is recommended to modify this distribution by assuming that it covers only 90% of the entire range of variability of the evaluated parameter. The remaining 10% can be distributed symmetrically between the lower and upper areas of variation of the parameter [6], or asymmetrically, with 2% around lower values and 8% around upper values [13]. Figure 1 shows an example of the Expanded Triangle Distribution (ETD) concept (based on [7] and [12]).
d) Some authors recommend that experts provide additional percentile values for the assessed parameter to verify the plausibility of the assessment and check its compliance with the assumed triangular distribution.
e) Provide experts with the opportunity to access the results of the entire elicitation process and organise an additional session with all experts to critically analyse both the process procedure itself and its results.
f) Documentation in full of all stages of the elicitation process, including a description of their progress, analysis of the results obtained and archiving of the whole so that each element of the process can be reproduced at any time in the future.

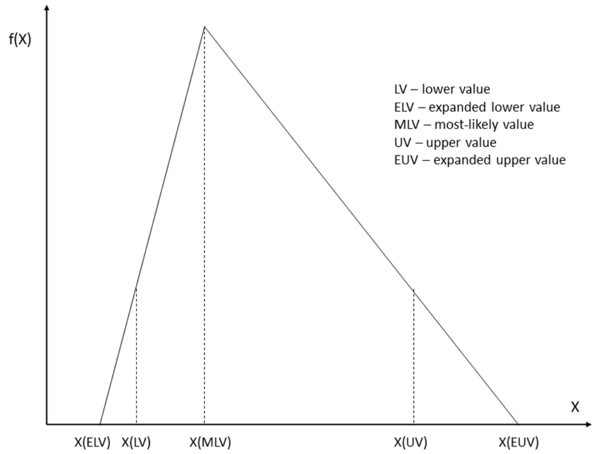


Fig. 1. The Expanded Triangle Distribution (ETD) concept – an example

Practical advice on the implementation of points b), c) and d) can be found e.g., in the publication on risk analysis of dams [32] in the form of suggested questions to experts:

- What is the lowest reasonably plausible number you can imagine the likelihood to be?
- What is the highest reasonably plausible number you can imagine the likelihood to be?
- Is it more likely to be somewhere in between these values?
- If so, what is the most likely value?
- The probability is not likely to be less than x? (e.g., 10th percentile)
- The Probability is not likely to be more than y? (e.g., 90th percentile)
- It cannot be less than v? (e.g., 0th percentile) nor more than z? (100th percentile)
- It is equally likely to be more or less than m? (50th percentile)

The above-described methodology, based on the ETD concept, has been used successfully in several cases, e.g., in cost risk analysis [12]. However, in many cases, such as estimating the expected life of new technical facilities, it has proved unreliable. We see the main reasons for this situation in the following limitations of the ETD concept:

- The assumption that the range of a random variable X is restricted to a closed interval between ELV and EUV is contrary to maintenance experience on the durability of machinery and equipment.
- The values of 8 and 2% define the skewness of the probability distribution, but these are not universal values, and their adoption has not been sufficiently justified anywhere.
- In many practical situations it is crucial to determine probabilities for values of variable X outside the ELV to EUV range, which is impossible when using the ETD method.

In view of the above-mentioned limitations of the ETD method, the authors propose an alternative method devoid of these deficiencies. The assumptions of this method and the general procedure for its application is presented in Section 3.

3. Modified procedure for expert elicitation of a probability distribution for a continuous random variable

Based on the literature analysis conducted in Section 2 and our own experience, we propose a modified procedure for expert elicitation of a probability distribution for random variables, those of a continuous nature (e.g., expressed in units of time). The general procedure for the practical application of this method, consisting of eight steps, is shown in Figure 2.

Step one requires a clear, precise, and unambiguous formulation of the problem to be addressed by the experts. The experts should have all the relevant information for the evaluation, but not be burdened with unnecessary details that add little or nothing to the subject of the evaluation. The proper formulation of the task is the basis for the selection of appropriate experts who are authorities in the relevant field of knowledge.

The creation of as numerous and competent a group of experts is the objective of phase two. This is a difficult task, because usually these two criteria conflicts with each other – the more numerous the expert group, the greater the chance that it will also include less competent representatives. This step should also include selecting and adding to the expert team (or selecting from among them) an experienced facilitator, responsible for the harmonious work of the whole team – the group leader.

The next step is to develop an elicitation implementation plan, considering both organizational and scheduling aspects. All constraints (e.g., time, financial, etc.) should be considered, as well as possible disruptions that may occur during the elicitation process (e.g., threats and hazards). The plan should be as detailed as possible, but at the same time flexible (e.g., considering the possibility of one of the experts being indisposed). An important part of the plan is the preparation of appropriate forms for collecting data from experts, which should easily allow further computer processing of the information obtained.

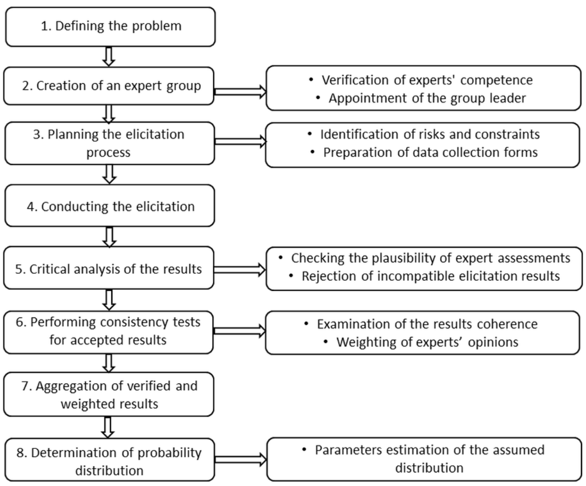


Fig. 2. General procedure for modified expert elicitation procedure of the lifetime distribution

Step four is a key part of the evaluation process, so it should proceed as quickly and smoothly as possible. To avoid possible mistakes of anchoring and suggesting the opinions of other team members each expert should perform the evaluation without contacting other experts.

The discussion on the assessment and arguing for or against certain opinions can take place in step five, after the work of step four has been completely closed. In case of significant divergence between expert opinions, it is recommended to carry out an in-depth analysis, which should provide a conclusive answer to the question: are the results plausible? If the answer to this question is positive, you can proceed to step seven, which is to aggregate the results from all expert evaluators. If, on the other hand, the answer is negative, additional tasks must be taken to reach a compromise among the experts or to eliminate the opinions of those experts who could not convincingly justify their decisions.

In the first part of step six, additional verification of the consistency of the expert judgements should be carried out using a consistency test. The results of this test can be used as a basis for assessing the credibility of the individual experts and for assigning appropriate weights to their opinions in the second part of this step. This will allow the quality of individual elicitation to be considered in the process of aggregating the opinions of different experts.

The next step is to aggregate the verified elicitation results. The aggregation process uses the ratings of all the experts, considering the weights estimated in the previous step, in order to obtain unambiguous data allowing the estimation of the parameters of the assumed probability distribution.

The last step of the procedure is to create a parametric model of the lifetime probability distribution sought and to use it for practical purposes, e.g., determination of the expected lifetime of new technical equipment, for which the lack of operational data precludes the use of statistical methods.

The innovation of the proposed model is that the first 5 steps have been developed by modifying best practice in different areas of application of expert assessments used for critical infrastructures. The sixth and seventh steps, which aim to objectivize the assessments of individual experts, are fully innovative. We propose that verification of the consistency of the expert judgements should be carried out using a consistency test. The results of this test can be used as a basis for assessing the credibility of the individual experts and for assigning appropriate weights to their opinions in the second part of this step. This will allow the quality of individual elicitation to be considered in the process of aggregating the opinions of different experts. The aggregation process uses the ratings of all the experts, considering the weights estimated in the previous step, to obtain unambiguous data allowing the estimation of the parameters of the assumed probability distribution.

4. Formal construction of the expert lifetime elicitation procedure

We use the quantile method in the proposed procedure of the Expert Elicitation of Lifetime (EEL) of a Technical Device (TD). This method is often used in engineering research. For example, in the article [3], the quantile method was used to identify the costliest damage to parts of fleet vehicles. On the pages of Transport Topics [19] Evan Lockridge wrote "Engine makers are providing customers a gauge to help them determine how dependable and durable an engine is supposed to be, called a B-life rating." The construction of this lifetime measure is also based on a quantile function. The BX% rating in Weibull ++ is used to estimate the time when the probability of failure reaches a certain point (X%). Industry specialists consider this measure as a standard for measuring the life expectancy of technical products. For example, in predicting engine life, the most frequently heard ratings are B10% and B50% of life rating [19]. In this case B10% life is the expected engine durability expressed in kilometres of operation, before 10% of all operated engines of a specific type will require a major overhaul, renovation, or replacement. Thus, such information is very useful in giving customers a good idea of engine life expectations for a specific engine family. In practice BX% ratings are based on the durability data that engine manufacturers have on file and operating data [35]. So, a research problem appeared: *How to build an equivalent of this measure of lifetime for new technical* *devices for which operational data will appear only in the future*? Our research is an attempt to solve this problem.

In our research, we do not have operational data or there is very little data, so we cannot use statistical methods to estimate parameters. Hence the need to develop an expert method for the assessment of unknown TD lifetime parameters. The primary role of the Reliability Engineering Expert (REE) is to identify hazards and manage the risks associated with the reliability of assets that may adversely affect the operations of a facility or company investing in new equipment. In such a case, we believe that the method of determining the lifetime of these equipment, developed in this article, may be useful. In the presented research, the BX% lifetime estimates are replaced with 100*p*% percentiles obtained from REE. Based on Expert Data (ED), the lifetime parameters of a predetermined family distributions are determined.

The likelihood of a system failure can be assessed under different circumstances using the REE group's opinion. It provides an applicable method for a facile computational prediction of future performances that aims to replace the usage of failure rates by a combination of instructed REE elicitation [28]. Due to the lack of historical data, expert judgment is used regarding the probability of the system failure in the planned operating conditions. Data on selected parameters of the lifetime are obtained using an appropriately designed questionnaire. In the designed survey, experts are asked to express their opinion on the potential lifetimes t_p at certain levels $p_1,\ldots,p_l \in (0,1)$ of the unreliability in the assumed process of use and service for given TD. The originality of the developed lifetime parameter estimation procedure results from the application of this expert information for a specific lifetime model, instead of historical data. Such an approach to the issue of parameter evaluation has not yet been developed in the reliability theory.

Potential lifetime t_p at the unreliability level p is the quantile determined from the one of the equations $F(t_p)=p$ or $R(t_p)=1-p$. We assume that the potential lifetime is continuous, so t_p lifetime is derived from the quantile equation $t_p = R^{-1}(1-p)$, where R^{-1} is the Invers Reliability Function (IRF). Potential lifetime t_p is the time during which the new TD will not fail with probability $r=1-p$. The potential lifetime t_p plays a fundamental role in developing the EEL procedure of TD. In the proposed EEL procedure, we use the fact that it is enough to know as many different potential lifetimes as there are parameters for the assumed Lifetime Family (LF) distributions. The characterization of the LF parameters of a given TD with the elaborated EEL procedure relies only on the potential lifetimes reported by a group of *k* independent REE experts.

Let TD be a new device (equipment) whose lifetime is to be estimated by a group of k REEs. Moreover, let $\mathrm{LF}(\alpha_1,\ldots,\alpha_s)$ denote the s parametric lifetime family of this device determined based on the knowledge of damage physics. The lack of historical data does not allow the use of statistical estimation of these parameters. In such a situation, we suggest using the EEL procedure to determine their value. As already indicated, the general idea of the expert elicitation is to use a potential lifetime. Experts from the REE group make individually elicitation the potential lifetimes $t_{i1},\ldots,t_{is}$ for $i=1,\ldots,k$ and s different levels of reliability $r_1,\ldots r_s$ or dually levels of unreliability $p_1=1-r_1,\ldots,p_r=1-r_s$. Moreover, they provide at least one location parameter as control values. Let $l_{i1},\ldots,l_{iq}$ denote the control parameters of the i-th expert. The control parameters should be different from the selected potential lifetimes. Thus, we obtain ED as a two-block input matrix (1):

$$\tilde{M}_k(r_1;\ldots;r_s)=\left[\begin{array}{ccc|ccc}\tilde{t}_{11} & \ldots & \tilde{t}_{1s} & \tilde{l}_{11} & \ldots & \tilde{l}_{1q}\\ \tilde{t}_{21} & \ldots & \tilde{t}_{2s} & \tilde{l}_{21} & \ldots & \tilde{l}_{2q}\\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots\\ \tilde{t}_{k1} & \ldots & \tilde{t}_{ks} & \tilde{l}_{k1} & \ldots & \tilde{l}_{kq}\end{array}\right] \tag{1}$$

Before we proceed to identifying lifetime distributions from the obtained ED, the group leader examines the plausibility of this data. At this stage, not plausible experts are rejected, and new experts are appointed in their place. The procedure is repeated until a fixed number of experts remain. The result of the work of the group leader is to establish a group of k experts and obtain an updated expert plausible data matrix $\tilde{M}_k(r_1;\ldots;r_s)$. Only the data of the first block is needed to determine the LF parameters. The data contained in the second block we will use to determine weights for individual experts. To determine the LF parameters $\alpha_{i1},\ldots,\alpha_{is}$ for the i-th expert, a system of equations (2) is solved:

$$\begin{cases} R_{\mathrm{LF}(\alpha_1,\ldots,\alpha_s)}(\tilde{t}_{i1})=r_1 & \\ \qquad\ldots & \text{for } i=1,\ldots,k \\ R_{\mathrm{LF}(\alpha_1,\ldots,\alpha_s)}(\tilde{t}_{is})=r_s & \end{cases} \tag{2}$$

where $R_{\mathrm{LF}(\alpha_1,\ldots,\alpha_s)}$ is RF of the $\mathrm{LF}(\alpha_1,\ldots,\alpha_s)$. If there exist Invers Reliability Function (IRF) $R^{-1}_{\mathrm{LF}(\alpha_1,\ldots,\alpha_s)}$, the parameters of potential expert lifetime can be determined by solving equivalent systems of equations (3):

$$\begin{cases} R^{-1}_{\mathrm{LF}(\alpha_1,\ldots,\alpha_s)}(r_1)=\tilde{t}_{i1} & \\ \qquad\ldots & \text{for } i=1,\ldots,k \\ R^{-1}_{\mathrm{LF}(\alpha_1,\ldots,\alpha_s)}(r_s)=\tilde{t}_{is} & \end{cases} \tag{3}$$

Thus, for the i-th expert we obtain a random lifetime $\tilde{T}_i$, the probability distribution of which has the form (4):

$$\tilde{T}_i \sim \mathrm{LF}(\tilde{\alpha}_{i1},\ldots,\tilde{\alpha}_{is}), i=1,\ldots,k \tag{4}$$

Based on the first block of the ED matrix, we obtained expert parameters $\tilde{\alpha}_{i1},\ldots,\tilde{\alpha}_{is}$ of the given LF distribution for all k experts. The obtained random lifetimes $\tilde{T}_1,\ldots,\tilde{T}_k$ are necessary to perform consistency tests. In this step, we proceed to determine the theoretical values $l_{i1},\ldots,l_{iq}$ of the control parameters for all k experts. In this way, we obtain the matrix (5) of the theoretical values of the control parameters for all k experts:

$$M_k=\begin{bmatrix} l_{11} & \ldots & l_{1q}\\ l_{21} & \ldots & l_{2q}\\ \ldots & \ldots & \ldots\\ l_{k1} & \ldots & l_{kq}\end{bmatrix} \tag{5}$$

The data consistency test is carried out for each expert separately. It consists in comparing the control parameters $\tilde{l}_{i1},\ldots,\tilde{l}_{iq}$ given by the i-th expert and recorded in the second block of the ED matrix, with their theoretical equivalents $l_{i1},\ldots,l_{iq}$ determined from the obtained lifetimes $\tilde{T}_1,\ldots,\tilde{T}_k$. If the control parameters given by a certain expert do not meet the conditions specified by the group leader, the data of that expert is omitted, and a new expert is appointed in his place.

If the ED matrix is plausible and consistent, then we proceed to the next step of the EEL procedure. In this step, based on the selected control parameter, the weights of the obtained lifetimes $\tilde{T}_1,\ldots,\tilde{T}_k$ are determined. These weights are measures of the quality of the expert information contained in the ED matrix. The quality of the opinion of the i-th expert is assessed based on the relative measures of deviations dev_1 or dev_2 of the expert value $\tilde{\theta}$ of a given control parameter from the theoretical value θ of this parameter. To determine the quality measures of expert opinions, we propose the formulas (6) and (7):

$$dev_1(\tilde{\theta}) \stackrel{\text{def}}{=} \frac{\tilde{\theta}-\theta}{\theta} \tag{6}$$

$$dev_2(\tilde{\theta}) \stackrel{\text{def}}{=} \frac{|\tilde{\theta}-\theta|}{\theta} \tag{7}$$

The obtained measures of relative deviations of expert values of control parameters from their theoretical values are used to determine the weights of the obtained lifetimes $\tilde{T}_1,\ldots,\tilde{T}_k$. If $\tilde{\theta}_i \neq \theta$ for $i=1,\ldots,k$, then the weights are determined separately for the parameters as follows:

$$w_i(\tilde{\theta}) = \frac{\frac{1}{dev_2(\tilde{\theta}_i)}}{\sum_{j=1}^{k}\frac{1}{dev_2(\tilde{\theta}_j)}}, i=1,\ldots,k \tag{8}$$

If $\tilde{\theta}_i = \theta$ for a certain expert, then as the difference $|\tilde{\theta}-\theta|$ we take a small value, e.g., $0{,}000001$. Then the obtained weights are used to determine the aggregated parameters $\breve{\alpha},\ldots,\breve{\alpha}_s$ of the TD lifetime $\breve{T}$. Lifetime $\breve{T}$ parameterized in this way finalizes the presented EEL procedure, and its result is the weighted probability distribution (9):

$$\breve{T} \sim \mathrm{LF}(\breve{\alpha}_1,\ldots,\breve{\alpha}_s) \tag{9}$$

The obtained lifetime $\breve{T}$ can be used to determine the functional and numerical both unconditional and conditional reliability characteristics of a TD.

However, it should be remembered that determining the LF parameters and its functional and numerical characteristics based on the EEL procedure is not always an easy task, as it may be necessary to know the specific properties of the families of lifetime distributions.

In the next section, we will do this for the family of Weibull lifetime distribution. Weibull lifetime can be applied to many situations. The main advantage of using this probability distribution is that it is flexible enough to accommodate different types of TD lifetimes and its well-known properties. Some of them that are useful for the EEL procedure are also presented in the next section.

5. Application of the EEL procedure to the Weibull lifetime distribution

Starting with a three-parameter Weibull lifetime distribution, the general Weibull model is given by the following Probability Density Function (PDF) [30]:

$$f_{wbl(\beta;\eta;\gamma)}(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} e^{-\left(\frac{t-\gamma}{\eta}\right)^{\beta}} \mathbb{I}_{[\gamma,\infty)}(t), \beta>0, \eta>0, -\infty<\gamma<\infty \tag{10}$$

where β is the shape parameter, η is the scale parameter, γ is the location parameter and $\mathbb{I}_{(\gamma,\infty)}$ is the indicator function (11):

$$\mathbb{I}_{(\gamma,\infty)}(t) := \begin{cases} 1 & \text{if} \quad t\in(\gamma,\infty) \\ 0 & \text{if} \quad t\notin(\gamma,\infty) \end{cases} \tag{11}$$

Since the main properties of the Weibull lifetime distribution is determined by the scale and shape parameters, we will focus further on the one- and two-parameter family of Weibull lifetime.

5.1. One-parameter Weibull lifetime distribution

This part of the publication presents the results of research on the properties of the Weibull distribution depending only on the shape parameter. These properties allow for a better eliciting information of the location characteristics and hence, the one-parameter Weibull lifetime plays a special role in our study. This special case occurs when the scale parameter is one and the location parameter is zero. In this case, one can only speak of a relative lifetime without entering unit names. PDF $f_{wbl(\beta)}$ for the one-parameter Weibull lifetime $wbl(\beta)$ reduces to (12):

$$f_{wbl(\beta)}(t) = \beta t^{\beta-1} e^{-t^{\beta}} \mathbb{I}_{[0,\infty)}(t), \beta>0 \tag{12}$$

Now let's look at the effects of the beta shape parameter. The Fig. 3 shows the effect of different values of the shape parameter, β, on the shape of the PDF, independently of the other parameters. As you can see, the shape can take on a variety of forms based on the value of β.

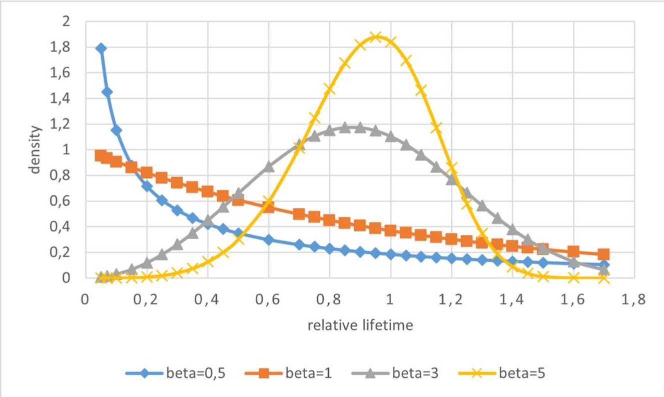


Fig. 3. One-parameter Weibull density curves for $\beta = 0{,}5$; 1; 3; and 5

As can be found in [34] for $\beta<2{,}6$ the Weibull PDF is positively skewed, for $2{,}6<\beta<3{,}7$ coefficient of skewness approaches zero and consequently, it may approximate the normal PDF. For $\beta>3{,}7$ it is negatively skewed. If $1<\beta\le 2$, then density function is concave downward and then upward, with inflection point given in (13):

$$t = \left(\frac{3(\beta-1)+\sqrt{(5\beta-1)(\beta-1)}}{2\beta}\right)^{\frac{1}{\beta}} \tag{13}$$

If $\beta>2$ density function is concave upward, then downward, then upward again, with inflection points at (14):

$$t = \left(\frac{3(\beta-1)\pm\sqrt{(5\beta-1)(\beta-1)}}{2\beta}\right)^{\frac{1}{\beta}} \tag{14}$$

The Fig. 4 shows the effects of these varied values of β on the reliability plot. From the Fig. 4 it is clear, that all the reliability curves intersect at the point $(1;0{,}368)$. The following is the plot of the Weibull failure rate with the same values of β as above.

In Fig. 5 we can see that the failure rate can take various shapes informing about the type of aging of the TD. If $\beta>2$, then the curve $\lambda(t)$ is convex and its slope increases with the increase of t . Conse-

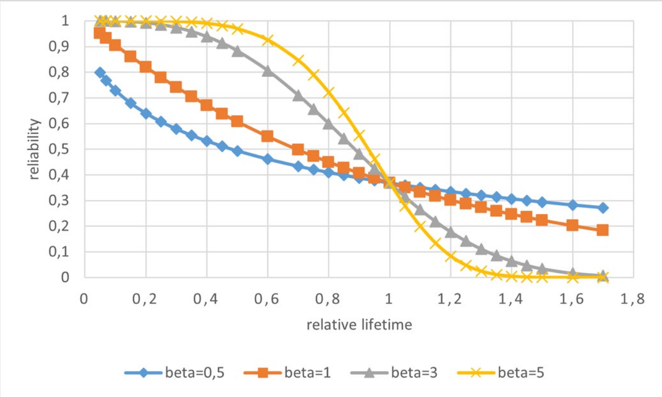


Fig. 4. One-parameter Weibull reliability curves for $\beta = 0{,}5$; 1; 3; and 5

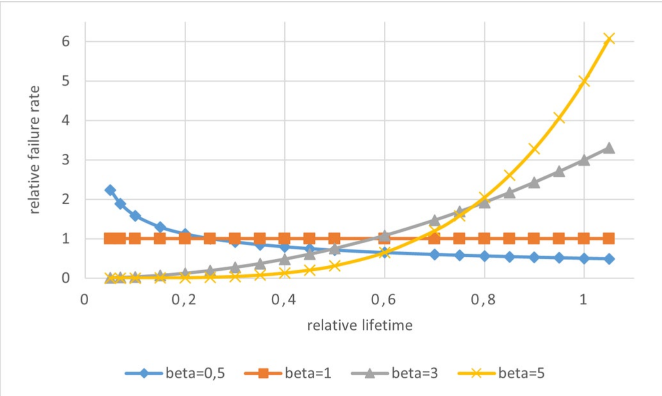


Fig. 5. One-parameter Weibull failure rate for β $= 0{,}5$; 1; 3; and 5

quently, the failure rate increases at an increasing rate as t increases, indicating wear out life. Depending on how skewness is measured we have different values of β giving a value of zero for the measure of skewness chosen [30]:

$\beta \approx 3{,}60235$ for skewness = zero,

$\beta \approx 3{,}43954$ for mean = median,

$\beta \approx 3{,}31247$ for mean = mode,

$\beta \approx 3{,}25889$ for mode = median.

Regarding the kurtosis, we have two values of β ($\beta \approx 2{,}25200$ and $\beta \approx 5{,}77278$) giving kurtosis $= 3$. The standardized normal and Weibull distributions have the same mean hazard rate $= 0{,}90486$ when $\beta \approx 3{,}43927$, which is neartly the value of shape parameter such that the mean is equal to the median. The effect of β can be translated into various modes of failures, as given in Table 1.

Table 1. Type of failures corresponding to β values

β value	type of failure	meaning
$\beta<1$	infant mortality	high probability of failing at early stages
$\beta=1$	random failures	failures are independent of time
$1<\beta<4$	early wear out	can be due to generic failure modes, such as corrosion
$\beta \geq 4$	rapid wear out	steep curve with fast wear out at some point

As we can see, the shape parameter provides important information about the aging process of the TD for which we do not have statistical data yet. Determination of this parameter based on ED plays a key role in predictive research.

5.2. Two-parameter Weibull lifetime distribution

We now assume that the expert elicitation of the potential lifetimes refers to TD, whose lifetime T belongs to the two-parameter family of Weibull distributions $wbl(\beta;\eta)$, where β is the shape parameter and η is the scale parameter. The Weibull lifetime with its two parameters permits the modelling of different regions of the bathtub curve in the lifecycle of a great number of components [37]. PDF $f_{wbl(\beta;\eta)}$ of the two-parameter Weibull's lifetime takes the form (15):

$$f_{wbl(\beta;\eta)}(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} \mathbb{I}_{(0,\infty)}(t), \ \beta>0, \eta>0 \quad (15)$$

Scale parameter η is life characteristic because it is the time T such that $\Pr(T \leq \eta) = 0{,}632$. For two-parameter family $wbl(\beta;\eta)$, if $\beta = 1$, then failure rate is constant $\lambda_{wbl(1;\eta)}(t) = \frac{1}{\eta}$ and LF $wbl(1,\eta)$ is the exponential LF. For $\beta = 2$ the family $wbl(2;\eta)$ is the Rayleigh LF with a linearly increasing failure rate. For $0<\beta<1$ Weibull lifetime are characterized by decreasing failure rate. Thus, depending on the shape parameter, the Weibull distribution belongs to one of the classes: IFR, DFR or CFR, denoting, respectively, classes of increasing, decreasing or constant failure rate. For more details on this distribution and application, see the work of [30].

Potential lifetime t_p of the Weibull lifetime in engineering terminology defined as [23] takes the form (16):

$$t_p = \eta \cdot \left(\ln\left(\frac{1}{1-p}\right)\right)^{\frac{1}{\beta}}, 0<p<1 \quad (16)$$

The experts' task is to assess potential lifetimes t_p for two given probability levels $p_1, p_2 \in (0,1)$. The data comes from the k REE group with comparable knowledge and sufficient experience in the management, maintenance, and design departments. As opinions differ, aggregation is performed to produce a single Weibull lifetime model. For this purpose, a weighting factor is calculated for each expert so that a weighted average of the opinions can be calculated. In summary, the steps to be taken to create an effective aggregate potential lifetime $\breve{T}$ of a TD using the EEL procedure for family $wbl(\beta;\eta)$ are as follows:

a) Appointment of a group of k experts and a group leader to assess the durability of a new TD designed to operate under established operating conditions.

b) Obtaining a plausible and consistent ED matrix of input data composed of potential lifetimes $\tilde{t}_{i;1}, \tilde{t}_{i;2}$ for two reliability levels $r_1 = 1-p_1$, $r_2 = 1-p_2$ and additional location parameters $\tilde{l}_{i;1},\ldots,\tilde{l}_{i;q}$ for control purposes.

c) Determination of Weibull's lifetime $\tilde{T}_i \sim wbl(\tilde{\beta}_i;\tilde{\eta}_i)$ of the i-th expert, for $i = 1,\ldots,k$.

d) Calculation of the theoretical values of the control parameters $l_{i;1},\ldots,l_{i;q}$ for the obtained expert lifetime $\tilde{T}_i, i = 1,\ldots,k$. The control parameters can be a mode, median, expected value, or other numeric localization measures.

e) Selection of a control parameter as a weighting criterion and calculation of weights for individual expert opinions.

f) Determination of the weighted Weibull potential lifetime $\breve{T} \sim wbl(\breve{\beta};\breve{\eta})$ for the selected criterion and two different reliability levels r_1, r_2 .

g) Finally, it remains to use the obtained lifetime $\breve{T}$ to calculate the unconditional or conditional probabilities of survival of the TD and its functional and numerical characteristics useful in reliability tests.

Using the presented EEL procedure for determining the aggregated lifetime, we move to the formal calculation side. Let t_{i1} and t_{i2} for

$i=1,\ldots,k$ be given the potential lifetimes for two different reliability levels r_1 and r_2 for the TD starting the mission at age zero be given. To determine the parameters $\tilde{\beta}_i$ and $\tilde{\eta}_i$ for the ED of the i-th expert, system of equations (17) should be solved:

$$\begin{cases} \tilde{\eta}_i\left(-\ln\left(r_1\right)\right)^{\frac{1}{\tilde{\beta}_i}}=t_{i1} \\ \tilde{\eta}_i\left(-\ln\left(r_2\right)\right)^{\frac{1}{\tilde{\beta}_i}}=t_{i2} \end{cases} \tag{17}$$

The aim is to determine the parameters $\tilde{\beta}_i$ and $\tilde{\eta}_i$ of the Weibull's lifetime $\tilde{T}_i$ as a function of the pairs $\left(t_{i1},r_1\right)$ and $\left(t_{i2},r_2\right)$ given by i-th expert. Solving the system (17) due to the scale parameter we get (18):

$$\begin{cases} \tilde{\eta}_i=\dfrac{t_{i1}}{\left(-\ln\left(r_1\right)\right)^{\frac{1}{\tilde{\beta}_i}}} \\ \tilde{\eta}_i=\dfrac{t_{i2}}{\left(-\ln\left(r_2\right)\right)^{\frac{1}{\tilde{\beta}_i}}} \end{cases} \tag{18}$$

After comparing the right sides of the (18), we get an equation with one unknown parameter $\tilde{\beta}_i$, which can be expressed as a function of the variables $\left(t_{i1},p_1\right)$ and $\left(t_{i2},p_2\right)$. Thus, the solution (19) for the parameter $\tilde{\beta}_i$ is obtained as a function of $\left(t_{i1},r_1\right),\left(t_{i2},r_2\right)$:

$$\tilde{\beta}_i=\log_{\frac{t_{i1}}{t_{i2}}}\left(\frac{\ln\left(r_1\right)}{\ln\left(r_2\right)}\right) \tag{19}$$

By inserting the determined shape parameter $\tilde{\beta}_i$ into the first equation (18) we get scale parameter $\tilde{\eta}_i$:

$$\tilde{\eta}_i=\frac{t_{i1}}{\left(-\ln\left(r_1\right)\right)^{\log_{\frac{\ln(r_1)}{\ln(r_2)}}\frac{t_{i1}}{t_{i2}}}} \tag{20}$$

The lifetime distribution $wbl\left(\tilde{\beta}_i;\tilde{\eta}_i\right)$ determined in this way is an expert distribution of the two-parameter Weibull lifetime $\tilde{T}_i$. The random lifetime $\tilde{T}_i$ obtained by the EEL procedure is denoted by $\tilde{T}_i\sim wbl\left(\left(t_{i1},r_1\right),\left(t_{i2},r_2\right)\right)$. For the obtained T_i, its functional and numerical characteristics can be determined. In such parameterization, the RF takes the form (21):

$$R_{wbl\left(\left(t_{i1},r_1\right),\left(t_{i2},r_2\right)\right)}(t)=e^{-\left(\frac{t\left(-\ln\left(r_1\right)\right)^{\log_{\frac{\ln(r_1)}{\ln(r_2)}}\frac{t_1}{t_2}}}{t_1}\right)^{\log_{\frac{t_1}{t_2}}\left(\frac{\ln(r_1)}{\ln(r_2)}\right)}} \tag{21}$$

Weibull's potential lifetime $t_{i;p}$, for $i=1,\ldots,k$, $r\in(0,1)$ and $p=1-r$ takes the form (22):

$$t_{i;p}=\frac{t_{i1}}{\left(-\ln\left(r_1\right)\right)^{\log_{\frac{\ln(r_1)}{\ln(r_2)}}\frac{t_{i1}}{t_{i2}}}}\left(-\ln(r)\right)^{\frac{1}{\log_{\frac{t_{i1}}{t_{i2}}}\left(\frac{\ln(r_1)}{\ln(r_2)}\right)}} \tag{22}$$

Thus, for the Weibull's potential lifetime, having the ED in the form $\left(t_{i1},r_1\right),\left(t_{i2},r_2\right)$, it is possible to determine the scale parameter $\tilde{\eta}_i$, and the shape parameter $\tilde{\beta}_i$, and then calculate the lifetime location parameters for the i-th expert, such as: expected value (ev), mode (mo) and quartiles, in particular the median (me) and measure of deviation or skewness. Of course, having an ED, we can directly use it to calculate these measures. Apart from the expert's index, the calculation formulas for them take the form (23), (24) or (25), respectively:

$$ev\left(wbl\left(\left(t_1,r_1\right),\left(t_2,r_2\right)\right)\right)=\frac{t_1}{\left(-\ln\left(r_1\right)\right)^{\log_{\frac{\ln(r_1)}{\ln(r_2)}}\frac{t_1}{t_2}}}\Gamma\left(1+\frac{1}{\log_{\frac{t_1}{t_2}}\left(\frac{\ln\left(r_1\right)}{\ln\left(r_2\right)}\right)}\right) \tag{23}$$

$$mo\left(wbl\left(\left(t_1,r_1\right),\left(t_2,r_2\right)\right)\right)=\frac{t_1}{\left(-\ln\left(r_1\right)\right)^{\log_{\frac{\ln(r_1)}{\ln(r_2)}}\frac{t_1}{t_2}}}\left(\frac{\log_{\frac{t_1}{t_2}}\left(\frac{\ln\left(r_1\right)}{\ln\left(r_2\right)}\right)-1}{\log_{\frac{t_1}{t_2}}\left(\frac{\ln\left(r_1\right)}{\ln\left(r_2\right)}\right)}\right)^{\left(\log_{\frac{t_1}{t_2}}\left(\frac{\ln(r_1)}{\ln(r_2)}\right)\right)^{-1}} \tag{24}$$

$$me\left(wbl\left(\left(t_1,r_1\right),\left(t_2,r_2\right)\right)\right)=\frac{t_1}{\left(-\ln\left(r_1\right)\right)^{\log_{\frac{\ln(r_1)}{\ln(r_2)}}\frac{t_1}{t_2}}}\left(\ln 2\right)^{\left(\log_{\frac{t_1}{t_2}}\left(\frac{\ln(r_1)}{\ln(r_2)}\right)\right)^{-1}} \tag{25}$$

These are the potential localization characteristics that are used in this article to construct quality measures of the EEL by comparing REE characteristics with their theoretical counterparts. The resulting Weibull's lifetime is used to determine the potential lifetime for a given reliability level r. Of course, complementary probability $p=1-r$ is the unreliability with which we want to determine the potential lifetime. The potential life t_p is calculated from the equation $R_{wbl(\beta;\eta;\gamma)}\left(t_p\right)=1-p$. The lifetime t_p at the percentile level $100\left(1-p\right)\%$ denotes that the TD will be operational during this time at the reliability $r=1-p$. For example, $t_{0,1}$ is the lifetime at which given TD will be operational with the probability $0,9$. Fig. 6 shows the relationship between potential lifetime and shape parameter β for various values of risk level p ($p=0,02;0,04;0,06;0,08;0,10$) and scale parameter $\eta=3500$. For $p=0,02$ potential life $t_{0,02}$ is the lifetime counted in adopted units of time, for which the TD will have a failure probability of $0,02$.

Larger the value of β, longer the potential lifetime for the same value of η. In the presented probabilistic concept of determining Weibull distribution parameters, the potential lifetime t_p of the TD for a given probability level p is assessed by experts, because of their specialist knowledge.

6. Exemplification of the presented EEL procedure

Assuming that, the lifetime of the tested TD belongs to the family $wbl\left(\beta;\eta\right)$, k REE assess potential lifetimes t_1 and t_2 for two different reliability levels $r_1,r_2\in\left(0,1\right)$ and basic location parameters: modal value mo, median me and expected value ev. Thus, the ED received from REE takes the form of the mapping (26):

$$\text{ED}:\left(0,1\right)^2\ni\left(r_1,r_2\right)\to\left(\tilde{t}_1,\tilde{t}_2\middle|\widetilde{mo},\widetilde{me},\widetilde{ev}\right)\in\mathbb{R}_+^5 \tag{26}$$

where $\mathbb{R}_+^5$ denotes the Cartesian product of positive real numbers.

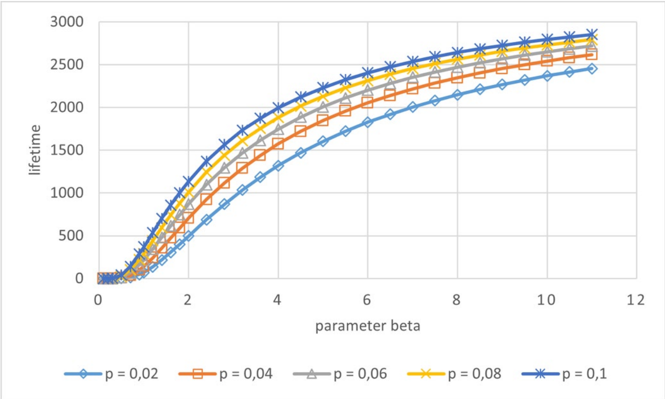


Fig. 6. Potential lifetime t_p versus β for $\eta = 3500$ and $p = 0{,}02$ (bright blue), $p = 0{,}04$ (light brown), $p = 0{,}06$ (gray), $p = 0{,}08$ (yellow), $p = 0{,}1$ (dark blue)

If k experts evaluate the location parameters of the potential lifetime of the same TD, based on the same two reliability levels r_1, r_2, then we obtain the set of ED in the form of five-dimensional vectors arranged in the matrix $\tilde{M}_k$ (27):

$$\tilde{M}_k(r_1;r_2) = \left[\begin{array}{cc|ccc} \tilde{t}_{11} & \tilde{t}_{12} & \widetilde{mo}_1 & \widetilde{me}_1 & \widetilde{ev}_1 \\ \tilde{t}_{21} & \tilde{t}_{22} & \widetilde{mo}_2 & \widetilde{me}_2 & \widetilde{ev}_2 \\ \dots & \dots & \dots & \dots & \dots \\ \tilde{t}_{k1} & \tilde{t}_{k2} & \widetilde{mo}_k & \widetilde{me}_k & \widetilde{ev}_k \end{array}\right] \quad (27)$$

The theoretical values of the location parameters are determined based on two potential lifetimes $\tilde{t}_{i1}$ and $\tilde{t}_{i2}$, given by REE for $i = 1,\dots,k$. In this way we obtain a matrix M_k of the theoretical values of the lifetime location parameters mo_i, me_i, ev_i:

$$M_k(r_1;r_2) = \begin{bmatrix} mo_1 & me_1 & ev_1 \\ mo_2 & me_2 & ev_2 \\ \dots & \dots & \dots \\ mo_k & me_k & ev_k \end{bmatrix} \quad (28)$$

Let's illustrate these matrices with a practical example for given reliability level $r_1 = 0{,}9$ and $r_2 = 0{,}1$. The opinions of the group of $k = 4$ REE on potential lifetime parameters for a certain TD used continuously, presented in the form of a matrix $M_4(0{,}9;0{,}1)$, are as (29), where the unit of time is the one day of using TD:

$$\tilde{M}_4(0{,}9;0{,}1) = \left[\begin{array}{cc|ccc} 3500 & 4500 & 4000 & 4000 & 4000 \\ 3200 & 4800 & 4000 & 4000 & 4000 \\ 3000 & 4500 & 3500 & 3500 & 3500 \\ 2800 & 4000 & 3500 & 3500 & 3500 \end{array}\right] \quad (29)$$

To assess the quality of the ED, we calculate the theoretical values of the control parameters for all four experts. This is how we get the matrix (30):

$$M_4(0{,}9;0{,}1) = \begin{bmatrix} 4175 & 4081 & 4032 \\ 4223 & 4099 & 4041 \\ 3959 & 3843 & 3788 \\ 3581 & 3843 & 3433 \end{bmatrix} \quad (30)$$

The Table 2 summarizes the parameters and potential lifetime t_p at the unreliability level $p = 0{,}01$ of the Weibull distribution determined for the given ED.

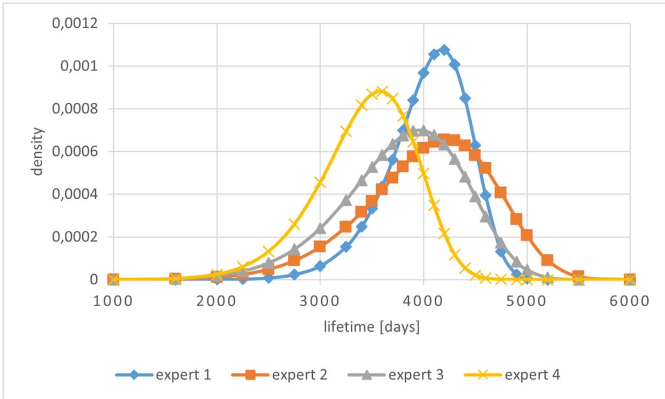
In all four cases, the beta parameter is greater than 4, which proves that all experts treated the tested TD in the same way as a high-quality object whose rapid wear occurs only after a longer period of use. For a graphical comparison, graphs of PDF curves (Fig. 7), reliability function (Fig. 8) and failure rate function (Fig. 9) were prepared for the obtained four expert Weibull lifetimes $\tilde{T}_1, \tilde{T}_2, \tilde{T}_3, \tilde{T}_4$.

Fig. 7. Two-parameter Weibull PDF curves for the first (blue), second (orange), third (gray) and fourth (yellow) expert

Figure 7 shows that the expert lifetimes are generally similar and are almost completely concentrated in the range of 1600 to 5600 days. As for Weibull distributions, they are characterized by high symmetry. This is due to the high values of the shape parameter. The mode values of the obtained lifetimes differ the most for the second and fourth experts, the difference being around 600 days. The lifetime by the first expert has the lowest dispersion, and the second and third experts have the greatest dispersion.

Table 2. Parameters of the Weibull distribution determined for given ED

Expert number	Shape parameter β	Scale parameter η	$t_{0,01}$ [days]
1	12,273071	4204,35587	2890
2	7,607066	4301,55567	2350
3	7,607066	4032,708436	2203
4	8,647649	3632,23519	2134



The presented graphs of the reliability function illustrate the differences of expert predictions. As can be seen from Fig. 8, the first and fourth expert are characterized by the maximum difference in the reliability value. This difference is achieved at 4000 days and is approximately $0{,}5$, but for 3000 and fewer days, this difference is already below $0{,}2$.

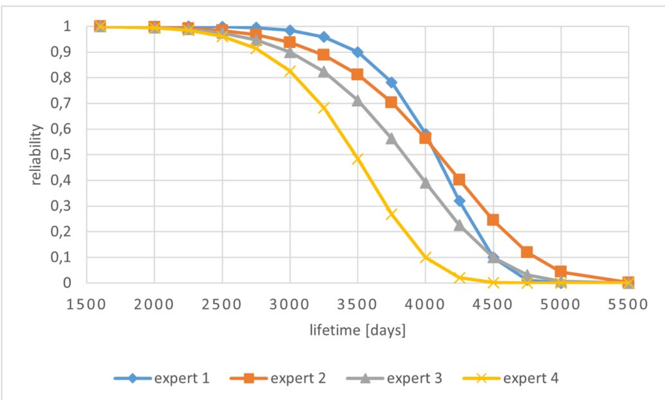


Fig. 8. Two-parameter Weibull RF for the first (blue), second (orange), third (gray) and fourth (yellow) expert

The last presented function for individual experts is the failure rate (Fig. 9). This function at any time characterizes the relative deterioration of the reliability of the TD per day. In engineering practice, historical data on the device or system under consideration is traditionally used to determine this function. Here we showed how to derive this function based on the ED. In all cases, the $\lambda(t)$ curves are convex, and their slopes increase with the increase of t. Consequently, the failure rates increase with the increase of t, which additionally indicates the wear of the TD.

Fig. 9. Two-parameter Weibull failure rate for the first (blue), second (orange), third (gray) and fourth (yellow) expert

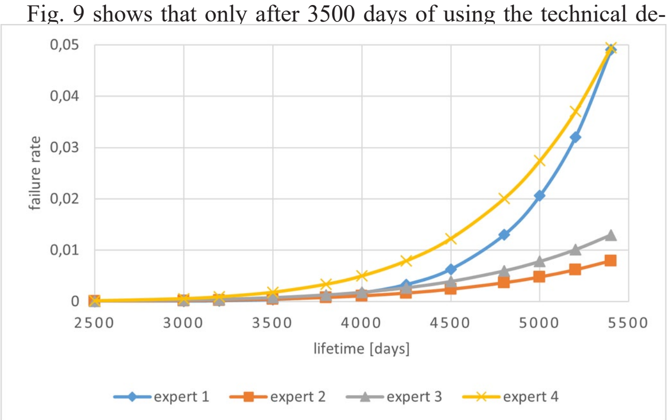


Fig. 9 shows that only after 3500 days of using the technical device, the failure rates for all experts are greater than $0,001$, and then their growth significantly accelerates. The greatest increase results from the data obtained from the first and fourth experts.

In the EEL procedure, we propose that the quality of the i-th expert eliciting information should be assessed based on relative deviation measure dev_1 of the expert value of control parameters, i.e., the mode, the median or the expected value from their theoretical values. Calculation results are summarized in the Table 3.

Table 3 shows that for the experts' elicitation based on the modal value, all four opinions were slightly underestimated and the opinion of the fourth expert was rated the highest. The fourth expert is also rated the highest in the median criterion, and this time this expert was the only one to provide a minimally overestimated value. In the case of the expected value criterion, except the fourth expert, the other experts again slightly lowered the expected value, and the first expert assessed this value most accurately.

Table 3. Expert deviation for the first measure of deviation

Expert number	$dev_1(\widetilde{mo})$	$dev_1(\widetilde{me})$	$dev_1(\widetilde{ev})$
1	$-0,04199$	$-0,01977$	$-0,00802$
2	$-0,05271$	$-0,02420$	$-0,01013$
3	$-0,11587$	$-0,08926$	$-0,07612$
4	$-0,02261$	$0,005312$	$0,019426$

The measure dev_2 of relative deviations of expert values of control parameters from their theoretical values are used to determine the weights of individual experts. The results of the weight calculations for all experts are presented in the Table 4.

The calculated weights of expert lifetime assessments confirm the expert opinion quality ranking. Taking a specific location parameter as a criterion, the obtained weights are used to calculate the aggregated shape $\breve{\beta}$ and scale $\breve{\eta}$ parameters. The calculation results are presented in the Table 5.

In this way, using the EEL procedure, we obtained the following aggregated lifetime distribution for the individual criteria:

Table 4. Weights of ED for individual location parameters

Expert number	$w_i(\widetilde{mo})$	$w_i(\widetilde{me})$	$w_i(\widetilde{ev})$
1	$0,25$	$0,17$	$0,43$
2	$0,20$	$0,14$	$0,34$
3	$0,09$	$0,04$	$0,05$
4	$0,46$	$0,65$	$0,18$

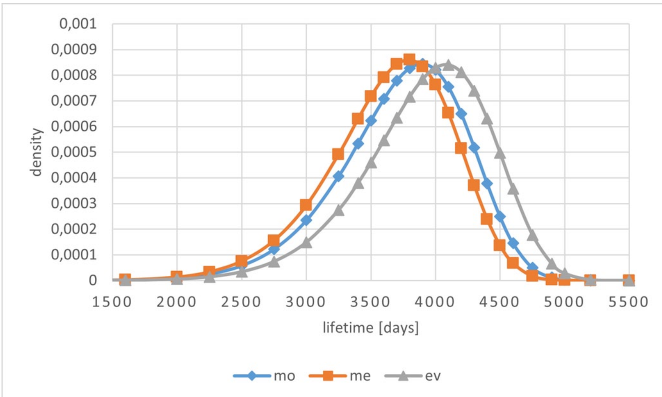
Table 5. List of the aggregated lifetime parameters for three criteria

Characteristics	Criterion *mo*	Criterion *me*	Criterion *ev*
Shape parameter $\breve{\beta}$	8,998	8,923	9,386
Scale parameter $\breve{\eta}$	3945	3843	4129

$$\begin{cases} \breve{T}_{mo} \sim wbl(8,998;3945) \\ \breve{T}_{me} \sim wbl(8,923;3843) \\ \breve{T}_{ev} \sim wbl(9,386;4129) \end{cases} \tag{31}$$

Density curves for the obtained aggregate distributions are presented in Fig. 10.

Fig. 10. Two-parameter aggregated Weibull density curves for the mo *(blue),* me *(orange) and* ev *(gray) criterion*



As can be seen from Fig. 10, the differences between the obtained distributions are relatively small. If we take the centrally located density curve as the criterion for selecting the aggregate lifetime, then in this case the lifetime mode should be selected.

Then, for the obtained aggregate lifetimes $\breve{T}_{mo}, \breve{T}_{me}, \breve{T}_{ev}$ the mode, the median and the expected value were calculated from the formulas (32), (33) and (34), and for the three criteria under consideration.

$$mo(T) = \eta\left(1 - \frac{1}{\beta}\right)^{\frac{1}{\beta}}, \beta > 1 \tag{32}$$

$$me(T) = \eta(\ln 2)^{\frac{1}{\beta}} \tag{33}$$

$$ev(T) = \eta\,\Gamma\left(1+\frac{1}{\beta}\right) \qquad (34)$$

The results of the calculations are presented in the Table 6.

As would be expected for the mode criterion, we obtained the intermediate values of the mode, the median and the expected value. The values of these localization parameters differ very little for all three criteria, which confirms the previously noted large PDF symmetry of the obtained aggregated lifetimes $\breve{T}_{mo}, \breve{T}_{me}, \breve{T}_{ev}$.

Table 6. List of the aggregated lifetime location parameters for three criteria

Parameter	Criterion *mo*	Criterion *me*	Criterion *ev*
mo	3894	3792	4080
me	3787	3688	3971
ev	3736	3637	3918

At the end of this article, the standard deviation (sd), the coefficient of variation (cv) and the skewness coefficient (cs) were calculated using the formulas (35), (36) and (37) for $T \sim wbl(\beta,\eta)$ and all three aggregated lifetimes:

$$sd(T) = \eta\left(\Gamma\left(1+\frac{2}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right)^{\frac{1}{2}} \qquad (35)$$

$$cv(T) = \left(\frac{\Gamma\left(1+\frac{2}{\beta}\right)}{\Gamma^2\left(1+\frac{1}{\beta}\right)} - 1\right)^{\frac{1}{2}} \qquad (36)$$

$$cs(T) = \frac{ev(T) - mo(T)}{sd(T)} \qquad (37)$$

The calculation results are summarized in Table 7.

The performed calculations show that considering the mode criterion, the standard deviation as well as the coefficients of variation and skewness have intermediate values compared to the other two criteria.

Table 7. List of lifetime characteristics for chosen criteria

Characteristics	Criterion mo	Criterion me	Criterion ev
sd	496	487	500
cv	0,1329	0,1339	0,1277
cs	-0,3182	-0,3168	-0,3245

Moreover, as would be expected in all cases, the skewness is negative. In the presented example, the mode criterion was adopted as the result of the performed EEL procedure. The lifetime $\breve{T}_{mo}$ obtained according to this criterion has a Weibull distribution with a shape parameter of $8{,}998$ and a scale parameter of 3945, i.e., $\breve{T}_{mo} \sim wbl(8{,}998;\, 3945)$. For the obtained lifetime $\breve{T}_{mo}$, graphs of the reliability function (Fig. 11) and the failure rate (Fig. 12) are prepared.

Potential lifetimes for selected failure probabilities, i.e., for $p = 0{,}01; 0{,}05; 0{,}1$ and $0{,}9$ are listed in the table 8. This information is very important in planning inspections of newly manufactured technical devices.

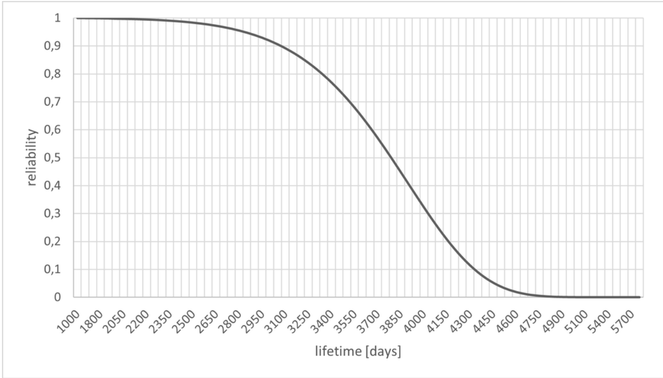


Fig. 11. Graph of the obtained reliability function

Using the formula (38), the failure rate function was determined (39) and then its graph was prepared (Fig. 12).

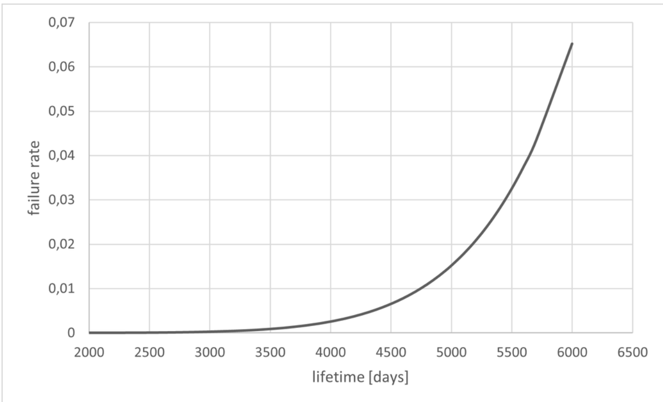
Table 8. List of the potential lifetimes

p	0,01	0,05	0,1	0,9
t_p [days]	2366	2836	3072	4328

$$\lambda_{wbl(\beta;\eta)}(t) = \frac{\beta}{\eta^{\beta}} t^{\beta-1} \qquad (38)$$

$$\lambda_{wbl(8{,}998;3945)}(t) = (3{,}95289\text{E}-32)\, t^{7{,}998},\ t > 0 \qquad (39)$$

Fig. 12. Graph of the predicted failure rate



Note that from the predicted failure rate obtained using the EEL procedure up to 4050 days of use of the TD in question, its failure rate will be less than $0{,}003$. Of course, the final verification of the obtained results will take place in the process of using the tested technical devices.

7. Summary, conclusions, and orientations for future work

Maintenance of machinery and technical equipment under conditions of uncertainty requires the use of probabilistic prediction models in the form of lifetime distributions. Estimation of the parameters of these distributions is carried out with the use of statistical methods based on data about real life realizations of these objects. However, in cases when completely new solutions are introduced into exploitation, we do not have such data and the only possible way of estimating the expected lifetime of these objects is the use of expert methods.

The paper proposes a modified method for estimating the probability distribution of the lifetime for new technical equipment based on expert assessments of parameters characterizing the potential lifetime of these objects. For the Weibull distribution, we use three parameters, two of which characterize the distribution and the third one to assess the quality of lifetime prediction by experts.

The innovation and originality of the developed lifetime parameter estimation procedure results from the application of this expert information for a specific lifetime model, instead of historical data. Such an approach to the issue of parameter evaluation has not yet been developed in the reliability theory.

The method is based on a subjective Bayesian approach to the problem of randomness and integrated with models of classical probability theory. A new procedure for expert elicitation of probabilities for any continuous random variable was developed, consisting of eight main steps. The first five steps have been developed based on good practices used in expert assessments of critical infrastructures. The sixth and seventh steps, which aim to objectivize the assessments of individual experts, are fully innovative. We propose that verification of the consistency of the expert judgements should be carried out using a consistency test. The results of this test can be used as a basis for assessing the credibility of the individual experts and for assigning appropriate weights to their opinions in the second part of this step. This will allow the quality of individual elicitation to be considered in the process of aggregating the opinions of different experts. The aggregation process uses the ratings of all the experts, taking into account the weights estimated in the previous step, in order to obtain unambiguous data allowing the estimation of the parameters of the assumed probability distribution, which is a novelty not previously published in the literature.

Verification of the developed model on practical numerical examples for Weibull distribution has shown that the proposed method eliminates the basic limitations of the methods so far known and used in engineering practice. The calculations carried out demonstrated that considering the mode criterion, the standard deviation as well as the coefficients of variation and skewness have intermediate values compared to the other two criteria. Moreover, as would be expected in all cases, the skewness is negative. In the presented example, the mode criterion was adopted as the result of the performed Expert Elicitation of Lifetime procedure.

Further work of the authors will aim to generalize the developed method also to other probability distributions and to integrate this method with Bayesian inference process in operational decision making. This will require, among other things, consideration of economic aspects, and above all of the costs arising from the unreliability of the system under consideration.

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