

Deflection estimation of glued laminated timber beams reinforced with CFRP fibre composites

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Abstract: Due to the increasing use of fibre composites in the construction industry, wooden beams reinforced with composite elements in various forms are becoming a popular solution. In order to repair the existing structure or to increase the load capacity of new elements, various types of reinforcing elements in the form of tapes and bars are introduced into the cross-section. For unconventional methods to have applicational significance, it should be possible to estimate the deflection of the designed structural element (it is necessary to check SLS with the use of non-complex analytical methods. The method used to estimate the deflection of wooden beams reinforced with composites gives good results for elements reinforced with bars and several tapes glued to the lower lamella.

Keywords: timber, reinforcement, fibre composites, stiffens

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Introduction

The interest in the use of fibre composites in construction is systematically growing. Composites are used both to strengthen existing structures as required, and to reinforce new elements. The use of composite bars and tapes for concrete reinforcement is a solution widely presented in the literature. Fibre composites can also be used to reinforce wood, in particular glued laminated timber. Currently, due to its natural origin, wood is a material that is experiencing a renaissance and is often used, especially due to its high aesthetic value. The form of the reinforcement used can be very diverse. In the case of new elements, the possibility of placing the reinforcement inside the section is important. Reinforcement glued into the cross-

section may take the form of: tapes (Yang et al., 2016), bars (Fossetti et al., 2015; Rajczyk & Jończyk, 2018; Yang et al., 2016) or tapes glued vertically to the last lamella (Lu et al., 2015). In order to use a given solution in engineering practice, it is necessary to define methods for estimating the load capacity and stiffness of the designed element. Currently, numerical programs are used to estimate the behavior of various construction elements, but this is laborious and requires good knowledge of the software used. Therefore, uncomplicated analytical methods are still widely used in structural design.

Various analytical methods for the calculation of wooden elements reinforced with composites can be found in the literature. One of the most popular methods for estimating the load capacity is the method based on the balance of forces in the beam cross-section (Borri et al., 2005; Yang et al., 2016, Corradi 2020). Apart from the aforementioned method, there are also other methods of determining the load capacity presented by (Nowak, 2007), which, however, have not gained much popularity or they are relatively new (Ling et al., 2020). Due to the necessity to check two limit states (ULS and SLS) when designing structural elements, apart from the resistance, the stiffness should also be determined. The method for determining the stiffness of composite reinforced elements is presented in (Fossetti et al., 2015).

The aim of the article is to assess the suitability of the method of determining the stiffness of structural elements, presented by (Fossetti et al., 2015), for various methods of strengthening wooden beams with composite reinforcement placed inside the cross-section. The results of analytical calculations of various variants were compared with the results of numerical analyzes presented by (Jończyk, 2019).

1. Materials and methods

1.1. Numerical analysis

This article uses the results of numerical research presented by (Jończyk, 2019). The methodology of numerical research has been discussed only visually on the basis of the article in which the results are used as comparative.

Numerical calculations were made for three variants of wooden beam reinforcement with the use of various forms of reinforcing elements (Fig. 1a). The analysis simulated a four-point bending test according to the scheme shown in Figure 1b. It was assumed that all reinforcement elements are made of CFRP (Carbon Fibre Reinforced Polymer) fibrous carbon composites and have the same cross-sectional area.

Beam models have been implemented in ANSYS as spatial elements. The connections between the reinforcing elements and the wood are modelled as rigid (line contact). The wood was modelled as an orthotropic material, assuming different material properties for the three considered directions (longitudinal, radial and tangential). The material properties were adopted as for uniform glued laminated timber of GL24h class. On the basis of PN-EN 14080: 2013-07, the value of Young's modulus for the longitudinal direction was determined, while the remaining values of the module were calculated on the basis of the dependencies given in *Wood Handbook. Wood as an Engineering Material* (2010). Material data for composite materials was taken from the article (Nowak et al., 2013). The material data used for the numerical simulations are presented in Table 1. The analysis was carried out in the linear range.



Fig. 1. Assumptions adopted for numerical simulations: a) analyzed cross-sections, b) static diagram adopted for numerical models (based on Jończyk, 2019)

 Table 1. Material data of wood and composite materials, adopted for numerical analysis (Jończyk, 2019)

	Young modulus			Kirchhoff modulus			Poisson ratio		
Materials	GPa			GPa			_		
	E_x	E_y	E_z	G_{xy}	G_{xz}	G_{yz}	v_{xy}	v_{xz}	v_{yz}
Timber	11.5	0.897	0.494	0.736	0.701	0.034	0.372	0.467	0.435
CFRP	165	10	10	5	5	0.5	0.3	0.3	0.03

1.2. Analytical calculations

Analytical calculations of the stiffness of the reinforced beams were performed on the basis of the methodology presented in (Fossetti et al., 2015). The methodology presented below is used to estimate the stiffness in the linear range, which is analogous to the methodology used to conduct numerical simulations (Jończyk, 2019).

The moment of inertia of a wooden beam [m⁴] is calculated according to the formula:

$$I_t = \frac{bh^3}{12} \tag{1}$$

where:

b – beam width [m], h – beam height [m].

The reinforcement position factor and the reinforcement ratio are calculated on the basis of the following formulas:

$$\lambda = \frac{c}{h} \tag{2}$$

$$\omega = \frac{A_r}{bh} \frac{E_r}{E_0} \tag{3}$$

where:

- λ reinforcement position factor,
- c distance from the centre of gravity of the reinforcement to the bottom of the beam [m],
- ω reinforcement factor,
- E_r Young's modulus of reinforcement [Pa],
- A_r reinforcement cross-sectional area [m²],
- E_0 Young's modulus of wood [Pa].

Using the coefficients calculated on the basis of formulas (2) and (3), it is possible to determine the position of the neutral axis in the reinforced element (formula (4)), and then the moment of inertia of the reinforced section (formula (5)).

$$x_{G.r} = \frac{h}{2} \frac{1+2\omega\lambda}{1+\omega} \tag{4}$$

$$I_{t,r} = I_t + bh \left(\frac{h}{2} - x_{G,r}\right)^2 + A_r \frac{E_r}{E_0} (x_{G,r} - c)^2$$
(5)

By dividing the formula (5) by I_t one can obtain the inertia coefficient:

$$\beta_r = 1 + \frac{3(1-2\lambda)^2}{\beta} \frac{\xi\omega}{\xi+\omega} \tag{6}$$

where: $\xi = n - (n - 1) \gamma$, $n = E_1/E_0$, $\gamma = h_1/h$,

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 E_1, E_0 – Young's modulus of different class of wood in case of heterogenic glulam [Pa],

 h_1 , h_2 - heights of different class of wood in case of heterogenic glulam [m].

The stiffness of the reinforced element can be calculated from the formula:

$$\rho_{t,r} = \beta_r \beta \rho_0 \tag{7}$$

assuming $\beta = 1$ for the used glued laminated timber, and the stiffness of the element without reinforcement ρ_0 according to formula (8).

$$\rho_0 = E_0 I_t \tag{8}$$

The first component of the deflection determines the bending effect:

$$f_{m.3} = \frac{1}{48} \frac{FL^3}{\rho_{t.r}} \left(1 - \frac{3}{2} \zeta^2 + \frac{1}{2} \zeta^3 \right) \tag{9}$$

taking into account the static diagram based on formula (10):

$$\zeta = \frac{L_1}{L} \tag{10}$$

where:

F – loading force [N], L – beam length [m],

 L_1 – spacing between supports [m].

However, the second component of the deflection takes into account the influence of the shear forces:

$$f_{m.3.T} = \chi \frac{F}{4} \frac{L}{GA} (1 - \zeta)$$
(11)

where:

 $\chi = 1.2$ (constant for rectangular sections), GA – product of shear modulus and cross section area [N].

The total deflections can be determined on the basis of the formula:

$$f_m = f_{m.3} + f_{m.3.T} \tag{12}$$

2. Results and discussion

On the basis of the presented algorithm, analytical calculations of the deflection values for the three types of reinforcement used in the cross-section were carried out, according to the scheme presented in Figure 1a. The comparison of the deflection values determined in numerical simulations (Jończyk, 2019) with the values determined in an analytical manner is shown in Figures 2-4.

Deflections determined analytically for beams reinforced with three bars and two tapes glued to the last lamella have similar values and in the linear range show a similar nature of the work of the reinforced elements (Figs. 2, 4). The deflection value of a beam reinforced with one tape glued between the last and penultimate lamella significantly differs from the value determined in numerical simulations (Fig. 3).



Fig. 2. Deflection vs. load diagram for a three-bar reinforced beam (own study)



Fig. 3. Deflection vs. load diagram for a beam reinforced with one tape (own study)



Fig. 4. Deflection vs. load diagram for a beam reinforced with two tapes glued to the last lamella (*own study*)

Conclusions

The article calculates the deflections of wooden beams reinforced with fiber composites. Three cases were analyzed: reinforcement with three bars between the last and penultimate lamella, reinforcement with one strip between the last and penultimate lamella and gluing two strips vertically into the last lamella. The performed analytical calculations were compared with the values obtained in numerical simulations (Jończyk, 2019).

The deflection calculation method presented by (Fossetti et al., 2015) obtains deflection values comparable to the results of numerical simulations for beams reinforced with several composite elements. In the case of using one belt, the analytical and numerical deflection values differ significantly. The above situation may suggest that the presented methodology is adequate to the elements in which the lamellas cooperate with each other through most of the wooden surface. However, the nature of the work of the beam reinforced with the tape, which is located between the lamellas, may be different. Most likely, this is due to differences in adhesion between the wood and the various reinforcing elements (tapes, bars). The presented results are only theoretical analyzes and appropriate experimental studies should be carried out in order to verify them correctly.

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