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# Average course approximation of measured subsidence and inclinations of mining area by smooth splines



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## ABSTRACT

The results of marking average courses of subsidence measured on the points of measuring line no. 1 of the “Budryk” Hard Coal Mine, set approximately perpendicularly to a face run of four consecutively mined longwalls in coal bed 338/2 have been presented in the article. Smooth splines were used to approximate the average course of measured subsidence after subsequent exploitation stages. The minimising of the sum of the squared differences between the average and forecasted subsidence, using J. Bialek’s formula, was used as a selection criterion of parameter values of smoothing an approximating function. The parameter values of this formula have been chosen in order to match forecasted subsidence with measured ones. The average values of inclinations have been calculated on the basis of approximated values of observed subsidence. It has been shown that by doing this the average values of extreme measured inclinations can be obtained in almost the same way as extreme observed inclinations. It is not necessary to divide the whole profile of a subsidence basin into parts. The obtained values of variability coefficients of a random scattering for subsidence and inclinations are smaller than their values which occur in the literature.

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## 1. Introduction

The random scattering (fluctuation) of the values of the indicators of mining area deformation measured in observation lines leads to their courses being irregular. This is mainly the result of random cracks of the surface rock mass layer (Kowalski & Jędrzejec, 2015).

The determination of the average course of deformation indicators is significant because it is one of the factors that facilitates the evaluation of the credibility of forecast impacts (Orwat & Mielimąka, 2016a; Orwat, 2016a). This determination may be conducted via the use of different mathematical functions, which allow the average course of a given deformation indicator to be obtained.

In their article Stoch, Niedojadło, Sopata and Moskała (2014) suggest the use of orthogonal polynomials to determine the average course of measured vertical dislocations of an area and its

inclinations. Their use, however, is related to difficulties in determining the optimal degree of a polynomial, not always satisfactory approximation results as well as the necessity to divide a subsidence trough profile into two slopes.

The authors of this article used smooth splines to obtain the average courses of measured subsidence and inclinations indirectly. They demonstrated the approximation procedure and compared the average measured values of these deformation indicators with their courses forecasted using J. Bialek’s formulas (Bialek, 1991). The subsidence and inclinations observed in line no. 1 of the “Budryk” Hard Coal Mine after the termination of the subsequent exploitation stages in coal bed 338/2 have been used as research material.

## 2. Material and methods

The results of the surveys, run on observation line no. 1, originate from the years 1994–1996, when the “Budryk” Coal Mine was running (in rock mass which was previously intact) the exploitation of coal bed 338/2 with four longwall workings marked as 001, 002, 005 and 007 (Fig. 1) (Mielimąka, 2009).

In the exploitation area, coal bed 338/2 was located at a depth of 580 m in the north (longwall 001) to 700 m in the south (longwall

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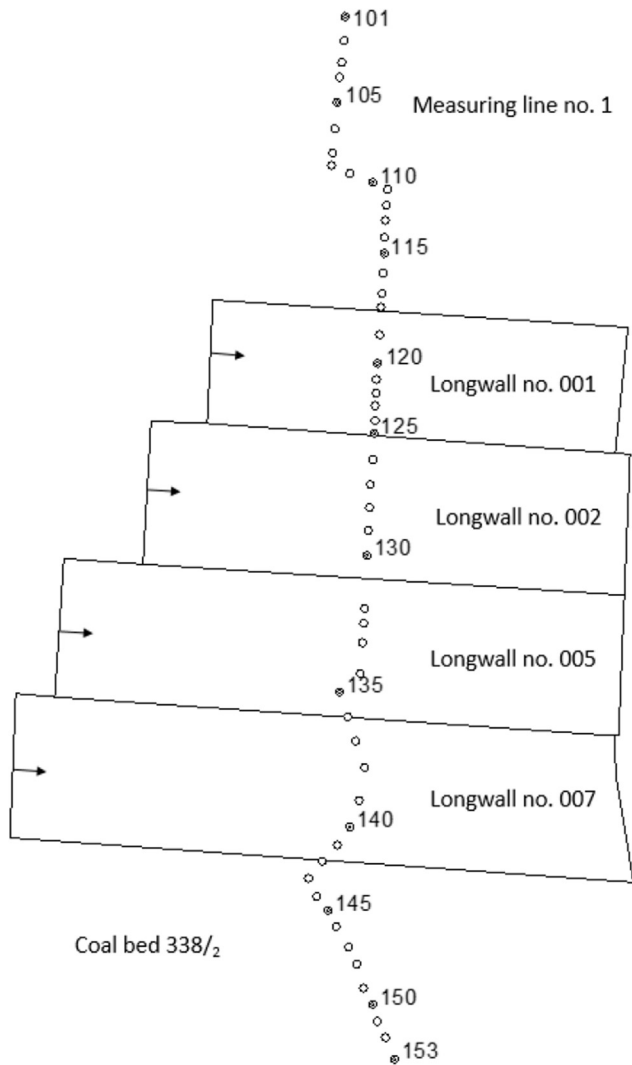


Fig. 1. Shape and position of measuring line towards the exploitation edges in coal bed 338/2 (Mielimąka, 2009).

007) and its inclination angle amounted to 7°. It was being exploited at a height of 2 m via a longitudinal system with roof fall. The average length of longwalls oscillated within 250 m, and their face runs: from 750 m (longwall 001) to 1080 m (longwall 007).

The overburden thickness amounted to 60 m and it was mainly created by Quaternary formations (approx. 50 m) and Triassic ones (approx. 10 m).

The impact of the exploitation of longwalls 001, 002, 005 and 007 in coal bed 338/2 were observed in measuring line no. 1 which was set perpendicularly to their face runs. It consisted of 53 points with an average distance from each other of 37 m.

The measurements of the points' heights and the segments' lengths were conducted in measuring line no. 1, at different time intervals, of between one to four months. Four observation cycles, which demonstrate almost static subsidence troughs, were designated. They were formed after the termination of subsequent exploitation stages in coal bed 338/2.

The approximation of the average course of mining area subsidence measured after the termination of exploitation in the subsequent longwalls marked as 001, 002, 005 and 007 was conducted using the *R* – *project* computer programme, with the use of *smooth.spline* function. The most significant arguments of this function comprise (Lis, 2011):

- *df* – the number of the degree of freedom assuming values from 0 to *n*, where *n* is the number of measuring points;
- *spar* – smoothing parameter of an approximating function assuming values from 0 to 1 where value 0 denotes a total lack of smoothing – then an approximating function assumes the form of an approximated function;
- *cv* – cross validation, when *cv = truth*, then we deal with *leave-one-out*, if *cv = falsehood*, then we deal with general cross validation – *GCV*;
- *nknots* – number of knots, if *all.knots = falsehood*, a number smaller than *n*, where *n* is the number of measuring points.

The approximation of subsidence was conducted for different values of the smoothing parameter and for general cross validation. In such a case the remaining parameters are automatically determined by the programme.

The approximation best describing the average course of the values of the measured subsidence is the approximation conducted for the value of the smoothing parameter, for which the general cross validation reaches a minimum. It occurs, however, that a general cross validation reaches a minimum of some range of value of a smoothing parameter (Fig. 2) or for a smoothing parameter equalling 0 (then we deal with an interpolation).

The solution in such cases will be the visual selection of a smoothing parameter, however, this is frequently inaccurate and subjective (Fig. 3).

The clear-cut selection of the value of a smoothing parameter guarantees an assumption of the minimalisation criterion of a loss function between approximated and forecasted subsidence values:

$$\sum_{i=1}^n (W_{iapprox} - W_{ifore})^2 = minimum \quad (1)$$

where: *n* – the number of measuring points, *W<sub>iapprox</sub>* – approximated subsidence values in *i*-th point [mm], *W<sub>ifore</sub>* – forecasted subsidence values in *i*-th point [mm].

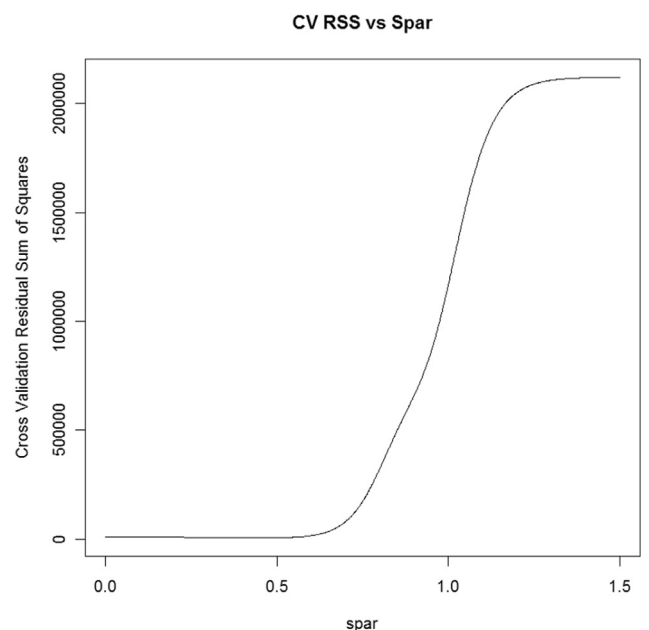
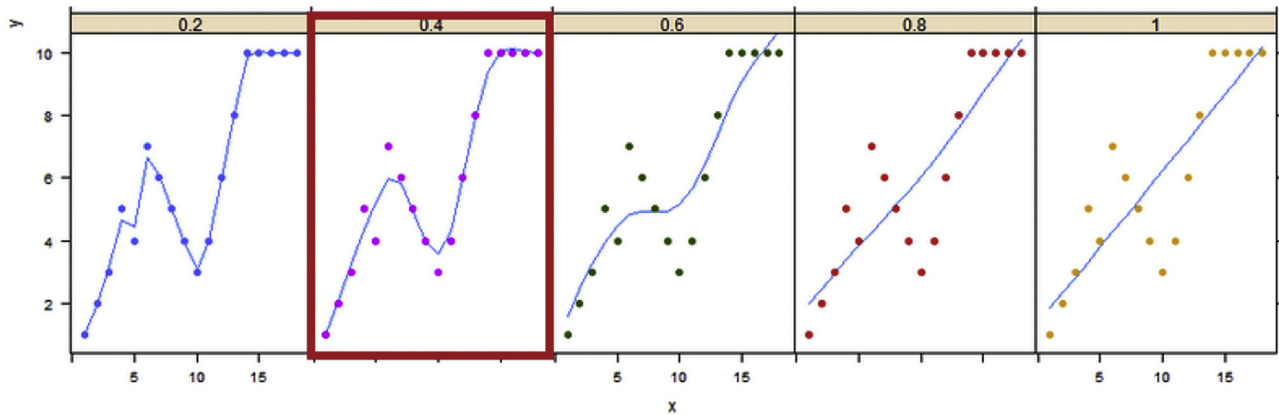
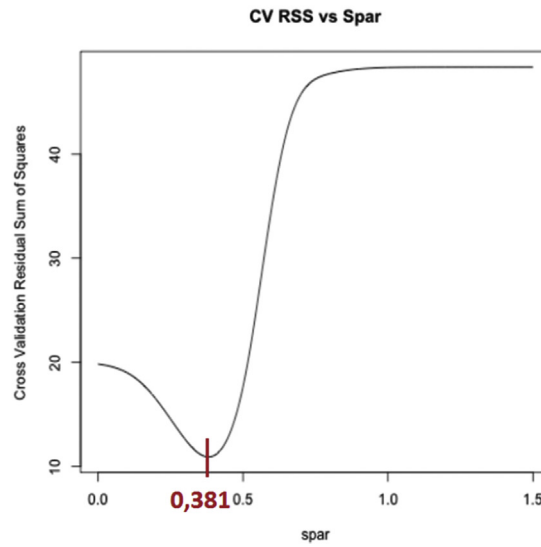


Fig. 2. Cross validation minimum in some range of smoothing parameter (Orwat, 2016b).



a)



b)

Fig. 3. Visual selection of smoothing parameter value (a) and its corresponding analytical value (b) (Orwat, 2016b).

In addition, the assumed approximation criterion formula (1) enables the average course of measured subsidence which is more approximated to the model course to be obtained (Orwat, 2016b).

The method for determining the average course of the measured deformation indicator, demonstrated above, has only been used for subsidence. The research of authors of this article indicate that its use in the case of inclinations (particularly in the case of curvatures) enables extreme average values significantly smaller than extreme measured values to be obtained.

For this reason the average course of measured inclinations was obtained from the difference of subsidence approximated in neighbouring points divided by the length measured between these points:

$$T_{i-1,i}^k = \frac{W_{iapprox}^k - W_{i-1approx}^k}{L_{i-1,i}^k} \quad (2)$$

where:  $k$  – measuring cycle,  $i$  – measuring point,  $T_{approx}$  – average value of inclination [mm/m],  $W_{approx}$  – average value of subsidence [mm],  $L$  – measured length of segment [m].

### 3. Results

The approximation of measured subsidence, in accordance with the procedure presented, requires the reforecasting of the subsidence in the points of the measuring line. This reforecasting was done via the EDBJ programme (Bialek, 2003) by the use of formulas taking into account the existence of an exploitation margin and far impacts. The theory behind the parameter values of selected impacts was determined via the TGB programme. This programme calculates parameter values:  $a$ ,  $tg\beta$  and  $A_{obr}$  of the J. Bialek subsidence formula (Bialek, 1991), based on the data about the geometry of excavations (a free shape) and the location of measuring points. The selection criterion of these parameters' values is the minimum of the residual sum of the squares between measured and reforecasted subsidence (Bialek & Mielimąka, 2001). In Table 1 the determined parameters values, values of correlation coefficients  $R$  between measured and reforecasted profiles of subsidence troughs and standard deviations  $\sigma_W$  between these subsidence were juxtaposed.

The approximation of the subsidence measured in line no. 1 after subsequent stages of exploitation was conducted many times.

**Table 1**

Results of matching reforecasted subsidence to subsidence measured in line no. 1 after the subsequent stages of exploitation.

Exploitation range	<i>a</i>	<i>tgβ</i>	<i>A<sub>obr</sub></i>	<i>R</i>	<i>σ<sub>w</sub></i> [mm]
On longwall 001	0.625	2.567	0.167	0.9946	18.9
On longwalls 001 and 002	0.759	2.833	0.183	0.9974	35.3
On longwalls 001, 002 and 005	0.795	2.833	0.183	0.9950	55.9
On longwalls 001, 002, 005 and 007	0.801	2.467	0.167	0.9899	85.9

**Table 2**

Values of approximation parameters carried out with reference to the forecasted subsidence by J. Bialek's formulas.

Exploitation range	<i>spar</i>	<i>GCV</i>	<i>PC</i> [mm <sup>2</sup> ]
On longwall 001	0.37	75.56	685.50
On longwalls 001 and 002	0.40	245.75	2862.79
On longwalls 001, 002 and 005	0.46	604.967	12 348.94
On longwalls 001, 002, 005 and 007	0.55	1541.094	42 512.81

It was done according to the assumptions of formula (1) which allowed for the determination of the optimal values of smoothing parameters.

The values of the smoothing parameters obtained in this way, as well as the values of general cross validation and the residual sum of squares (penalized criterion – *PC*) between measured and approximated values of subsidence are shown in Table 2.

The above table indicates that the smoothing parameter value of the function approximating the average courses of measured mining area subsidence increases with the increase of the exploitation range: from 0.37 (on the first longwall) to 0.55 (on four longwalls). This may be related to the increase of random factor involvement in the values of measured subsidence with the development of exploitation. Thus, the values of general cross validation increase as well as the residual sums of squares between the average and measured values of subsidence.

The courses of mining area subsidence: measured (continuous line), average (line dash – dot) and forecasted (dotted line), from J. Bialek's formulas, after the termination of given stages of exploitation (on the first longwall – in black, on two longwalls – in blue,

on three longwalls – in green and on four longwalls – in red) are shown in Fig. 4.

Table 3 comprises maximum measured, average and forecasted values of the subsidence of mining area and standard deviations between average and measured subsidence, as well as the coefficients of the variability of subsidence defined as:

$$M_W = \frac{\sigma_W}{|W_{approx}^{max}|} \cdot 100\% \tag{3}$$

where:  $W_{approx}^{max}$  – maximum average value of subsidence [mm],  $\sigma_W$  – standard deviation between average and measured values of subsidence [mm].

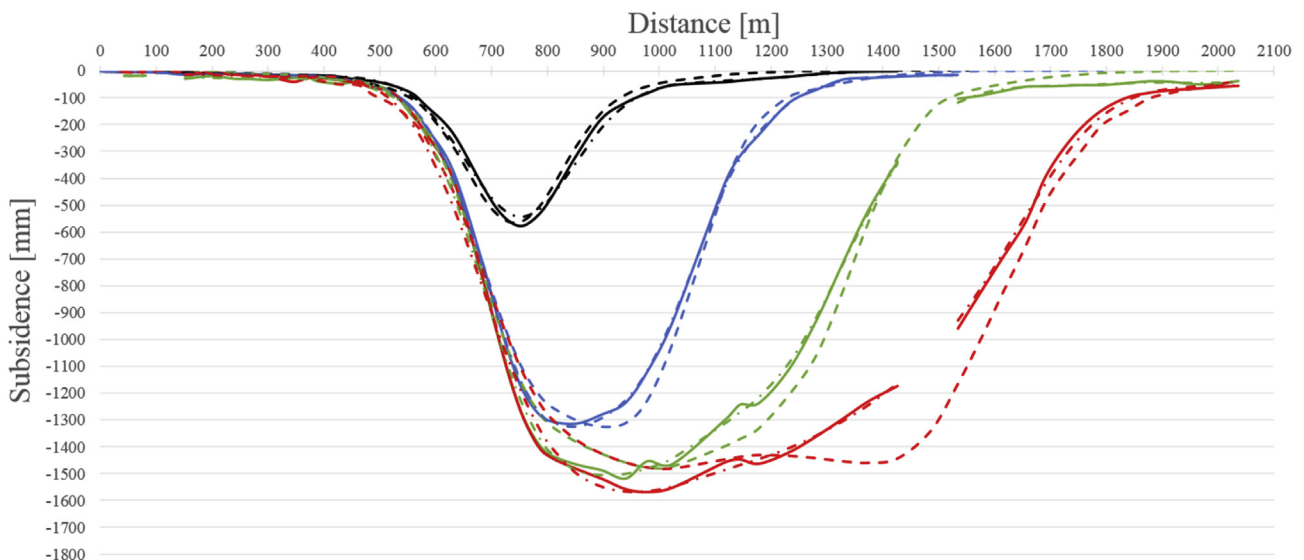
Average values of inclinations were calculated from formula (2) as a quotient of differences of the average approximations of the values of the measured subsidence in neighbouring observation points and the length of segments between these points measured in given cycles. It must be emphasised that in order to obtain maximum values of average curvatures approximated with the measured ones, calculations must be performed on the basis of the average values of measured inclinations (Orwat, & Mielimąka, 2016b; Mielimąka, & Orwat, 2016).

Fig. 5 presents the courses of mining area inclinations: measured (continuous line), average (line dash – dot) and forecasted (dotted line), using J. Bialek's formulas, after the termination of the exploitation in the subsequent longwalls (of the first longwall – in black, two longwalls – in blue, three longwalls – in green colour and four longwalls – in red).

Table 4 juxtaposes the values: maximum measured, average and forecasted mining area inclinations as well as standard deviations between average and measured inclinations  $\sigma_T$ , as well as coefficients of their variability  $M_T$ .

#### 4. Discussion

Based on Fig. 4 and Table 3 it may be stated that the course of the average values of subsidence describe measured subsidence. This is confirmed by the small values of the standard deviations between average and measured subsidence (after the first and second longwall) and their maximum average values approximate to the maximum measured subsidence.

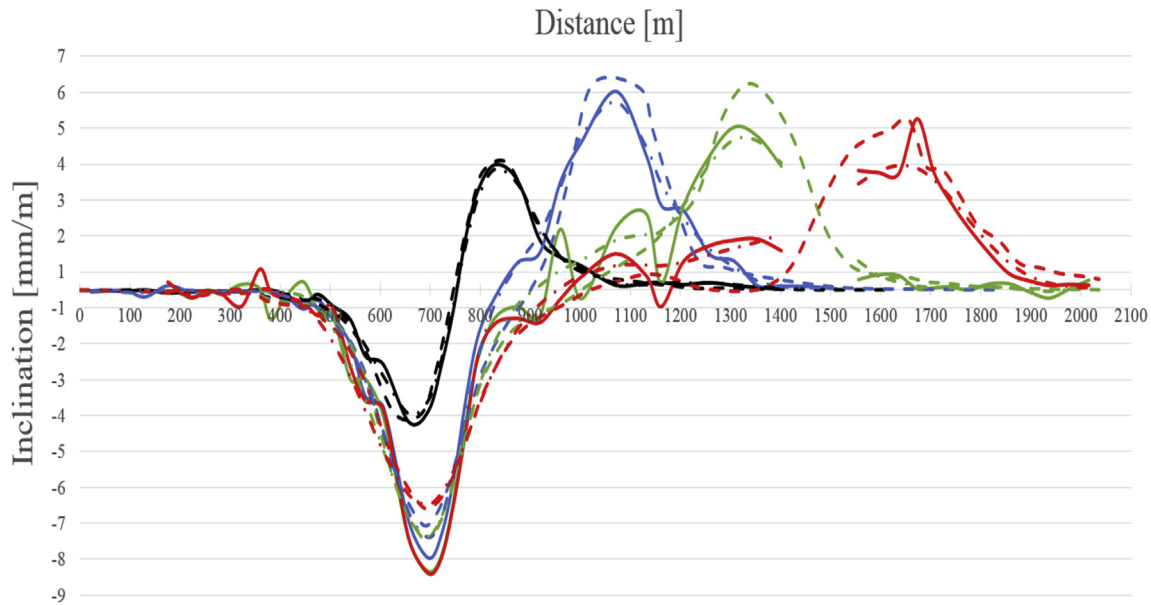


**Fig. 4.** Courses of measured, average and forecasted subsidence, from J. Bialek's formulas.

**Table 3**

Maximum values of measured, approximated and forecasted subsidence, their standard deviations and coefficients of variability.

Exploitation range	$W_{meas}^{max}$ [mm]	$W_{approx}^{max}$ [mm]	$W_{fore}^{max}$ [mm]	$\sigma_W$ [mm]	$M_W$ [%]
On longwall 001	-575	-563.86	-564	4.30	0.76
On longwalls 001 and 002	-1315	-1326.10	-1324	8.57	0.65
On longwalls 001, 002 and 005	-1518	-1505.74	-1478	15.88	1.05
On longwalls 001, 002, 005 and 007	-1568	-1567.82	-1479	29.46	1.88
Average	-	-	-	<b>14.55</b>	<b>1.09</b>

**Fig. 5.** Courses of measured, average and forecasted inclinations using J. Bialek's formulas.**Table 4**

Maximum values of measured, average and forecasted inclinations, their standard deviations and coefficients of variability.

Exploitation range	$T_{meas}^{max}$ [mm/m]	$T_{approx}^{max}$ [mm/m]	$T_{fore}^{max}$ [mm/m]	$\sigma_T$ [mm/m]	$M_T$ [%]
On longwall 001	-3.68	-3.47	-3.49	0.14	3.97
On longwalls 001 and 002	-7.48	-6.90	-6.53	0.28	4.04
On longwalls 001, 002 and 005	-7.85	-6.86	-6.89	0.53	7.72
On longwalls 001, 002, 005 and 007	-7.92	-5.98	-6.04	0.68	11.30
<b>Average</b>	-	-	-	<b>0.41</b>	<b>6.76</b>

The value of standard deviations  $\sigma_W$  and coefficients of variability  $M_W$ , obtained after subsequent stages of exploitation, increased with the shaping of the full basin. The maximum value of the coefficient of variability  $M_W = 1.88\%$  and the maximum value of standard deviation  $\sigma_W = 29.46$  mm were determined for subsidence after the exploitation of four longwalls. It follows that the participation of a random factor in the measured values of subsidence increases with the development of mining exploitation.

The determined average value of the variability coefficient of subsidence  $M_W = 1.09\%$  is significantly smaller than the ones determined by A. Kowalski at 2.90% (Kowalski, 2007).

As Fig. 5 and Table 4 indicate, the maximum average values of inclinations obtained during the procedure are smaller than their measured values. This is due to the adopted type of approximation (the least squares method). The differences increase with the exploitation of subsequent longwalls from 0.21 mm/m after the first longwall to 1.94 mm/m after four longwalls.

The variability coefficients of inclinations  $M_T$  increase after the

exploiting of subsequent longwalls from 3.97% after the first longwall to 11.30% after four longwalls. Their average value equals  $M_T = 6.76\%$  and is smaller than the one given by A. Kowalski: 9.30% (Kowalski, 2007).

## 5. Conclusions

The method of determining the average course of inclinations measured in the segments of measuring line no. 1 of the "Budryk" Hard Coal Mine after the subsequent exploitation of four longwalls was demonstrated in this article. Smooth splines were used for this, which approximated the average course of subsidence measured after subsequent stages of exploitation. The minimising of the sum of the squared differences between average and forecasted subsidence, via J. Bialek's subsidence formula, was used as a selection criterion for the value of the smoothing parameter of the approximating function. The average values of measured inclinations were calculated on the basis of approximated values of measured subsidence.

The determinations of the average courses of measured subsidence and inclinations allow the following conclusions to be made:

- The approximation of the average course of measured mining area subsidence, by means of smooth splines, provides good results, provided that the coefficient values of smooth splines are determined based on a minimalisation criterion of standard deviation between average measured and reforecasted subsidence.
- Both in the case of subsidence and inclinations, the values of standard deviations, obtained after subsequent stages of exploitation, between the average and measured values of deformation indicators and the values of the variability of these indicators increase with the development of mining exploitation. This is the result of the increase of the participation of a random factor in the measured values of deformation indicators when approaching a full basin.
- The average values of the coefficient of the variability determined for subsidence and inclinations are significantly smaller than the ones given by A. Kowalski. It must be emphasised that the measurement results demonstrated the impacts of exploitation run in the intact rock mass.
- The proposed method of obtaining the average values of measured inclinations by the use of smooth splines does not require the splitting of the whole profile of the subsidence trough into parts, which is necessary in the case of orthogonal polynomials (Stoch et al., 2014).

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