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# THE LEVEL OF FRICTION FORCE REDUCTION IN SLIDING MOTION VERSUS THE AMPLITUDE AND FREQUENCY OF TANGENTIAL VIBRATIONS

## POZIOM REDUKCJI SIŁY TARCIA W RUCHU ŚLIZGOWYM W FUNKCJI AMPLITUDY I CZĘSTOTLIWOŚCI DRGAŃ STYCZNYCH

**Key words:**

reduction of friction force, vibrations

**Słowa kluczowe:**

redukcja siły tarcia, drgania

**Abstract**

This work presents the results of comparative analyses of the influence of tangential longitudinal and tangential transverse vibrations on friction force in sliding motion. The analyses were conducted in the function of two principal vibrations parameters such as the frequency $f$ and amplitude $u_0$. It has been demonstrated that, during longitudinal vibrations, the amplitude $v_a$ of their velocity constitutes a key parameter on whose magnitude the initiation of reduction in friction force depends. However, the level of the reduction cannot be determined unequivocally based on the magnitude of this parameter alone.

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Analyses were carried out in the Matlab-Simulink environment with the use of our own original models and computational procedures utilizing Dahl's, Dupont's and LuGre dynamic friction models for the description of friction force. The results of simulating analyses were experimentally verified.

## INTRODUCTION

The phenomenon of friction force reduction in sliding motion has constituted the subject of interest of scientists at numerous global research centres, mainly due to its utilitarian significance. Until now, however, this phenomenon has not been comprehensively explored nor described, while published results of experimental investigations and simulating analyses are predominantly fragmented and focused on a narrow range of the variability of analysed parameters and, moreover, are frequently conflicting. In addition, no versatile model is yet available facilitating the conduct of simulating analyses for optional spatial orientation of excited vibrations in relation to the direction of sliding motion.

This work addresses the level of the reduction of friction forces in sliding motion during tangential vibrations of the support in relation to their direction, frequency $f$, and amplitude $u_0$. Simulating analyses of friction force reduction as a function of these parameters were carried out in the Matlab-Simulink environment with the use of original computational models described in detail in works **[L. 1–3]**. The models are based on dynamic motion equations of a shifting body. Dahl's **[L. 4]**, Dupont's **[L. 5]**, and LuGre **[L. 6]** dynamic friction models were utilized for the description of friction force. Experimental investigations were conducted using our original test rigs described in **[L. 1, 2, 7]**. An excellent fit of simulating analyses with experimental results has been achieved.

This work confirmed the fact, already known from our own earlier work and that of other authors, that in longitudinal vibrations the amplitude $v_a$ of vibrations velocity is the key parameter on whose magnitude the ability of reducing the friction force depends. For this phenomenon to occur, $v_a$ must be greater than the sliding velocity. This limitation does not exist in transverse vibrations. It has also been shown that, although the magnitude of vibrations velocity $v_a$ in relation to drive velocity is the dominant factor regarding occurrence of friction force reduction in longitudinal vibrations, it cannot be used for controlling this force, because, at the identical magnitude of $v_a$ available under various sets of vibrations parameters $f$ and $u_0$, the level of friction force reduction is not the same.

## SIMULATING ANALYSES

As highlighted above, simulating analyses carried out by the current authors and other researchers clearly indicated that, in sliding motion occurring in the presence of tangential longitudinal vibrations of support, the magnitude of a dimensionless coefficient $\gamma$(the quotient of amplitude of vibrations velocity $v_a$ and sliding velocity $v_n$) constitutes a decisive factor determining the possibility of reducing the friction force. For such a reduction to occur, the magnitude of this factor must be greater than 1, as expressed in the following formula:

$$\gamma = \frac{v_a}{v_n} > 1 \tag{1}$$

If the above condition is not satisfied, the reduction of friction force will not occur. At transverse vibrations, no such limitation exists. The influence of vibrations on friction force in sliding motion is typically illustrated by graphs presenting the variability of the driving force $F_n$ as a function of coefficient $\gamma$, as defined above. This force is perceived as the average friction force.

In harmonic motion, this is described by the following relationship:

$$u = u_0 \sin(\omega t) \tag{2}$$

The amplitude $v_a$ of vibrations velocity, which appears in the definition of coefficient $\gamma$, is described by the following relationship:

$$v_a = u_0 \cdot \omega = 2\pi u_0 f \tag{3}$$

It is a function of both principal parameters of vibrations: the amplitude $u_0$, as well as frequency $f$. The knowledge of the influence of each of these parameters on the level of the reduction of friction force becomes important information in the analyses of sliding motion that is taking place in the presence of excited vibrations. However, there is no such information in the quantitative sense available in the currently available literature. This void becomes partially filled by the outcomes of simulating analyses and experimental investigations presented in this paper.

Simulating analyses were carried out at the frequency $f$ of excited vibrations of support within the range of 50–6000 Hz and their amplitude $u_0$ in the range of 0.01–2.0 μm. It has been assumed that the mass of the shifted body was $m$ = 0.650 kg, while surface stresses at their contact with support $p_n$ = 0.022 N/mm$^2$. The drive velocity was $v_n$ = 0.5 mm/s. Analyses were carried out both at longitudinal and transverse tangential vibrations. Some results are presented in **Figures 1 – 4**.

**Figure 1** presents the results of simulating analyses on the influence of amplitude $u_0$ of vibrations on driving force $F_n$ which is essential for initiating sliding motion of the body and for sustaining this motion along the support undergoing vibrations at defined frequency *f*, and **Figure 2** presents the results of simulating analyses concerning the influence on the magnitude of this force of vibrations frequency at a precisely determined amplitude $u_0$.

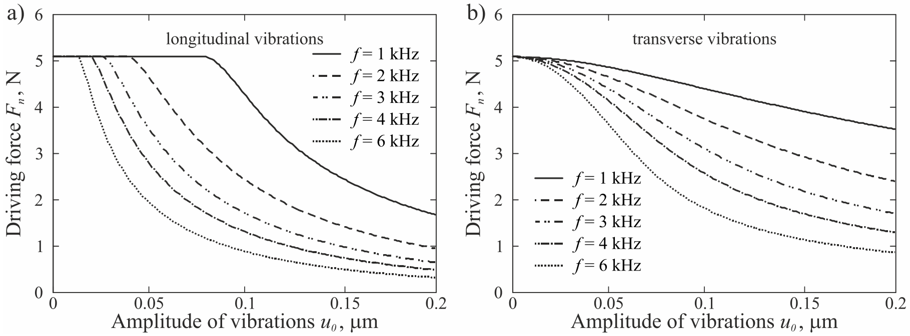


**Fig. 1. Variability of driving force $F_n$ vs. amplitude $u_0$ of vibrations at their different frequency *f*: a) longitudinal vibrations, b) transverse vibrations; $v_n = 0.5$ mm/s, $p_n = 0.022$ N/mm$^2$**

Rys. 1. Zmiana siły napędu $F_n$ w funkcji amplitudy $u_0$ drgań przy różnej ich częstotliwości *f*: a) drgania wzdłużne, b) drgania poprzeczne; $v_n = 0{,}5$ mm/s, $p_n = 0{,}022$ N/mm$^2$

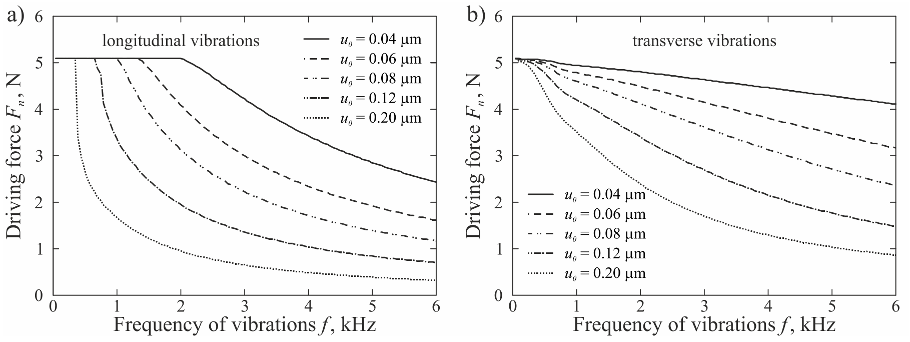


**Fig. 2. Variability of driving force $F_n$ vs. frequency *f* of vibrations at their different amplitude $u_0$: a) longitudinal vibrations, b) transverse vibrations; $v_n = 0.5$ mm/s, $p_n = 0.022$ N/mm$^2$**

Rys. 2. Zmiana siły napędu $F_n$ w funkcji częstotliwości *f* drgań przy różnej ich amplitudzie $u_0$: a) drgania wzdłużne, b) drgania poprzeczne; $v_n = 0{,}5$ mm/s, $p_n = 0{,}022$ N/mm$^2$

As illustrated in **Figs. 1** and **2**, both the amplitude $u_0$, as well as the frequency *f* of support vibrations exert a non-linear influence on the driving force essential for sustaining the motion of shifting body. It is seen that the reduction of this force under longitudinal vibrations occurs only for the magnitude of parameters $u_0$ and $f$ for which the condition $v_a > v_n$ is satisfied. At transverse vibrations, this limitation does not occur. A comparison of graphs for transverse vibrations with those presented for longitudinal vibrations indicate that, upon fulfilling this condition, the reduction of drive force under longitudinal vibrations is much greater than that occurring under transverse vibrations.

Based on the sets of graphs, such as those in **Figs. 1** and **2**, the 3-dimensional (3-D) master curves were prepared presenting the variability of the driving force $F_n$ as a function of the amplitude and frequency of vibrations of the support along which the sliding motion was performed. These 3-D master curves are presented in **Figures 3** and **4**.

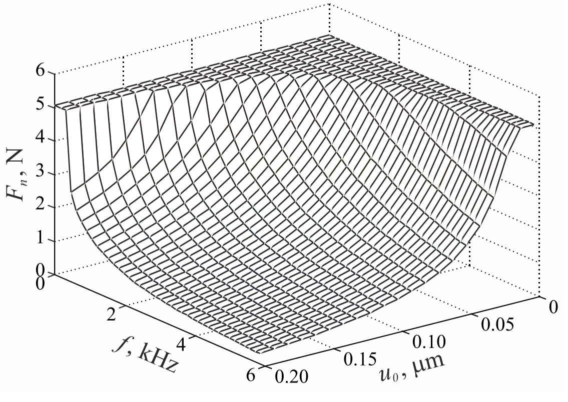


**Fig. 3. Driving force $F_n$ vs. amplitude $u_0$ and frequency $f$ of longitudinal vibrations of the support; $v_n$ = 0.5 mm/s, $p_n$ = 0.022 N/mm$^2$**

Rys. 3. Zależność siły napędu $F_n$ od amplitudy $u_0$ i częstotliwości $f$ drgań przy drganiach wzdłużnych podłoża; $v_n$ = 0,5 mm/s, $p_n$ = 0,022 N/mm$^2$

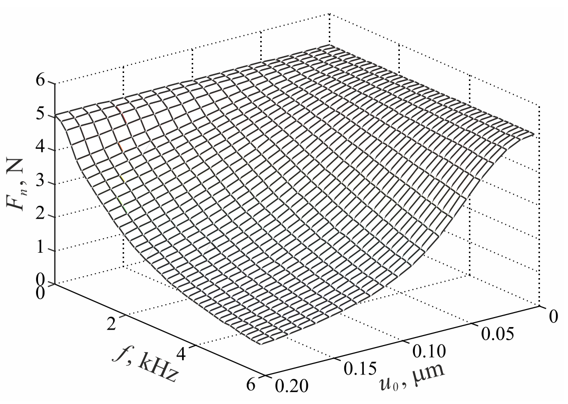


**Fig. 4. Driving force $F_n$ vs. amplitude $u_0$ and frequency $f$ of transverse vibrations of the support; $v_n$ = 0.5 mm/s, $p_n$ = 0.022 N/mm$^2$**

Rys. 4. Zależność siły napędu $F_n$ od amplitudy $u_0$ i częstotliwości $f$ drgań przy drganiach poprzecznych podłoża; $v_n$ = 0,5 mm/s, $p_n$ = 0,022 N/mm$^2$

## EXPERIMENTAL VERIFICATION OF SIMULATING ANALYSES RESULTS

The results of simulating analyses were experimentally verified using test rigs described in works **[L. 1, 2, 7]**. The experiments involved the determination of changes in the driving force that essential for initiating and sustaining sliding motion of a body, which are caused by initiation of the support vibrations. Measurements were conducted under both modes of vibrations, i.e. tangential longitudinal and tangential transverse. These were conducted under two variants of parameters variability of excited vibrations.

The first variant involved measuring the variability of the driving force $F_n$ as a function of amplitude $u_0$ of vibrations at their fixed frequency $f$. The measurements were carried out at various frequencies of excited vibrations in the range of 400–5000 Hz, at the drive velocity $v_n$ = 0.5 mm/s, and surface pressures $p_n$ = 0.022 N/mm$^2$.

The second variant involved measurements of changes in the driving force as a function of vibrations frequency $f$ at selected amplitude $u_0$ in the range of 0.1 – 2.0 µm. The drive velocity and surface pressures were identical with those in the first variant. Examples of experimental results for both variants of measurements are presented in **Figures 5** and **6**, while a comparison of simulating analyses with experimental results is presented in **Figures 7** and **8**.

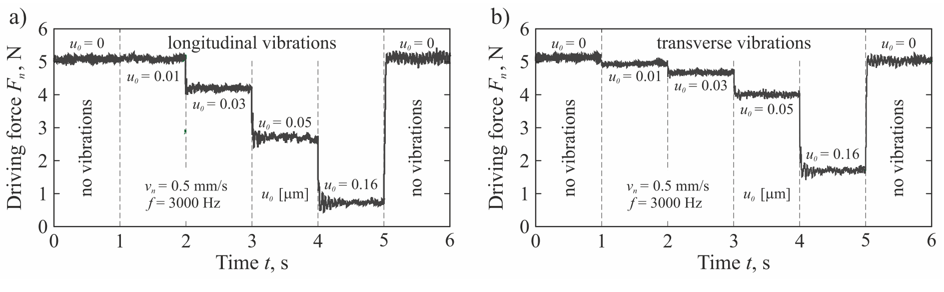


**Fig. 5. Variability of the driving force $F_n$ in relation to the amplitude $u_0$ of support vibrations, at frequency $f$ = 3000 Hz: a) longitudinal vibrations, b) transverse vibrations; $v_n$ = 0.5 mm/s, $p_n$ = 0.022 N/mm$^2$**

Rys. 5. Zmiana siły napędu $F_n$ w zależności od amplitudy $u_0$ drgań przy ich częstotliwości $f$ = 3000 Hz: a) drgania wzdłużne, b) drgania poprzeczne; $v_n$ = 0,5 mm/s, $p_n$ = 0,022 N/mm$^2$

The excellent agreement of these results can be seen, which demonstrates the adequacy of developed computational model. This, in turn, demonstrates that graphs, as presented in **Figures 3** and **4**, can be utilised when controlling the friction force in sliding motion through the use of tangential vibrations.

However, the amplitude $v_a$, of vibrations velocity cannot be used for controlling the friction force. Although the magnitude of it constitutes a criterion for the occurrence of friction force reduction in longitudinal vibrations, the level of consequent reduction at identical magnitudes of $v_a$ is different, in a dependence on vibration parameters $u_0$ and $f$, as illustrated in **Figures 9** and **10**. These Figures present profiles of driving force $F_n$ under predetermined value of $v_a$ ($v_a > v_n$) but under varying magnitudes of frequency $f$ and amplitude $u_0$. These analyses were conducted at $v_n$ = 0.5 mm/s and $v_a$ = 2 mm/s.

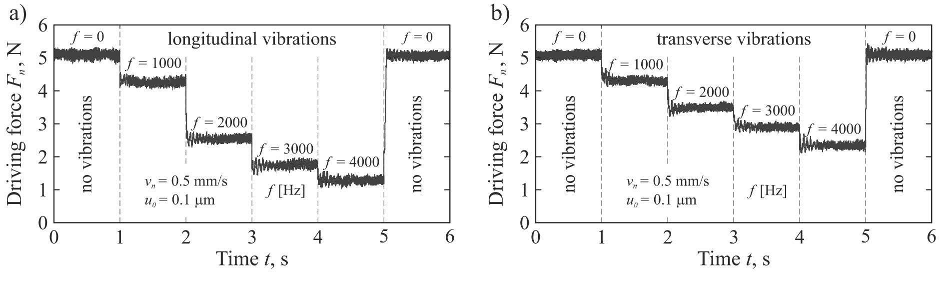


**Fig. 6. Variability of the driving force $F_n$ in relation to frequency $f$, at fixed amplitude $u_0$ = 0.1 µm: a) longitudinal vibrations, b) transverse vibrations; $v_n$ = 0.5 mm/s, $p_n$ = 0.022 N/mm²**

Rys. 6. Zmiana siły napędu $F_n$ w zależności od częstotliwości $f$ drgań przy ich amplitudzie $u_0$ = 0,1 µm: a) drgania wzdłużne, b) drgania poprzeczne; $v_n$ = 0,5 mm/s, $p_n$ = 0,022 N/mm²

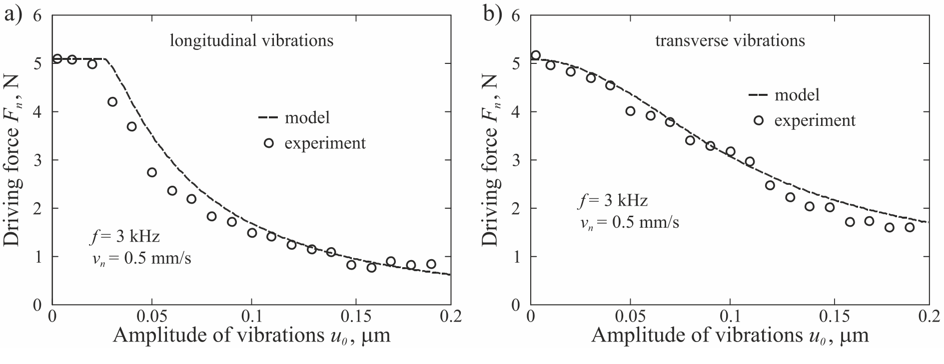


**Fig. 7. Comparison of experimental and simulating analyses results of driving force reduction in relation to the amplitude of vibrations: a) longitudinal vibrations, b) transverse vibrations; $v_n$ = 0.5 mm/s, $p_n$ = 0.022 N/mm²**

Rys. 7. Porównanie wyników badań symulacyjnych i doświadczalnych redukcji siły napędu w zależności od amplitudy drgań: a) drgania wzdłużne, b) drgania poprzeczne; $v_n$ = 0,5 mm/s, $p_n$ = 0,022 N/mm²

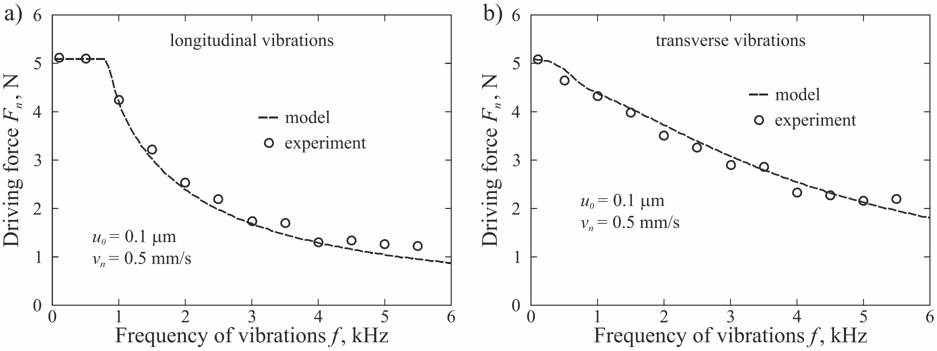


**Fig. 8. Comparison of experimental and simulating analyses results of driving force reduction in relation to frequency of vibrations: a) longitudinal vibrations, b) transverse vibrations; $v_n$ = 0.5 mm/s, $p_n$ = 0.022 N/mm$^2$**

Rys. 8. Porównanie wyników badań symulacyjnych i doświadczalnych redukcji siły napędu w zależności od częstotliwości drgań: a) drgania wzdłużne, b) drgania poprzeczne; $v_n$ = 0,5 mm/s, $p_n$ = 0,022 N/mm$^2$

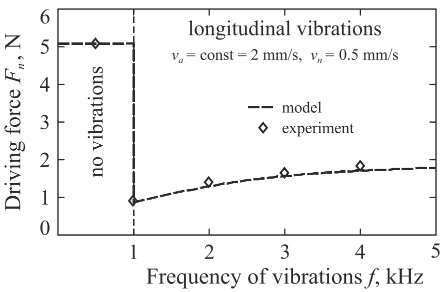


**Fig. 9. Variability of driving force at fixed amplitude of vibration velocity ($v_a$ = constant) and different frequency $f$ of vibrations**

Rys. 9. Zmiana siły napędu przy ustalonej amplitudzie prędkości drgań ($v_a$ = const) i różnej ich częstotliwości

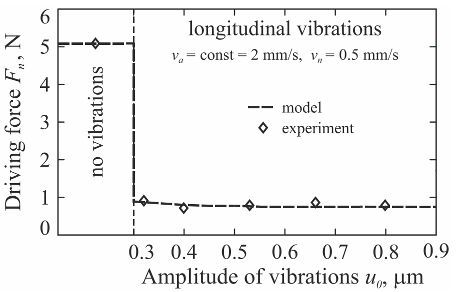


**Fig. 10. Variability of driving force at fixed amplitude of vibration velocity ($v_a$ = constant) and different amplitude $u_0$ of vibrations**

Rys. 10. Zmiana siły napędu przy ustalonej amplitudzie prędkości drgań ($v_a$ = const) i różnej ich amplitudzie

At pre-determined magnitude of the amplitude $v_a$ of vibration velocity, the reduction of friction force will occur upon satisfying the boundary condition $v_a > v_n$ as shown in **Figures 9** and **10**. The level of this reduction depends on the value of vibrations parameters $u_0$ and $f$. Under increasing frequency $f$, however, satisfying the requirement for $v_a$ = constant demands a reduction of amplitude $u_0$. Such action results in lowering the magnitude of the reduction of the drive force as shown in **Figure 9**. Satisfying the requirement for $v_a$ = constant under increasing amplitude of vibrations requires a reduction of vibrations frequency. In this case, an increased magnitude of reduction in the drive force $F_n$ will be attained as shown in **Figure 10**.

## CONCLUSIONS

An excellent agreement of simulating analyses and experimental investigations demonstrates that the computational model utilised in our numerical analyses is adequate and correctly reflects changes in friction forces happening in sliding motion conducted over a vibrating support. This means, in turn, that the model can be utilised for controlling friction force in such motion through adequate selection of vibration parameters of support, such as $u_0$ and $f$, over which the motion is conducted. The magnitude of amplitude $v_a$ of vibrations velocity alone is insufficient for determining the level of friction force reduction in sliding motion.

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## Streszczenie

**W pracy przedstawiono wyniki analiz porównawczych oddziaływania drgań stycznych wzdłużnych i drgań poprzecznych na siłę tarcia w ruchu ślizgowym. Analizy przeprowadzono w funkcji dwóch podstawowych pa-**

**rametrów drgań, jakimi są częstotliwość $f$ i amplituda $u_0$. Wykazano, że przy drganiach wzdłużnych amplituda $v_a$ ich prędkości jest parametrem kluczowym, od wartości którego zależy wystąpienie redukcji siły tarcia. Jednak poziomu tej redukcji nie można określić jednoznacznie tylko na podstawie wartości tego parametru. Analizy przeprowadzono w środowisku Matlab/Simulink, przy wykorzystaniu oryginalnych, własnych modeli i procedur obliczeniowych wykorzystujących do opisu siły tarcia dynamiczne modele tarcia Dahla, Duponta i LuGre. Wyniki analiz symulacyjnych zweryfikowano doświadczalnie.**