

Jerzy JAKUBIEC, Beata KRUPANEK
SILESIA UNIVERSITY OF TECHNOLOGY
10 Akademicka St., Gliwice

Application of delta function to probabilistic modeling of communication delays in wireless networks – introduction and mathematical basis

Abstract

The paper presents the mathematical basis of a new method for building probabilistic models of communication delays in wireless networks in the case if the sent data are not correct and have to be retransmitted. The method is based on using a delta function sequence to describe delays in retransmissions between a transmitter and a receiver [1, 2] under assumption that the access time of the transmitter is taken as random and described by a probability density function. The retransmissions are caused by passive or active external disturbances influencing the communication channel established in the wireless medium [3, 4, 5]. Theoretical considerations are illustrated by examples using both measured and simulated data.

Keywords: delta function, wireless networks, probabilistic model of communication delays.

1. Introduction

In times of common usage of wireless networks for data transmission in measuring and control systems, determination of quality of services provided by such networks is an important problem because the knowledge of transmission channel quality parameters allows adequate design of the system components [6], [7, 8]. From measurement and functional points of view, the essential quality parameters of data transmission in the system are connected with delays which arise in a communication channel during sending a message from a transmitter to a receiver [9].

There are many mathematical means which can be used for description of communication delays – generally, they can be divided into two groups: deterministic and probabilistic. The deterministic dependencies are widely used to calculate maximum delays in networks, for example in task scheduling [10]. Moreover, the deterministic approach is commonly applied in planning assignment of resources in computer systems by using the queuing theory [11] which enables, among other things, calculation of delays in realization of tasks. The application of Petri's nets to analysis of networks [12] can be included in this category of description, too.

Deterministic delay models usually allow obtaining simpler mathematical dependencies than the probabilistic ones. However, nowadays one can observe the increasing role of this second kind of description. One can point out several areas of the probabilistic modeling application in description of network properties. The first one is connected with the analysis of networks aiming at better planning their structure and efficiency [13, 14]. The second one concerns the problems of communication in networks, the analysis of which by using probabilistic description enables improving its parameters such as energy consumption [10], decreasing delays in networks [16] and the like [17, 18, 19]. The third area deals with the constantly increasing applications of wireless networks as communication interfaces in measuring and control systems. Delays in such systems should be described in a probabilistic way because they can cause specific measurement errors for signals varying in time and other errors arising during the transmission of measured data [20]. All measurement errors should be described in probabilistic categories because they are composed with other errors in the process of uncertainty calculation of measurement results [21]. Some of the communications errors are caused by external disturbances [22] which are of random character. Therefore the delays connected with them should be described in the probabilistic way. Moreover,

in the last years, one can observe the increasing number of publications which concern sensor networks, i.e. such a kind of wireless networks which are used for coupling small size measurement instruments [23, 24, 25, 26]. It is specific for these networks that the same probabilistic mathematical means both are applied for analyzing transmission procedures and can be the basis of measurement data processing. Description of the data processing in sensor networks as a stochastic process and use of the Kalman filter to work out both transmission and processing procedures is a characteristic example of this approach [27, 28, 29].

The method described in the paper represents a probabilistic approach to modeling the properties of wireless networks. Its novelty consists in treating the Dirac's delta function as a probability density function describing delays which occur when the wireless transmission is disturbed by external factors [2, 5, 22] and it is necessary to retransmit the data. The delay model obtained in this way is useful for simulative analysis of wireless networks [30, 31].

2. Mathematical basis of the proposed description of delays

2.1. The model of transmission delays between two nodes

Let us consider a simple situation shown in Fig. 1, where **A** and **B** denote the nodes of a wireless network which communicate directly.

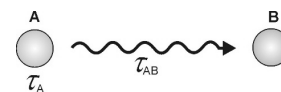


Fig. 1. Sources of delays during transmission from node A to B, τ_A – time necessary to obtain the access to a communication medium, τ_{AB} – time of the message transmission

Let us take then that the transmission of a message from the node **A** to **B** is activated at the moment t_0 and this message is received by the node **B** at moment t_B . The difference between these two moments:

$$\tau_{\text{tot}} = t_B - t_0 \quad (1)$$

is the total time of the transmission and can be interpreted as the communication delay of the message transmitted from **A** to **B**.

One can determine two main sources of partial delays being components of the total delay τ_{tot} . The first one τ_A is associated with activity inside the node **A** and in the considered situation can be qualified as the time necessary to obtain the access to the communication medium. The second component τ_{AB} describes the time necessary to transmit a message from **A** to **B** through the medium. Denoting the moment when **A** gets the access by t_A , the total delay (1) can be written as the sum of partial delays:

$$\tau_{\text{tot}} = t_B - t_A + t_A - t_0 = \tau_{AB} + \tau_A \quad (2)$$

A model of the total delay has to contain all possible values of the delays occurring in the described conditions. If one takes into account that access procedures commonly used in wireless

networks are of random character [32], the total delay τ_{tot} should be described in probabilistic categories, too. To obtain a compact model of delays, there is assumed that all the delays in Eq. (2) are treated as uncorrelated random variables, which enables determining the probability density function of the total delay as:

$$g_{tot}(\tau_{tot}) = g_A(\tau_A) \otimes g_{AB}(\tau_{AB}), \quad (3)$$

where \otimes is the symbol of convolution and $g_A(\tau_A)$ and $g_{AB}(\tau_{AB})$ are the probability density functions of the partial random delays τ_A and τ_{AB} .

Example 1. The total delay τ_{tot} of communication between two ZigBee modules [32] was measured by using the experiment system described in [22]. The distance between the modules was about 1m, the message contained 1 byte of data and there were no disturbances affecting the transmission. The measuring experiment was performed 10 000 times. The obtained results are shown in Fig. 2 in the form of the histogram.

Basing on the experiment results, the hypothesis that the set of the delay measurement results can be described by the normal distribution was tested by using Kolmogorov-Smirnov and χ^2 tests [33]. The effect of the test was positive at the confidence level equal to 0.95. On the basis of the measurement results, the mean value of the delay and its experimental standard deviation were calculated. The obtained values are: $E(\tau_{tot}) = 8.2$ ms and $\sigma = 1$ ms, respectively. Therefore, the total delay distribution can be described by the normal probability density function $N(8.2, 1)$ ms presented in Fig. 2 as the continuous line.

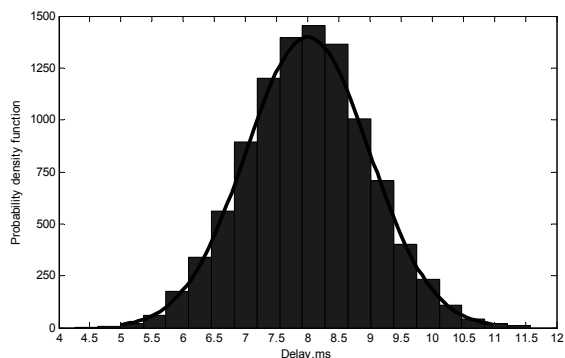


Fig. 2. Histogram of the results of the experiment described in Example 1

The next part of the experiment consisted in determination of the partial delays. As every message contains the same portion of data, the delay caused by its transmission delay is constant and its measured value was $d_{tr} = 2$ ms. Taking Eq. (2) into account, the mean value of the access delay is: $E(\tau_A) = E(\tau_{tot}) - d_{tr} = 8.2 - 2 = 6.2$ ms. Therefore, the experimentally identified distribution of the transmitter access delay can be described as the normal probability density function $g_A(\tau_A) = N(6.2, 1)$ ms and the transmission delay as $\tau_{AB} = d_{tr} = 2$ ms. The knowledge about the delays in the communication channel created between the transmitter A and the receiver B can be written as:

$$\mathbf{Delay}_{AB} \{ N((E(\tau_A), \sigma_A), d_{tr}) \} = \{ N(6.2, 1), 2 \} \text{ ms.} \quad (4)$$

The expression (4) and Eq. (2) form together the model of the total delay in the channel AB. To perform convolution (3), it is necessary to describe the constant delay τ_{AB} in probabilistic categories. Such a description can be obtained by using the Dirac's delta [34] as a probability density function. Assuming b to be a constant value, the delta function for the delay τ can be written as:

$$\delta(\tau - b) = \begin{cases} \infty & \text{for } \tau = b \\ 0 & \text{for } \tau \neq b. \end{cases} \quad (5)$$

Moreover, there is:

$$\int_{-\infty}^{+\infty} \delta(\tau - b) d\tau = 1. \quad (6)$$

For the measured results obtained in Experiment 1, one can describe the probability density function of the transmission delay as $g_{AB}(\tau_{AB}) = \delta(\tau_{AB} - b_{tr})$, where $b_{tr} = 2$ ms.

As it is proved in [22], the convolution of the probability density function $g(\tau)$ with the delta function (5) gives:

$$g(\tau) \otimes \delta(\tau - b) = g(\tau - b), \quad (7)$$

which means that this convolution shifts the function $g(\tau)$ in the horizontal axis by the value b . It allows simplifying transformations by making such displacements instead of performing the convolutions. Basing on Eq. (6), one can write the probability density function of the access delay in the form:

$$g_A(\tau_A) = g_{Apat}(\tau_A - \tau_0), \quad (8)$$

where τ_0 is the expected value of $g_A(\tau_A)$, i.e.: $\tau_0 = E[g_A(\tau_A)]$.

The probability density function $g_{Apat}(\cdot)$, called a pattern, has the same shape as $g_A(\tau_A)$ but its mean value is equal to zero. The results described in Example 1 and shown in Fig. 1 enable obtaining such a pattern for a ZigBee module as the normal probability density function $g_{Apat}(\tau_A) = N(0, 1)$ ms properly truncated because $\tau_A > 0$.

2.2. The delay model of communication with retransmissions

The electromagnetic field being the communication medium in wireless networks is exposed to many different disturbances both of passive and active nature [22]. The passive disturbances are caused by terrain obstacles, walls, etc., while the active ones result from influences having electromagnetic and electrostatic character. Generally, the appearance of disturbances cause that some received message are not correct. Information about this fact is sent back to the transmitter which usually tries to send the message once more, i.e. retransmit it. The number of retransmissions depends on the construction of the wireless module and can be determined by its user [22, 32].

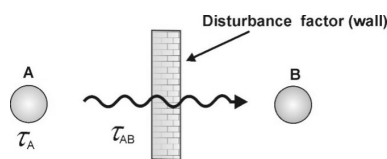


Fig. 3. Transmission between two nodes disturbed by a wall

In Fig. 3 there is shown the situation when two nodes A and B communicate directly but a disturbance, represented in this figure by a wall, causes that some transmissions have to be repeated, i.e. the message must be retransmitted. It means that the partial delay τ_{AB} describing the correct transmission of a message is not constant as it has been considered previously but depends on the number of retransmissions. In the work [22], it has been experimentally proved that in this case τ_{AB} can be represented by the following delta function sequence:

$$g_{AB}(\tau_{AB}) = a_0 \delta(\tau_{AB} - b_0) + a_1 \delta(\tau_{AB} - b_1) + \dots + a_k \delta(\tau_{AB} - b_k) \quad (9)$$

where $\delta(\cdot)$ denotes the delta function defined by (3), k is the number of retransmissions (in practice less than 6 [32]), Both a_0, a_1, \dots, a_k and b_0, b_1, \dots, b_k are constant coefficients with non-negative values.

According to the general assumption, the sequence (9) is treated as a probability density function, therefore, it has to fulfill the following normalizing condition:

$$\int_{-\infty}^{+\infty} g_{AB}(\tau_{AB}) d\tau_{AB} = 1. \quad (10)$$

After introducing Eq. (9) to (10) and using (6), one obtains:

$$\begin{aligned} & \int_{-\infty}^{+\infty} [a_0 \delta(\tau_{AB} - b_0) + a_1 \delta(\tau_{AB} - b_1) + \dots + a_k \delta(\tau_{AB} - b_k)] d\tau_{AB} = \\ & = a_0 \int_{-\infty}^{+\infty} \delta(\tau_{AB} - b_0) d\tau_{AB} + a_1 \int_{-\infty}^{+\infty} \delta(\tau_{AB} - b_1) d\tau_{AB} + \dots + \\ & + a_k \int_{-\infty}^{+\infty} \delta(\tau_{AB} - b_k) d\tau_{AB} = a_0 + a_1 + \dots + a_k = 1. \end{aligned} \quad (11)$$

Taking into account that the coefficients $a_i, i = 0, 1, \dots, k$, are non-negative, it results from Eq. (11) that $0 \leq a_i \leq 1$ and if $i = 0$ then $a_0 = 1$.

To obtain the total communication delay τ_{tot} , the realizations of partial delays τ_A and τ_{AB} are added up accordingly to Eq. (2). It means that if they are treated as uncorrelated random variables, the probability density function of the total delay can be obtained as convolution (3). After introducing Eq. (9) to (3) and taking into account that convolution is a linear transformation, one can write (3) as the sequence of partial convolutions:

$$g_{tot}(\tau_{tot}) = a_0 g_A(\tau_A) \otimes \delta(\tau_{AB} - b_0) + \dots + a_k g_A(\tau_A) \otimes \delta(\tau_{AB} - b_k). \quad (12)$$

Using Eq. (7) to transform (12), one obtains the sequence:

$$g_{tot}(\tau_{tot}) = a_0 g_A(\tau_{tot} - b_0) + a_1 g_A(\tau_{tot} - b_1) + \dots + a_k g_A(\tau_{tot} - b_k) \quad (13)$$

which is the sum of duplicates of the probability density function $g_A(\cdot)$ properly moved in time by the constant values b_0, b_1, \dots, b_k and multiplied by the constant coefficients a_0, a_1, \dots, a_k , which describe the probability of occurrence of the succeeding retransmissions.

Eq. (13) is the probabilistic model of the total communication delay in a situation when disturbances affect the transmission, which causes the necessity of retransmissions. Instead of using the function $g_A(\cdot)$ in this model, it is better to introduce the pattern $g_{Apat}(\cdot)$ described by (8), the expected value of which is equal to 0. In this case the time displacements b_0, b_1, \dots, b_k can be determined in relation to the vertical axis as constant delays $\tau_0, \tau_1, \dots, \tau_k$, which allows calculating them on the basis of measurement results (see Example 3). Taking this into account, one can write the parameters of the model describing the delays of the disturbed communication on the way between the nodes A and B as:

$$\text{Delay}_{AB} \{ g_{Apat}(\cdot), (a_0, \tau_0), (a_1, \tau_1), \dots, (a_k, \tau_k) \}. \quad (14)$$

Example 2. To illustrate the properties of the delay model, let us use the pattern from Example 1 to obtain a histogram of the total delay by using the Monte Carlo method when assuming that:

- disturbances cause only one retransmission, i.e. $k = 1$,
- the pattern $g_{Apat}(\cdot)$ has the normal distribution $N(0, 1)$ ms, i.e. its expected value is equal to 0 and standard deviation is equal to 1 ms,

- $a_0 = 0.75$ which means that 75% transmissions is not repeated, i.e. the probability of the correct first transmission is equal to 0.75,
- the time displacement (delay) of the first transmission $\tau_0 = 13$ ms,
- $a_1 = 0.25$, i.e. 25% messages is retransmitted,
- the time displacement (delay) of the second transmission (retransmission) $\tau_1 = 20$ ms.

The above parameters can be written in the form (14) as:

$$\text{Delay}_{AB} \{ N(0, 1) \text{ ms}, (0.75, 13 \text{ ms}), (0.25, 20 \text{ ms}) \}. \quad (15)$$

The results of the simulation experiment realized in 10^5 steps are presented in Fig. 4 in the form of the histogram.

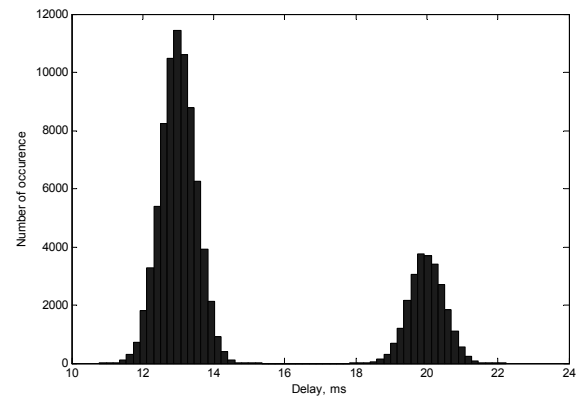


Fig. 4. Exemplary delay histogram obtained for the model (15) by using the Monte Carlo method

The obtained histogram can be treated as a multi-modal distribution describing the total delay. The first mode has the maximum for $\tau_0 = 13$ ms and the second one for $\tau_1 = 20$ ms. Basing on the histogram from Fig. 4, one can say that the model (13) can be generally interpreted as the weighted sum (weights: a_0, a_1, \dots, a_k) of the patterns $g_{Apat}(\cdot)$ displaced in time by $\tau_0, \tau_1, \dots, \tau_k$, respectively. The experimental results of verification of the thesis that this model is a good description of delays in wireless networks exposed to disturbances, one can find in the work [22].

3. Conclusions

The essence of the method of modeling delays in wireless networks presented in the paper consists in using the delta function sequence treated as the probability density function of the delay in a communications chain with retransmissions. The chain must be divided into elementary channels consisting of two nodes, a transmitter and a receiver, communicating directly. The transmission between them can be identified in a measurement way presented in [22] and should be described in probabilistic categories as the delta function sequence. The total delay in a composed chain can be obtained by using convolution of the partial descriptions.

The partial model describing the delay of the transmission between two nodes is relatively simple. Its important feature is the possibility of including the influence of disturbances on the delay. The obtained results allow using the proposed probabilistic model in the analysis of disturbed wireless networks by using simulative programs such as OPNET Modeler [22, 30].

4. References

- [1] Jakubiec J., Krupane B.: Model of communication delays in wireless networks. Problems and progress in metrology. PPM'15, Kościelisko, 07-10 June 2015, pp. 45-48, 2015.

- [2] Krupanek B., Bogacz R.: Applications of wireless transmission model in indoor environment. IMEKO XXI World Congress, Prague, August 30 - September 4, 2015.
- [3] Krupanek B., Bogacz R.: Investigations of transmission delays in ZigBee networks. Prz. Elektrot., nr 1, pp. 70-75, 2014.
- [4] Krupanek B., Bogacz R.: Modelling of communication delays in wireless networks. Advances measurement tools in technical diagnostics for systems' reliability and safety. 13th IMEKO TC10 Workshop on Technical Diagnostics, Warsaw, Poland, June 26-27, 2014. Proceedings, pp. 21-26, 2014.
- [5] Jakubiec J., Krupanek B.: Probabilistyczny model opóźnień transmisji w jednorodnym systemie bezprzewodowym poddawanym zaburzeniom. Prz. Elektrot., nr 11, s. 20-22, 2014.
- [6] Yuan L., Zhu Y.: Modeling and Simulating Wireless Sensor Transportation Monitoring Network. The Sixth World Congress on Intelligent Control and Automation, Vol. 2, 2006, pp. 8640-8644.
- [7] Chaudhary D. D., Waghmare L. M.: Quality of service analysis in wireless sensor network by controlling end-to-end delay. 7th IEEE Conference on Industrial Electronics and Applications (ICIEA), 2012, pp. 703-708.
- [8] Xiong X., Tan J., Lin X.: Study on Communication Architecture Design of Wide-Area Measurement System. IEEE Transactions on Power Delivery, Vol. 28, Issue 3, 2013, pp. 1542-1547.
- [9] Topór-Kamiński T., Krupanek B., Homa J.: Delays Models of Measurement and Control Data Transmission Network. Advanced Technologies for Intelligent Systems of National Border Security, Studies in Computational Intelligence, 440, pp. 257-279.
- [10] Miczulski W., Powroźnik P.: A new elastic scheduling task model in the node of a control and measurement system. Metrol. Meas. Syst., Vol. XX (2013), No. 1, pp. 87-98.
- [11] Chitre, M., Motani, M., Shahabudeen, S.: Throughput of Networks With Large Propagation Delays. IEEE Journal of Oceanic Engineering, Vol. 37, Issue 4, 2012, pp. 645-658.
- [12] Bobbio A.: System Modelling with Petri Nets. Instituto Elettrotecnico Nazionale Galileo Ferraris, System Reliability Assessment, Kluwer p.c., 1990, p. 102-143.
- [13] Yang S., Kuipers, F.A.: Traffic uncertainty models in network planning. IEEE Communications Magazine, Vol. 52, Issue 2, 2014, pp. 172-177.
- [14] Nistor M., Lucani D. E., Vinhoza T. T. V., Costa R. A.: On the Delay Distribution of Random Linear Network Coding. IEEE Journal on Selected Areas in Communications, Vol. 29, Issue 5, 2011, pp. 1084-1093.
- [15] Liu Y., Ni L., Chuanping H.: A Generalized Probabilistic Topology Control for Wireless Sensor Networks. IEEE Journal on Selected Areas in Communications, Vol. 30, Issue 9, 2012, pp. 2776-2780.
- [16] Angrisani L., Capriglione D., Ferrigno L., Miele G.: An Internet Protocol Packet Delay Variation Estimator for Reliable Quality Assessment of Video-Streaming Services. IEEE Transactions on Instrumentation and Measurement, Vol. 62, Issue 5, 2013, pp. 914-923.
- [17] Bao D., De Vito L., Rapuano S.: A Histogram-Based Segmentation Method for Wideband Spectrum Sensing in Cognitive Radios. IEEE Transactions on Instrumentation and Measurement, Vol. 62, Issue 7, 2013, pp. 1900-1908.
- [18] Priebe S., Kurner T.: Stochastic Modeling of THz Indoor Radio Channels. IEEE Transactions on Wireless Communications, Vol. 12, Issue 9, 2013, pp. 4445-4455.
- [19] Quer G., Meenakshisundaram H., Tamma B. R., Manoj B. S.: Using Bayesian Networks for Cognitive Control of Multi-hop Wireless Networks. Military Communications Conference MILCOM, 2010, pp. 201 - 206.
- [20] Zhang Z.: Theory and Applications of Network Error Correction Coding. Proceedings of the IEEE, Vol. 99, Issue 3, 2011, pp. 406-420.
- [21] Jakubiec J.: A New Conception of Measurement Uncertainty Calculation. Acta Physica Polonica A. Vol. 124 (2013), No. 3, pp. 436-444.
- [22] Krupanek B.: Modeling of transmission delays caused by disturbances in the wireless networks in IEEE 802.15.4 standard (in Polish), PhD thesis, Gliwice 2012.
- [23] Akyildiz I. F., Sankarasubramaniam Y., Cayirci E.: A survey on sensor networks. IEEE Communication Magazine, 2002, 40(8), pp. 102-114.
- [24] Gallo D., Landi C., Pasquino N.: Multi-sensor network for urban electromagnetic field monitoring. IEEE Transactions on Instrumentation and Measurement, 2009, 58(9), pp. 3315-3322.
- [25] Moustpha A. I., Selmic R. R.: Wireless sensor network modeling using modified recurrent neural networks: application to fault detection. IEEE Transactions on Instrumentation and Measurement, 2008, 57(5), pp. 981-988.
- [26] Oliver R., Fohler G.: Probabilistic estimation of end-to-end path latency in wireless sensor networks. IEEE 6th International Conference on Mobile Adhoc and Sensor Systems MASS '09, 2009, pp. 423 - 431.
- [27] Sun S., Deng Z.: Multi-sensor optimal information fusion Kalman filter. Automatica, 2004, 40(6), pp. 1017-1023.
- [28] Ribeiro A., Schizas I., Roumeliotis S., Giannakis G.: Kalman filtering in wireless sensor networks. IEEE Control Systems, 2010, 30(2), pp. 66-86.
- [29] Cui P., Zhang H., Lam J., Ma L.: Real-time Kalman filtering based on distributed measurements. International Journal of Robust and Nonlinear Control, 2013, 23(14), pp. 1597-1608.
- [30] Krupanek B.: Simulations of experimental set up based on ZigBee standard using Opnet. International PhD Workshop OWD 2009, Wisła, 17-20.10.2009, pp. 262-267.
- [31] Topór-Kamiński T., Grygiel M.: Probabilistic Modeling of Delays in Data Transmission System with Wireless Network Interfaces Employing Random String Functions. Acta Physica Polonica A. Vol. 124 (2013), No. 3, pp. 578-585.
- [32] Eady F.: Hands-On ZigBee: Implementing 802.15.4 with Microcontrollers. Elsevier Inc, 2007.
- [33] Porter F.: Testing Consistency of Two Histograms, California Institute of Technology Lauritsen Laboratory for High Energy Physics, Pasadena, 2008.
- [34] Gupta S.C.: Delta Function. IEEE Transactions on Education, vol. 7, iss. 1. 1964, pp. 16-22.

Received: 02.07.2015

Paper reviewed

Accepted: 03.08.2015

Prof. Jerzy JAKUBIEC, PhD, DSc, eng.

Jerzy Jakubiec received the M.S. degree in 1971 and Ph. D in 1978 both in electrical engineering from the Silesian University of Technology. He joined the Electrical Engineering Faculty at the Silesian University of Technology in Gliwice where he presently is a Full Professor in Institute of Metrology, Electronics and Automatics. His research activities cover the field in metrological and functional analysis of measuring systems. Prof. Jakubiec is the author of more than 160 research papers and 6 monographs.

e-mail: jerzy.jakubiec@polsl.pl**Beata KRUPANEK, DSc, eng.**

Beata Krupanek is a professor assistant in Institute of Measurement Science, Electronics and Control at Silesian University of Technology. She received the M.S. degree in 2008 in optoelectronics in Faculty of Mathematics and Physics. The PhD thesis was based on delay modelling in wireless networks especially ZigBee communication systems. The area of research interests is connected with measurements and simulations of computer networks, programming microcontrollers and determination of Quality of Services parameters.

e-mail: beata.krupanek@polsl.pl