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Reliability assessment for micro inertial measurement unit based on accelerated degradation data and copula theory

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Bingjia Chi [a,b], Yashun Wang [a,b,]*, Jingwen Hu [a,b], Shufeng Zhang [a,b], Xun Chen [a,b]

[a] National University of Defense Technology, Laboratory of Science and Technology on Integrated Logistics Support, College of Intelligence Science and Technology, Yanwachi str., Changsha, 410073, Hunan, China
[b] National University of Defense Technology, Laboratory of Science and Technology on Integrated Logistics Support, Changsha, 410073, China

Highlights

- The reliability evaluation of MIMU is carried out by using accelerated degradation test.
- The MIMU's marginal ADT model is obtained by the general Wiener process.
- A multivariate-dependent ADT model of MIMU is established based on copula theory.
- Sufficient experiments and result analysis verify the effectiveness of proposed methods.

Abstract

With its extensive use in industry, assessing the reliability of the micro inertial measurment unit (MIMU) has become a pressing need. Unfortunately, the MIMU is made up of several components, and the degradation processes of each are intertwined, making it difficult to assess the MIMU's reliability and remaining useful life. In this research, we offer a reliability assessment approach for the MIMU, which has long-lifetime and multiple performance characteristics (PCs), based on accelerated degradation data and copula theory.Each PC model of MIMU is constructed utilizing drift Brownian motion to depict accelerated degradation process. The copula function is used to model the multivariate dependent accelerated degradation test data and to describe the dependency between multiple MIMU performance parameters. The particle swarm optimization algorithm is used to estimate the unknown parameters in the multi-dependent ADT model. Finally, the storage test and simulation example on MIMU's accelerated degradation data verify the feasibility and effectiveness of the proposed method.

Keywords

MIMU, Wiener process, accelerated degradation test, copula, reliability assessment.

1. Introduction

MIMU is a new inertial measurement micro-system based on micro-electro-mechanical system (MEMS) technology. It combines micro-gyroscope, micro-accelerometer, micro-signal conversion processing circuit and signal correction circuit to collect object motion information, with benefits such as small size, light weight, low power consumption, and long life. MIMU has been widely used in industry in a variety of scenarios, including aerospace, autonomous driving, and so on [8]. MIMU's reliability and safety have a direct impact on the equipment's performance and lifespan. To ensure the safe and dependable operation of MIMU as the core of MEMS inertial navigation system during its service life, it is required to investigate the reliability evaluation technique of MIMU. However, due to rapid technological advancements and rising demand, MIMU now has excellent reliability and long-life properties, posing new challenges in terms of life and reliability evaluation.

Simulation tests make it difficult to observe MIMU failure data in a short period of time, and long-term tests will increase the budgetary cost. Accelerated testing (AT) extrapolates the life and performance information obtained under high stress level tests through reasonable statistical methods to obtain reliability or life estimates under conventional stress conditions, under the premise of ensuring that the failure mechanism of the product is not changed [12, 32]. The failure process of the product is significantly accelerated and the test time is greatly shortened due to the application of accelerated stress [18]. Therefore, accelerated testing provides a feasible way for efficient reliability assessment of MIMU. Accelerated testing is classified into two types based on the different failure modes of the tested products: accelerated life testing (ALT) and accelerated degradation testing (ADT) [31]. For MIMU, only a small amount of failure may occur or no failure may occur at all when ALT is performed. In this case, if ALT is still used to evaluate the reliability and lifetime of MIMU, more samples and time are needed to conduct the test. On the one hand, there is a lack of sufficient failure data, and on the other hand, the reliability and life estimates derived from this will be lose authenticity. As a result,

(*) Corresponding author.

E-mail addresses: B. Chi (ORCID: 0000-0003-4589-3511): bjchi1997@163.com, Y. Wang (ORCID: 0000-0001-6766-6139): wangyashun@nudt.edu.cn, J. Hu (ORCID: 0000-0002-3708-8654): hjw1028@163.com, S. Zhang (ORCID: 0000-0002-9252-2986): sfzhang@nudt.edu.cn, X. Chen (ORCID: 0000-0003-2657-518X): chenxun@nudt.edu.cn

using an accelerated degradation test to assess MIMU reliability is an unavoidable option.

The ability to appropriately depict the performance degradation trajectory of a MIMU accelerated degradation model is crucial. The performance degradation of MIMU is mainly affected by external environmental stress and its own material properties, and its degradation process is a typical random process. The real performance degradation data of the MEMS gyroscope as a component of the MIMU also demonstrates that the main performance degradation process tends to rise or decrease in a relatively brief length of time [15]. As a result, the Wiener process with random, non-monotonic independent increments can be utilized to describe the degradation path of MIMU performance. The Wiener process is widely utilized for degradation path modeling and analysis because there is no constraint of monotonic degradation path [35]. Lin et al. [13] used the Wiener process model to conduct a comprehensive investigation on the degradation of composite materials under high-cycle bending fatigue (HCBF) pressures. Pan et al. [24] introduced an expectation maximization algorithm-based Wiener process reliability estimation technique that takes measurement error into account. Wang et al. [30] developed a generalized Wiener process model that incorporates nonlinearity, time uncertainty, and the unit-to-unit variation.

At the moment, research on MIMU reliability and life prediction is primarily focused on its sub-components, such as the MEMS component. A lifetime prediction approach for MEMS gyroscopes is offered out by Liu et al. [15] ,which is based on accelerated degradation testing and an acceleration factor model. A method combining wavelet analysis and support vector machine (SVM) for MEMS gyroscope was proposed by Miao et al. [21]. Wavelet analysis is used to preprocess the performance data of gyroscope, and SVM is used to model the processed data, so as to realize the prediction of residual life of gyroscope. MEMS accelerometers for space applications were subjected to a reliability study by I. Marozau et al. [28]. More relevant literature can be referred to [19, 20, 22, 23].

As a complex electronic product, the failure of MIMU is often caused by multiple performance degradation, any of which can cause MIMU failure if it reaches the failure threshold. There is frequently a link between the multiple MIMU performance degradation processes because of the similar operating environment, stress, structural design, and other aspects. If the correlation between different performance degradation processes is ignored, it will definitely cause the loss of reliability information, which makes the results of system reliability analysis deviate from the actual application. Therefore, one of the challenging issues in developing the MIMU multivariate ADT model is how to characterize the association between performance degradation. The joint distribution method and copula method are common methods for describing the relationship between multivariate performance parameters. [2, 6, 17, 26]. In practice, the joint distribution method assumes that the edge distributions all conform to a certain distribution, which has great limitations and is prone to discrepancies with the actual situation. While the Copula function acts as a bridge between marginal distribution and the joint distribution, and there is no restriction on the form of the marginal distribution function, it has become an important and convenient tool for solving the correlation problem. Hao et al. [7] employed the binary nonlinear diffusion process to simulate the degradation of the binary performance parameters of LED lamps, defined the dependence of the binary performance parameters using the Frank Copula function, and used the Monte Carlo (MCMC) algorithm to optimize the parameters, providing for the evaluation of degraded equipment's reliability. Peng et al. [25] developed a binary degradation model based on an inverse gaussian process, a joint distribution established with the copula function, and a two-stage Bayesian method for the degradation process and copula function parameter estimation.

Regarding the issue above, this paper proposes a reliability assessment method of MIMU based on accelerated degradation data and copula theory. First, the functional principle and performance degradation characteristics of MIMU are analyzed, and the degradation model of MIMU is established by using Wiener process. Meanwhile, the copula function is introduced to obtain the MIMU reliability assessment model under multiple performance degradation, and then the MIMU reliability assessment and life prediction under normal stress were realized, in order to make full use of the reliability information in accelerated degradation data and accurately describe the correlation between performance degradation processes. The following are the major contributions of our work: 1) The accelerated degradation test is used to evaluate the reliability of MIMU, with high reliability and high reliability, which improves the reliability and remaining useful life evaluation efficiency of MIMU; 2) The Copula joint probability model is used to solve the reliability evaluation problem in the case of MIMU with multi-component interdependent competition failure, which improves the accuracy of the results; 3) Sufficient experiments and result analysis are carried out, which can provide reference for the reliability evaluation of similar products.

The rest of the paper is carried out as follows. The functional principle and degradation mechanism of MIMU are examined in Section 2, and the MIMU degradation model and reliability evaluation model are established based on Wiener process and Copula theory. Section 3 includes a storage test and a simulation example based on MIMU's accelerated degradation data. Finally, the conclusions are drawn in section 4.

2. Reliability evaluation model of MIMU

2.1. MIMU structure and performance degradation characteristic

MIMU is composed of power module, micro-inertial device module, main control module, communication module, shell module, etc. It's a typical example of a multi-component system. Fig. 1 depicts the MIMU function structure chart.

The power module is mainly composed of a voltage conversion chip and related circuits. It is responsible for converting the externally provided power supply into a stable, low-noise multi-channel power supply required by the components in the system.

The micro-inertial device module is mainly composed of the three-axis MEMS gyroscope, three-axis MEMS accelerometer, temperature sensor, mechanical mount and peripheral circuits. It is the core part of the MIMU and the basis for the system's sensitive functions and performance. The three-axis gyroscope measures the angular velocity signals along the three axes of the carrier coordinate system, and the three-axis accelerometer measures the acceleration signals along the three axes of the carrier coordinate system. The temperature sensor is responsible for monitoring the internal temperature of the sensor and compensating the error of the inertial sensor through the temperature compensation circuit and the corresponding algorithm to ensure the output accuracy of MIMU. If combined with the three-axis geomagnetic sensor, barometer, etc., the MEMS sensor can be dynamically corrected in real time to compensate for the accumulated error and further improve the detection accuracy.

The main control module is composed of the micro-processing unit chip and corresponding auxiliary circuit. It samples the sensor signal in real time through the communication interface, and then completes the calculation of the carrier attitude and position information.

The communication module is responsible for the communication between MIMU and external devices. The housing module provides the sensor installation reference, and protects and seals the internal sensors and circuits of the MIMU.

The MEMS inertial devices (the gyroscopes and accelerometers) and electronic devices inside MIMU will suffer cumulative damage under long-term environmental stress, which will lead to the decline or even failure of MIMU-related performance indicators. The performance degradation of MIMU is mainly reflected in the performance degradation of the internal gyroscope and accelerometer. Most of

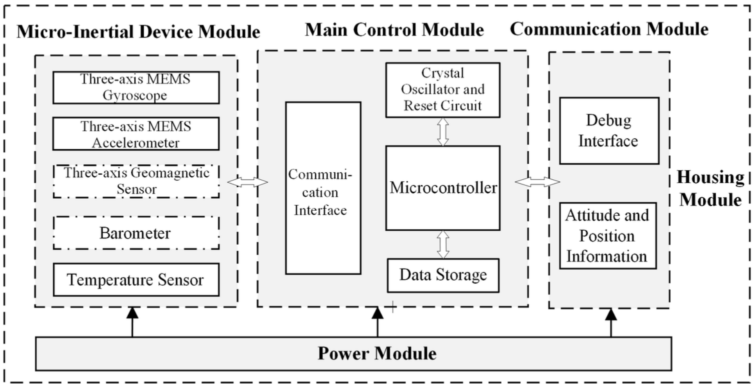

Fig. 1. The function structure chart of MIMU

the existing studies only use the performance state of one MEMS inertial device to characterize the performance state of the entire MIMU micro-inertial system, but for the MIMU, which consists of multiple inertial devices, it is very unreliable to judge the performance state of the whole device by only relying on the performance index of a single device. Therefore, this study proposes to characterize the MIMU micro-inertial system's performance state by taking into account both the gyroscope and accelerometer's performance states.

2.2. Model assumptions

We suppose there are L accelerated stress levels in the MIMU constant stress accelerated degradation test (CSADT), with S_l being the l stress level. At each accelerated stress level S_l, N samples are tested. For all samples, M measurements are taken, with K PCs observed at the sequencing $t_1<t_2<\ldots t_j\ldots<t_M$, and degradation measurements $y_{lik}(t_j)$ taken until the ending time t_M, $l = 1, 2, ..., L$, $i = 1, 2, ..., N$, $k = 1, 2, ..., K$, $j = 1, 2, ..., M$, where K is the total number of MIMU PCs, N is the number of samples in the experiment and M is the number of inspection times for each accelerated stress level S_l. In general, the MIMU accelerated degradation data at the l stress level can be stated in the following way in general:

$$Y_{lNK\times M}=\begin{pmatrix} Y_{l1}\\ Y_{l2}\\ \vdots\\ Y_{lK}\end{pmatrix}=\begin{pmatrix} y_{l11}(t_1) & y_{l11}(t_2) & \cdots & y_{l11}(t_M)\\ \vdots & \vdots & \ddots & \vdots\\ y_{lN1}(t_1) & y_{lN1}(t_2) & \cdots & y_{lN1}(t_M)\\ \vdots & \vdots & \ddots & \vdots\\ y_{l1K}(t_1) & y_{l1K}(t_2) & \cdots & y_{l1K}(t_M)\\ \vdots & \vdots & \ddots & \vdots\\ y_{lNK}(t_1) & y_{lNK}(t_2) & \cdots & y_{lNK}(t_M)\end{pmatrix} \tag{1}$$

The following assumptions are made to fit MIMU degradation data at accelerated stress levels.

Assumption 1: Wiener process assumption

Assume MIMU has K degradation PCs, each of which is driven by a Brownian motion mechanism. To build the degradation model of each PC, we use a linear stochastic model based on Brownian motion. The model is written as follows:

$$y_{lik}\left(t_j\right)=y_{lik}\left(0\right)+\mu_{lk}t_{lj}+\sigma_{lk}B(t_{lj}) \tag{2}$$

where $y_{lik}(t_j)$ is the kth degradation parameter of the ith sample at the lth stress level at the inspection time t_j, $y_{lik}(0)$ is the initial value of kth degradation parameter of the ith sample at the lth stress level, and μ_{lk} is the drift parameter of the kth degradation parameter at the lth stress level, reflecting the rate of degradation. σ_{lk} is the diffusion parameter of the kth degradation parameter at the lth accelerated stress, reflecting the effect of random factors on MIMU performance during the test, and $B(\bullet)$ is the standard Brownian motion, $B(t)\sim N(0,t)$.

The difference is derived by Eq. (3) as:

$$\Delta y_{lik}\left(t_j\right)=\mu_{lk}t_{lj}+\sigma_{lk}B(t_{lj}) \tag{3}$$

Assumption 2: acceleration equation assumption

To assess the MIMU's reliability under normal operating conditions, an acceleration model should be built initially. We suppose that P accelerated stress exists (e.g., vibration, temperature, and voltage), and sets $\boldsymbol{S} = (s_1, s_2, \ldots, s_p)$, $\boldsymbol{\eta} = (\eta_1, \eta_2, \ldots, \eta_p)$, where s_p is the pth accelerated stress type and η_p is the pth constant coefficient, $p = 1, 2, \ldots, P$.

MIMU's stress-acceleration model is represented as follows for single and multiple acceleration variables:

$$\mu(\boldsymbol{S};\eta)=\eta_0\cdot\prod_{p=1}^{P}\left[\phi(s_p)\right]^{\eta_p} \tag{4}$$

where $\mu(\boldsymbol{S};\boldsymbol{\eta})$ is the drift parameter, $\Phi(\bullet)$ is the function of the accelerating stress s_p. Meanwhile, we assume that σ_{lk} is not affected by the accelerated stress.

2.3. Failure time distribution

The kth PC $\Delta y_{lk}(t_j)$ follows a normal distribution at the lth stress level, as can be obtained [10], and $\Delta y_{lk}(t_j) \sim N(\mu_{lk}t_{lj}, \sigma t_{lj})$.

The pdf. of $\Delta y_{lk}(t_j)$ is:

$$f\left(\Delta y_{lk}\left(t_j\right)\right)=\frac{1}{\sigma_k\sqrt{2\pi t_{lj}}}\exp\left(-\frac{\left(\Delta y_{lk}\left(t_j\right)-\mu_{lk}t_{lj}\right)^2}{2\sigma_k^2 t_{lj}}\right) \tag{5}$$

and the corresponding cdf. of $\Delta y_{lk}(t_j)$ is:

$$F\left(\Delta y_{lk}\left(t_j\right)\right)=\phi\left(-\frac{\Delta y_{lk}\left(t_j\right)-\mu_{lk}t_{lj}}{\sigma_k\sqrt{t_{lj}}}\right) \tag{6}$$

We define D_k as the kth PC of MIMU's failure threshold. The time when y_k first crosses the associated failure threshold D_k (i.e., the first passage time (FPT)) is known as the failure time T_k:

$$T_k=\inf\left\{t:y_k(\mathrm{t})\geq D_k\right\} \tag{7}$$

The life distribution of MIMU degradation failure is depicted by the distribution of the first passage time, lifetime information about MIMU degradation failure can be acquired from this. At the lth accelerated stress, the life distribution function of the first passage time can be derived from the relation [27]:

$$F_{lk}\left(t\right)=\phi\left(\frac{\mu_{lk}t-D_k}{\sigma_k\sqrt{t}}\right)+\exp\left(\frac{2\mu_{lk}D_k}{\sigma_k^2}\right)\phi\left(\frac{-D_k-\mu_{lk}t}{\sigma_k\sqrt{t}}\right) \tag{8}$$

As a conclusion, the product reliability function can be written as:

$$R_{lk}\left(t\right)=\phi\left(\frac{D_k-\mu_{lk}t}{\sigma_k\sqrt{t}}\right)-\exp\left(\frac{2\mu_{lk}D_k}{\sigma_k^2}\right)\phi\left(-\frac{D_k+\mu_{lk}t}{\sigma_k\sqrt{t}}\right) \tag{9}$$

where $R_{lk}(t)$ is the kth degradation parameter's reliability at the lth stress level, and D_k is the kth degradation parameter's failure threshold. μ_{lk} is the drift parameter of the kth degradation parameter at the lth stress level, σ_k is the diffusion parameter, $\Phi(\cdot)$ is a normal distribution cumulative probability function.

The probability density function of FPT is:

$$f_{lk}(t) = -\frac{dR_{lk}(t)}{dt} = \frac{D_k}{\sigma_k\sqrt{2\pi t^3}}\exp\left(-\frac{(D_k-\mu_{lk}t)^2}{2\sigma_k^2 t}\right) \tag{10}$$

2.4. Multivariate dependent accelerated degradation modeling of MIMU

2.4.1. Model building

A copula is a multivariate probability distribution with uniform marginal probability distributions for all variables in probability theory and statistics. Copulas are used to depict the dependence relationship between random variables [34]. Assume that the MIMU's performance degradation parameters (Y_1, Y_2, …, Y_K) are a random vector. Assume that the marginal CDFs $F_k(y) = P(Y_k \le y)$ are continuous functions. Sklar's theorem [2] states that every multivariate cumulative distribution function $H(y_1, y_2, …, y_K) = P(Y_1 \le y_1, Y_2 \le y_2, …, Y_K \le y_K)$ of the random vector (Y_1, Y_2, …, Y_K) can be expressed in terms of its marginals $F_k(y) = P(Y_k \le y)$ and a copula C:

$$H(y_1,y_2,...,y_K) = C(F_1(y_1),F_2(y_2),...,F_K(y_K)) \tag{11}$$

Set $f(y_1, y_2, …, y_K)$ is the joint distribution probability density function of the performance degradation of MIMU, it can be derived from the Eq. (11), then:

$$\begin{aligned} f(y;\theta_1,\theta_2,...\theta_K,\delta) &= \frac{\partial^K H(y_1,y_2,...,y_K)}{\partial y_1\partial y_2\cdots\partial y_K} \\ &= \frac{\partial^K C(F_1(y_1;\theta_1),F_2(y_2;\theta_2),...,F_K(y_K;\theta_K);\delta)}{\partial F_1(y_1;\theta_1)\partial F_2(y_2;\theta_2)\cdots\partial F_K(y_K;\theta_K)}\prod_{k=1}^{K} f_k(y_k;\theta_k) \\ &= c(y_1,y_2,...,y_K;\delta)\prod_{k=1}^{K} f_k(y_k;\theta_k) \end{aligned} \tag{12}$$

where:

$$c(y_1,y_2,...,y_K;\delta) = \frac{\partial^K C(F_1(y_1;\theta_1),F_2(y_2;\theta_2),...,F_K(y_K;\theta_K);\delta)}{\partial F_1(y_1;\theta_1)\partial F_2(y_2;\theta_2)\cdots\partial F_K(y_K;\theta_K)}$$

is the copula probability density function, $F_K(y_K;\theta_K)$ and $f_K(y_K;\theta_K)$ are the marginal distribution function and marginal probability density function of the MIMU performance parameter y_k, respectively. $\boldsymbol{\delta}$ are the parameters of the copula function.

The CDF of the time to failure for the K degradation processes in the MIMU micro-system can be expressed as $F_k(t) = 1\text{-}R_k(t)$ $k = 1, 2, …, K$. The joint CDF of T_1, T_2, …, T_K is written as:

$$P(T_1 \le t_1, T_2 \le t_2,\ldots,T_K \le t_K) = H(t_1,t_2,...,t_K) = C(F_1(t_1),F_2(t_2),...,F_K(t_K);\delta) \tag{13}$$

Correspondingly, the marginal reliability for MIMU is expressed as:

$$P(T_1 > t_1, T_2 > t_2,\ldots,T_K > t_K) = \overline{H}(t_1,t_2,...,t_K) = C(R_1(t_1),R_2(t_2),...,R_K(t_K);\delta) \tag{14}$$

The relationship between $C(F_1(t_1), F_2(t_2), ..., F_K(t_K))$ and $C(R_1(t_1), R_2(t_2), ..., R_K(t_K))$ is represented as:

$$\begin{aligned} &C(R_1(t_1),R_2(t_2),\ldots,R_K(t_K)) \\ &= 1-\sum_{k=1}^{K}F_i(t_i) + \sum_{1\le i<j\le K} C(F_i(t),F_j(t),...) - \sum_{1\le i<j<h\le K} C(F_i(t_i),F_j(t_j),F_h(t_h)...) \\ &+\cdots+(-1)^K C(F_1(t_1),F_2(t_2),...,F_K(t_K)) \end{aligned} \tag{15}$$

Therefore, the MIMU's reliability function with K degradation processes at time t can be expressed as:

$$\begin{aligned} R(t) &= P(T_1 > t, T_2 > t,\ldots,T_K > t) \\ &= P(Y_1(t) < D_1, Y_2(t) < D_2,\ldots,Y_K(t) < D_K) \\ &= C(R_{Y_1}(t),R_{Y_2}(t),...,R_{Y_K}(t)) \end{aligned} \tag{16}$$

where $RY_K(t)$ denotes MIMU's marginal reliability function for the kth degradation trajectory at time t.

Table 1 shows CDFs and PDFs for common bivariate copulas in particular.

2.4.2. Determination of the Copula function

The Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) concepts are utilized in this research to choose the copula function model that best fits the original data. The copula type with the lowest AIC/BIC is the superior choice. According to AIC/BIC principles. Eq. (17) and Eq. (18) are the AIC and BIC formulas [1, 9]:

$$AIC = -2\ln MLE + 2u \tag{17}$$

$$BIC = -2\ln MLE + u\ln N \tag{18}$$

Table 1. The CDFs and PDFs for common bivariate copulas

Copula function	$C(u,v;\delta)$	$c(u,v;\delta)$	Range of parameters
Frank	$-\frac{1}{\delta}\ln(1+\frac{(e^{-\delta u}-1)(e^{-\delta v}-1)}{(e^{-\delta}-1)})$	$\frac{-\delta e^{-\delta(u+v)}(e^{-\delta}-1)}{((e^{-\delta}-1)+(e^{-\delta u}-1)(e^{-\delta v}-1))^2}$	$(-\infty,+\infty)/\{0\}$
Gumbel	$\exp(-((-\ln u)^{\delta}+(-\ln v)^{\delta})^{1/\delta})$	$\frac{C(u,v;\delta)(-\ln C(u,v;\delta)+\delta-1)[(\ln u)(\ln v)]^{\delta-1}}{uv[(-\ln u)^{\delta}+(-\ln v)^{\delta}]^{2-1/\delta}}$	$(1,+\infty)$
Clayton	$\max\{(u^{-\delta}+v^{-\delta}-1)^{-1/\delta},0\}$	$(1+\delta)(uv)^{-\delta-1}(u^{-\delta}+v^{-\delta}-1)^{-2-1/\delta}$	$(0,+\infty)$

where MLE is the maximum value of likelihood function, u is the number of parameters in the model, and N is sample size.

2.5. Parameter estimation

The log-likelihood function can be derived as follows using the joint probability density Eq. (12):

$$\ln L(\theta_1,\theta_2,\ldots,\theta_K,\delta)=\underbrace{\sum_{l=1}^{L}\sum_{i=1}^{N}\sum_{j=1}^{M}\ln c\left(F_1\left(y_{lij1};\theta_1\right),F_2\left(y_{lij2};\theta_2\right),\ldots,F_K\left(y_{lijK};\theta_K\right);\delta\right)}_{\text{Dependency Structure } L_c} + \underbrace{\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{i=1}^{N}\sum_{j=1}^{M} f_k\left(y_{klij};\theta_k\right)}_{\text{Marginals } L_{Mar}} \tag{19}$$

where $F_1(y_1;\theta_1)$, $F_2(y_2;\theta_2)$, …, $F_K(y_K;\theta_K)$ are the marginal failure distribution function for the MIMU performance parameter $\boldsymbol{y}$, and $f_1(y_1;\theta_1)$, $f_2(y_2;\theta_2)$, …, $f_K(y_K;\theta_K)$ are the marginal pdf, $\boldsymbol{\delta}$ are the parameters of the copula function.

Because the log-likelihood function Eq. (19) has so many unknown parameters, it is impossible to maximize it directly. The copula function efficiently distinguishes between the marginal and joint distributions of random variables. To estimate the parameters of the reliability model with multivariate performance degradation, we introduce a two-step parameter estimation method based on particle swarm optimization (PSO), which substantially minimizes the complexity of parameter estimation. The following are the specific implementation steps.

A. Parameter estimation of marginal ADT models

Combined with the probability density Eq. (5), the likelihood function of marginal ADT models can be derived:

$$\begin{aligned} L_{kMar} &= \prod_{l=1}^{L}\prod_{i=1}^{N_l}\prod_{j=1}^{M_l}\left\{\frac{1}{\sqrt{2\pi\sigma_k^2 t_{lj}}}\exp\left(-\frac{\left(\Delta y_{lik}\left(t_j\right)-\mu_k(s)t_{lj}\right)^2}{2\sigma_k^2 t_{lj}}\right)\right\} \\ &= \frac{(2\pi)^{\frac{-M_l\cdot N_l\cdot L}{2}}}{\sigma_k^{M_l\cdot N_l\cdot L}}\cdot\prod_{l=1}^{L}\prod_{j=1}^{M_l}\left(t_{lj}\right)^{\frac{-N_l}{2}}\cdot\exp\left[-\frac{1}{2\sigma_k^2}\sum_{l=1}^{L}\sum_{i=1}^{N_l}\sum_{j=1}^{M_l}\frac{\left(\Delta y_{lik}\left(t_j\right)-\mu_k(s)t_{lj}\right)^2}{t_{lj}}\right] \end{aligned} \tag{20}$$

Furthermore, the log-likelihood function is:

$$\begin{aligned} \ln L_{kMar} &= -N_l M_l L\ln\sqrt{2\pi} - N_l M_l L\ln\sigma_k - N_l\sum_{l=1}^{L}\sum_{j=1}^{M_l}\ln\sqrt{t_{lj}} \\ &\quad - \frac{1}{2\sigma_k^2}\sum_{l=1}^{L}\sum_{i=1}^{N_l}\sum_{j=1}^{M_l}\frac{\left(\Delta y_{lik}\left(t_j\right)-\mu_k(s)t_{lj}\right)^2}{t_{lj}} \end{aligned} \tag{21}$$

Therefore, the problem of estimating the parameters of the marginal distribution function using maximum likelihood can be turned into an extreme value problem. PSO is introduced in this article to change multi-parameter estimating challenges into multi-parameter optimum. The method features a flexible and simple structure, as well as a decent global search and a quick convergence speed.

The likelihood function is the objective function of the multi-parameter estimate method, and the goal of optimization is to obtain the largest likelihood function. As a particle, the parameters to be estimated $(\sigma, \eta_1, \eta_2, \ldots, \eta_P)$ are defined. In general, the higher the number of particles, the greater the objective function's convergence. However, too many particles will increase the calculation time. The higher the number of iterations, the more accurate the convergence to the best value will be, but it will also take longer to compute. The literature describes the particle optimization procedure [16].

B. Copula parameter estimation of joint distribution function

Based on the estimated value of the marginal distribution function parameter $\theta_{Mar} = [\sigma, \eta_0, \eta_1, \eta_2, \ldots, \eta_P]_{K\times(2+P)}$, the estimated value of the unknown parameter of the copula function is obtained by using the maximum likelihood estimation method:

$$\hat{\theta} = \arg\max\left[\sum_{l=1}^{L}\sum_{i=1}^{N}\sum_{j=1}^{M}\ln c\left(F_1\left(y_{lij1};\widehat{\theta_1}\right),F_2\left(y_{lij2};\widehat{\theta_2}\right)\cdots,F_K\left(y_{lijK};\widehat{\theta_K}\right);\delta\right)\right] \tag{22}$$

where $F_1(y_{lij1};\widehat{\theta_1})$, $F_2(y_{lij2};\widehat{\theta_2})$, …, $F_K(y_{lijK};\widehat{\theta_K})$ are the marginal distribution function for the MIMU performance degradation parameters $\boldsymbol{y}$. $\boldsymbol{\delta}$ are the parameters of the copula function.

3. Numerical examples

To illustrate the efficiency of the proposed modeling method, we use examples from the MIMU storage degradation test and constant stress accelerated degradation simulation in this study.

3.1. MIMU storage degradation test

Related scholars have carried out the sensitive stress analysis of micro gyroscopes and micro accelerometers and obtained that temperature stress is an important factor affecting their performance [3–5, 11]. Therefore, the temperature stress was selected to carry out a constant stress accelerated storage test on MIMU, and the temperature stress was controlled to be 60°C during the test. The test sample experimental procedure is shown in Fig. 2.

The bias, bias stability, and bias instability of the MEMS gyroscope and MEMS accelerometer in the MIMU were tested for performance every 24 hours during the test, which is one of the primary indicators used to assess the MIMU's performance. The overall test period was 1176 hours, and there were 49 performance degradation data recorded.

The effective parameter of MIMU Y-axis bias is selected as the performance degradation index of MIMU, and the variation curves of bias of gyroscope and bias of accelerometer with time are obtained by processing the experimental data, as shown in Fig. 3. The MIMU performance degradation curve in Fig. 3 tends to increase or decrease in a short period of time, which has the characteristics of a stochastic process. It preliminarily verifies the validity of the Wiener process modeling assumption.

Fig. 4 depicts a two-dimensional scatter plot of the MIMU's performance data. It's clear that the top tails are linked, whereas the bottom tails haven't changed much. It can be preliminarily judged that there is a correlation between the MIMU performance degradation amounts. Therefore, issues related to multi-component degradation must be taken into account while evaluating MIMU's reliability.

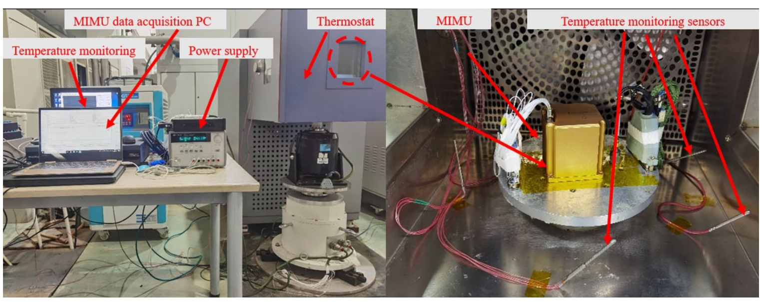

Fig. 2. MIMU storage degradation test

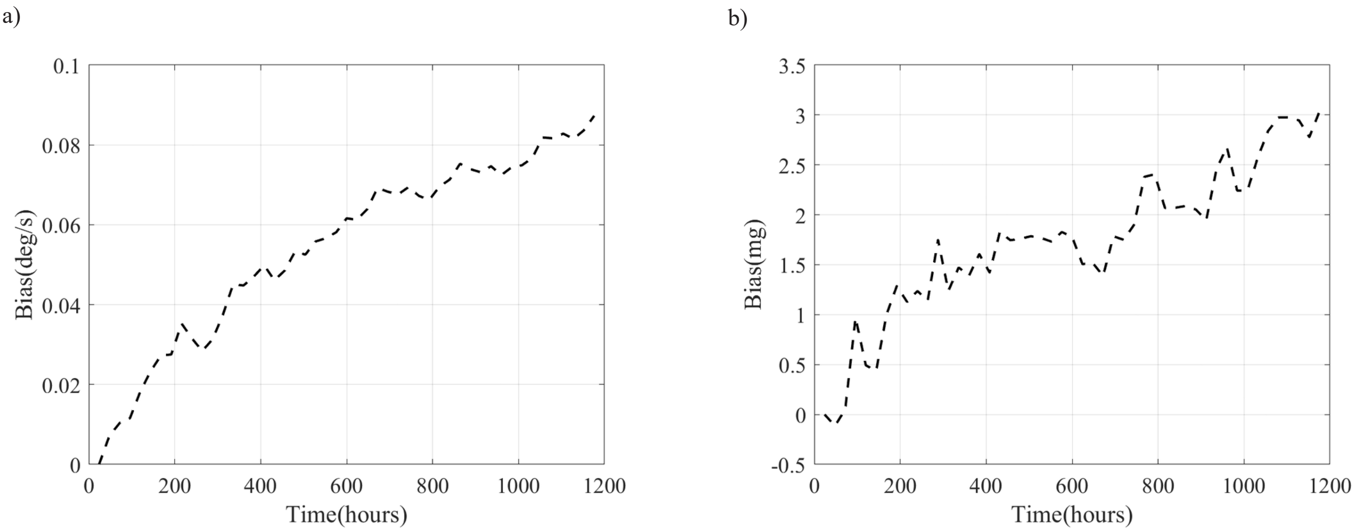

Fig. 3. Storage degradation data of MIMU: a) storage degradation data of gyroscope bias, b) storage degradation data of accelerometer bias

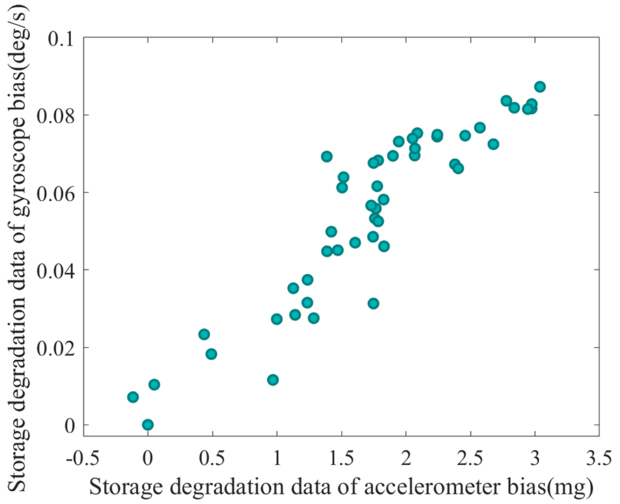

Fig. 4. Scatter plot of the MIMU's original data distribution

3.2. MIMU accelerated degradation simulation test

3.2.1. Simulation test

The MIMU constant stress accelerated degradation simulation test is performed in this part using the performance degradation data from the MIMU storage degradation test in Section 3.1. Table 2 shows the parameter settings of the marginal ADT model of each MIMU's PC, as well as the test acceleration stress temperature T. The CSADT's temperature stress levels were set at 60°C, 70°C, and 80°C, with 70, 50, and 30 inspections at each level, respectively. Every 24 hours, assess MIMU's performance. For each accelerating stress, the MIMU sample size is 4. The performance degradation is set to 0 at the start. The degradation curve of MIMU simulation performance (Bias) is shown in Fig. 5.

According to Fig. 5, we can clearly see that the degradation trajectories of the two PCs fluctuate over time, proving that the MIMU performance degrades as a typical random process. For each PC, the performance degradation trajectory at high stress is more likely to reach the failure threshold and the degradation trend is more pronounced than at low stress, which is consistent with the accelerated testing assumptions.

Table 2. The parameter setting of marginal ADT models

PCs	Parameter	set value
Gyroscope bias	η_0	5.89
	η_1	3768.09
	σ	5.8966×10^{-4}
Accelerometer bias	η_0	60.52
	η_1	3133.3
	σ	0.0571

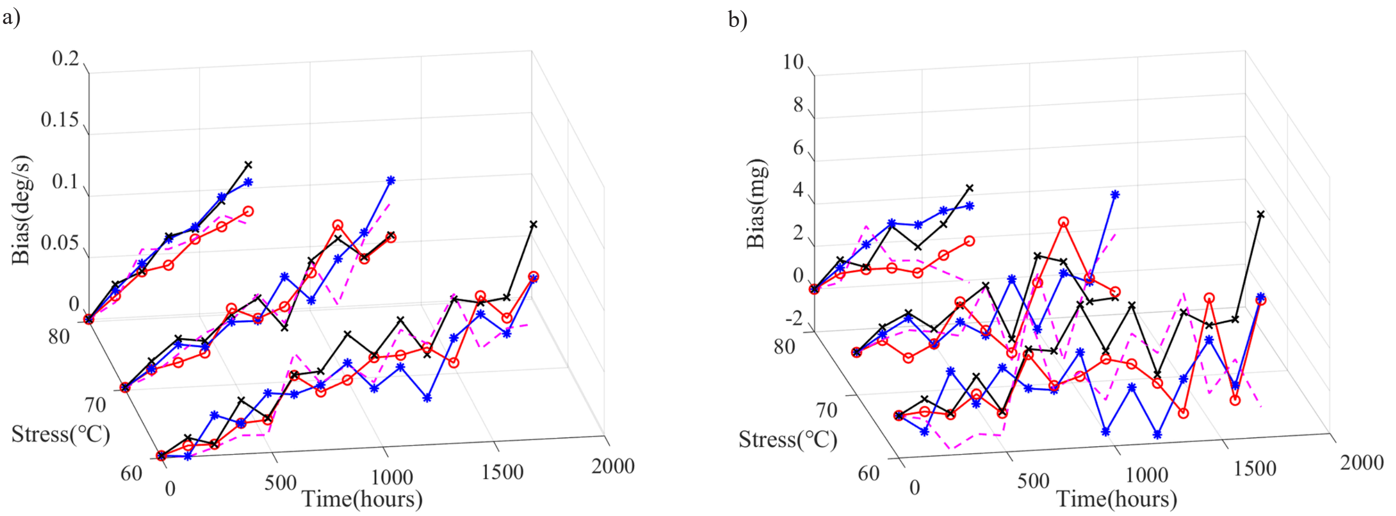

Fig. 5. CSADT data of MIMU's PCs: a) storage degradation data of gyroscope bias, b) storage degradation data of accelerometer bias

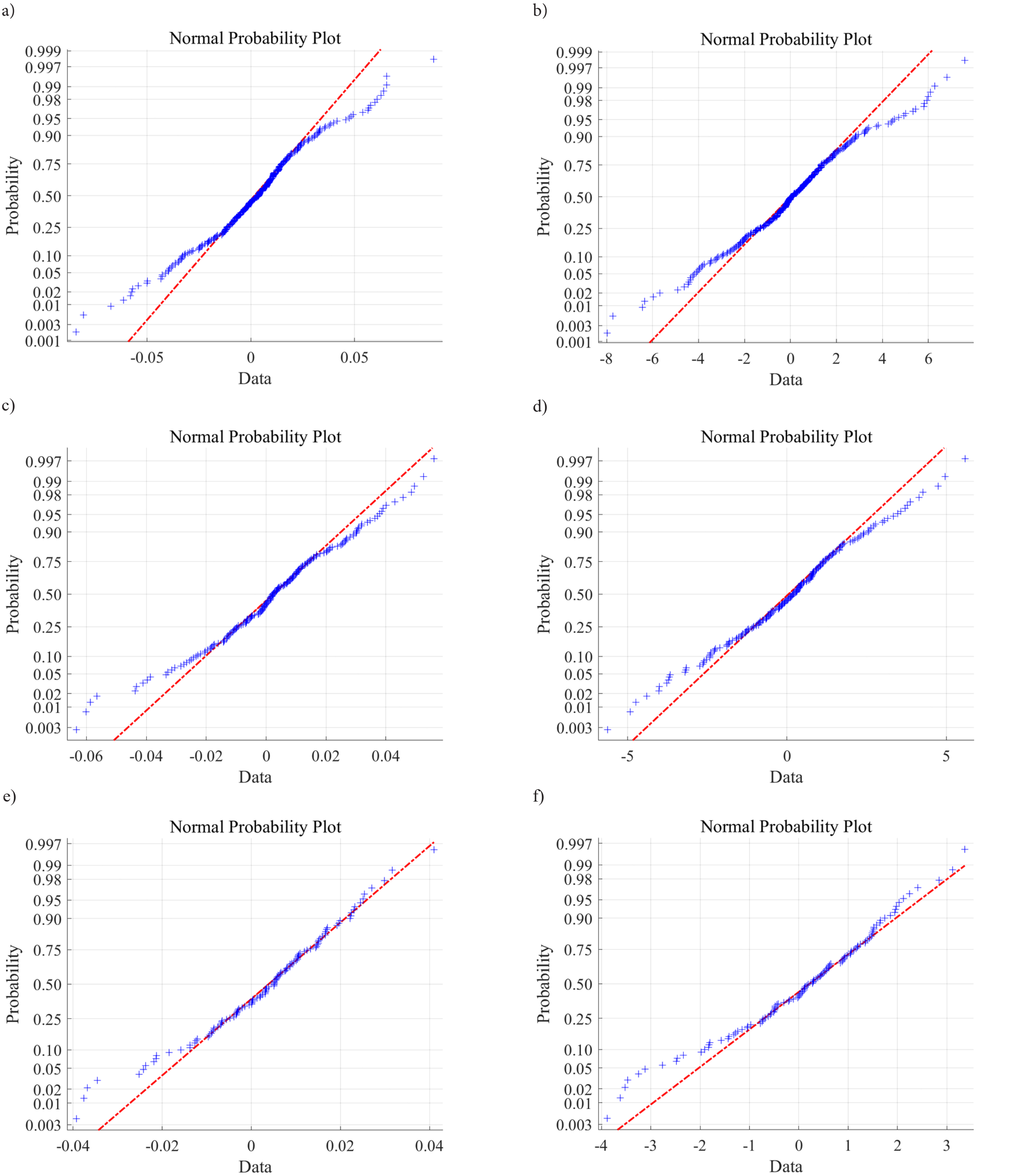

Fig. 6. Goodness-of-fit test of normal distribution of MIMU's degradation incremental data: a) goodness-of-fit test of normal distribution of 60℃ gyroscope bias degradation incremental data, b) goodness-of-fit test of normal distribution of 60℃ accelerometer bias degradation incremental data, c) goodness-of-fit test of normal distribution of 70℃ gyroscope bias degradation incremental data, d) goodness-of-fit test of normal distribution of 70℃ accelerometer bias degradation incremental data, e) goodness-of-fit test of normal distribution of 80℃ gyroscope bias degradation incremental data, f) goodness-of-fit test of normal distribution of 80℃ accelerometer bias degradation incremental data

3.2.2. Degradation model checking

We can conclude that performance degradation increment y follows a normal distribution due to the nature of the Wiener process [14, 33], $\Delta y = y(t+\Delta t) \sim N(\mu\Delta t, \sigma^2\Delta t)$. Therefore, the effectiveness of the MIMU performance degradation model can be verified by using the normal probability distribution diagram. If the incremental performance degradation data obeys a normal distribution, the curve should be approximated as a straight line. Fig. 6 depicts the normal distribution probability curve of the MIMU performance degradation increment under each acceleration stress. In the normal probability distribution diagram, the performance degradation increment under each temperature stress is almost a straight line, demonstrating the efficiency of the Wiener process for modeling MIMU performance degradation.

3.2.3. Parameter estimation based on PSO algorithm

To fit the accelerated degradation data of MIMU's each PC independently, the random-effect general Wiener process model, i.e. Eq. (2), is being used. Table 3 shows the estimators of marginal ADT model parameters for each PC based on the statistical inference process described in section 2.5.

The multivariate parameter estimation method based on PSO is used to estimate the parameters of the marginal distribution function. Set the simulation to 300 particles and 200 iterations, then choose 6 different paths at random to construct the objective function. Use the average of the estimated values from the 6 separate paths as the estimated parameters. Fig.7 depicts the iterative procedure and estimated parameter vectors for six possible pathways. The estimated values under different paths are quite close, as can be seen, and the average of the parameters obtained by six different paths is used to determine the ideal value of each parameter. The relative square error (RSE) [29], $RSE = (\theta_{Estimated} - \theta_{Setting})^2/(\theta_{Setting})^2$, is calculated to compare the error between the estimated value and the real value, as shown in Table 2. The results show that the PSO estimation method is accurate and effective.

Table 3. Parameter estimation

PCs	Parameter	Estimation value	RSE
Gyroscope bias	η_0	8.7282	23.2%
	η_1	3899.2	0.12%
	σ	5.9501×10^{-4}	0.00%
Accelerometer bias	η_0	81.7891	12.35%
	η_1	3542.9	1.7%
	σ	0.0569	0.00%

3.2.4. Determination of the Copula function

The associated parameters of each Copula function are estimated using the maximum likelihood estimation method, and the AIC and BIC principles are utilized to identify the best Copula. The results of parameter estimates, the Kendall coefficient, AIC values, and BIC values are shown in Table 3. Gumbel copula has the minimum AIC and BIC values, indicating that it has the best fitting effect on degradation-related data, as shown in Table 4.

3.2.5. Reliability assessment

Combined with the specific application conditions of MIMU in practice, the failure thresholds of its performance parameters are determined to be gyroscope bias ≥ 0.15 deg/s and accelerometer bias ≥ 10 mg. Substituting the estimated value of the model parameters and the failure thresholds of the two performance parameters into equations (8) and (10), the reliability curves of MIMU under normal stress 25°C without considering the parameter correlation and considering the parameter correlation can be obtained, as shown in Fig. 8.

a)

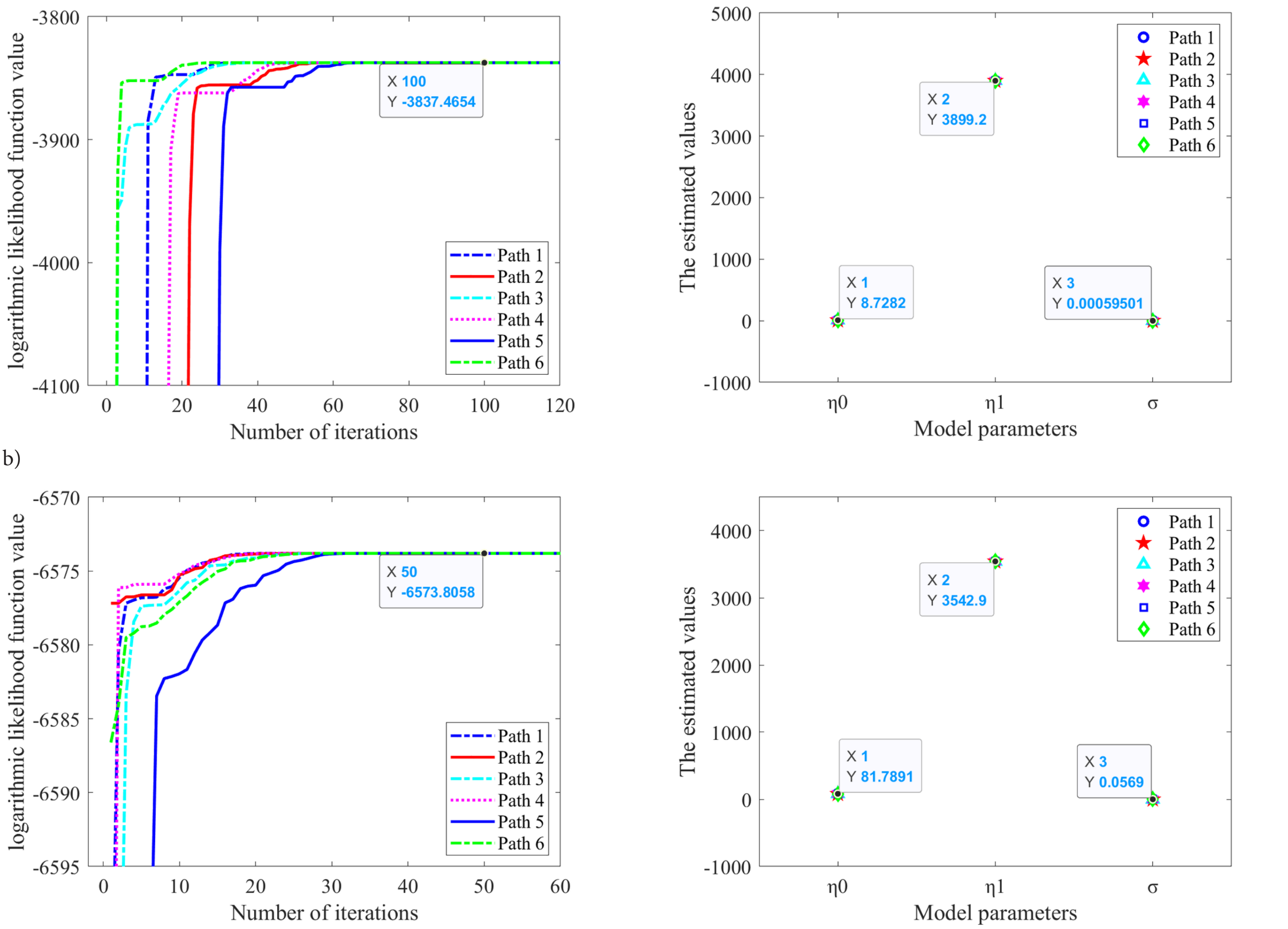

Fig. 7. Variation curve of objective function with iteration times and estimated parameters under different paths: a) estimation results of gyroscope bias parameters, b) estimation results of accelerometer bias parameters

Table 4. Copula function selection

Copula	δ	Kendall τ	AIC	BIC
Frank	17.5248	0.7931	-1264.084	-1259.688
Gumbel	4.8819	0.7951	-1412.894	-1408.499
Clayton	5.9737	0.7491	-1305.590	-1301.195

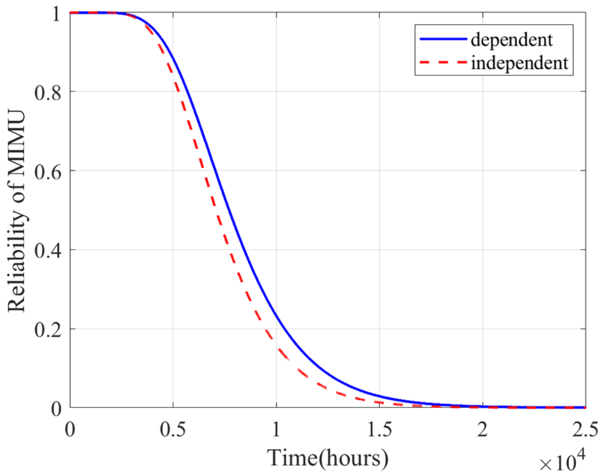

Fig. 8. Reliability evaluation of MIMU under normal stress

According to Fig. 8, the reliability assessment results obtained independently by considering the MIMU performance degradation are lower than those obtained based on the proposed method, which indicates that the reliability assessment results related to ignoring the system performance degradation are more conservative. According to the product life calculation formula $MTBF = \int_0^{\infty} R(t)dt$, we can obtain the MIMU lifetime of 7486.3 hours without considering degradation correlation and 8181.2 hours when considering performance degradation correlation. The result has a 9.3% longer lifespan when independency is taken into account. Considering the two comprehensively, the modeling method that takes into account degradation correlation is closer to the actual application in some ways, and can give a more solid foundation for the evaluation results.

4 Conclusion

A MIMU reliability assessment approach is proposed in this study, which is based on accelerated degradation data and copula theory. To describe the MIMU ADT data and build the univariate ADT model for each PC, we use the general Wiener process. By using copula function, the MIMU reliability assessment model under multiple performance degradation was obtained. The suggested MIMU reliability assessment approach is demonstrated by the MIMU storage test and accelerated degradation simulation example. The results reveal that treating the relationship between various performance factors as independent of each other for a highly reliable product like MIMU with many performance parameters is not in accordance with the actual situation and tend to underestimate the product's reliability.

References

1. Aho K, Derryberry D, Peterson T. Model selection for ecologists: the worldviews of AIC and BIC. Ecology 2014; 95(3): 631–636, https://doi.org/10.1890/13-1452.1.
2. Busababodhin P, Amphanthong P. Copula modelling for multivariate statistical process control: a review. Communications for Statistical Applications and Methods 2016; 23(6): 497–515, https://doi.org/10.5351/CSAM.2016.23.6.497.
3. Chen B, Liu H, Wang H et al. Thermal shock induced nanocrack as high efficiency surface conduction electron emitter. Applied Surface Science 2011; 257(21): 9125–9128, https://doi.org/10.1016/j.apsusc.2011.05.111.
4. Dong X, Huang Q, Xu W et al. Research on temperature characteristic of parasitic capacitance in MEMS capacitive accelerometer. Sensors and Actuators A: Physical 2019; 285: 581–587, https://doi.org/10.1016/j.sna.2018.12.001.
5. Duong Q-H, Buchaillot L, Collard D et al. Thermal and electrostatic reliability characterization in RF MEMS switches. Microelectronics Reliability 2005; 45(9–11): 1790–1793, https://doi.org/10.1016/j.microrel.2005.07.095.
6. Gu Y-K, Fan C-J, Liang L-Q, Zhang J. Reliability calculation method based on the Copula function for mechanical systems with dependent failure. Annals of Operations Research 2022; 311(1): 99–116, https://doi.org/10.1007/s10479-019-03202-5.
7. Hao H, Su C. Bivariate Nonlinear Diffusion Degradation Process Modeling via Copula and MCMC. Mathematical Problems in Engineering 2014; 2014: 1–11, https://doi.org/10.1155/2014/510929.
8. Iannacci J. Reliability of MEMS: A perspective on failure mechanisms, improvement solutions and best practices at development level. Displays 2015; 37: 62–71, https://doi.org/10.1016/j.displa.2014.08.003.
9. Jordanger L A, Tjøstheim D. Model selection of copulas: AIC versus a cross validation copula information criterion. Statistics & Probability Letters 2014; 92: 249–255, https://doi.org/10.1016/j.spl.2014.06.006.
10. Kang J, Lu Y, Zhao B et al. Remaining useful life prediction of cylinder liner based on nonlinear degradation model. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2021; 24(1): 62–69, https://doi.org/10.17531/ein.2022.1.8.
11. Le X L, Kim K, Choa S-H. Analysis of Temperature Stability and Change of Resonant Frequency of a Capacitive MEMS Accelerometer. International Journal of Precision Engineering and Manufacturing 2022; 23(3): 347–359, https://doi.org/10.1007/s12541-021-00602-1.
12. Li S, Chen Z, Liu Q et al. Modeling and Analysis of Performance Degradation Data for Reliability Assessment: A Review. IEEE Access 2020; 8: 74648–74678, https://doi.org/10.1109/ACCESS.2020.2987332.
13. Lin W, Chai Y, Liu Q. Remaining Useful Life Prediction of Electronic Products Based on Wiener Degradation Process. IFAC-PapersOnLine 2019; 52(24): 24–28, https://doi.org/10.1016/j.ifacol.2019.12.372.
14. Liu D, Wang S, Zhang C. Reliability estimation from two types of accelerated testing data based on an artificial neural network supported Wiener process. Applied Mathematics and Computation 2022; 417: 126757, https://doi.org/10.1016/j.amc.2021.126757.
15. Liu Y, Wang Y, Fan Z et al. Lifetime prediction method for MEMS gyroscope based on accelerated degradation test and acceleration factor model. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2020; 22(2): 221–231, https://doi.org/10.17531/ein.2020.2.5.
16. Liu Y, Wang Y, Fan Z et al. A new universal multi-stress acceleration model and multi-parameter estimation method based on particle swarm optimization. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability 2020; 234(6): 764–778, https://

doi.org/10.1177/1748006X20918793.
17. Lu S, Lu H, Kolarik W J. Multivariate performance reliability prediction in real-time. Reliability Engineering & System Safety 2001; 72(1): 39–45, https://doi.org/10.1016/S0951-8320(00)00102-2.
18. Lu X, Chen X, Wang Y, Tan Y. Consistency analysis of degradation mechanism in step-stress accelerated degradation testing. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2017; 19(2): 302–309, https://doi.org/10.17531/ein.2017.2.19.
19. Marozau I, Auchlin M, Pejchal V et al. Reliability assessment and failure mode analysis of MEMS accelerometers for space applications. Microelectronics Reliability 2018; 88–90: 846–854, https://doi.org/10.1016/j.microrel.2018.07.118.
20. Medjaher K, Skima H, Zerhouni N. Condition assessment and fault prognostics of microelectromechanical systems. Microelectronics Reliability 2014; 54(1): 143–151, https://doi.org/10.1016/j.microrel.2013.09.013.
21. Miao J, Li X, Ye J. Predicting research of mechanical gyroscope life based on wavelet support vector. 2015 First International Conference on Reliability Systems Engineering (ICRSE), Beijing, China, IEEE: 2015: 1–5, https://doi.org/10.1109/ICRSE.2015.7366508.
22. Mulloni V, Barbato M, Meneghesso G. Long-term lifetime prediction for RF-MEMS switches. Journal of Micromechanics and Microengineering 2016; 26(7): 074004, https://doi.org/10.1088/0960-1317/26/7/074004.
23. Mulloni V, Lorenzelli L, Margesin B et al. Temperature as an accelerating factor for lifetime estimation of RF-MEMS switches. Microelectronic Engineering 2016; 160: 63–67, https://doi.org/10.1016/j.mee.2016.03.023.
24. Pan D, Wei Y, Fang H, Yang W. A reliability estimation approach via Wiener degradation model with measurement errors. Applied Mathematics and Computation 2018; 320: 131–141, https://doi.org/10.1016/j.amc.2017.09.020.
25. Peng W, Li Y-F, Yang Y-J et al. Bivariate Analysis of Incomplete Degradation Observations Based on Inverse Gaussian Processes and Copulas. IEEE Transactions on Reliability 2016; 65(2): 624–639, https://doi.org/10.1109/TR.2015.2513038.
26. Pourhassan M R, Raissi S, Hafezalkotob A. A simulation approach on reliability assessment of complex system subject to stochastic degradation and random shock. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2020; 22(2): 370–379, https://doi.org/10.17531/ein.2020.2.20.
27. Si X-S, Wang W, Hu C-H et al. Remaining Useful Life Estimation Based on a Nonlinear Diffusion Degradation Process. IEEE Transactions on Reliability 2012; 61(1): 50–67, https://doi.org/10.1109/TR.2011.2182221.
28. Skima H, Medjaher K, Varnier C et al. A hybrid prognostics approach for MEMS: From real measurements to remaining useful life estimation. Microelectronics Reliability 2016; 65: 79–88, https://doi.org/10.1016/j.microrel.2016.07.142.
29 Sun F, Fu F, Liao H, Xu D. Analysis of multivariate dependent accelerated degradation data using a random-effect general Wiener process and D-vine Copula. Reliability Engineering & System Safety 2020; 204: 107168, https://doi.org/10.1016/j.ress.2020.107168.
30. Wang X, Jiang P, Guo B, Cheng Z. Real-time Reliability Evaluation with a General Wiener Process-based Degradation Model: real-time reliability evaluation. Quality and Reliability Engineering International 2014; 30(2): 205–220, https://doi.org/10.1002/qre.1489.
31. Wang Y, Fang X, Zhang C et al. Lifetime prediction of self-lubricating spherical plain bearings based on physics-of-failure model and accelerated degradation test. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2016; 18(4): 528–538, https://doi.org/10.17531/ein.2016.4.7.
32. Wang Y, Zhang C, Chen X, Tan Y. Lifetime prediction method for electron multiplier based on accelerated degradation test. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2014; 16(3): 484–490.
33. Xu D, Xing M, Wei Q et al. Failure behavior modeling and reliability estimation of product based on vine-copula and accelerated degradation data. Mechanical Systems and Signal Processing 2018; 113: 50–64, https://doi.org/10.1016/j.ymssp.2017.07.035.
34. Zhang Y, Ma Y, Ouyang L, Liu L. A novel reliability model for multi-component systems subject to multiple dependent competing risks with degradation rate acceleration. Eksploatacja i Niezawodnosc - Maintenance and Reliability 2018; 20(4): 579–589, https://doi.org/10.17531/ein.2018.4.9.
35. Zhang Z, Si X, Hu C, Lei Y. Degradation data analysis and remaining useful life estimation: A review on Wiener-process-based methods. European Journal of Operational Research 2018; 271(3): 775–796, https://doi.org/10.1016/j.ejor.2018.02.033.