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Multi-state approach to food packaging material quality and consumption safety analysis

Keywords

food packaging, permeability, tensile strength, swelling, transmittance, biofilm formation, safety function

Abstract

The food packaging material and its quality assessment based on permeability, tensile strength, swelling, transmittance, and biofilm formation are discussed in this chapter. The semi-Markov model of food packaging material quality change process is introduced and its characteristics are determined. Next, the safety and resilience indicators are proposed for multi-state analysis, identification, prediction and optimization of packaged food product consumption safety.

1. Introduction

The general approach to the analysis of the safety and resilience of packaged food product consumption is proposed. The methodology commonly used in the analysis of critical infrastructure safety and reliability and other complex technical or industrial systems and processes (Bogalecka, 2020; Dąbrowska, 2020; Grabski, 2015; Iosifescu, 1980; Kołowrocki, 2004, 2014, 2021; Kołowrocki & Soszyńska-Budny, 2011; Korolyuk et al., 1976; Limnios & Oprisan, 2001; Torbicki & Raith, 2021) is adapted to modelling, identification, prediction and optimization of packaged food product consumption safety and reliability.

The chapter is organized into five parts, this Introduction as Section 1, Sections 2–7 and Conclusion as Section 8.

Section 2 is devoted to the problems of kinds and methods of food packaging material quality assessment based on its factors such as permeability, thermal properties, tensile strength and mechanical properties changes, swelling and mass changes, and biofilm formation.

In Section 3, the general semi-Markov model of food packaging material quality change process is introduced and presented. The proposed process is described by defining the food packaging material quality states and fixing their number. Other parameters of the food packaging material quality change process introduced in this section are its initial probabilities at particular food packaging material quality states, the probabilities of transitions between the particular food packaging material quality states, the distribution functions, and the density functions of the conditional sojourn times at the food packaging material quality states and their mean values. Next, the basic characteristics of the food packaging material quality change process are determined, i.e., the unconditional mean values of the conditional sojourn times at the food packaging material quality states, the limit values of the food packaging material quality change process transient probabilities at the particular food packaging material quality states and the food packaging material quality change process total sojourn time at the particular food packaging material quality states during the fixed time.

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Section 4 is devoted to the safety of the consumption of packaged food products. There are introduced the notions of packaged food product consumption basic indicators such as: the packaged food product consumption safety function, the packaged food product consumption risk function, the packaged food product consumption fragility curve, the moment when the packaged food product consumption risk function exceeds a permitted level, the mean values of the packaged food product's lifetimes in the consumption safety state subsets and the intensities of a packaged food product's shelf life degradation, i.e. the intensities of the packaged food product's shelf life departure from the consumption safety state subsets.

In Section 5, safety and resilience indicators for the packaged food product consumption are proposed in the case that the packaged food product is impacted by its packaging quality. The safety of consumption of the packaged food product impacted by the food packaging material quality change process is considered. The safety and resilience indicators introduced in Section 4 are modified and the coefficients of the food packaging material quality impact on the packaged food product's shelf life degradation and the packaged food product's shelf life degradation resilience indicators, i.e. the coefficients of the packaged food product's shelf life degradation resilience to the food packaging material quality impact are defined. These packaged food product consumption safety and resilience indicators are determined for the packaged food product's lifetimes in the safety state subsets that have piecewise exponential safety functions.

The general approach to optimization of packaged food products' safety consumption is proposed in Section 6.

In Section 7, preliminary remarks and approaches are proposed for the identification of food packaging material quality and the packaged food products' safety consumption.

The conclusions are made about the context of the chapter, and the perspective for future research is formulated and applications in the field considered in this chapter are suggested in Section 8.

2. Quality assessment of food packaging material

The main function of the packaging is to protect the packed food against external factors such as light, dust, microorganisms, water and water vapour, odours, gases and mechanical damage. Maintaining the packaging protective functions is possible due to the appropriate physicochemical parameters of the packaging materials. Incorrectly selected packaging material may lead to physicochemical and microbiological changes (fat oxidation, lipolysis, proteolysis, bacterial growth, destruction of vitamins, changes in taste and smell) during food storage. Furthermore, changes in the properties of the packaging material during storage, caused by contact with the ingredients of the packed food, can reduce the quality of the product. The selected parameters of packaging materials, the behaviour of which is crucial for the shelf life of food, are presented below. Until now, changes in the physicochemical parameters of packaging materials have not been treated as indicators that describe the packaging-product interaction. However, the assessment of these parameters can be a useful tool for describing changes in the material.

2.1. Permeability

Water vapour high barrier properties are required for dry food products, whose texture could change under the influence of penetrating moisture. Furthermore, hydrated products with a long shelf life require packaging with a high value of the water vapour barrier. Products sensitive to oxidation require packaging with an appropriate barrier to oxygen. Packaging intended for specific food products should be selected individually, considering all characteristics of the product and the physicochemical properties of the packaging material. The penetration of molecules into the polymer matrix depends on the structure of the polymer. Food components can change the structure of polymers and hence their barrier to gases (Drieskens et al., 2009).

The permeability of the water vapour is determined by the weighing method with the use of a moisture analyzer. The sampler to determine the level of water vapour permeability is filled with water and closed with a sample of the tested material. This measurement is based on the precise determination of the loss of water mass from the sampler, which evaporates from the interior of the sampler through the tested packaging material. Water vapour penetration occurs as a result of an increase in vapour pressure as a result of increasing the water temperature inside the sampler. The permeability of the water vapour is expressed in $mg/cm^2 \cdot h$.

2.2. Thermal properties

Polymers form two-phase structures: amorphous and crystalline. The amorphous phase in the structure of polymers is responsible for the penetration of gas molecules. The probability of the appearance of “holes” in these areas, corresponding to the size of the diffusing particles, is greater than in the crystalline areas. Polymers with a higher content of the crystalline phase provide a better barrier to gases compared to polymers with a predominantly amorphous phase (Drieskens et al., 2009). The degradation of materials under the influence of ingredients in food products can cause a disturbance of the ordered structures of the material.

The thermal properties of polymeric materials are determined by differential scanning calorimetry. This technique allows determination of the crystallinity of the polymers, which may change during their degradation. The degree of crystallinity can be determined from the thermal effects of phase changes during the melting of the polymer and is expressed in J/g.

2.3. Mechanical properties

Packaging requires certain properties to fulfill its protective function. Changes in the mechanical parameters of packaging materials, such as tensile strength and elongation at break, can be indicators of changes in the structure of the material caused by interactions between the packaging material and the food product.

The contact with ingredients such as water, fats, acids and many chemical compounds (which are metabolites of microorganisms present on the surface of the product) can cause changes in packaging mechanical properties during storage. Moreover, in the case of using biodegradable materials, contact with the mentioned factors can cause degradation of the material during the product storage stage (Steinka et al.).

The measurement of the strength properties of packaging materials is most often based on a static tensile test and the measurement of the resulting forces and elongation at break. Tensile strength is expressed in MPa and elongation at break in %.

2.4. Swelling and mass changes

Polymer packaging materials are swollen when they absorb moisture or fat from packaged food. This phenomenon can cause many unfavourable changes in the properties of the packaging. During absorption, food ingredients can leach low-molecular-weight compounds contained in polymeric materials. If substances are extracted from the material that modify the mechanical properties of polymers (e.g. plasticizers) or substances added to protect them (e.g. antioxidants or UV stabilizers), the physicochemical properties of the material may change during storage. The soaking of packaging material with food ingredients can also cause delamination of multilayer materials, caused by leaching or degradation of adhesives (Rhim et al., 2007; Garrido-López et al., 2010).

Swelling and mass changes are determined by the weighing method. Both parameters are expressed as a percentage of the increase or loss of mass.

2.5. Biofilm formation

Microorganisms do not have the ability to penetrate the polymer structure. However, eluted low-molecular-weight compounds from the polymer structure can provide a carbon source, causing further multiplication and the formation of biofilms on the surface of the packaging material. When microbes multiply on the surface of packaging materials, they can produce pigments. Pigments that diffuse into the structure of the material may cause a colour change or a matting of the material. The formation of biofilms is determined by microbiological methods, and changes in colour or transparency are measured by the transmittance of the material (Fotopoulou & Karapanagioti, 2017). Transmittance is defined as the power of radiation passed through the sample in relation to the power of radiation incident on the sample and is expressed in % in the value from the interval $\langle 0\%, 100\% \rangle$.

3. Modelling food packaging material quality change process

To construct the food packaging material quality change process, the set of $n, n \in N$, kinds of food packaging material properties is distinguished and these kinds of properties are denoted by $m_1, m_2, \dots, m_n$.

This way the set

$$\mathcal{M} = \{m_1, m_2, \dots, m_n\}$$

is the set of kinds of food packaging material's properties. These properties mentioned above may attain different levels. Namely, the food packaging material's property m_i, $i = 1,2, \dots, n$, may reach l_i levels

$$m_{i1}, m_{i2}, \dots, m_{il_i}, i = 1,2, \dots, n,$$

that are called the states of this food packaging material's property.
The set

$$m_i = \{m_{i1}, m_{i2}, \dots, m_{il_i}\}, i = 1,2, \dots, n,$$

is called the set of states of food packaging material's property m_i.
Under these assumptions, the food packaging material quality change process is introduced as a vector

$$Q(t) = [m_1(t), m_2(t), \dots, m_n(t)], t \in \langle 0, +\infty),$$

where

$$m_i(t), t \in \langle 0, +\infty), i = 1,2, \dots, n,$$

are the particular food packaging material properties' change processes defined on the time interval $t \in \langle 0, +\infty)$ and having their values in the food packaging material property's state sets m_i, $i = 1,2, \dots, n$.
The vector

$$q_i = [b_1, b_2, \dots, b_n], \quad (1)$$

where

$$b_i = \begin{cases} 0, & \text{if a food packaging material's property } m_i \text{ is not changed or improved insignifcantly (is in the typical range),} \\ m_{ij}, & \text{if a food packaging material's property } m_i \text{ is changed and is in the untipical range } m_{ij}, \\ & j = 1,2,\dots,l_i \end{cases} \quad (2)$$

for $i = 1,2, \dots, n$, is called the food packaging material quality state. Further, the vectors that cannot occur may be eliminated and the remaining food packaging material quality states, defined by (1)–(2), are marked by q_k for $k = 1,2, \dots, v$, and the set is formed

$$Q = \{q_k, k = 1,2, \dots, v\}, \quad (3)$$

where

$$q_k \neq q_l, k \neq l, k, l \in \{1,2, \dots, v\}.$$

The set Q, given by (3) is called the set of food packaging material quality states, while v is called the number of food packaging material quality states.
A function

$$Q(t), t \in \langle 0, +\infty), \quad (4)$$

having values in the food packaging material quality states set Q given by (3) is called the food packaging material quality change process.
Next, a semi-Markov model of food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$ is assumed. Its random conditional sojourn time at the food packaging material quality state q_k while the next transition will be done to the state q_l, $k, l = 1,2, \dots, v$, $k \neq l$ is denoted by θ_{kl}. Then, the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$ is described by the following parameters that can be evaluated by experts or identified statistically using the methods given in (Bogalecka, 2020, 2021; Grabski, 2015; Iosifescu, 1980; Kołowrocki, 2014; Limnios & Oprisian, 2005; Smith, 1955):

- the vector

  $$[p_k(0)]_{1\text{x}v} = [p_1(0), p_2(0), \dots, p_v(0)], \quad (5)$$

  of initial probabilities,

  $$p_k(0) = P(Q(0) = q_k), k = 1,2, \dots, v, \quad (6)$$

  of the process $Q(t)$, $t \in \langle 0, +\infty)$ staying at its particular states at the moment $t = 0$, according to the formula

  $$p_k(0) = \frac{n_k(0)}{n(0)}, k = 1,2, \dots, v, \quad (7)$$

where

$$n(0) = \sum_{k=1}^{v} n_k(0), \tag{8}$$

is the number of the process $Q(t)$, $t \in \langle 0, +\infty)$ realizations starting at the initial moment $t = 0$,

- the matrix

$$[p_{kl}]_{v\mathrm{x}v} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1v} \\ p_{21} & p_{22} & \cdots & p_{2v} \\ \vdots & \vdots & \ddots & \vdots \\ p_{v1} & p_{v2} & \cdots & p_{vv} \end{bmatrix} \tag{9}$$

of realizations of probabilities p_{kl}, $k, l = 1,2, \dots, v$, of the process $Q(t)$, $t \in \langle 0, +\infty)$ transitions between the food packaging material quality states q_k and q_l, according to the formula

$$p_{kl} = \frac{n_{kl}}{n_k}, k, l = 1,2, \dots, v, k \neq l, \tag{10}$$

where by formal agreement

$$\forall k = 1,2, \dots, v, p_{kk} = 0,$$

and

$$n_k = \sum_{l \neq k}^{v} n_{kl}, k = 1,2, \dots, v, \tag{11}$$

is the realisation of the total number of the process $Q(t)$, $t \in \langle 0, +\infty)$ transitions from the state q_k during the experimental time,

- the matrix

$$[M_{kl}]_{v\mathrm{x}v} = \begin{bmatrix} M_{11} & M_{12} & \dots & M_{1v} \\ M_{21} & M_{22} & \dots & M_{2v} \\ \vdots & \vdots & \ddots & \vdots \\ M_{v1} & M_{v2} & \dots & M_{vv} \end{bmatrix} \tag{12}$$

of mean values M_{kl} of conditional sojourn times θ_{kl}

$$M_{kl} = E[\theta_{kl}] = \int_0^\infty t dH_{kl}(t)$$

$$= \int_0^\infty t h_{kl}(t) dt, \tag{13}$$

for

$$k, l = 1,2, \dots, v, k \neq l,$$

of the process $Q(t)$, $t \in \langle 0, +\infty)$ at the food packaging material quality state q_k, $k = 1,2, \dots, v$, when the next state is q_l, $k, l = 1,2, \dots, v$, where

$$H_{kl}(t) = P(\theta_{kl} < t), t \in \langle 0, +\infty), \tag{14}$$

for

$$k, l = 1,2, \dots, v, k \neq l$$

are given in the matrix

$$[H_{kl}(t)]_{v\mathrm{x}v}$$

$$= \begin{bmatrix} H_{11}(t) & H_{12}(t) & \dots & H_{1v}(t) \\ H_{21}(t) & H_{22}(t) & \dots & H_{2v}(t) \\ \vdots & \vdots & \ddots & \vdots \\ H_{v1}(t) & H_{v2}(t) & \dots & H_{vv}(t) \end{bmatrix} \tag{15}$$

of conditional distribution functions of sojourn times θ_{kl} of the process $Q(t)$, $t \in \langle 0, +\infty)$ in the food packaging material quality state q_k while the next transition will be done to the state q_l, where by formal agreement

$$\forall k = 1,2, \dots, v, H_{kk} = 0.$$

The matrix (15) is complied with the another one

$$[h_{kl}(t)]_{v\mathrm{x}v} = \begin{bmatrix} h_{11}(t) & h_{12}(t) & \dots & h_{1v}(t) \\ h_{21}(t) & h_{22}(t) & \dots & h_{2v}(t) \\ \vdots & \vdots & \ddots & \vdots \\ h_{v1}(t) & h_{v2}(t) & \dots & h_{vv}(t) \end{bmatrix} \tag{16}$$

of conditional densities of sojourn times θ_{kl} of the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$ in the food packaging material quality state q_k while the next transition will be done to the state q_l, $k, l = 1,2, \dots, v$, $k \neq l$,

$$h_{kl}(t) = \frac{dH_{kl}(t)}{dt}, t \in \langle 0, +\infty), \tag{17}$$

for

$$k, l = 1,2, \dots, v, k \neq l,$$

where by formal agreement

$$\forall k = 1,2, \dots, v, h_{kk} = 0.$$

Using above defined parameters of the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, the following its main characteristics can by predicted, using the procedure adopted from (Kołowrocki & Soszyńska-Budny, 2011). Namely, taking into account the formula for the total probability, the considered process $Q(t)$, $t \in \langle 0,+\infty)$, may be characterized by:

- the vector

$$[H_k(t)]_{1xv} = [H_1(t), H_2(t), \ldots, H_v(t)], \quad (18)$$

of unconditional distribution functions of sojourn times θ_k, $k = 1,2,\ldots,v$ at particular food packaging material quality states q_k of the process $Q(t)$, $t \in \langle 0,+\infty)$

$$H_k(t) = \sum_{l=1}^{v} p_{kl} H_{kl}(t), \; k = 1,2,\ldots,v, \quad (19)$$

where p_{kl} and $H_{kl}(t)$ are defined by (10) and (14) respectively,

- the vector

$$[h_k(t)]_{1xv} = [h_1(t), h_2(t), \ldots, h_v(t)], \quad (20)$$

of their corresponding density functions

$$h_k(t) = \sum_{l=1}^{v} p_{kl} h_{kl}(t), \; k = 1,2,\ldots,v, \quad (21)$$

where p_{kl} and $h_{kl}(t)$ are defined by (10) and (17) respectively,

- the vector

$$[M_k]_{1xv} = [M_1, M_2, \ldots, M_v], \quad (22)$$

of mean values of the process $Q(t)$, $t \in \langle 0,+\infty)$ unconditional sojourn times θ_k, $k = 1,2,\ldots,v$, at the particular food packaging material quality states calculated from the formula

$$M_k = E[\theta_k] = \sum_{l=1}^{v} p_{kl} M_{kl}, \quad (23)$$

for

$$k = 1,2,\ldots,v,$$

where p_{kl} and M_{kl} are defined by (10) and (13) respectively,

- the vector

$$[p_k]_{1xv} = [p_1, p_2, \ldots, p_v] \quad (24)$$

of limit values of transient probabilities

$$p_k(t) = P(Q(t) = q_k), \quad (25)$$

for

$$t \in \langle 0,+\infty), \; k = 1,2,\ldots,v,$$

of the process $Q(t)$, $t \in \langle 0,+\infty)$ at the food packaging material quality states q_k, $k = 1,2,\ldots,v$, calculated from the formula

$$p_k = \lim_{t\to\infty} p_k(t) = \frac{\pi_k M_k}{\sum_{l=1}^{v} \pi_l M_l}, \quad (26)$$

for

$$k = 1,2,\ldots,v,$$

where M_k, $k = 1,2,\ldots,v$, are given by (23), and the probabilities π_k, $k = 1,2,\ldots,v$, satisfy the system of equations

$$\begin{cases} [\pi_k] = [\pi_k][p_{kl}] \\ \sum_{l=1}^{v} \pi_l = 1 \end{cases} \quad (27)$$

where

$$[\pi_k] = [\pi_1, \pi_2, \ldots, \pi_v],$$

and $[p_{kl}]$ is given by (9).

Finally, other interesting characteristics of the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, possible to obtain is the vector

$$[\hat{M}_k]_{1xv} = [\hat{M}_1, \hat{M}_2, \ldots, \hat{M}_v], \quad (28)$$

of the total sojourn times $\hat{\theta}_k$, $k = 1,2,\ldots,v$, at the particular process's states for the sufficiently large time θ that have approximately normal distributions with the expected value given by

$$\hat{M}_k = E[\hat{\theta}_k] \cong p_k \theta, \quad (29)$$

where p_k are given by (26).

4. Safety consumption of packaged food – multi-state approach

The properties of packaging materials change with time as a result of interactions between packaging and products during storage. Thus, the safety of packaged food products depends on the food packaging material quality change process' states degrade with the time t, $t \in \langle 0, +\infty)$.

In the multi-state analysis of packaged food products' safety consumption with these products degrading consumption safety states, it is assumed that:

- the packaged food product has its consumption safety state set $\{0,1,\dots,z\}$, $z \geq 1$,
- the consumption safety states are ordered, and the safety state 0 means the worst one (the packaged food product does not fit to consumption) whereas the consumption safety state z is the best one (the packaged food product is free of threatening changes and it fits to consumption),
- r, $r \in \{1,2,\dots,z\}$, is the critical consumption safety state (the packaged food product is characterized by dangerous symptoms for consumer, it means that packaged food product staying in the consumption state less than this consumption state is highly dangerous for the food and its consumers),
- $T(u)$, $u = 1,2,\dots,z$, is a random variable representing the lifetime of the packaged food product in the consumption safety state subset $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, while it was in the consumption safety state z at the moment $t = 0$,
- the consumption safety states degrade with time t, $t \in \langle 0, +\infty)$,
- $s(t)$, $t \in \langle 0, +\infty)$, is the packaged food product consumption safety state at the moment t, $t \in \langle 0, +\infty)$, given that it was in the consumption safety state z at the moment $t = 0$.

The above assumptions mean that the properties of food packaging material and the consumption safety of the inside food products may be changed over time only from better to worse (Kołowrocki, 2004, 2014; Kołowrocki & Soszyńska-Budny, 2011; Xue, 1985; Xue & Yang, 1995 a, b) as it is illustrated in Figures 1–2.

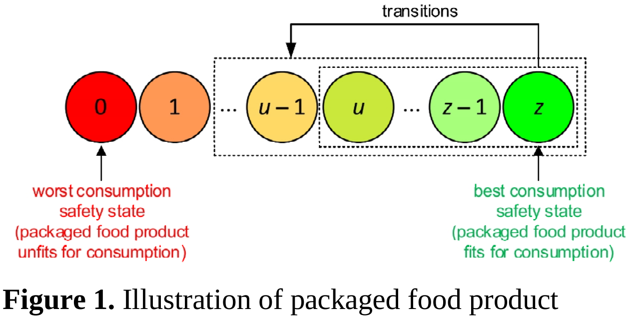

Figure 1. Illustration of packaged food product consumption safety states changing.

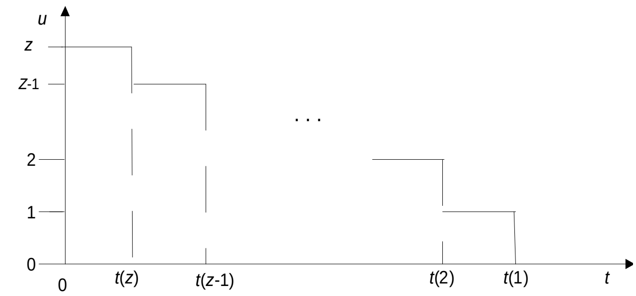

Figure 2. Relationship between realizations $t(u)$, $u = 1,2,\dots,z$, of packaged food product lifetime $T(u)$, $u = 1,2,\dots,z$, in consumption safety state subsets $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$.

The packaged food product unconditional lifetime in the consumption safety state subset $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, is denoted by $T(u)$ and the packaged food product consumption safety function is defined by the vector (Kołowrocki & Soszyńska-Budny, 2011)

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1),\dots,\boldsymbol{S}(t,z)],\ t \in \langle 0,+\infty), \quad (30)$$

where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u | s(0) = z)$$

$$= P(T(u) > t), \quad (31)$$

for

$t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$,

is the probability that the packaged food product is in the consumption safety state subset $\{u, u+1,\dots z\}$, $u = 1,2,\dots,z$, at the moment t, $t \in \langle 0,+\infty)$, while it was in the best packaged food product consumption safety state z at the moment $t = 0$ ($\boldsymbol{S}(t,0) = 1$).

The packaged food product consumption safety functions $\boldsymbol{S}(t,u)$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, defined by (31) are called the coordinates of the packaged food product consumption safety function $\boldsymbol{S}(t,\cdot)$, $t \in \langle 0,+\infty)$, given by (30). Thus, the relationship between the distribution function $\boldsymbol{F}(t,u)$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, of the packaged food product lifetime $T(u)$, $u = 1,2,\dots,z$, in the consumption safety state subset $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, and defined by (31) the coordinate $\boldsymbol{S}(t,u)$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, of its consumption safety function (30) is given by

$$\boldsymbol{F}(t,u) = P(T(u) \le t) = 1 - P(T(u) > t)$$

$$= 1 - \boldsymbol{S}(t,u),$$

for

$t \in \langle 0,+\infty), u = 1,2,\dots,z.$

The exemplary graph of a three-state ($z = 2$) packaged food product consumption safety function

$$\boldsymbol{S}(t,\cdot) = [1, \boldsymbol{S}(t,1), \boldsymbol{S}(t,2)],$$

where

$t \in \langle 0,+\infty),$

is given in Figure 3.

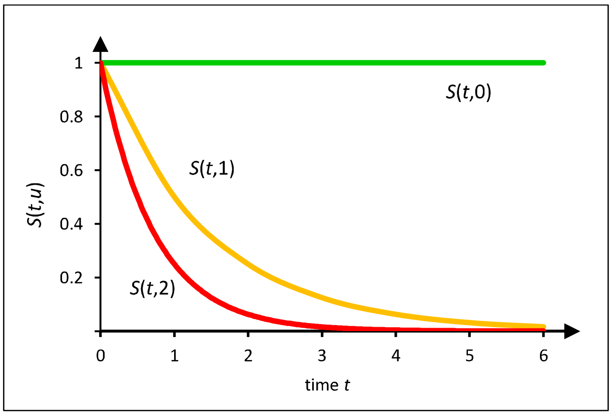

Figure 3. Graphs of packaged food product three-state consumption safety function $\boldsymbol{S}(t,\cdot)$ coordinates.

If r is the critical consumption safety state, then the packaged food product consumption risk function

$$\boldsymbol{r}(t) = P(s(t) < r | s(0) = z)$$

$$= P(T(r) \le t), t \in \langle 0,+\infty), \quad (32)$$

is defined as the probability that the packaged food product is in the subset of consumption safety states worse than the critical consumption safety state r, $r \in \{1,\dots,z\}$, while it was in the best consumption safety state z at the moment $t = 0$, and given by

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r), t \in \langle 0,+\infty), \quad (33)$$

where $\boldsymbol{S}(t,r)$, $t \in \langle 0,+\infty)$, is the coordinate of the packaged food product consumption safety function given by (31) for $u = r$.

The graph of the exemplary packaged food product consumption risk function $\boldsymbol{r}(t)$, $t \in \langle 0,+\infty)$, defined by (33), also called the fragility curve (Gouldby et al., 2010), is given in Figure 4.

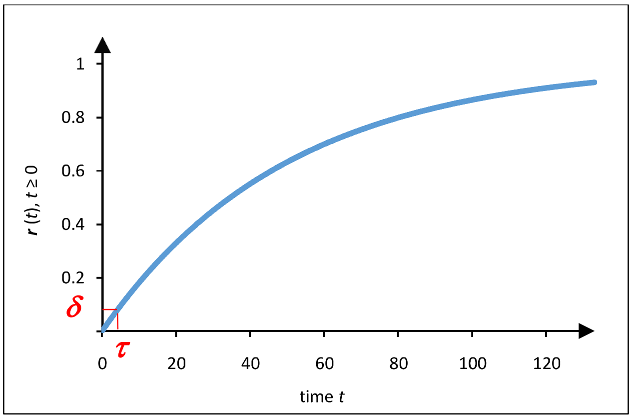

Figure 4. Graph of the exemplary packaged food product consumption risk function $\boldsymbol{r}(t)$, so-called fragility curve.

The moment $\boldsymbol{\tau}$, when the packaged food product consumption risk function exceeds a permitted level δ, $\delta \in (0,1)$, is defined by

$$\boldsymbol{\tau} = \boldsymbol{r}^{-1}(\delta), \quad (34)$$

where $\boldsymbol{r}^{-1}(t)$, $t \in \langle 0,+\infty)$, is the inverse function of the risk function $\boldsymbol{r}(t)$, $t \in \langle 0,+\infty)$, given by (33).

The mean values of the packaged food product's lifetimes in the consumption safety state subsets $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, are defined by

$$\boldsymbol{\mu}(u) = \int_0^\infty [\boldsymbol{S}(t,u)]dt, u = 1,2,\dots,z, \quad (35)$$

where $\boldsymbol{S}(t,u)$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, are the coordinates of the packaged food product consumption safety function (30) given by (31).

The intensities of a packaged food product's shelf life degradation, i.e. the intensities of the packaged food product's shelf life departure from the consumption safety state subsets $\{u, u+1,\dots,z\}$, $u = 1,2,\dots,z$, are defined by

$$\boldsymbol{\lambda}(t,u) = \frac{-\frac{d\boldsymbol{S}(t,u)}{dt}}{\boldsymbol{S}(t,u)}, t \in \langle 0,+\infty), u = 1,2,\dots,z, (36)$$

where $\boldsymbol{S}(t,u)$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, are the coordinate of the packaged food product consumption safety function (30) given by (31).

In the case, when the packaged food product's lifetimes in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, have piecewise exponential safety functions, i.e. the coordinates $\boldsymbol{S}(t,u)$, $t \in \langle 0, +\infty)$, $u = 1,2, \dots, z$, of the packaged food product consumption safety function (31) are given by

$$\boldsymbol{S}(t,u) = \exp[-\boldsymbol{\lambda}(u)t], \ t \in \langle 0, +\infty),$$

$$\boldsymbol{\lambda}(u) \geq 0, \ u = 1,2, \dots, z, \qquad (37)$$

where $\boldsymbol{\lambda}(u)$, $u = 1,2, \dots, z$, are the intensities of a packaged food product's shelf life degradation, i.e. the intensities of the intensities of the packaged food product's shelf life departure from the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, the above packaged food product consumption safety indicators, defined by (32)–(36), take the following forms:

- the packaged food product consumption risk function

  $$\boldsymbol{r}(t) = 1 - \exp[-\boldsymbol{\lambda}(r)t], \ t \in \langle 0, +\infty), \qquad (38)$$

  where

  $$\boldsymbol{\lambda}(r) \geq 0,$$

- the moment $\boldsymbol{\tau}$, when the packaged food product consumption risk function exceeds a permitted level δ, $\delta \in (0,1)$

  $$\boldsymbol{\tau} = -\frac{1}{\boldsymbol{\lambda}(r)} ln(1-\delta), \qquad (39)$$

- the mean values of the packaged food product's lifetimes in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$,

  $$\boldsymbol{\mu}(u) = \frac{1}{\boldsymbol{\lambda}(u)}, \ u = 1,2, \dots, z, \qquad (40)$$

- the intensities of a packaged food product's shelf life degradation, i.e. the intensities of the packaged food product's shelf life departure from the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$,

  $$\boldsymbol{\lambda}(u) = \frac{1}{\boldsymbol{\mu}(u)}, \ u = 1,2, \dots, z, \qquad (41)$$

  where $\boldsymbol{\mu}(u)$, $u = 1,2, \dots, z$, are the mean values of the packaged food product's lifetimes in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, given by (40).

Furthermore, in the case where the packaged food product's shelf life is impacted by outside conditions, the following resilience indicators can be defined:

- the coefficients of the outside conditions impact on the packaged food product's shelf life degradation

  $$\boldsymbol{\rho}(t,u) = \frac{\boldsymbol{I\lambda}(u)}{\boldsymbol{\lambda}(u)} = \frac{\boldsymbol{\mu}(u)}{\boldsymbol{I\mu}(u)}, \ u = 1,2, \dots, z, \qquad (42)$$

  where $\boldsymbol{\lambda}(u)$ and $\boldsymbol{I\lambda}(u)$, $u = 1,2, \dots, z$, are the intensities of packaged food product's shelf life degradation without and with outside impacts respectively, determined according to (36) and (55) or (41) and (64) respectively, as well as $\boldsymbol{\mu}(u)$ and $\boldsymbol{I\mu}(u)$, $u = 1,2, \dots, z$, are the mean values of the packaged food product's lifetimes in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, without and with outside impacts, determined according to (35) and (53) or (40) and (61) respectively,
- the packaged food product's shelf life degradation resilience indicators, i.e. the coefficients of the packaged food product's shelf life degradation resilience to its outside impacts

  $$\boldsymbol{RI}(t,u) = \frac{1}{\boldsymbol{\rho}(t,u)}, \qquad (43)$$

  for

  $$t \in \langle 0, +\infty), \ u = 1,2, \dots, z,$$

  where $\boldsymbol{\rho}(t,u)$, $u = 1,2, \dots, z$, are the coefficients of the impact on the packaged food product's shelf life degradation defined by (42).

5. Safety and resilience consumption of packaged food impacted by packaging material properties

The packaged food product's conditional lifetime in the consumption safety state subset $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, while the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, is at the food packaging material quality state q_k, $k = 1,2, \dots, v$, is denoted by

$[T(u)]^{(k)}$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$, and the conditional packaged food product consumption safety function related to the process $Q(t)$, $t \in \langle 0,+\infty)$, by the vector

$$[\boldsymbol{S}(t,\cdot)]^{(k)}$$

$$= [1, [\boldsymbol{S}(t,1)]^{(k)}, [\boldsymbol{S}(t,2)]^{(k)}, \ldots, [\boldsymbol{S}(t,z)]^{(k)}], \quad (44)$$

for

$t \in \langle 0,+\infty)$, $k = 1,2,\ldots,v$,

with the coordinates defined by

$$[\boldsymbol{S}(t,u)]^{(k)} = P([T(u)]^{(k)} > t | Q(t) = q_k), \quad (45)$$

for

$t \in \langle 0,+\infty)$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$.

The safety function $[\boldsymbol{S}(t,u)]^{(k)}$, $t \in \langle 0,+\infty)$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$, defined by (45) is the conditional probability that the packaged food product's conditional lifetime $[T(u)]^{(k)}$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$, in the consumption safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, is greater than t, $t \in \langle 0,+\infty)$, while the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, is at the food packaging material quality state q_k, $k = 1,2,\ldots,v$.
Next, the unconditional lifetime in the consumption safety state subset $\{u, u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, of the packaged food product impacted by the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, is denoted by $IT(u)$, $u = 1,2,\ldots,z$, and the unconditional packaged food product consumption safety function impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$, by the vector

$$\boldsymbol{IS}(t,\cdot) = [\boldsymbol{IS}(t,1), \boldsymbol{IS}(t,2), \ldots, \boldsymbol{IS}(t,z)], \quad (46)$$

for

$t \in \langle 0,+\infty)$,

with the coordinates defined by

$$\boldsymbol{IS}(t,u) = P(IT(u) > t), \quad (47)$$

for

$t \in \langle 0,+\infty)$, $u = 1,2,\ldots,z$.

In the case when the packaged food product storage time T in the fixed package is large enough, the coordinates of the unconditional packaged food product consumption safety function related to the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, defined by (4), are evaluated by

$$\boldsymbol{IS}(t,u) \cong \sum_{k=1}^{v} p_k [\boldsymbol{S}(t,u)]^{(k)}, \quad (48)$$

for

$t \in \langle 0,+\infty)$, $u = 1,2,\ldots,z$,

where $[\boldsymbol{S}(t,u)]^{(k)}$, $t \in \langle 0,+\infty)$, $u = 1,2,\ldots,z$, $k = 1,2,\ldots,v$, are the coordinates of the conditional packaged food product consumption safety function related to the process $Q(t)$, $t \in \langle 0,+\infty)$, defined by (44)–(45) and p_k, $k = 1,2,\ldots,v$, at the process $Q(t)$, $t \in \langle 0,+\infty)$, limit transient probabilities at the food packaging material quality states q_k, $k = 1,2,\ldots,v$, given by (47).
If r is the critical consumption safety state, then the packaged food product consumption safety indicator impacted by the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, the packaged food product consumption risk function

$$\boldsymbol{Ir}(t) = P(s(t) < r | s(0) = z)$$

$$= P(IT(r) \le t), \; t \in \langle 0,+\infty), \quad (49)$$

is defined as a probability that the packaged food product impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$ is in the subset of consumption safety states worse than the critical consumption safety state r, $r \in \{1,\ldots,z\}$, while it was in the best packaged food product consumption safety state z at the moment $t = 0$ and given by

$$\boldsymbol{Ir}(t) = 1 - \boldsymbol{IS}(t,r), \; t \in \langle 0,+\infty), \quad (50)$$

where $\boldsymbol{IS}(t,r)$, $t \in \langle 0,+\infty)$, is the coordinate of the unconditional packaged food product consumption safety function related to the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$, given by (48) for $u = r$.

The graph of the packaged food product consumption risk function $\boldsymbol{Ir}(t)$, $t \in \langle 0, +\infty)$, defined by (50), is the packaged food product consumption safety indicator called the fragility curve of the packaged food product impacted by the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$.

Other practically useful safety and resilience indicators of packaged food product consumption safety impacted by the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, are:

- the moment of exceeding an acceptable value of the packaged food product, impacted by the process $Q(t)$, $t \in \langle 0, +\infty)$, consumption risk function level δ given by

$$\boldsymbol{I\tau} = \boldsymbol{Ir}^{-1}(\delta), \tag{51}$$

where $\boldsymbol{Ir}^{-1}(t)$, $t \in \langle 0, +\infty)$, is the inverse function of the packaged food product consumption risk function $\boldsymbol{Ir}(t)$, $t \in \langle 0, +\infty)$, given by (50),

- the mean values of the packaged food product's, impacted by the process $Q(t)$, $t \in \langle 0, +\infty)$, lifetimes in the consumption safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2, \ldots, z$, given by

$$\boldsymbol{I\mu}(u) = \int_0^\infty [\boldsymbol{IS}(t,u)]dt$$

$$\cong \sum_{k=1}^{v} p_k [\boldsymbol{\mu}(u)]^{(k)}, \; u = 1,2, \ldots, z, \tag{52}$$

where $[\boldsymbol{\mu}(u)]^{(k)}$, $u = 1,2, \ldots, z$, $k = 1,2, \ldots, v$, are the mean values of packaged food product's conditional lifetimes $[T(u)]^{(k)}$, $u = 1,2, \ldots, z$, $k = 1,2, \ldots, v$, in the consumption safety state subsets $\{u, u+1, \ldots, z\}$ at the food packaging material quality state q_k, $k = 1,2, \ldots, v$, are given by

$$[\boldsymbol{\mu}(u)]^{(k)} = \int_0^\infty [\boldsymbol{S}(t,u)]^{(k)} dt, \tag{53}$$

for

$u = 1,2, \ldots, z$, $k = 1,2, \ldots, v$,

and $[\boldsymbol{S}(t,u)]^{(k)}$, $t \in \langle 0, +\infty)$, $u = 1,2, \ldots, z$, $k = 1,2, \ldots, v$, are defined by (45) and p_k, $k = 1,2, \ldots, v$, are given by (26),

- the mean values $\boldsymbol{I\bar{\mu}}(u)$, $u = 1,2, \ldots, z$, of the packaged food product's, impacted by the process $Q(t)$, $t \in \langle 0, +\infty)$, lifetimes in the particular consumption safety states are given by

$$\boldsymbol{I\bar{\mu}}(u) = \boldsymbol{I\mu}(u) - \boldsymbol{I\mu}(u+1), \tag{54}$$

for

$u = 0,1, \ldots, z-1$,

and

$\boldsymbol{I\bar{\mu}}(z) = \boldsymbol{I\mu}(z)$,

where $\boldsymbol{I\mu}(u)$, $u = 1,2, \ldots, z$, are given by (52),

- the intensities of packaged food product's shelf life degradation of the packaged food product, impacted by the process $Q(t)$, $t \in \langle 0, +\infty)$ / the intensities of the packaged food product impacted by the process $Q(t)$, $t \in \langle 0, +\infty)$ departure from the consumption safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2, \ldots, z$,

$$\boldsymbol{I\lambda}(t,u) = \frac{-\frac{d\boldsymbol{IS}(t,u)}{dt}}{\boldsymbol{IS}(t,u)}, \tag{55}$$

for

$t \in \langle 0, +\infty)$, $u = 1,2, \ldots, z$,

- the coefficients of the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, impact on the packaged food product's shelf life intensities of degradation / the coefficients of the process $Q(t)$, $t \in \langle 0, +\infty)$ impact on the packaged food product's shelf life intensities of departure from the consumption safety state subsets $\{u, u+1, \ldots, z\}$, $u = 1,2, \ldots, z$,

$$\boldsymbol{I\rho}(t,u) = \boldsymbol{I\lambda}(t,u)/\boldsymbol{\lambda}(t,u), \tag{56}$$

for

$t \in \langle 0, +\infty)$, $u = 1,2, \ldots, z$,

where $\boldsymbol{\lambda}(t,u)$, $t \in \langle 0, +\infty)$, $u = 1,2, \ldots, z$, are the intensities of degradation of the packaged food product's shelf life without the impact of the process $Q(t)$, $t \in \langle 0, +\infty)$ defined by (36) and $\boldsymbol{I\lambda}(t,u)$, $t \in \langle 0, +\infty)$, $u = 1,2, \ldots, z$, are the intensities of degradation of the packaged food product's shelf life impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$, defined by (55),

- the resilience indicators of the packaged food product's shelf life to the process $Q(t)$, $t \in \langle 0,+\infty)$ impact are defined by

$$\boldsymbol{IRI}(t,u) = 1/\boldsymbol{I\rho}(t,u), \quad (57)$$

for

$$t \in \langle 0,+\infty), u = 1,2,\dots,z,$$

where $\boldsymbol{I\rho}(t,u)$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, are the coefficients of the process $Q(t)$, $t \in \langle 0,+\infty)$ impact on the packaged food product's shelf life intensities of degradation given by (56).

In the case when the fixed packaged food product's conditional lifetimes in the consumption safety state subsets $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$, have piecewise exponential consumption safety functions, i.e. the coordinate $[\boldsymbol{S}(t,u)]^{(k)}$, $t \in \langle 0,+\infty)$, $u = 1,2,\dots,z$, defined by (45), of the packaged food product consumption safety function (44) are given by

$$[\boldsymbol{S}(t,u)]^{(k)} = \exp[-[\boldsymbol{\lambda}(u)]^{(k)}t], [\boldsymbol{\lambda}(u)]^{(k)} \geq 0,$$

for

$$t \in \langle 0,+\infty), u = 1,2,\dots,z, k = 1,2,\dots,v,$$

where $[\boldsymbol{\lambda}(u)]^{(k)}$, $u = 1,2,\dots,z$, $k = 1,2,\dots,v$, are the packaged food product's shelf life impacted by the food packaging material quality change process $Q(t)$, $t \in \langle 0,+\infty)$ intensities of degradation at the food packaging material quality states q_k, $k = 1,2,\dots,v$, the above defined indicators take the following forms:

- the coordinates of the unconditional packaged food product consumption safety function of the packaged food product impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$,

$$\boldsymbol{IS}(t,u) \cong \sum_{k=1}^{v} p_k \exp[-[\boldsymbol{\lambda}(u)]^{(k)}t], \quad (58)$$

for

$$t \in \langle 0,+\infty), u = 1,2,\dots,z,$$

- the packaged food product consumption risk function

$$\boldsymbol{Ir}(t) \cong 1 - \sum_{k=1}^{v} p_k \exp[-[\boldsymbol{\lambda}(r)]^{(k)}t], \quad (59)$$

for

$$t \in \langle 0,+\infty),$$

- the moment of exceeding an acceptable value of the packaged food product, impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$, consumption risk function level δ given by the value of the inverse to risk function $\boldsymbol{Ir}(t)$, given by (59) for $t = \delta$

$$\boldsymbol{I\tau} =$$

$$(1 - \sum_{k=1}^{v} p_k \exp[-[\boldsymbol{\lambda}(r)]^{(k)}(\cdot)])^{-1}(\delta), \quad (60)$$

- the mean values of the packaged food product's, impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$, lifetimes in the consumption safety state subsets $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$, are given by

$$\boldsymbol{I\mu}(u) \cong \sum_{k=1}^{v} \frac{p_k}{[\lambda(u)]^{(k)}}, u = 1,2,\dots,z, \quad (61)$$

- the mean value $\boldsymbol{I\bar{\mu}}(u)$, $u = 1,2,\dots,z$, of the packaged food product's, impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$, lifetimes in the particular consumption safety states are given by

$$\boldsymbol{I\bar{\mu}}(u) = \sum_{k=1}^{v} p_k \cdot \left(\frac{1}{[\lambda(u)]^{(k)}} - \frac{1}{[\lambda(u+1)]^{(k)}}\right), \quad (62)$$

for

$$u = 0,1,\dots,z-1,$$

and

$$\boldsymbol{I\bar{\mu}}(z) = \sum_{k=1}^{v} \frac{p_k}{[\lambda(z)]^{(k)}}, \quad (63)$$

- the intensities of packaged food product's shelf life degradation impacted by the process $Q(t)$, $t \in \langle 0,+\infty)$,

$$\boldsymbol{I\lambda}(t,u) = \frac{\sum_{k=1}^{v} p_k [\lambda(u)]^{(k)} \exp[-[\lambda(u)]^{(k)}t]}{\sum_{k=1}^{v} p_k \exp[-[\lambda(u)]^{(k)}t]}, \quad (64)$$

for

$t \in \langle 0, +\infty), u = 1,2, \dots, z,$

- the coefficients of the process $Q(t)$, $t \in \langle 0, +\infty)$ impact on the packaged food product's shelf life intensities of degradation

$$\boldsymbol{I\rho}(t,u) = \frac{\sum_{k=1}^{v} p_k [\boldsymbol{\lambda}(u)]^{(k)} \exp[-([\boldsymbol{\lambda}(u)]^{(k)})t]}{\boldsymbol{\lambda}(u) \sum_{k=1}^{v} p_k \exp[-[\boldsymbol{\lambda}(u)]^{(k)} t]}, \quad (65)$$

for

$t \in \langle 0, +\infty), u = 1,2, \dots, z,$

where $\boldsymbol{\lambda}(u)$, $u = 1,2, \dots, z$, are the intensities of packaged food product's shelf life degradation without impact of the process $Q(t)$, $t \in \langle 0, +\infty)$,

- the resilience indicators of the packaged food product's shelf life to the process $Q(t)$, $t \in \langle 0, +\infty)$ impact are defined by

$$\boldsymbol{IRI}(t,u) = \frac{\boldsymbol{\lambda}(u) \sum_{k=1}^{v} p_k \exp[-[\boldsymbol{\lambda}(u)]^{(k)} t]}{\sum_{k=1}^{v} p_k [\boldsymbol{\lambda}(u)]^{(k)} \exp[-([\boldsymbol{\lambda}(u)]^{(k)})t]}, \quad (66)$$

for

$t \in \langle 0, +\infty), u = 1,2, \dots, z,$

where $\boldsymbol{\lambda}(u)$, $u = 1,2, \dots, z$, are the intensities of packaged food product's shelf life degradation without impact of the process $Q(t)$, $t \in \langle 0, +\infty)$.

6. Optimizing consumption safety of packaged food

Considering the packaged food product consumption safety function $\boldsymbol{IS}(t,\cdot)$, $t \in \langle 0, +\infty)$, defined by (46)–(47) and related to the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, coordinate given by (48), it is natural to assume that the process $Q(t)$, $t \in \langle 0, +\infty)$, has a significant influence on the packaged food product safety. This influence is also expressed by the equation (52) for the mean values of the packaged food product's lifetimes in the consumption safety state subsets. From the linear equation (52), it can be seen that the mean value of the packaged food product's lifetime $\boldsymbol{I\mu}(u)$, $u = 1,2, \dots, z$, in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, is determined by the limit values of transient probabilities p_k, $k = 1,2, \dots, v$, of the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, at their states q_k, $k = 1,2, \dots, v$, and the mean values $[\boldsymbol{I\mu}(u)]^{(k)}$, $u = 1,2, \dots, z$, $k = 1,2, \dots, v$, of the packaged food product's conditional lifetimes in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, at these food packaging material quality states. Therefore, the packaged food product's lifetime optimization approach based on the linear programming can be proposed (Klabjan, 2006). Namely, it can be looked for the corresponding optimal values $\dot{p}_k$, $k = 1,2, \dots, v$, of the transient probabilities p_k, $k = 1,2, \dots, v$, of the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, at the state q_k, $k = 1,2, \dots, v$, to maximize the mean value $\boldsymbol{I\mu}(u)$ of the packaged food product's lifetime in the consumption safety state subsets $\{u, u+1, \dots, z\}$, $u = 1,2, \dots, z$, under the assumption that the mean values $[\boldsymbol{I\mu}(u)]^{(k)}$, $k = 1,2, \dots, v$, $u = 1,2, \dots, z$, of packaged food product's conditional lifetimes in the consumption safety state subsets are fixed. As a special case of the above formulated packaged food product's lifetime optimization problem, if r, $r = 1,2, \dots, z$, is a critical consumption safety state, the optimal values $\dot{p}_k$, $k = 1,2, \dots, v$, of the transient probabilities p_k, $k = 1,2, \dots, v$, of the food packaging material quality change process $Q(t)$, $t \in \langle 0, +\infty)$, at its states are wanted to be found to maximize the mean value $\boldsymbol{I\mu}(r)$ of the packaged food product's lifetime in the consumption safety state subset $\{r, r+1, \dots, z\}$, $r = 1,2, \dots, z$, under the assumption that the mean values $[\boldsymbol{I\mu}(r)]^{(k)}$, $k = 1,2, \dots, v$, of the packaged food product's conditional lifetimes in this consumption safety state subset are fixed. The optimization problem is formulated more exactly as a linear programming model with the objective function of the following form

$$\boldsymbol{I\mu}(r) = \sum_{k=1}^{v} p_k [\boldsymbol{I\mu}(r)]^{(k)}, \quad (67)$$

for a fixed $r \in \{1,2, \dots, z\}$, and with the following bound constraints

$$\check{p}_k \le p_k \le \hat{p}_k, \sum_{k=1}^{v} p_k = 1, \quad (68)$$

where

$$[\boldsymbol{I\mu}(r)]^{(k)},\ [\boldsymbol{I\mu}(r)]^{(k)} \geq 0,\ k = 1,2,\dots,v, \quad (69)$$

are fixed mean values of the packaged food product's conditional lifetimes in the consumption safety state subset $\{r, r+1, \dots, z\}$, and

$$\check{p}_k,\ 0 \leq \check{p}_k \leq 1, \quad (70)$$

and

$$\hat{p}_k,\ 0 \leq \hat{p}_k \leq 1, \quad (71)$$

where

$$\check{p}_k \leq \hat{p}_k,\ k = 1,2,\dots,v,$$

are lower and upper bounds of the unknown transient probabilities p_k, $k = 1,2,\dots,v$, respectively. Now, the optimal solution of the linear programming problem formulated by (67)–(71) can be obtained, i.e. the optimal values $\dot{p}_k$ of the transient probabilities p_k, $k = 1,2,\dots,v$, that maximize the objective function given by (67) can be found. The maximizing procedure is described in (Kołowrocki & Magryta, 2020; Magryta-Mut, 2020).

Finally, after applying this procedure, the maximum value of the packaged food product's total mean lifetime in the consumption safety state subset $\{r, r+1, \dots, z\}$, defined by the linear form (67), can be got in the following form

$$\dot{\boldsymbol{I\mu}}(r) = \sum_{k=1}^{v} \dot{p}_k [\boldsymbol{I\mu}(r)]^{(k)}, \quad (72)$$

for a fixed $r \in \{1,2,\dots,z\}$.

Further, by replacing the limit transient probabilities p_k, $k = 1,2,\dots,v$, existing in the formulae (48) by their optimal values $\dot{p}_k$, $k = 1,2,\dots,v$, the optimal form of the packaged food product consumption safety is got and the expressions for all remaining safety indicators considered in Section 4 as well.

7. Application

Going into details and considering findings given in Section 2–3, the general approach to modelling the food packaging material quality change process, the quality of the packaging material during contact with the packed food is periodically assessed based on $n = 3$ types of factors of the physicochemical properties of the packaging material:

m_1 – permeability [g/(m^2·24 h)],
m_2 – tensile strength [MPa],
m_3 – swelling [%].

These factors are selected to evaluate changes in the properties of packaging materials after contact with food products. Tensile strength, swelling, and water vapour permeability are easy to control, and the measuring techniques are uncomplicated. The ranges of variability of particular factors are specific according to the type of polymer used for the packaging material. Taking into account experts' opinion, each factor may reach $l_i = 3$, $i = 1,2,3$ levels as it is shown in Table 1.

Table 1. Levels of factors of physicochemical packaging material properties

Factor m_i, $i = 1,2,3$	Level of factors of physicochemical material properties		
	$l_1 = 1$ (typical)	$l_2 = 2$ (disturbed)	$l_3 = 3$ (exceeded)
m_1 (permeability)	$\langle 0, X)$	$\langle X, 1.4X)$	$\langle 1.4X, +\infty)$
m_2 (tensile strength)	$\langle Y, +\infty)$	$\langle 0.4Y, Y)$	$\langle 0.4Y, 0\rangle$
m_3 (swelling)	$\langle 0, Z)$	$\langle Z, 1.1Z)$	$\langle 1.1Z, +\infty)$

X, *Y*, *Z* express the value of particular factors at the moment $t = 0$. It is possible that values of *X*, *Y*, *Z* improved insignificantly on the first days of food contact with packaging material.

Considering data from Table 1 and (1)–(3), the number of food packaging material quality states is

$$v = 3 \cdot 3 \cdot 3 = 27,$$

and they are numbered as follows

$$q_1 = [0,0,0],\ q_2 = [0,0,1],\ \dots,\ q_{27} = [2,2,2].$$

8. Conclusion

This chapter presents the first general approach to investigating the influence of time-changing properties of packaging materials on the safety of the consumption of packaged food products, based on mathematical modelling. Presented theoretical tools are devoted to statistical identification, prediction and optimization of consumption safety of packaged foods.

Further practical identification of the investigated relations using the proposed model and real data could be possible as the next steps of the research with respect to the selected packaging materials. The authors believe that this approach can provide a useful and valuable theoretical tool to analyse the safety of food packaging materials and determining the consumer's safety.

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