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Approximating curve by a single segment of B-Spline or Bézier curve directly in CAD environment

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Abstract

The paper presents the method of approximating curves with a single segment of the B-Spline and Bézier curves. The method for determining a single curve segment using the optimization methods in the CATIA environment is shown. The algorithms of simulated annealing and design of experiment are used for optimization. For the same purpose, a new original procedure for determining the distance between the given curves using explicit parameters in the CATIA environment was also used. This approximation of the cyclic curves results in the curve oscillation as shown in the examples. The results show that the approximation method with Bézier curve using control points as "free" points can be applied to obtain the best results of approximation.

Keywords

Bézier curve, curve approximation, design of experiment, algorithms of simulated annealing

1. Introduction

Among all curves implemented in CAD environment, the Bézier curve is characterized by the best quality of division, hence it is often used in CAD systems and in engineering applications [1]. This curve is recommended for CAD modeling when the curve is divided into segments [2]. The sum of the segments is geometrically equivalent to the curve before the division. This is especially important in industrial practice [3]. It happens that geometry is imported from other CAD systems than currently used or is created as a different type of curve. Therefore, some curves should be replaced with Bézier curves, which are used for further modeling. The indication of the accuracy of the approximation is the distance between the curves, which should be as small as possible. It should be mentioned that certain curves can be approximated by Bézier curves with some error (e.g., circle arc) [4–7]. The methods of automatic curve conversion implemented in the CAD environment do not always give a satisfactory result. This is the result of the assumption of maintaining the directions of tangent vectors at the ends of the approximated curve [2]. The user does not have access to the mathematical model of the curve in the CAD environment, so another solution should be used [8].

The article presents the methodology for approximating the curve by the Bézier curve using optimization methods implemented in the CATIA environment, such as the simulated annealing algorithm, design of experiment, and the "mixed" method. The optimization consists of such matching of Bézier curve that the distance between the curves is as small as possible. The optimization uses the explicit procedure for determining the distance of curves in the CATIA environment proposed by the authors because the procedure of curves distance analysis implemented in the CAD environment can not be used due to its implicit nature.

A single curve segment in the CAD environment is most often a geometric representation of the third-degree polynomials. Depending on the limitations, it assumes the mathematical form of the Hermite curve or the Bézier curve (Figure 1) [1–3]. In both cases, the four conditions are required to describe the curve segment. For the Hermite curve, there are two points (begin and end point) and two vectors tangent at the starting and ending points. For the Bézier curve, there are four control points, where $P_{1\text{ and }}P_2$ define the beginning and the end of the segment and the sections P_1P_2 and P_3P_4 determine the directions of tangents at the beginning and the end of the segment.

In the CATIA environment, in the generative shape design (GSD) module, there is available a spline in form similar to Hermite, where a starting and ending point, senses and magnitudes of the tangential vectors at these points and additionally sides and radii of curvature can be specified (Figure 2a).

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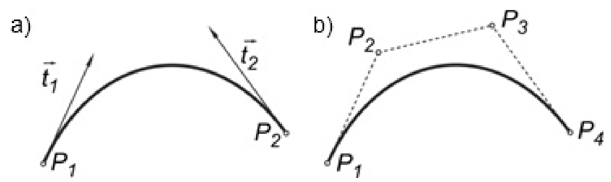

Figure 1. The Hermite curve (a) and the Bézier curve (b).

Additionally, there is possibility to select the surface or the plane, where the spline has to be modeled. In the FreeStyle module, the spline is available in the form of the Bézier curve (3D curve) with an active option of creating with the control points (Figure 2b).

2. Curve Approximation Method

The method of curve approximating by the spline is presented on the example of the circle arc approximation. In the CATIA environment, a circular arc with a radius of 100 mm, an angle of 90°, a center at point of 0.0.0 lying in the *x,y* plane was created (Figure 3).

Two parameters of a real-number type named *t*1 and *t*2 were created and initial values *t*1 =1 and *t*2 = 1 were assigned. These are the initially assumed values of the tangent vectors at the beginning and the end of the spline, respectively.

In the GSD module the spline was created, which the beginning is one of the circle arc end, and the second one is the other end of the circle arc (Figure 4). At the start and the end point, the tangential vector of the spline was determined by selecting tangency of the circle arc as a defining element (type: From curve), and the vector magnitudes (tangent tension) were determined by parameters respectively at the beginning of the spline *t*1 and at the end of *t*2. The parameter relations *t*1and *t*2 to spline tangents appeared in the model tree (Figure 5).

The task with the curve approximating consists of selecting such magnitudes of the tangent vectors of the spline, at the beginning and the end, that the spline approximates the curve as accurate as possible. The maximum distance of approximating spline to the curve can be a measure of approximation accuracy. In the CATIA environment, there is a tool to measure the distance of curves (distance analysis), but the measurement result is not an evident parameter

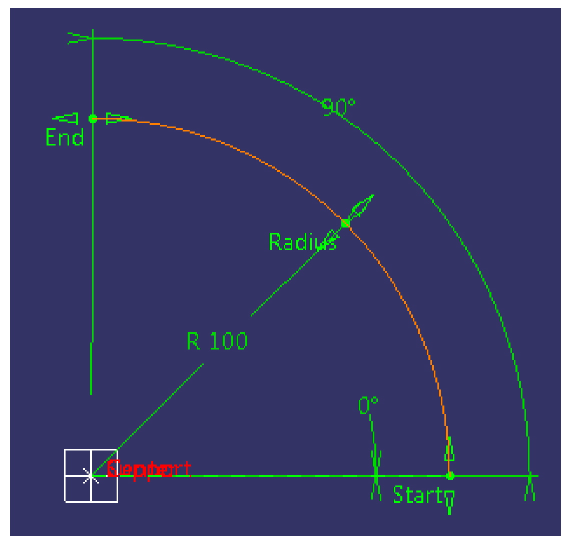

Figure 3. **Circular arc that will be approximated with the spline.**

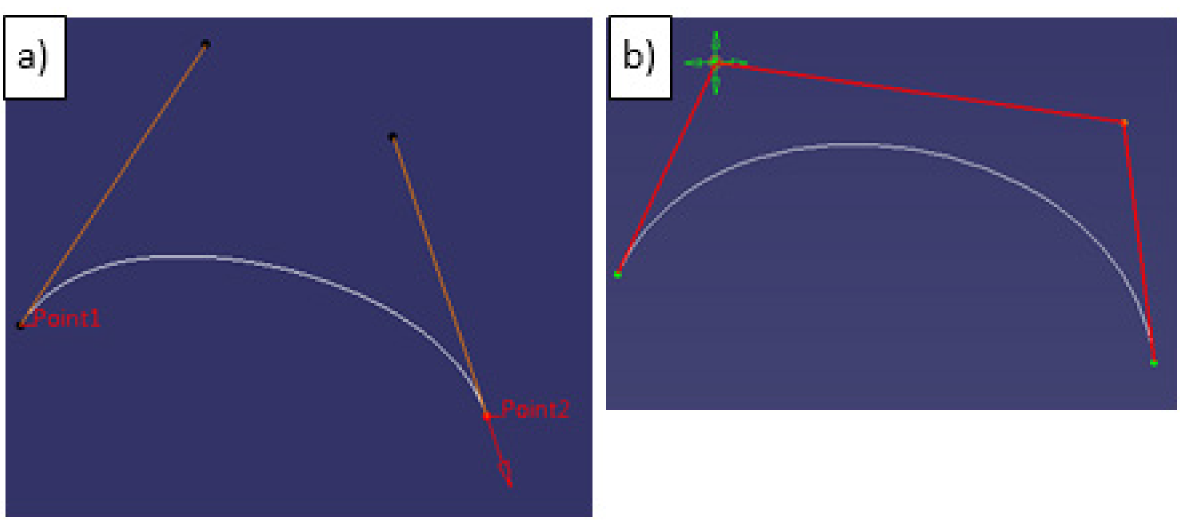

Figure 2. Single curve segments: (a) spline in the GSD module and (b) Bézier curve in the free-style module.

(that can be used by the user as a parameter). Therefore, it is necessary to prepare a new, specific measurement procedure where the maximum curves distance can be parameterized. The procedure requires repeating operations, therefore a programming language can be used, but it is not shown here. Distances measurement should be discretized. The points, which the distances are measured, are created with the Points and Planes Repetition command (Figure 6). In the considered example, the division into 10 segments was adopted, which requires to use 9 points. If the programming language is used, the number of points can be much greater. At each point dividing the approximated curve, a normal plane to this curve must be additionally created (the active option: Create normal planes also). The measurement of the distance from the approximating curve is realized in these planes.

Afterwards, the points that are created as a result of intersection of the spline with the created planes should be determined. If the option Create in a new body was selected in the previous command, points and planes were created in the new geometrical set (Figure 7). That allows selecting all planes in the Intersection command by choosing the geometrical set where they are located (Figure 8).

The distances between points in the individual planes should be described by variables. For this purpose, the distance parameters (length) are created from *l1* to *l9*, which the distance measurement formulas are given as: distance (point on the approximated curve, point on the approximating curve). The relevant pairs of points lying in the same planes are indicated in brackets. The task is to minimize the distance; hence, it is needed to choose the largest value from among specified parameters *l1*÷*l9*. For this purpose, a distance parameter *lmax* is created with assigned formula (Figure 9):

$$max(l1/1mm,l2/1mm,l3/1mm,l4/1mm,l5/1mm,l6/1mm,l7/1mm,l8/1mm,l9/1mm)*1mm$$

As the max function is in operation only for real-type parameters, the values of parameters *l1* ÷*l9* are divided by 1 mm, and finally the maximum value is multiplied by 1 mm to give back the length unit.

Now, by changing the values of parameters *t1* and *t2* (magnitudes of tangent vectors), it is possible to change the maximum distance between curves. This distance should be minimized. This can be realized using the optimization command in the Product Engineering Optimizer module (Figure 10).

The minimization has to be selected as the type of optimization, the *lmax* variable as the optimized parameter, *t1* and *t2* as parameters influencing the value *lmax*. For parameters *t1* and *t2*, their range can be specified. The simulated annealing

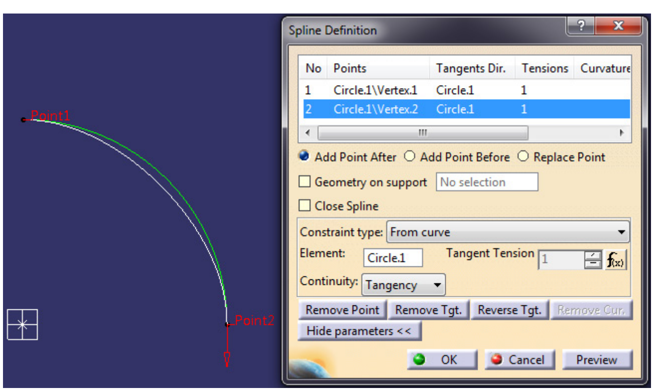

Figure 4. Defining the spline.

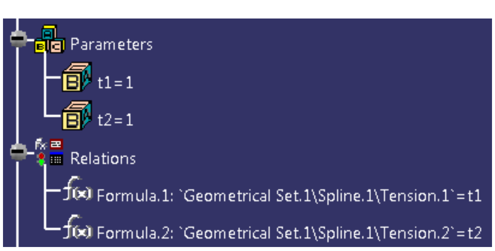

Figure 5. A fragment of the model tree in the CATIA environment with the assumed parameters t1 and t2 and their relations with the tangent vectors.

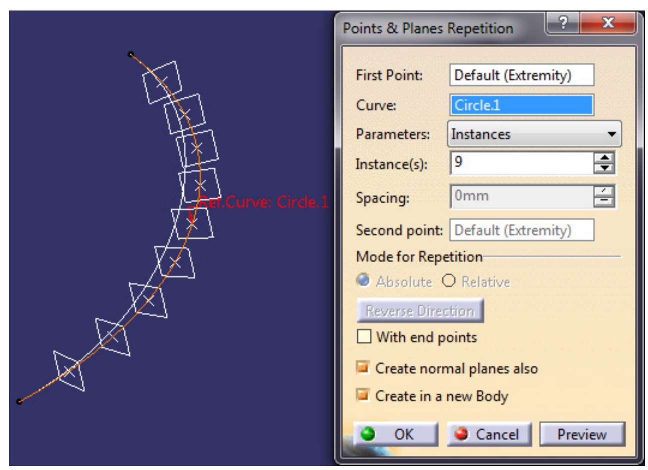

Figure 6. The points and planes creation.

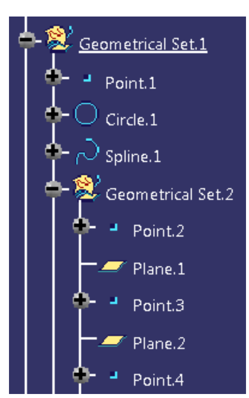

Figure 7. A fragment of the model tree with points and planes in a new geometrical set.

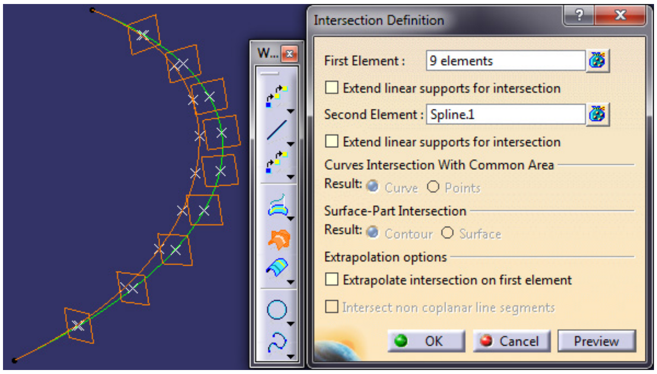

Figure 8. Determination of points as intersection result of the spline with a set of planes.

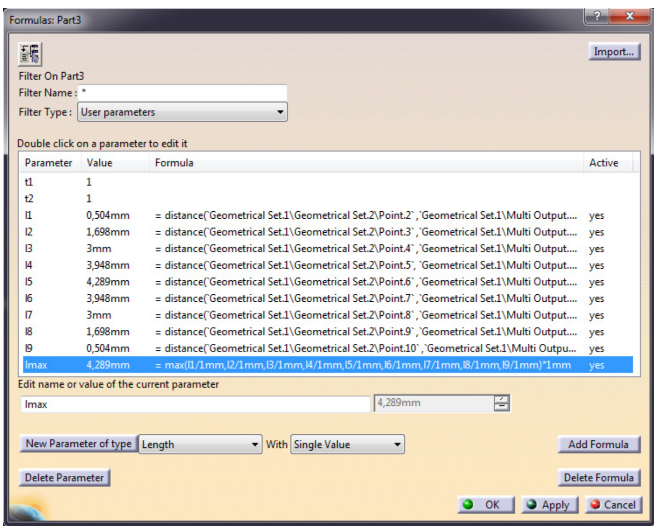

Figure 9. Created parameters and formulas.

algorithm was chosen. Because this algorithm may be unstable, the criteria for interrupting the calculation are assumed: the maximum number of updates 200, the number of consecutive updates without improvements 50, the maximum calculation time 5 minutes. After determining the optimization parameters, it can be run. After performing calculations in the computations results tab, the optimization results in the form of a list are available. The solutions that are closest to the expected one should be chosen from the list. Sometimes, it is not the last solution on the list; hence, it is convenient to use the Excel spreadsheet. If the save optimization, data option was selected before optimization, the program asked for a file name where the results are saved in the Excel format. In this file, it is possible to sort by *lmax* and select the smallest value *lmax*, read parameter values for *t1* and *t2*. This is the first approximation but not the most appropriate that can be obtained. At this stage, it is possible to limit the change range of *t1* and *t2* and then perform the optimization again.

The following results were obtained for the approximated arc: *t1*= 1.168367213, *t2*= 1.168367209. The values are very

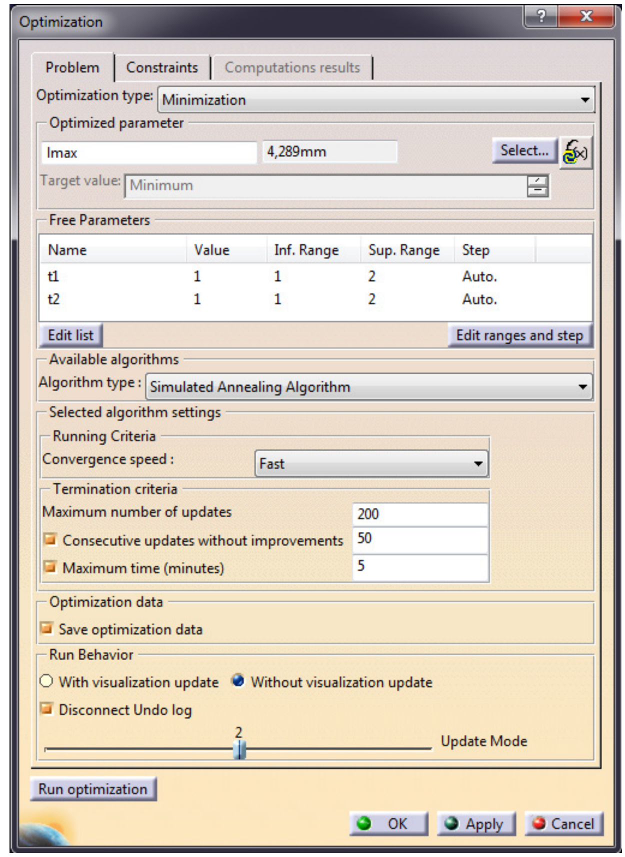

Figure 10. The distance lmax minimization.

similar (the circle arc is symmetrical). In this case, it was possible to use one parameter *t1* = *t2* , however, in the case of asymmetrical curves, they should be independent.

A comparison of both curves is shown in Figure 11. The distance analysis tool was used. The maximum distance between the curves is 0.081 mm. To visualize the distance, comb box analysis with displayed envelope was activated.

The spline oscillation is seen. This is due to the fact that the spline is a parametric representation of the curve with third-degree polynomials. This shows that the circle cannot be approximated very precisely with a spline. Unfortunately, this note also applies to other cyclic curves such as involute, extended involute, cycloid, and so on. Therefore, some curves in CAD environments are normally represented in a different way than with parametric cubic curves.

In a similar way, the curve segment can be approximated by the Bézier curve. In this case, two parameters d_1 and d_2 as the length type should be created. Then at the beginning and the end of the approximated curve, the tangent segment with the appropriate lengths d_1 and d_2 should be modeled (Figure 12). The control points P_2 and P_3 are the endpoints of these segments.

Having the points P_1, P_2, P_3, P_4 it is possible to create a Bézier curve segment (3D Curve command in the FreeStyle module).

The type of creation by the control points should be chosen and appropriate points should be selected (Figure 13).
The curve approximation is realized in a similar way to that shown earlier for the B-Spline curve. The variable parameters in this case are the lengths of the tangent segments d_1 and d_2. Figure 14 shows the comparison of a circle fragment with an approximating Bézier curve. In this case, the oscillation of the approximation curve also occurred.
The Design of Experiment (DOE) method can be used to determine the curve approximation by a spline. This method is beneficial if there are more variables affecting the result. The method will be shown on the example of approximation of a circle segment by the Bézier curve. This time the points are not fixed with tangent straight lines at the beginning and the end of the curve. It is assumed that the coordinates of points P_2 and P_3 are the variables.
In the CATIA environment, the variables *x*2, *y*2, *x*3, *y*3 are created and they are corresponding to the coordinates of the points P_2 and P_3 (Figure 15). The values of variables *x*2, *y*2, *x*3, *y*3 are initially assumed. Control points P_2 and P_3 are created on the plane where the section of the approximated circle was located (Figure 16).
Having the points P_1, P_2, P_3, P_4 the Bézier curve segment is created similarly as in the previous example (3D Curve command in the FreeStyle module). The type of creation by the control points should be chosen and the appropriate points should be selected.
After the Bézier curve was created, the matching procedure

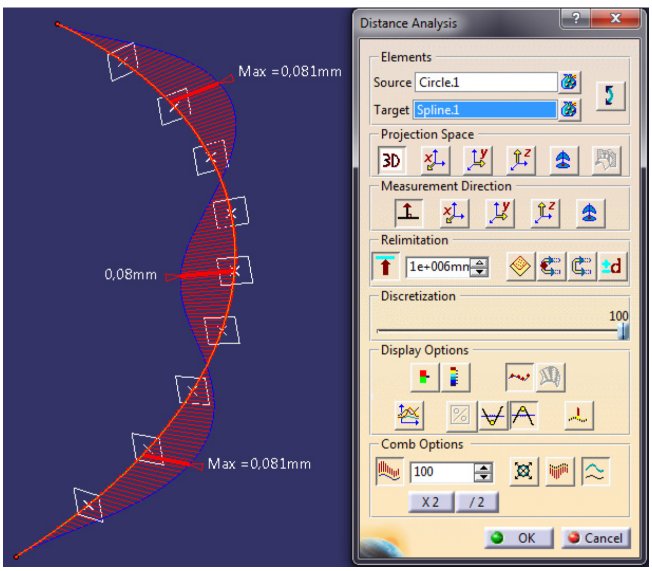

Figure 11. Analysis of the curves distance.

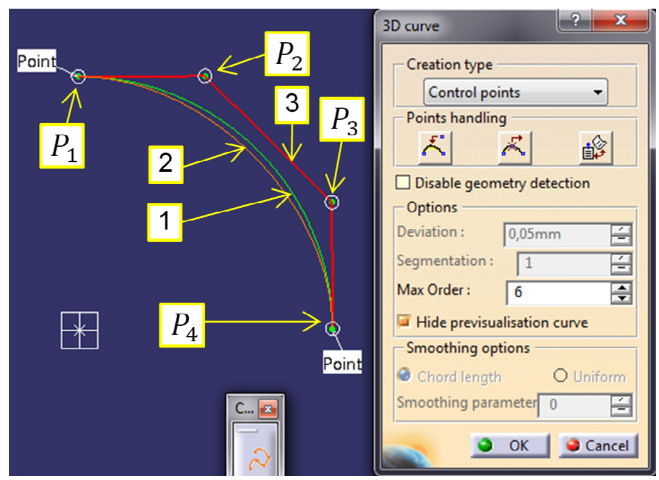

Figure 13. The Bézier curve creating; 1—approximated circular arc, 2—Bézier curve, and 3—Bézier polygon.

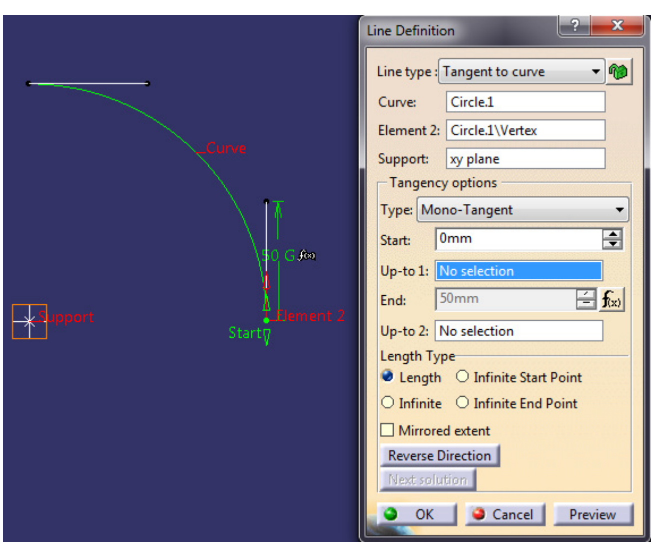

Figure 12. Tangent segment to the curve.

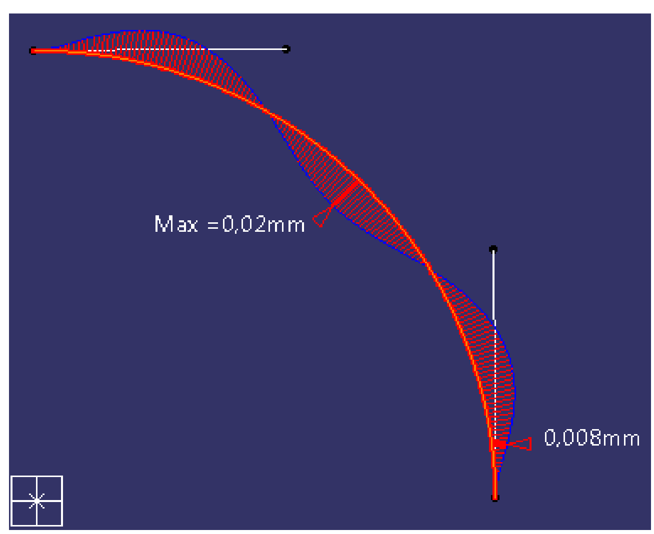

Figure 14. Comparison of the circle arc with the approximating Bézier curve.

is identical to the B-Spline curve up to the optimization stage. A set of planes which intersect curves is created. Variables corresponding to the distance of points in individual planes and a formula determining the maximum distance of lmax from among are defined.
To determine the parameters values *x*2, *y*2, *x*3, *y*3, the design method using the DOE experiment (Design of Experiment,

Figure 15. Variables corresponding to coordinates of control points P_2 and P_3

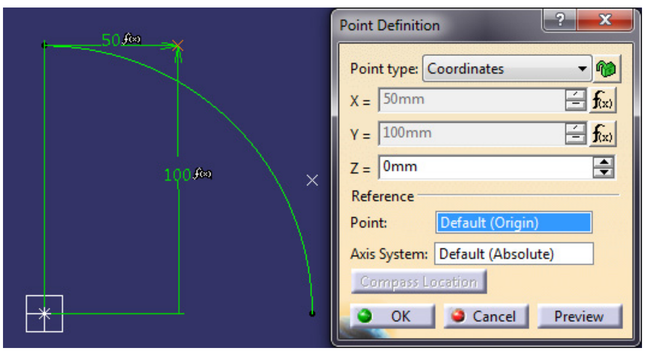

Figure 16. Creating a control point.

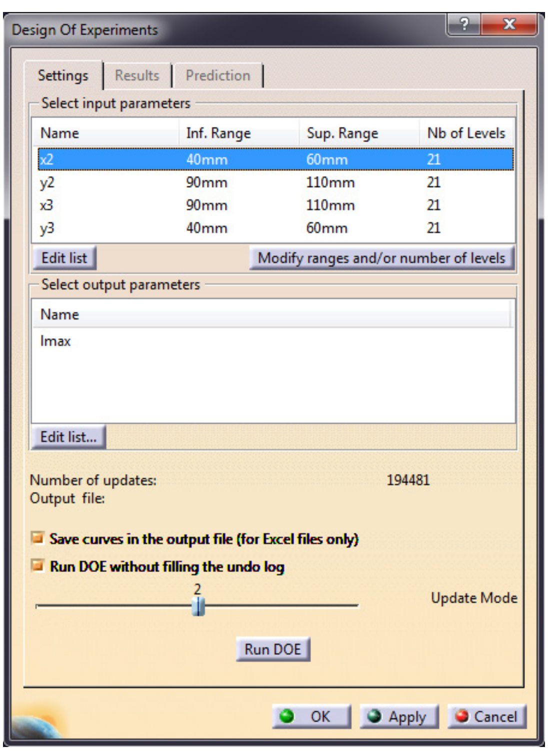

Figure 17. **Design of Experiment (DOE).**

Product Engineering Optimizer module) should be used. The input parameters are the values of coordinates *x*2, *y*2, *x*3, *y*3 (Figure 17). For each coordinate, the minimum value (Inf. Range), the maximum (Sup. Range), and the number of levels (Nb of Levels) are declared. The number of levels defines the discretization step. The lmax distance is the output parameter. After starting the experiment (Run DOE), a table of all possible combinations of dimensions is created. The obtained table in Excel format can be sorted according to the lmax column. Then the values of the parameters *x*2, *y*2, *x*3, *y*3 for the smallest lmax can be read. Because many solutions are created (in the shown example for 21 levels for each variable *x*2, *y*2, *x*3, *y*3 194481 combinations are generated), it is strongly advised to assume not to many levels in this step. After determining the first approximation, the initial data can be modified by narrowing the search.
The results *x*2 = 55.29, *y*2 = 99.92, *x*3 = 99.92, *y*3 = 55.29 were obtained. This means that the points P_2 and P_3 do not lie on the lines that are tangent to the circle arc at the beginning and the end. The Bézier curve in this case approximates the circle arc with a smaller error, but the number of oscillations increased (Figure 18).

3. Examples of Curve Approximation

As examples of approximation the involute and fillet curve were presented. These curves can be found in the flank profile of gears (Figure 19).
Involute and fillet curve were created by parametric equations directly in the CAD environment [9, 10]. Modeling with parametric equations in CATIA is not the subject of this article, so it is not presented.

3.1. An example of involute approximation (B-Spline)

The involute was created for the data: module m_n = 5 mm, number of teeth *z* = 20, pressure angle α_n = 20°, spur gear.

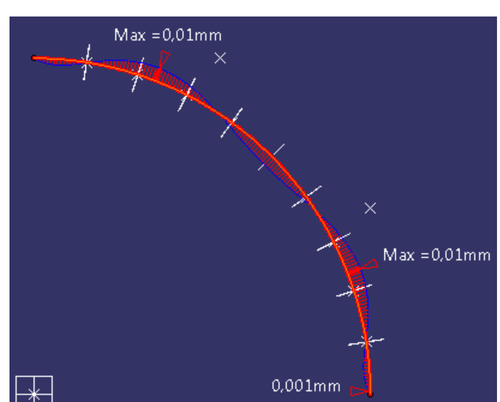

Figure 18. Comparison of the circle arc with the approximating Bézier curve.

The involute was approximated by a single spline segment. The values of tangent vectors were determined: at the beginning of involute at the basic diameter *t1* = 0.386, on the addendum diameter: *t2* = 1.568 (Figure 20). The distance analysis of the involute and approximated curve was executed (Figure 21).

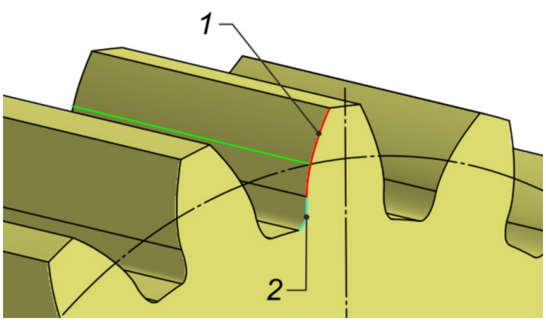

Figure 19. Gear flank profile; 1—involute and 2—fillet curve.

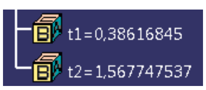

Figure 20. Values of tangent vectors in the model tree.

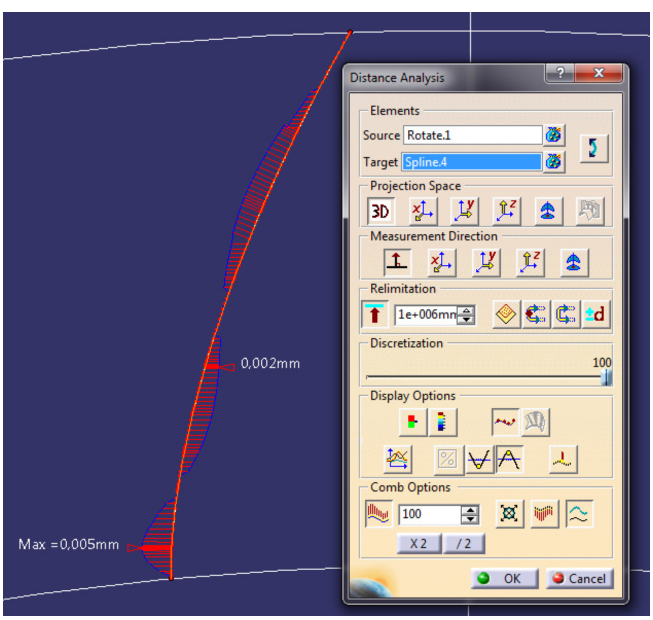

Figure 21. The distance analysis of the involute and approximated curve.

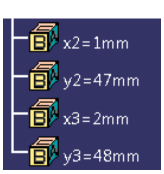

Figure 22. Coordinates of points P2 and P3 in the model tree.

The comparison shows that the maximum distance is ca. 0.005 mm, so in the industrial practice it is not satisfactory. Oscillations of the spline are visible. In order to increase accuracy, the involute should be approximated by several segments of the spline.

3.2. An example of fillet curve approximation (Bézier curve)

The fillet curve was modeled in the CATIA environment using the parametric equation of the extended involute in the plane *x*,*y*. To approximate fillet curve with the Bézier curve, the variables *x*2, *y*2, *x*3, *y*3 corresponding to the coordinates of the control points P_2 and P_3 were created and the initial values were assigned (Figure 22). The starting and ending point of the previously created curve is the control point P_1 and P_4 of the Bézier curve, respectively.

Then the points with these coordinates were created (Figure 23).

Having the points P_1, P_2, P_3, P_4 the Bézier curve segment (3D Curve command in the FreeStyle module) was created. The type of creation by the control points was chosen and the corresponding points were selected (Figure 24).

On the approximated fillet curve, a discrete set of nine points was created with planes perpendicular to the curve. The Bézier curve was intersected with these planes and a set of points was obtained (Figure 25).

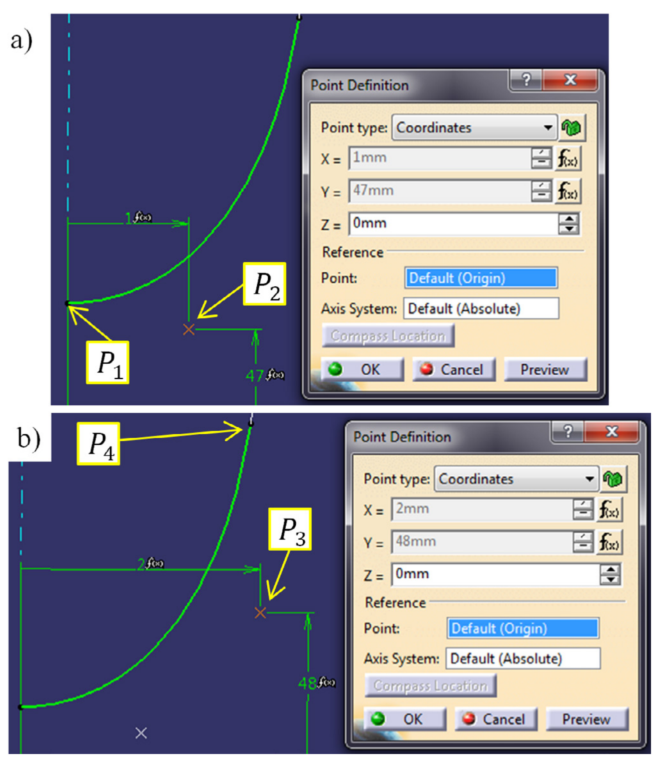

Figure 23. Creating points: (a) P_2 and (b) P_3.

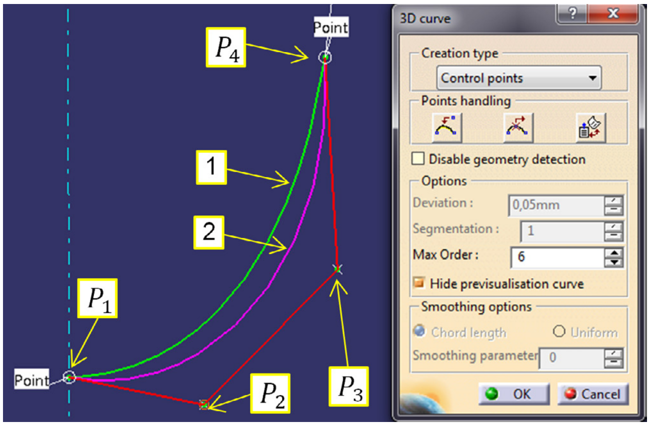

Figure 24. Modeling of the Bézier curve; 1—approximated fillet curve and 2—Bézier curve.

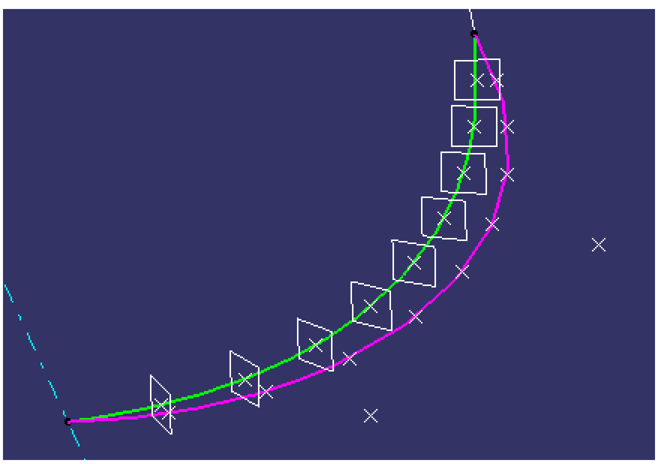

Figure 25. The points preparation for the distance measuring; 1—approximated fillet curve and 2—Bézier curve.

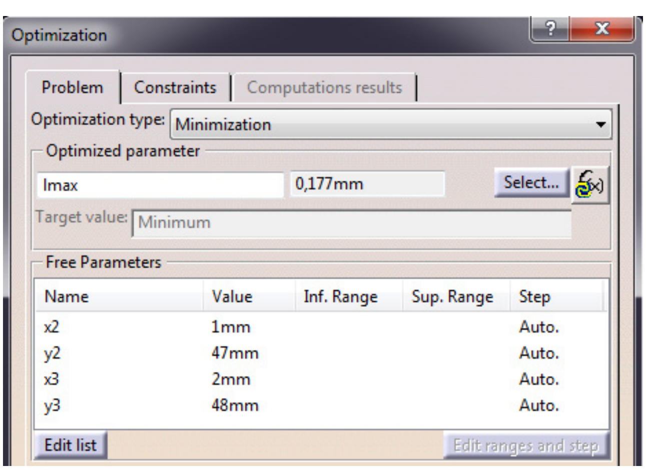

Figure 26. A fragment of the optimization window.

Variables l1÷ l9 were created and the formulas for the distance points measuring were assigned. Next the formula determining the maximum distance was defined (lmax). Pre-values of the parameters *x*2, *y*2, *x*3, *y*3 were determined using the simulated annealing algorithm (Figure 26).

Figure 27 shows preliminary results of the optimization.

Next step of the approximation was made using the DOE method, assuming appropriate values ranges of variables *x*2, *y*2, *x*3, *y*3 near the values determined in the previous stage (Figure 28).

Figure 29 shows a fragment of an Excel spreadsheet with sorted results. It is seen that for the assumed number of levels the same maximum distance of curves was obtained for the first five sets of parameters. Values of parameters were changed every 0.01 mm. Figure 30 shows the comparison of

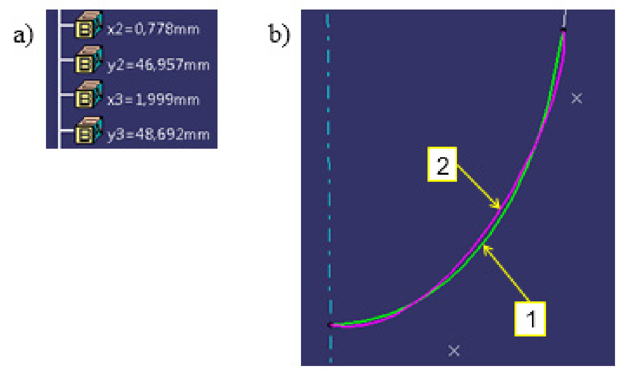

Figure 27. Preliminary results of the optimization: (a) parameters in the model tree and (b) curves: 1—fillet and 2—Bézier.

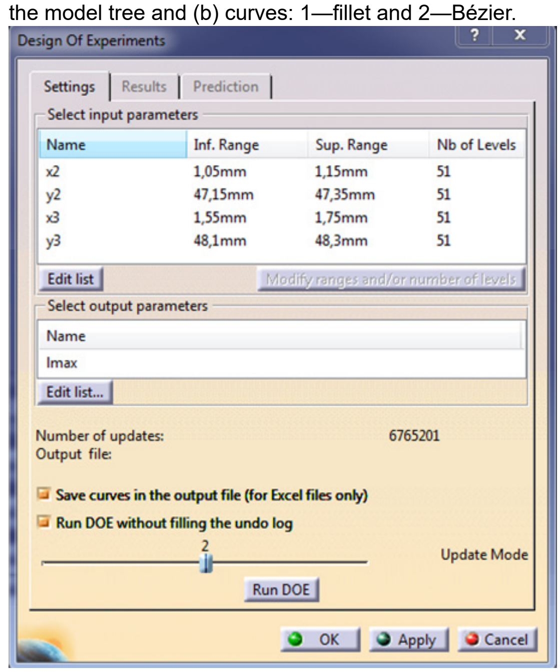

Figure 28. The preparation of the experiment.

	A	B	C	D	E	F	G
1	`Nb Eval`	x2 (mm)	y2 (mm)	x3 (mm)	y3 (mm)	Imax (mm)	
2	8999	1,05	47,23	1,71	48,22	0,002	0
3	9000	1,06	47,23	1,71	48,22	0,002	0
4	10332	1,07	47,23	1,71	48,24	0,002	0
5	11664	1,08	47,23	1,71	48,26	0,002	0
6	12997	1,1	47,23	1,71	48,28	0,002	0
7	6216	1,05	47,23	1,69	48,18	0,003	0
8	7548	1,06	47,23	1,69	48,2	0,003	0
9	10331	1,06	47,23	1,71	48,24	0,003	0

Figure 29. A fragment of an Excel spreadsheet with sorted results.

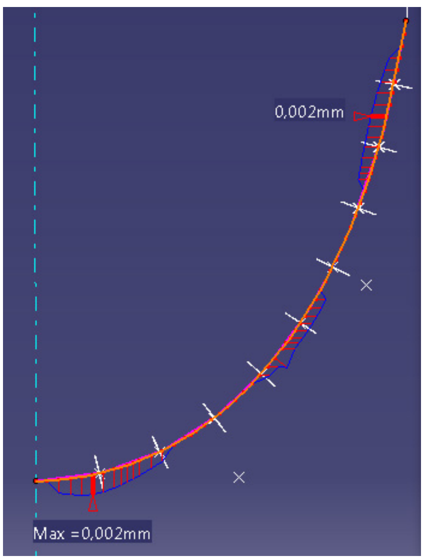

Figure 30. Comparison of the fillet curve and the approximating Bézier curve.

the fillet curve and approximating Bézier curve.

4. Conclusions

The presented examples show that to approximate the curve, the approximating method with Bézier curve using control points P_2 and P_3 as "free" points can be applied. That means that it should be assumed that the points are not located on the tangents at the beginning and the end of the approximate curve.

The approximation of involute or fillet curve with the single segment curve should not be used if very high accuracy is needed. Industrial practice indicates that the accuracy of the profile modeling in a CAD environment should be greater than obtained in machining process. The accuracy in CAD system should be at least one order of magnitude greater. Thus, this approximation can be used in CAD demonstrative models. The presented method of involute curves modeling by B-Spline or Bézier curves in the CAD environment causes the curve's oscillation. This is due to the fact that the spline function is described by the third-degree polynomial. To increase accuracy, the approximated curve should be divided into segments. It should be mentioned that oscillation will always occur, but the amplitude will be adequately lower. In workshop practice, the fillet curve as well as the involute should be divided into several segments. In the example under consideration, accuracy of 0.001 mm was achieved by dividing the curve for involutes into four segments and for the fillet curve into nine segments, respectively. The continuity of curve contact must be ensured between these segments.

References

[1] G. FARIN: Curves and surfaces for computer aided geometric design: a practical guide. 3rd ed. Academic Press, Boston, MA 1993.

[2] H. PRAUTZSCH, W. BOEHM, M. PALUSZNY: Bézier and B-spline techniques. Springer-Verlag, Heidelberg 2002.

[3] H.N. FITTERA, A.B. PANDEY, D.D. PATEL, J.M. MISTRY: A review on approaches for handling Bézier curves in CAD for manufacturing. *Procedia Eng.*, **97**(2014), 1155–1166.

[4] F. HIGUCHI, S. GOFUKU, T. MAEKAWA, H. MUKUNDAN, N.M. PATRIKALAKIS: Approximation of involute curves for CAD-system processing. *Eng. Comput.*, **23**(2007), 207–214.

[5] Y.J. AHN, C. HOFFMANN, Y.S. KIM: Curvature continuous offset approximation based on circle approximation using quadratic Bézier biarcs. *Comput.-Aided Des.*, **43**(2011), 1011–1017.

[6] S.H. KIM, Y.J. AHN: An approximation of circular arcs by quartic Bézier curves. *Comput.-Aided Des.*, **39**(2007), 490–493.

[7] J. SÁNCHEZ-REYES: Bézier representation of epitrochoids and hypotrochoids. *Comput.-Aided Des.*, **31**(1999), 747–750.

[8] M. BATSCH, T. MARKOWSKI, S. LEGUTKO, G.M. KROLCZYK: Measurement and mathematical model of convexo-concave Novikov gear mesh. *Measurement*, **125**(2018), 516–525.

[9] S. BABU, K.M. ABUBACKER: Development of involute profiled spur gear model with excel spreadsheet, solidworks and CAD technique. *Int. J. Mech. Eng.*, **5**(2018), 5–11.

[10] G.G. REY, G.B. GORDILLO: Parametric geometric modeling of a spur gear using solidworks. *Gear Solutions.*, **15**(2017), 30–36.