

Examination of the safety of a port oil terminal

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Abstract

Modeling the safety of critical infrastructure free of outside impacts is presented, and basic safety indicators are defined. The safety of the port oil terminal critical infrastructure free of any outside impacts is examined based on critical infrastructure safety statistical data provided by the operators. Its safety function and other safety indicators are identified and predicted. Furthermore, the safety and resilience indicators for the critical infrastructure of the port oil terminal impacted by its operations are determined, and the results are compared to its indicators obtained without considering operation impacts.

Introduction

The safety models used in this paper combine the multistate approach used to analyze the reliability of aging systems (Xue, 1985; Xue & Yang, 1995a, 1995b) with the reliability and safety analysis of systems whose own operation influence the degradation of their components (Kołowrocki & Soszyńska-Budny, 2011/2015; Kołowrocki, 2014). Such an analytical approach is applied practically in the paper to allow new solutions to be identified to examine the safety of critical infrastructure that is impacted by its own operation (Kołowrocki & Soszyńska-Budny, 2017, 2018, 2019a, 2019b). The results increased the accuracy of the safety analysis of real critical infrastructure since their operation is influenced by the aging of their components.

Critical infrastructure is defined as a complex system and its operating environment with inside-system dependencies and outside-system dependencies, such as degradation, that have a significant destructive influence on the health, safety, security, and economic and social conditions of large human communities and territories (Guldby et al., 2010; Lauge, Hernantes & Sarriegi, 2015).

The safety indicators for such a system, which are crucial for its operators, can be obtained using an original and innovative probabilistic approach to modeling operation process impacts on its safety (Kołowrocki & Soszyńska-Budny, 2017). In the first step of the proposed approach, starting from the simplest pure safety model without considering outside impacts, we can define the critical infrastructure and its assets which are practically useful safety indicators, SafI1-SafI8 (Kołowrocki & Soszyńska-Budny, 2018, 2019a). This set of safety indicators can be completed by linking the pure model safety with a model of the critical infrastructure operation process (Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2011/2015, 2017, 2019b). This method created a joint safety model of the critical infrastructure related to its operation process and can offer two additional resilience indicators, ResI1-ResI2, which are measures of the impact of critical infrastructure operation on its safety and resilience to operation (Kołowrocki & Soszyńska-Budny, 2017, 2019b). The paper is devoted to the practical application of this joint model to a safety and resilience examination of the critical infrastructure of a port oil terminal.

Critical infrastructure safety

In the multistate safety analysis used to define the critical infrastructure with degrading/aging components/assets, we assume that (Kołowrocki, 2014; Kołowrocki & Soszyńska-Budny, 2011/2015, 2018):

- n is the number of critical infrastructure assets;
- $A_i, i = 1, 2, \dots, n$, are the critical infrastructure assets;
- all assets and the critical infrastructure have the safety state set $\{0, 1, \dots, z\}$, $z \geq 1$, the safety states are ordered, the safety state 0 is the worst, and the safety state z is the best;
- $r, r \in \{1, 2, \dots, z\}$, is the critical safety state (critical infrastructure and its assets remaining in the safety states less than the critical state are highly dangerous for both the assets and their operating environments);
- $T_i(u), i = 0, 1, 2, \dots, n$, are independent random variables representing the asset lifetimes (A_i), $i = 1, 2, \dots, n$, in the safety state subset $\{u, u+1, \dots, z\}$, $u = 1, 2, \dots, z$, while they were in the safety state z at moment $t = 0$;
- $T(u)$ is a variable representing the lifetime of the critical infrastructure in the safety state subset $\{u, u+1, \dots, z\}$, $u = 0, 1, 2, \dots, z$, while it was in the safety state z at moment $t = 0$;
- the assets and the critical infrastructure safety states degrade with time t (measured in years);
- $s_i(t)$ is the asset A_i safety state at moment $t, t \geq 0$, given that it was in the safety state z at moment $t = 0$;
- $s(t)$ is the critical infrastructure safety state at moment $t, t \geq 0$, while it was in the safety state z at moment $t = 0$.

The above assumptions mean that the safety states of critical infrastructure with degrading assets may only become worse over time (Kołowrocki & Soszyńska-Budny, 2011/2015, 2018). The way in which the assets and the critical infrastructure safety states change is illustrated in Figures 1 and 2.

We denote the critical infrastructure unconditional lifetime in the safety state subset $\{u, u+1, \dots, z\}$, $u = 1, 2, \dots, z$, by $T(u)$ and define the critical infrastructure safety function by the vector (Kołowrocki & Soszyńska-Budny, 2017, 2018):

$$S(t, \cdot) = [S(t, 0), S(t, 1), \dots, S(t, z)], \quad t \geq 0,$$

with the coordinates defined by:

$$\begin{aligned} S(t, 0) &= P(T(0) > t) = 1, \\ S(t, u) &= P(T(u) > t) = 1 - F(t, u) \\ &\text{for } t \geq 0, u = 1, 2, \dots, z \end{aligned} \quad (1)$$

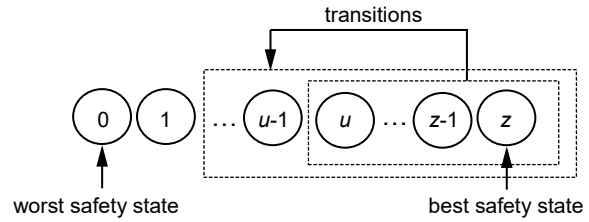


Figure 1. Illustration of critical infrastructure and changes in its assets states

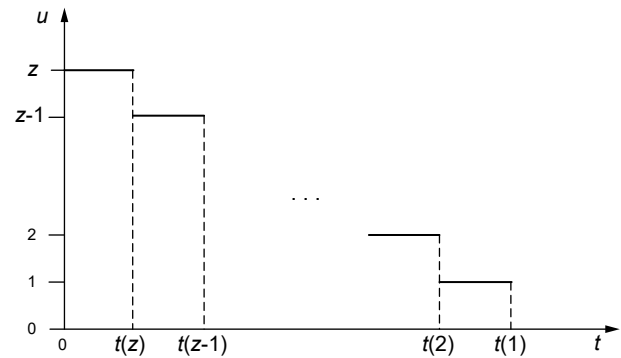


Figure 2. The relationship between the realizations $t(u)$ of the critical infrastructure lifetime $T(u)$ in the safety state subsets $\{u, u+1, \dots, z\}$, $u = 1, 2, \dots, z$

where $F(t, u), t \geq 0, u = 1, 2, \dots, z$ is the distribution function of the lifetime $T(u), u = 1, 2, \dots, z$.

The exemplary graph of a five-state ($z = 4$) critical infrastructure safety function,

$$S(t, \cdot) = [1, S(t, 1), S(t, 2), S(t, 3), S(t, 4)], \quad t \geq 0,$$

is shown in Figure 3.

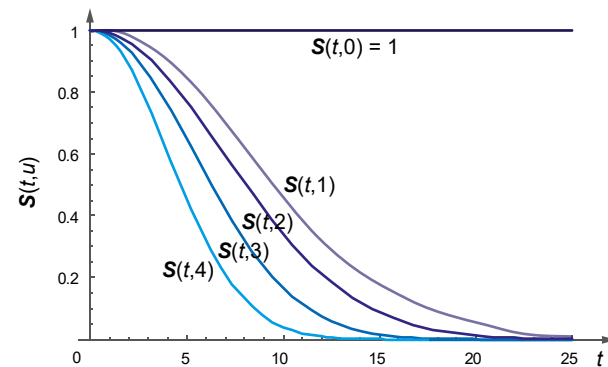


Figure 3. The graphs of a five-state critical infrastructure safety function coordinates

If $r, r \in \{1, \dots, z\}$, is the critical safety state, then the critical infrastructure risk function,

$$r(t) = P(s(t) < r | s(0) = z) = P(T(r) \leq t), \quad t \geq 0,$$

is defined as the probability that the critical infrastructure is in the subset of safety states worse than the critical safety state r , while it was in the best

safety state z at moment $t = 0$ and given by (Kołowrocki & Soszyńska-Budny, 2011/2015, 2018):

$$r(t) = 1 - S(t, r), \quad t \geq 0 \quad (2)$$

where $S(t, r)$ is the coordinate of the critical infrastructure unconditional safety function given by (1) for $u = r$. A graph of the exemplary critical infrastructure risk function, called the fragility curve (Guldbly et al., 2010), is presented in Figure 4.

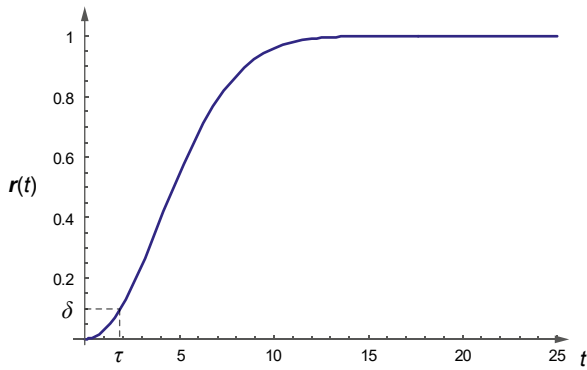


Figure 4. The fragility curve of an exemplary critical infrastructure risk function

Safety of port oil terminal critical infrastructure

Assets

We consider the port oil terminal critical infrastructure free from outside impacts. The considered port terminal placed at the Baltic seaside was designated for receiving oil products from ships, storage, and sending products by carriages or trucks. The terminal operates in a reverse way as well. The considered terminal is composed of three parts, A, B, and C, which are linked by a piping transportation system with the pier.

The main technical assets of the port oil terminal critical infrastructure are:

- A_1 – port oil piping transportation system,
- A_2 – internal pipeline technological system,
- A_3 – supporting pump station,
- A_4 – internal pump system,
- A_5 – port oil tanker shipment terminal,
- A_6 – loading railway carriage station,
- A_7 – loading road carriage station,
- A_8 – unloading railway carriage station,
- A_9 – oil storage reservoir system.

The scheme of the asset A_1 , the port oil piping transportation system, is presented in Figure 5.

The asset A_1 operating at the critical infrastructure of the port oil terminal consists of three subsystems:

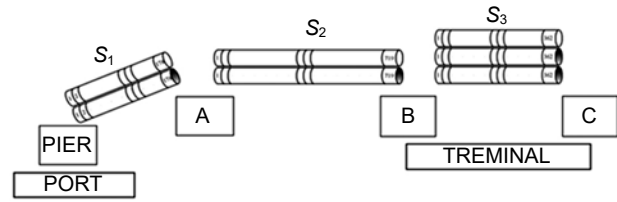


Figure 5. Schematic of the terminal and port oil piping transportation system

- the subsystem S_1 , composed of two pipelines, each composed of 176 pipe segments and 2 valves;
 - the subsystem S_2 , composed of two pipelines, each composed of 717 pipe segments and 2 valves;
 - the subsystem S_3 , composed of three pipelines, each composed of 360 pipe segments and 2 valves.
- Its operation is the main function of the oil terminal involving the remaining assets $A_2 - A_9$.

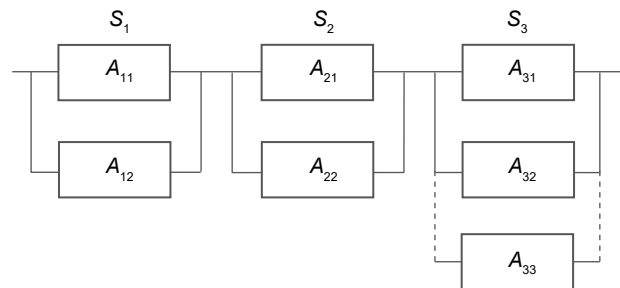


Figure 6. General scheme of the safety structure of the port oil piping transportation system

In the port oil transportation system presented in Figure 6, the asset A_1 is a series system composed of series-parallel subsystem S_1 , containing two pipelines (assets A_{11}, A_{12}), a series-parallel subsystem S_2 , containing two pipelines (assets A_{21}, A_{22}), and one series-“2 out of 3” subsystem S_3 containing 3 pipelines (assets A_{31}, A_{32}, A_{33}). The subsystems S_1, S_2 , and S_3 form a general series port oil pipeline system safety structure.

Safety parameters

After considering the comments and opinions of experts and taking into account the effectiveness and safety aspects of the operation of the port oil terminal critical infrastructure and its assets, we fixed all parameters (Kołowrocki & Soszyńska-Budny, 2011/2015, 2019a), the number safety states 3 ($z = 2$), and the following safety states:

- safety state 2 – an asset and the critical infrastructure of the port oil terminal are fully safe;
- safety state 1 – an asset and the port oil terminal critical infrastructure are less-safe because due to the possibility of environmental pollution;

- safety state 0 – an asset and the port oil terminal critical infrastructure are destroyed.

We also assume that:

- the assets and the port oil terminal critical infrastructure safety states can only worsen;
- the critical safety state of an asset and the port oil terminal critical infrastructure is $r = 1$;
- the port oil terminal critical infrastructure risk function permitted level is $\delta = 0.05$.

The approximate mean values of the lifetime of asset A_1 in the safety state subsets $\{1, 2\}$, $\{2\}$, calculated based on the safety data of its components from experts are (Kołowrocki & Soszyńska-Budny, 2019a):

$$\mu_1^0(1) \approx 63 \text{ years}, \mu_1^0(2) = 46 \text{ years}.$$

The mean values of the lifetimes of the remaining assets $A_2 - A_9$ in the safety state subsets $\{1, 2\}$, $\{2\}$, approximately evaluated by experts are:

$$\mu_i^0(1) = 80 \text{ years}, \mu_i^0(2) = 50 \text{ years}, i = 2, 3, \dots, 9.$$

Hence, applying (15) from (Kołowrocki & Soszyńska-Budny, 2019a), it follows that the intensities of an asset's departure from the safety states subsets $\{1, 2\}$, $\{2\}$, are respectively:

- for asset A_1

$$\lambda_1^0(1) = 0.015873, \lambda_1^0(2) = 0.021739 \quad (3)$$

- for assets $A_2 - A_9$

$$\lambda_i^0(1) = 0.0125, \lambda_i^0(2) = 0.02, i = 2, 3, \dots, 9 \quad (4)$$

Safety Indicators

Since the port oil terminal critical infrastructure is a three-state ($z = 2$) series system and assuming that the assets have exponential safety functions, its safety function (SafI1) determined after the application of (GMU, 2018) is given by:

$$\mathcal{S}^0(t, \cdot) = [1, \mathcal{S}^0(t, 1), \mathcal{S}^0(t, 2)], t \geq 0,$$

where, according to the formula for series critical infrastructure given in Corollary 1 (Kołowrocki & Soszyńska-Budny, 2019a):

$$\begin{aligned} \mathcal{S}^0(t, 1) &= \exp[-0.015873t] \exp[-0.0125t] \\ &\exp[-0.0125t] \exp[-0.0125t] \exp[-0.0125t] \\ &\exp[-0.0125t] \exp[-0.0125t] \exp[-0.0125t] \\ &\exp[-0.0125t] = \exp[-0.115873t], t \geq 0 \end{aligned} \quad (5)$$

$$\begin{aligned} \mathcal{S}^0(t, 2) &= \exp[-0.021739t] \exp[-0.02t] \\ &\exp[-0.02t] \exp[-0.02t] \exp[-0.02t] \\ &\exp[-0.02t] \exp[-0.02t] \exp[-0.02t] \\ &\exp[-0.02t] = \exp[-0.181739t], t \geq 0 \end{aligned} \quad (6)$$

The graph of the safety function of the port oil terminal critical infrastructure is shown in Figure 7.

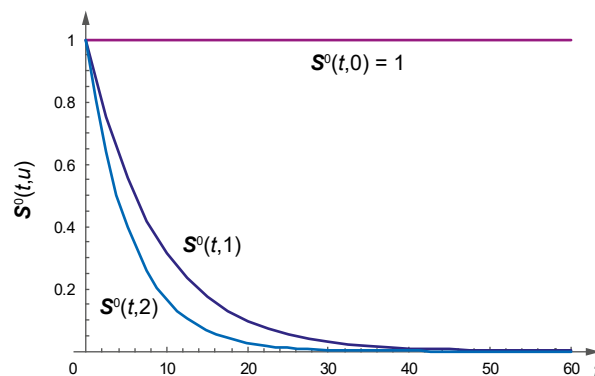


Figure 7. The graph of the port oil terminal critical infrastructure safety function coordinates

Applying (10) from (Kołowrocki & Soszyńska-Budny, 2019a) to (5)–(6), the expected lifetimes of the oil terminal critical infrastructure in the safety state subsets $\{1, 2\}$, $\{2\}$ (SafI4), respectively are:

$$\mu^0(1) \cong 8.63 \text{ years}, \mu^0(2) \cong 5.50 \text{ years} \quad (7)$$

It further follows from (13) from (Kołowrocki & Soszyńska-Budny, 2019a), that the mean lifetimes of the oil terminal critical infrastructure in particular safety states (SafI8) are:

$$\bar{\mu}^0(1) \cong 3.13 \text{ years}, \bar{\mu}^0(2) \cong 5.50 \text{ years} \quad (8)$$

Thus, according to (7), the oil terminal critical infrastructure mean lifetime up to but not exceeding the critical safety state $r = 1$ (SafI4) is

$$\mu^0(1) \cong 8.63 \quad (9)$$

Since the critical safety state is $r = 1$, then by (2) and (5), the port oil terminal critical infrastructure risk function (SafI2), is given by:

$$\begin{aligned} r^0(t) &= 1 - \mathcal{S}^0(t, 1) = 1 - \exp[-0.115873t] \\ &\text{for } t \geq 0 \end{aligned} \quad (10)$$

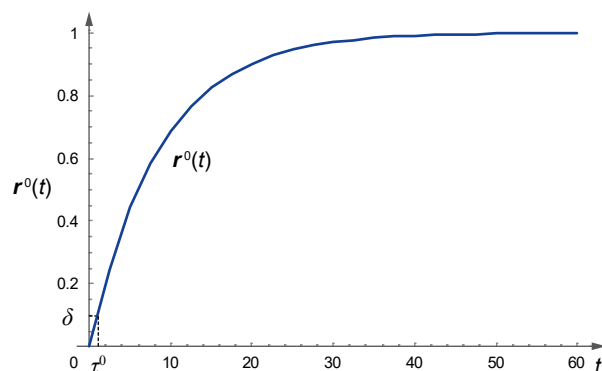


Figure 8. The graph of the risk function $r(t)$ (fragility curve) of the port oil terminal critical infrastructure

The graph of the risk function $r^0(t)$ of the critical infrastructure of the oil terminal (SafI3), called the fragility curve (Gouldby et. al., 2010), is shown in Figure 8.

According to (8) from (Kołowrocki & Soszyńska-Budny, 2019a) and (10), the moment when the oil terminal critical infrastructure risk function exceeds a permitted level $\delta = 0.05$ (SafI6), is

$$\tau^0 = -\frac{1}{0.115873} \ln(1 - 0.05) \cong 0.44 \text{ years} \quad (11)$$

and considering (7), the oil terminal critical infrastructure intensities of aging (SafI7) are:

$$\lambda^0(1) = 0.115873, \lambda^0(2) = 0.181739 \quad (12)$$

Safety of port oil terminal critical infrastructure impacted by operation processes

Operation processes

Asset A_1 presented in Figure 6 and the safety parameters of its components depend on its operation processes changing over time (Kołowrocki & Soszyńska-Budny, 2019b). Moreover, the operation of asset A_1 is the main activity of the port oil terminal involving the remaining assets $A_2 - A_9$ and determining their operation processes.

Based on expert opinions, it is possible to fix the basic parameters of the oil terminal critical infrastructure operation process (Kołowrocki & Soszyńska-Budny, 2019b), the number of operation process states $v = 7$, and the following operation process states:

- the operation state z_1 – transport of one kind of medium from the terminal part B to part C using two of the three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_6, A_7, A_9 ;
- the operation state z_2 – transport of one kind of medium from the terminal part C to part B using one of the three pipelines of the subsystem S_3 of the asset A_1 and assets A_2, A_4, A_8, A_9 ;
- the operation state z_3 – transport of one kind of medium from the terminal part B through part A to the pier using one of the two pipelines of the subsystem S_1 and one of the two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_4, A_5, A_9 ;
- the operation state z_4 – transport of one kind of medium from the pier through parts A and B to part C using one of the two pipelines of the subsystem S_1 , one of the two pipelines in subsystem S_2 and two of the three pipelines of the subsystem

S_3 of the asset A_1 and assets $A_2, A_3, A_4, A_5, A_6, A_7, A_9$;

- the operation state z_5 – transport of one kind of medium from the pier through part A to B using one of the two pipelines of the subsystem S_1 and one of the two pipelines of the subsystem S_2 of the asset A_1 and assets A_2, A_3, A_4, A_5, A_9 ;
- the operation state z_6 – transport of one kind of medium from the terminal part B to C using two of the three pipelines of the subsystem S_3 , and the simultaneous transport of another kind of medium from the pier through part A to B using one of the two pipelines of the subsystem S_1 and one of the two pipelines of the subsystem S_2 of the asset A_1 and assets $A_2, A_3, A_4, A_5, A_6, A_7, A_9$;
- the operation state z_7 – transport of one kind of medium from the terminal part B to C using one of the three pipelines of the subsystem S_3 , and the simultaneous transport of a second kind of medium from the terminal part C to B using one of the three pipelines of the subsystem S_3 of the asset A_1 and assets $A_2, A_4, A_6, A_7, A_8, A_9$.

The main characteristics of the port oil terminal critical infrastructure operation process $Z(t)$ fixed in (Kołowrocki & Soszyńska-Budny, 2019b) were the limit values of the transient probabilities of the operation process $Z(t)$ at particular operation states z_b , $b = 1, 2, \dots, 7$:

$$p_1 = 0.395, p_2 = 0.060, p_3 = 0.003, p_4 = 0.002, p_5 = 0.20, p_6 = 0.058, p_7 = 0.282 \quad (13)$$

Operation process impact

The coefficients of the impact of operation processes on the ageing of port oil terminal critical infrastructure at the operation states z_b , $b = 1, 2, \dots, 7$ are as follows (GMU, 2018; Kołowrocki & Soszyńska-Budny, 2019b):

- for asset A_1

$$\begin{aligned} [\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, \\ & b = 1, 2, 7, i = 1, \\ [\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, \\ & b = 3, 5, i = 1, \\ [\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, \\ & b = 4, 6, i = 1; \end{aligned}$$

- for asset A_2

$$\begin{aligned} [\rho_i^1(1)]^{(b)} &= 1.10, [\rho_i^1(2)]^{(b)} = 1.10, \\ & b = 1, 2, 7, i = 2, \\ [\rho_i^1(1)]^{(b)} &= 1.20, [\rho_i^1(2)]^{(b)} = 1.20, \\ & b = 3, 5, i = 2, \\ [\rho_i^1(1)]^{(b)} &= 1.30, [\rho_i^1(2)]^{(b)} = 1.30, \\ & b = 4, 6, i = 2; \end{aligned}$$

- for asset A_3

$$[\rho_i^1(1)]^{(b)} = 1, [\rho_i^1(2)]^{(b)} = 1,$$

$$b = 1,2,3,7, i = 3,$$

$$[\rho_i^1(1)]^{(b)} = 1.20, [\rho_i^1(2)]^{(b)} = 1.20,$$

$$b = 5, i = 3,$$

$$[\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30,$$

$$b = 4,6, i = 3;$$
- for asset A_4

$$[\rho_i^1(1)]^{(b)} = 1.10, [\rho_i^1(2)]^{(b)} = 1.10,$$

$$b = 1,2,7, i = 4,$$

$$[\rho_i^1(1)]^{(b)} = 1.20, [\rho_i^1(2)]^{(b)} = 1.20,$$

$$b = 3,5, i = 4,$$

$$[\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30,$$

$$b = 4,6, i = 4;$$
- for asset A_5

$$[\rho_i^1(1)]^{(b)} = 1, [\rho_i^1(2)]^{(b)} = 1,$$

$$b = 1,2,7, i = 5,$$

$$[\rho_i^1(1)]^{(b)} = 1.20, [\rho_i^1(2)]^{(b)} = 1.20,$$

$$b = 3,5, i = 5,$$

$$[\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30,$$

$$b = 4,6, i = 5;$$
- for asset A_6

$$[\rho_i^1(1)]^{(b)} = 1, [\rho_i^1(2)]^{(b)} = 1,$$

$$b = 2,5, i = 6,$$

$$[\rho_i^1(1)]^{(b)} = 1.10, [\rho_i^1(2)]^{(b)} = 1.10,$$

$$b = 1,7, i = 6,$$

$$[\rho_i^1(1)]^{(b)} = 1.20, [\rho_i^1(2)]^{(b)} = 1.20,$$

$$b = 3, i = 6,$$

$$[\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30,$$

$$b = 4,6, i = 6;$$
- for asset A_7

$$[\rho_i^1(1)]^{(b)} = 1, [\rho_i^1(2)]^{(b)} = 1,$$

$$b = 2,3,5, i = 7,$$

$$[\rho_i^1(1)]^{(b)} = 1.10, [\rho_i^1(2)]^{(b)} = 1.10,$$

$$b = 1,7, i = 7,$$

$$[\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30,$$

$$b = 4,6, i = 7;$$
- for asset A_8

$$[\rho_i^1(1)]^{(b)} = 1, [\rho_i^1(2)]^{(b)} = 1,$$

$$b = 1,3,4,5,6, i = 8,$$

$$[\rho_i^1(1)]^{(b)} = 1.10, [\rho_i^1(2)]^{(b)} = 1.10,$$

$$b = 2,7, i = 8;$$
- for asset A_9

$$[\rho_i^1(1)]^{(b)} = 1.10, [\rho_i^1(2)]^{(b)} = 1.10,$$

$$b = 1,2,7, i = 9,$$

$$[\rho_i^1(1)]^{(b)} = 1.20, [\rho_i^1(2)]^{(b)} = 1.20,$$

$$b = 3,5, i = 9,$$

$$[\rho_i^1(1)]^{(b)} = 1.30, [\rho_i^1(2)]^{(b)} = 1.30,$$

$$b = 4,6, i = 9 \tag{14}$$

Safety parameters impacted by operation process

We assume that the port oil terminal critical infrastructure assets $A_i, i = 1,2,\dots,9$ at the critical infrastructure operation process $Z(t)$ states $z_b, b = 1,2,\dots,7$, conditional safety functions

$$[S_i^1(t,\cdot)]^{(b)} = [1, [S_i^1(t,1)]^{(b)}, [S_i^1(t,2)]^{(b)}],$$

$$t \geq 0, b = 1,2,\dots,7, i = 1,2,\dots,9,$$

are exponential with the coordinates

$$[S_i^1(t,u)]^{(b)} = \exp[-[\lambda_i^1(u)]^{(b)} t],$$

$$t \geq 0, u = 1,2, b = 1,2,\dots,7, i = 1,2,\dots,9 \tag{15}$$

where

$$[\lambda_i^1(u)]^{(b)} = [\rho_i^1(u)]^{(b)} \cdot \lambda_i^0(u),$$

$$u = 1,2, b = 1,2,\dots,7, i = 1,2,\dots,9 \tag{16}$$

and

$$[\rho_i^1(u)]^{(b)}, u = 1,2, b = 1,2,\dots,7, i = 1,2,\dots,9$$

are the coefficients of the impact of operation processes on the degradation of the critical infrastructure assets $A_i, i = 1,2,\dots,9$, at operation states $z_b, b = 1,2,\dots,7$, defined by (14) and

$$\lambda_i^0(u), u = 1,2, i = 1,2,\dots,9$$

are the degradation of the port oil critical infrastructure assets without the impact of the operation process, defined by (3)–(4).

Under assumption (16), and considering (3)–(4) and (14), it follows that the intensities of asset departure from the safety state subsets $\{1,2\}, \{2\}$, with considering operation impact on their safety are:

- for asset A_1

$$[\lambda_i^1(1)]^{(b)} = 0.017460, [\lambda_i^1(2)]^{(b)} = 0.023913,$$

$$b = 1,2,7, i = 1,$$

$$[\lambda_i^1(1)]^{(b)} = 0.019048, [\lambda_i^1(2)]^{(b)} = 0.026087,$$

$$b = 3,5, i = 1,$$

$$[\lambda_i^1(1)]^{(b)} = 0.020635, [\lambda_i^1(2)]^{(b)} = 0.028261,$$

$$b = 4,6, i = 1;$$
- for asset A_2

$$[\lambda_i^1(1)]^{(b)} = 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022,$$

$$b = 1,2,7, i = 2,$$

$$[\lambda_i^1(1)]^{(b)} = 0.015, [\lambda_i^1(2)]^{(b)} = 0.024,$$

$$b = 3,5, i = 2,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026,$$

$$b = 4,6, i = 2;$$

- for asset A_3

$$[\lambda_i^1(1)]^{(b)} = 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, \\ b = 1,2,3,7, i = 3,$$

$$[\lambda_i^1(1)]^{(b)} = 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, \\ b = 5, i = 3,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b = 4, i = 3;$$

- for asset A_4

$$[\lambda_i^1(1)]^{(b)} = 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b = 1,2,7, i = 4,$$

$$[\lambda_i^1(1)]^{(b)} = 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, \\ b = 3,5, i = 4,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b = 4,6, i = 4;$$

- for asset A_5

$$[\lambda_i^1(1)]^{(b)} = 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, \\ b = 1,2,7, i = 5,$$

$$[\lambda_i^1(1)]^{(b)} = 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, \\ b = 3,5, i = 5,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b = 4,6, i = 5;$$

- for asset A_6

$$[\lambda_i^1(1)]^{(b)} = 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, \\ b = 2,5, i = 6,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b = 1,7, i = 6,$$

$$[\lambda_i^1(1)]^{(b)} = 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, \\ b = 3, i = 6,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b = 4,6, i = 6;$$

- for asset A_7

$$[\lambda_i^1(1)]^{(b)} = 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, \\ b = 2,3,5, i = 7,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b = 1,7, i = 7,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b = 4,6, i = 7;$$

- for asset A_8

$$[\lambda_i^1(1)]^{(b)} = 0.0125, [\lambda_i^1(2)]^{(b)} = 0.02, \\ b = 1,3,4,5,6, i = 8,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b = 2,7, i = 8;$$

- for asset A_9

$$[\lambda_i^1(1)]^{(b)} = 0.01375, [\lambda_i^1(2)]^{(b)} = 0.022, \\ b = 1,2,7, i = 9,$$

$$[\lambda_i^1(1)]^{(b)} = 0.015, [\lambda_i^1(2)]^{(b)} = 0.024, \\ b = 3,5, i = 9,$$

$$[\lambda_i^1(1)]^{(b)} = 0.01625, [\lambda_i^1(2)]^{(b)} = 0.026, \\ b = 4,6, i = 9 \quad (17)$$

Safety indicators impacted by operation process

Since the coordinates of the conditional safety functions for the port oil terminal critical infrastructure assets A_i , $i = 1,2,\dots,9$, take the form (15) with the ageing intensities at the operation states z_b , $b = 1,2,\dots,7$, given respectively by (17), as the oil terminal critical infrastructure is a three-state ($z = 2$) series system, then by Corollary 1 from (Kołowrocki & Soszyńska-Budny, 2019a), they are given for $t \geq 0$ by:

$$[\mathbf{S}^1(t,\cdot)]^{(1)} = [1, [\mathbf{S}^1(t,1)]^{(1)}, [\mathbf{S}^1(t,2)]^{(1)}], \\ [\mathbf{S}^1(t,1)]^{(1)} = \exp[-0.12371t], \\ [\mathbf{S}^1(t,2)]^{(1)} = \exp[-0.193913t]$$

$$[\mathbf{S}^1(t,\cdot)]^{(2)} = [1, [\mathbf{S}^1(t,1)]^{(2)}, [\mathbf{S}^1(t,2)]^{(2)}], \\ [\mathbf{S}^1(t,1)]^{(2)} = \exp[-0.12246t], \\ [\mathbf{S}^1(t,2)]^{(2)} = \exp[-0.191913t];$$

$$[\mathbf{S}^1(t,\cdot)]^{(3)} = [1, [\mathbf{S}^1(t,1)]^{(3)}, [\mathbf{S}^1(t,2)]^{(3)}], \\ [\mathbf{S}^1(t,1)]^{(3)} = \exp[-0.131548t], \\ [\mathbf{S}^1(t,2)]^{(3)} = \exp[-0.206087t];$$

$$[\mathbf{S}^1(t,\cdot)]^{(4)} = [1, [\mathbf{S}^1(t,1)]^{(4)}, [\mathbf{S}^1(t,2)]^{(4)}], \\ [\mathbf{S}^1(t,1)]^{(4)} = \exp[-0.146885t], \\ [\mathbf{S}^1(t,2)]^{(4)} = \exp[-0.230261t];$$

$$[\mathbf{S}^1(t,\cdot)]^{(5)} = [1, [\mathbf{S}^1(t,1)]^{(5)}, [\mathbf{S}^1(t,2)]^{(5)}], \\ [\mathbf{S}^1(t,1)]^{(5)} = \exp[-0.131548t], \\ [\mathbf{S}^1(t,2)]^{(5)} = \exp[-0.206087t];$$

$$[\mathbf{S}^1(t,\cdot)]^{(6)} = [1, [\mathbf{S}^1(t,1)]^{(6)}, [\mathbf{S}^1(t,2)]^{(6)}], \\ [\mathbf{S}^1(t,1)]^{(6)} = \exp[-0.146885t], \\ [\mathbf{S}^1(t,2)]^{(6)} = \exp[-0.230261t];$$

$$[\mathbf{S}^1(t,\cdot)]^{(7)} = [1, [\mathbf{S}^1(t,1)]^{(7)}, [\mathbf{S}^1(t,2)]^{(7)}], \\ [\mathbf{S}^1(t,1)]^{(7)} = \exp[-0.12496t], \\ [\mathbf{S}^1(t,2)]^{(7)} = \exp[-0.195913t] \quad (18)$$

Hence, applying (10) from (Kołowrocki & Soszyńska-Budny, 2019b), the expected lifetimes of the port oil terminal critical infrastructure in the safety state subsets $\{1, 2\}$, $\{2\}$ at the operation states z_b , $b = 1,2,\dots,7$ are respectively:

$$\begin{aligned}
 [\mu^1(1)]^{(1)} &\cong 8.08, [\mu^1(2)]^{(1)} \cong 5.16 \text{ years,} \\
 [\mu^1(1)]^{(2)} &\cong 8.17, [\mu^1(2)]^{(2)} \cong 5.21 \text{ years,} \\
 [\mu^1(1)]^{(3)} &\cong 7.60, [\mu^1(2)]^{(3)} \cong 4.85 \text{ years,} \\
 [\mu^1(1)]^{(4)} &\cong 6.81, [\mu^1(2)]^{(4)} \cong 4.34 \text{ years,} \\
 [\mu^1(1)]^{(5)} &\cong 7.60, [\mu^1(2)]^{(5)} \cong 4.85 \text{ years,} \\
 [\mu^1(1)]^{(6)} &\cong 6.81, [\mu^1(2)]^{(6)} \cong 4.34 \text{ years,} \\
 [\mu^1(1)]^{(7)} &\cong 8.00, [\mu^1(2)]^{(7)} \cong 5.10 \text{ years} \quad (19)
 \end{aligned}$$

Applying (7) from (Kołowrocki & Soszyńska-Budny, 2019b) to the results from (13) and (18), the port oil terminal critical infrastructure unconditional safety function (SafI1) is given by

$$S^1(t, \cdot) = [1, S^1(t, 1), S^1(t, 2)], \quad t \geq 0,$$

where

$$\begin{aligned}
 S^1(t, 1) &= 0.395\exp[-0.12371t] + \\
 &+ 0.060\exp[-0.12246t] + 0.003\exp[-0.131548t] + \\
 &+ 0.002\exp[-0.146885t] + 0.200\exp[-0.131548t] + \\
 &+ 0.058\exp[-0.146885t] + 0.282\exp[-0.12496t] \quad (20)
 \end{aligned}$$

$$\begin{aligned}
 S^1(t, 2) &= 0.395\exp[-0.193913t] + \\
 &+ 0.060\exp[-0.191913t] + 0.003\exp[-0.206087t] + \\
 &+ 0.002\exp[-0.230261t] + 0.200\exp[-0.206087t] + \\
 &+ 0.058\exp[-0.230261t] + 0.282\exp[-0.195913t] \quad (21)
 \end{aligned}$$

The graph of the three-state oil terminal critical infrastructure safety function is shown in Figure 9.

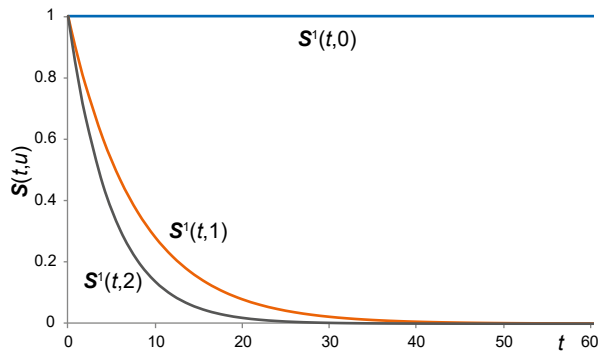


Figure 9. The graph of the oil terminal critical infrastructure safety function $S^1(t, \cdot)$ coordinate

Considering (19) and applying (9) from (Kołowrocki & Soszyńska-Budny, 2019b) for $r = u$, $u = 1, 2$, the expected lifetimes of the port oil terminal critical infrastructure in the safety state subsets $\{1, 2\}$, $\{2\}$ are respectively:

$$\mu^1(1) \cong 7.89 \text{ years, } \mu^1(2) \cong 5.03 \text{ years} \quad (22)$$

Furthermore, using (12), it follows that the mean lifetimes of the oil terminal critical infrastructure in the particular safety states are:

$$\bar{\mu}^1(1) \cong 2.86, \quad \bar{\mu}^1(2) \cong 5.03 \text{ years} \quad (23)$$

Since the critical safety state is $r = 1$, then by (2) and (20), the port oil terminal critical infrastructure risk function (SafI2), is given for $t \geq 0$ by

$$\begin{aligned}
 r^1(t) &= 1 - \{0.395\exp[-0.12371t] + \\
 &+ 0.060\exp[-0.12246t] + 0.003\exp[-0.131548t] + \\
 &+ 0.002\exp[-0.146885t] + 0.200\exp[-0.131548t] + \\
 &+ 0.058\exp[-0.146885t] + 0.282\exp[-0.12496t]\} \quad (24)
 \end{aligned}$$

Applying (8) to (24), the moment when the oil terminal critical infrastructure risk function exceeds a permitted level $\delta = 0.05$ (SafI6), is

$$\tau^1 = (r^1)^{-1}(\delta) \cong 0.404 \text{ year} \quad (25)$$

By (22), the mean lifetime of the port oil terminal critical infrastructure up to, but not exceeding the critical safety state $r = 1$ (SafI4), is

$$\mu^1(1) \cong 7.89 \text{ years} \quad (26)$$

Applying (11) and (22), the aging intensity of the oil terminal critical infrastructure (SafI7) are:

$$\lambda^1(t, 1) \cong 0.126743, \quad \lambda^2(t, 2) \cong 0.198807 \quad (27)$$

Considering (12) and (27) and applying (15), the impact of the coefficients of the operation process on the aging intensity of the oil terminal critical infrastructure (ResI1), are:

$$\begin{aligned}
 \rho^1(t, 1) &= \frac{\lambda^1(t, 1)}{\lambda^0(t, 1)} \cong \frac{0.126743}{0.115873} \cong 1.094 \\
 \rho^1(t, 2) &= \frac{\lambda^1(t, 2)}{\lambda^0(t, 2)} \cong \frac{0.198807}{0.181739} \cong 1.094 \quad (28)
 \end{aligned}$$

Finally, by (28) and (16), the port oil terminal critical infrastructure resilience indicator (ResI2), i.e. the coefficient of the resilience of the port oil terminal critical infrastructure to the impact of operational processes is

$$RI(t) = 1/\rho^1(t, 1) \cong 0.914 \cong 91\%, \quad t \geq 0 \quad (29)$$

Conclusions

The comparison of safety indicators (20)–(27) and (5)–(12) shows that the operational processes have a significant influence on the port oil terminal safety, which was clearly expressed in the resilience indicators (28)–(29). The proposed critical infrastructure safety models without considering outside impacts and the critical infrastructure impacted by its operation processes can be applied to analyze

the safety and resilience of various critical infrastructures. These, along with the newest results on the reliability of systems with aging and dependent components presented in (Szymkowiak, 2018a, 2018b, 2019) and (Blokus, 2019; Blokus & Kołowrocki, 2019), respectively, can serve as the basis for analyzing the safety of critical infrastructures composed of aging and dependent assets. Further research may involve considering other impacts and solving the problems of critical infrastructure safety optimization and identifying the optimal values of safety and resilience indicators (Kołowrocki & Soszyńska-Budny, 2011/2014; Guze, 2019). These results can help mitigate the consequences of critical infrastructure accidents and enhance the resilience of critical infrastructure to operation and other impacts (Bogalecka, 2019). This research may also be used as a background for business continuity and cost-effectiveness analyses of critical infrastructures under operation and other impacts.

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