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POWER SYSTEM PARTICIPATION FACTORS FOR REAL AND COMPLEX EIGENVALUES CASES

This paper provides a brief overview on existing approaches for defining participation factor in modal analysis. We calculated participation factors using different approximations and specific examples, and then compared the obtained results with equations derived from the mode evolution. As a result, the existing methods for determining a state variable participation factor in mode are proved incorrect result for complex eigenvalue case. Modern software applications designed to analyze power system stability deploy old approaches that provide incorrect results of modal analysis and pose risks to the operation of real power systems. To solve this problem we presented a new broadened definition of participation factor that assures correct results for real and complex eigenvalues cases. We used a two–mass mechanics system to validate proposed approach and our findings confirm the proposed participation factor theory.

KEYWORDS**:** modal analysis, participation factor, static stability, power system

1. INTRODUCTION

The Selective Modal Analysis (SMA) is the most modern method currently used to analyse power system stability. This method involves decomposition of power system free oscillations to separate components. Considering complex systems, the process of merging power systems is the most difficult to simulate. First, the main problem is the size of such system as it consists of hundreds of generators connected with thousands of power lines, bushes, and hundreds of load centres. Secondly, complex nature of network physical processes causes problems due to physical values with different time dynamics (electrical changes usually occur faster than mechanical change of generator rotor position). Therefore, creating an acceptable model for analysis of power system stability requires several simplifications.

The SMA consists of two main concepts: eigenvalues and participation factors. The concept of participation factors is important for analysing power

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system and is now growing in popularity with both research and industry. Participation factor as a scalar value is used to identify the relationships among the state variables and eigenmodes: measuring the participation of the state variables in the eigenmodes, and vice versa. The SMA theory was developed by Perez-Arriaga et al. in articles [1, 2]. Since that time had started to intensity development of SMA and applied it to power system stability analysis.

In the articles [3, 4, 5] authors revisit the concept of participation factors and separated it into two different definitions: state–in–mode and mode–in–states. The authors used probabilistic description of the uncertainty in the initial condition. They proposed new formulae for calculating participation factors in the real and complex eigenvalues cases of linear–invariant system matrix.

2. PARTICIPATION FACTOR: STATE–IN–MODE

As we mentioned above, two different approaches to state–in–mode participation factors are known today. The first approach is proposed by Perez–Arriaga et al. (1982) [1, 2] and the second — by Verghese et al. (2009) [3, 4, 5].

First, we briefly review the SMA theory and the mathematical model of power system, and then investigate these approaches in detail.

We use a system of linearized differential equations that describe the behaviour of oscillations in the system during minor disturbances as a mathematical model to study oscillating static stability of power systems. This model describes well enough the power system electromechanical processes as it starts with generator models and proceeds with macro problems of a system that ensures accurate reproduction of physical processes.

The power system consists of N synchronous machines that are interconnected. We can describe such power system via differential equation system in linear approximation as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}, \tag{1}$$

where $\mathbf{x}$ is a column–vector of state variables, $\mathbf{A}$ is a characteristic matrix of differential system equations. The matrix $\mathbf{A}$ is a real matrix, so $\mathbf{A}^* = \mathbf{A}$. In common scenes the matrix $\mathbf{A}$ had N different eigenvalues, and some of them are conjugate:

$$\lambda_i = \sigma_i \pm j\omega_i, \tag{2}$$

where σ_i is a real part of eigenvalue which characterizes margin of power system stability state, ω_i – imaginary part of eigenvalue which defines frequency of power system state. Under margin of state stability we'll understand the module of real part of eigenvalue. The left and right eigenvectors are defined by the next equations:

$$\mathbf{A}\mathbf{r}_i = \lambda_i \mathbf{r}_i,$$
$$\mathbf{l}_i \mathbf{A} = \mathbf{l}_i \lambda_i, \tag{3}$$

where $\mathbf{r}_i$ is a right eigenvector–column, $\mathbf{l}_i$ is a left eigenvector–row. The left and right eigenvectors are normalized to Kronecker delta:

$$\mathbf{l}_j \mathbf{r}_i = \delta_{ij}. \tag{4}$$

The initial condition we define as $\mathbf{x}_0 = \mathbf{x}(0)$. Then the solution to (1) is:

$$\mathbf{x}(t) = \mathbf{R} \cdot e^{\mathbf{\Lambda} t} \cdot \mathbf{L} \cdot \mathbf{x}_0, \tag{5}$$

where $\mathbf{R}$ is right eigenvectors matrix and each column of matrix is a right eigenvector; $\mathbf{L}$ is left eigenvectors matrix and each row of matrix is a left eigenvector. The diagonal matrix $\mathbf{\Lambda}$ consists of eigenvalues of matrix $\mathbf{A}$. In this case we can write the normalization rule between $\mathbf{L}$ and $\mathbf{R}$ matrixes as:

$$\mathbf{L} \times \mathbf{R} = \mathbf{1}. \tag{6}$$

To determine the participation factor of k state variable in i mode we have to decompose i mode using state vector. So, i–mode evolution can be determined with the next formula:

$$\mathbf{z}(t) = \mathbf{L}\mathbf{x}(t). \tag{7}$$

The components of vector $\mathbf{z}(t)$ present the evolution of the mode associated with eigenvalue. After inserting (5) into (6) we receive the evolution of k modes:

$$\mathbf{z}_i(t) = \mathbf{l}_i \mathbf{x}_0 e^{\lambda_i t}. \tag{8}$$

Note that at present all authors believe that defining participation factor k state variables in i mode starts with the selection of an initial conditional.

Let us consider the approaches to participation factor proposed by Perez–Arriaga in [1]. The authors chose the initial condition as $\mathbf{x}_0 = \mathbf{r}_i$ and received

$$p_i^k = l_i^k r_i^k. \tag{9}$$

Here, the participation factor of k–state variable in i–mode defined via equation (9) is a complex value. Thus, this formula cannot be used for modal analysis, because complex values do not compare. And more importantly, in the article [4] the authors showed that formula (9) is inappropriate for real eigenvalue cases of input matrix **A.**

Abed et. al. in the article [4] used the set–theoretical formulation of initial condition for defining participation factor by measuring a relative influence of state variable k in mode i. They started with a probabilistic description of uncertainty in the initial condition and replace an average value over system initial condition by a mathematical expectation. Hence, the initial formula of the participation factor became:

$$p_i^k = E\left(\frac{(l_i^k + l_i^{k*})x_0^k}{z_i^0 + z_i^{0*}}\right). \tag{10}$$

Where z_i^0 means $z_i^0 = z_i(t=0)$ and asterisk denotes complex conjugation. This formula is suited for both cases: real and complex eigenvalues of the matrix **A** with a linear time–invariant continuous–time system. After calculations the mathematical expectation of the expression for state–in–mode participation factor can be written as:

$$p_i^k = \frac{\left[\mathrm{Re}(l_i^k)\right]^2}{\mathrm{Re}(l_i)\times\left(\mathrm{Re}(l_i)\right)^T}. \tag{11}$$

In this case the participation factor is always real value unlike in (9). Note that normalization equation is satisfied the participation factors measuring participation *k* state variable in *i* mode:

$$\sum_{k=1}^{N} p_i^k = 1. \tag{12}$$

This assumption is true for all state variables.

3. INADEQUACY OF PARTICIPATION FACTORS FORMULA

The matrix **A** is an arbitrary real matrix with size $N\times N$. The matrix values depend on system that we describe by differential linear equations. Let us check the correctness of proposed formula (11) for participation factor.

The eigenvalues of matrix **A** can be either real or complex. We will investigate correctness of each type separately.

A. Example 1: Real eigenvalues of matrix A

Let us investigate two–dimensional system [4]:

$$A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}. \tag{13}$$

The state vector of system is $\boldsymbol{x}(t) = [x_1(t), x_2(t)]$. Where *a*, *b*, and *d* are non–zero real constants with $a \neq d$. Equation (1) is written as follows:

$$\begin{pmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix}\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix}. \tag{14}$$

The eigenvalues of matrix **A** are real values $\lambda_1 = a$, $\lambda_2 = d$. Having calculated the eigenvectors we receive the following expressions:

$$\mathbf{r}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{r}_2 = \begin{pmatrix} 1 \\ (d-a)/b \end{pmatrix}. \tag{15}$$

$$\mathbf{l}_1 = \begin{pmatrix} 1 & \frac{b}{a-d} \end{pmatrix}, \quad \mathbf{l}_2 = \begin{pmatrix} 0 & \frac{-b}{a-d} \end{pmatrix}. \tag{16}$$

The right (15) and left (16) eigenvectors satisfy the normalization to Kronecker delta (4).

Then, we investigate the mode associated with eigenvalue λ_1. The participation factors of each state variable in mode using (11) are:

$$p_1^1 = \frac{(a-d)^2}{(a-d)^2 + b^2}, \quad p_1^2 = \frac{b^2}{(a-d)^2 + b^2}. \tag{17}$$

Now, we can prove that the state–in–mode participation factors (17) are normalized with 1.

This result can be verified using mode evolution expressions. Having calculated (8) we received the following result:

$$z_1(t) = \left(x_0^1 + \frac{b}{a-d} x_0^2 \right) e^{\lambda_1 t}. \tag{18}$$

The (18) shows that the participation factors of state variables in mode are non–zero and different. If we calculate participation factors via formulae (9) we will obtain incorrect results which were demonstrated by Hashlamoun et. al. in [4].

B. Example 2: Complex eigenvalues of matrix A

Let us consider the participation factor for complex eigenvalues of input matrix **A.** We accept that the matrix **A** is:

$$\mathbf{A} = \begin{pmatrix} 0 & -c \\ b & 0 \end{pmatrix}, \tag{19}$$

where b and c are real constants and $b > 0$, $c > 0$. Having calculated we receive a complex–conjugate eigenvalues of matrix **A**:

$$\lambda_1 = j\sqrt{bc}, \quad \lambda_2 = -j\sqrt{bc}. \tag{20}$$

Further in the article we investigate only the modes associated with eigenvalue λ_1. In this case the left and right eigenvectors of matrix **A** are the following:

$$\mathbf{l}_1 = \begin{pmatrix} -\frac{j}{2}\sqrt{b/c} & \frac{1}{2} \end{pmatrix}, \quad \mathbf{l}_2 = \begin{pmatrix} \frac{j}{2}\sqrt{b/c} & \frac{1}{2} \end{pmatrix}, \tag{21}$$

$$\mathbf{r}_1 = \begin{pmatrix} j\sqrt{c/b} \\ 1 \end{pmatrix}, \quad \mathbf{r}_2 = \begin{pmatrix} -j\sqrt{c/b} \\ 1 \end{pmatrix}. \tag{22}$$

Certainly, the eigenvectors satisfy the normalization rule (4).

First, write the expression for investigation of mode evolution. Having calculated (8) we received:

$$\mathbf{z}_1(t) = \left(-\frac{j\sqrt{b/c}}{2} x_0^1 + \frac{1}{2} x_0^2 \right) e^{\lambda_1 t}. \quad (23)$$

As can be seen from (23), the state variables x_0^1 and x_0^2 are entered unequal in expression. So, we can conclude that the participation factors of state variables x_0^1 and x_0^2 in mode have to be different and non–zero.

Then, we calculate participation factors state variables x_0^1 and x_0^2 via (11). Having calculated we obtain the following expression for state–in–mode participation factors:

$$p_1^1 = 0, \qquad p_1^2 = 2. \quad (24)$$

Note that participation factors produced by (24) do not satisfy the (12), although the right and left eigenvectors are normalized to (4):

$$\sum_k p_1^k \neq 1. \quad (25)$$

Following the fact that point to unused formulae (11) is zero value for participation factor state variable x_0^1, because it is inconsistent with conclusion from mode evolution.

C. Analytics that demonstrates incorrectness of state–in–mode participation factor formula

In this sub–section we prove that the incorrectness of formula for calculating state–in–mode participation factor. Another evidence of formulae (11) inaccuracy for cases of complex eigenvalues is the ambiguity of left and right eigenvectors as for phase. Let us consider this problem in details. We accept that an arbitrary complex number is:

$$z = e^{j\varphi}, \quad (26)$$

where j is an imaginary unit, φ is the argument of complex number z. Note that the module of complex number is $|z| = 1$.

Then, we consider the complex eigenvalue λ_i and according to left $\mathbf{l}_i$ and right $\mathbf{r}_i$ eigenvectors. We divide all components of left eigenvector l_i by z and multiply the right eigenvector components r_i by z. Now, the new left and right eigenvectors are $\mathbf{l}_i'$, $\mathbf{r}_i'$ respectively. After calculation we obtain the following result:

$$\mathbf{l}'_i \to \mathbf{l}_i / z = \left(\frac{l_i^1}{z}, \ldots, \frac{l_i^n}{z} \right), \tag{27}$$

$$\mathbf{r}'_i \to \mathbf{r}_i \times z = \left(z r_i^1, \ldots, z r_i^n \right). \tag{28}$$

Here, under divide and multiply operations we mean a vectorized operation. Therefore, we divide or multiply each vector component by value z. These transformations did not change the normalization condition (4). We can enough easy to belief in it:

$$\mathbf{l}'_i \times \mathbf{r}'_i = \mathbf{l}_i / z \times \mathbf{r}_i \times z = \mathbf{l}_i \times \mathbf{r}_i = 1. \tag{29}$$

In this case, the eigenvectors $\mathbf{l}'_i$, $\mathbf{r}'_i$ satisfy the same equations for defining eigenvalues and eigenvectors (3) as the $\mathbf{l}_i$, $\mathbf{r}_i$. The difference between $\mathbf{l}'_i$, $\mathbf{r}'_i$ and $\mathbf{l}_i$, $\mathbf{r}_i$ lies in the components of vectors: values of real and imaginary parts. That's why, is present inadequacy of participation factor define by formulae (11). It causes such transformation of participation factor defined by formula (11):

$$p_i^k = \frac{\left[\mathrm{Re}(l_i^k) \cos\varphi + \mathrm{Im}(l_i^k) \sin\varphi \right]^2}{\sum_k \left[\mathrm{Re}(l_i^k) \cos\varphi + \mathrm{Im}(l_i^k) \sin\varphi \right]^2}. \tag{30}$$

The result of participation factor calculation depends on phase φ: different values φ produce different magnitude of state variable participation factor. In the case when $\varphi = 0$, the formula (30) reproduce the expression (11).

So, we can conduce that the proposed formula for state–in–mode participation factor (11) is incorrect for the cases with complex eigenvalues.

4. A NEW APPROACH TO STATE-IN-MODE PARTICIPATION FACTORS

In this chapter we propose a new concept of participation factor for state–in–mode variables. As is known, the participation factors are non–dimensional scalars that measure the interaction level between mode and state variables of a linear system. To determine the relative participation of k–th state variable in the i–th mode, the authors in [1] selected the initial condition $\mathbf{x}_0$ as unit vector along the k–th coordinate axis. Later, other authors in [4] used the averaging relative contributions over an uncertain set of initial conditions.

As mentioned above, system modes are associated with eigenvalues of linear system and their evolution can be described via expression (8):

$$\mathbf{z}_i(t) = \mathbf{l}_i \mathbf{x}_0 e^{\lambda_i t}. \quad (31)$$

Let us examine the process of power system excitations in details. Imagine the situation when power system generators are disturbed with small excitations. Then, modes with different frequencies start to appear in the power system. Further, we will research the one of all modes with current frequency. The oscillation mode consists of all oscillations state variables each generator with excitation has on this frequency. The state–in–mode participation factor measure the relative participant state variable in mode evolution and display which part of perturbation of each state variable influences on mode evolution. If some state variable perturbation doesn't appear then its deposit in mode evolution equals zero. That's why the participation factor must depend on power system configuration, for example generator types, their parameters, buses and etc. and mustn't depend on initial condition. So, we can come to general conclusion: *the participation factors of state variables in mode do not depend on initial condition but depend on power system configuration.*

Let us consider the power system that can be described by *n* state variables. We accept the vector of state variables initial condition as $\mathbf{x}_0=(x_1, x_2, \ldots, x_n)$. Now, our task is to determine how each state variable of initial condition vectors influences on *i* mode evolution associated with *i* eigenvalue. So, we have *n*–dimension space where each axis is a state variable. Further, we decompose evolution of needed mode in that *n*–dimensional space:

$$\mathbf{z}_i(t) = \left(l_i^1 x_i^1 + l_i^2 x_i^2 + \ldots + l_i^n x_i^n \right) e^{\lambda_i t}. \quad (32)$$

As a result, we rewrite $x_k^1, \ldots, x_k^n$ as *n*–dimensional vectors

$$\mathbf{x}_i^1 = \left(x_i^1, 0, \ldots, 0 \right)^T, \ldots, \mathbf{x}_i^k = \left(0, \ldots, x_i^k, \ldots, 0 \right)^T, \cdots. \quad (33)$$

Note that the vectors $\mathbf{x}_i^1, \ldots, \mathbf{x}_i^n$ form an orthogonalized system of functions. For *n*–dimensional space the vector of mode evolution is the following expression:

$$\mathbf{z}_i(t) = \left(z_i^1(t), \ldots, z_i^k(t), \ldots, z_i^n(t) \right)^T. \quad (34)$$

How is seen from equation (34), we can interpreted $l_i^1, \ldots, l_i^n$ as an amplitude of weight of vectors of each state variable in vectors of mode evolution $\mathbf{z}_i(t)$. The vector weights of state variables are square of amplitude module. This approach is similar to one in quantum mechanics used for wave function [6]:

$$\psi(t) = \sum_k C_k \varphi_k(t), \quad (35)$$

where C_k is an amplitude of weight and satisfies the following normalization rule:

$$\sum_k \left|C_k\right|^2 = 1. \tag{36}$$

The functions for $\varphi_k(t)$ are a basic set that forms a closed system of functions $\{\varphi_k(t)\}$.

So, the weight of state variable i component in mode is associated with i eigenvalue is $\left|l_i^k\right|^2$. For convenience we normalize participation factor by unit and receive a new expression for calculation participation factor state variable–in–mode:

$$p_i^k = \left|l_i^k\right|^2 \Big/ \sum_k \left|l_i^k\right|^2 . \tag{37}$$

Now, this formula satisfies the normalization rule (12). Note that if we normalize left eigenvectors by/with unit, then we can omit the denominator of expression (37). Furthermore, the new proposed formula solves the ambiguity problems of left and right eigenvectors as for a phase, which was discussed in section III.C. Let us check it:

$$p_i^k = \frac{\left|l'^k_i\right|^2}{\sum_k \left|l'^k_i\right|^2} = \frac{\left|zl_i^k\right|^2}{\sum_k \left|zl_i^k\right|^2} = \frac{\left|l_i^k\right|^2 |z|^2}{\sum_k \left|l_i^k\right|^2 |z|^2} = \frac{\left|l_i^k\right|^2}{\sum_k \left|l_i^k\right|^2}. \tag{38}$$

Here, we accept that $|z|=1$. The formula (38) shows that the participation factor expression did not change.

Our formula is similar to the expression (11), but yet presents the difference. Let us apply the new participation factor expression to examples 1 and 2.

The new expression of participation factor is the same as old expression (11) for real eigenvalues cases, because right and left eigenvector components are real numbers. Thus, applying expression (37) to example 1 (see III.A) does not change the participation factor results. The results for a complex eigenvalues case will be different. Further we consider this case in details.

Example 2 Revisited: For example 2, used expression (21), (22) the participation factors state variables in mode associated with eigenvalue λ_1 using (37) are:

$$p_1^1 = \frac{b}{b+c}, \qquad p_1^2 = \frac{c}{b+c}. \tag{39}$$

As seen from (37), the participation factors of state variables x_0^1 and x_0^2 are unequal and non–zero. It verifies the conclusion derived from mode evolution.

5. A NUMERICAL EXAMPLE: TWO–MASS MECHANICS SYSTEM (REVISED [4])

In this section we compare different approaches to participation factor of state variable in mode using numerical example. As numerical example we investigate the mechanical system depicted in Fig. 1 that was examined in the article [4].

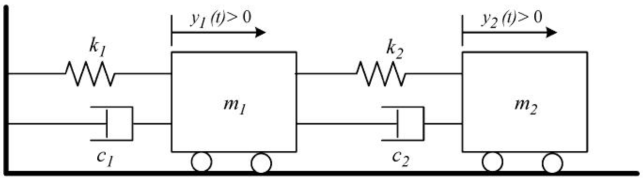

Fig. 1. Mechanical system

This mechanical system is represented by two degree–of–freedom linear time–invariant systems. The constant parameters are masses m_1 and m_2, damping coefficients c_1 and c_2, and spring coefficients k_1 and k_2. Note that we take constant values from [4]. The system dynamics is described by the linear differential equation system:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t), \tag{40}$$

where matrix **A** is the following:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -14.6 & -1.3 & 5 & 0.8 \\ 0 & 0 & 0 & 1 \\ 11.5 & 1.9 & -11.5 & -1.9 \end{bmatrix}. \tag{41}$$

The state vector is defined as

$$\mathbf{x}(t) = \left(x_1(t), x_2(t), x_3(t), x_4(t)\right)^T. \tag{42}$$

For more details, see article [4].

The matrix **A** eigenvalues are $\lambda_{1,2} = -0.217 \pm 2.315j$, $\lambda_{3,4} = -1.4203 \pm 4.2935j$. The mode evolution associated with eigenvalue $\lambda_{1,2}$ denotes $z_1(t)$ and $\lambda_{3,4}$ — $z_2(t)$.

Tables I–III show the magnitude of state variable participation factor in mode for different approaches: table I — calculated using the original definition (9) [1]; table II — revised definition (30) [4] for different phase multiplier; table III — new formula (37).

The participation factors given in Tables I–III are different. Moreover, they also have different state variable participates. Table II shows participation factor ambiguity as for a phase: different participation factor values for different phase

φ values and different state variables that are mostly present in mode 1 and 2. This serves as one more proof that the participation factor defined by formula (11) is not correct for cases of complex eigenvalues.

Table I. Participation factor, based on formula (9)

	mode 1	mode 2
x_1	0.2312	0.2804
x_2	0.2087	0.2666
x_3	0.2746	0.2210
x_4	0.2854	0.2320

Table II. Participation factor, based on formula (30)

	mode 1			mode 2		
φ	0°	45°	120°	0°	45°	200°
x_1	0.49	0.75	0.21	0.65	0.54	0.23
x_2	0.06	0.00	0.42	0.08	0.27	0.47
x_3	0.43	0.24	0.12	0.26	0.09	0.17
x_4	0.02	0.00	0.25	0.01	0.10	0.13

Table III. Participation factor, based on new formula (37)

	mode 1	mode 2
x_1	0.5916	0.7288
x_2	0.0892	0.0322
x_3	0.2660	0.2268
x_4	0.0532	0.0122

We can verify the correctness of different approaches to defining participation factor using the mode evolution of expression. For instance, let us consider the mode 1. To demonstrate that state variable participates more in mode 1 we use different initial conditions for state variables. We chose the same set of initial conditions as in article [4]: $x_0 = [0.1, 0, 0, 0]^T$, $x_0 = [0, 0.1, 0, 0]^T$, $x_0 = [0, 0, 0.1, 0]^T$, $x_0 = [0, 0, 0, 0.1]^T$.

Authors in [4] used to plot of $Re\{z_1(t)\}$ to determine which state variable is mostly present in the various initial conditions. They compared how all state variables contribute at the moment t = 0. The proposed approach isn't completely correct, because the oscillations of mode evolution are present in both directions: along real and imaginary axis. Thus, to evaluate the mode evolution, we have to evaluate their oscillation amplitude.

The simulation results are shown in Fig.2. As can be seen from curves state variable x_1 has the highest amplitude value, next are state variables x_3, x_2, x_4. This is consistent with conclusions to Table III and validates the new formula proposed for participation factor state-in-mode.

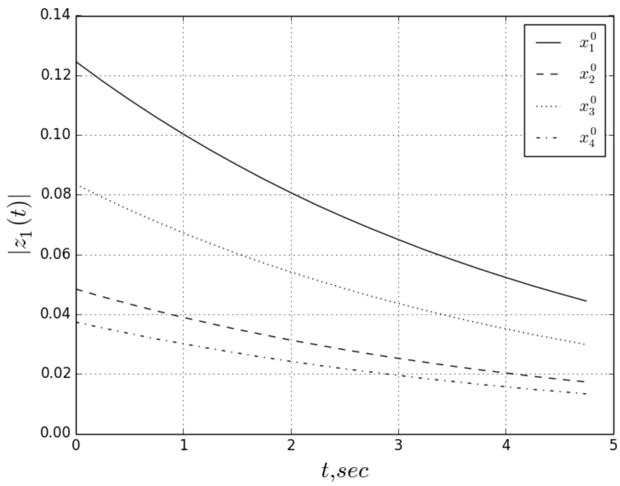

Fig. 2. Amplitude of evolution mode $z_1(t)$.

6. CONCLUSION

In this paper we made a brief overview of existing approaches to determining participation factor in modal analysis that characterizes the interaction between modes and state variables of power system. We performed calculations of participation factors with specific examples using different approaches and compared the results obtained with expressions derived from the mode evolution. As a result, the existing methods for determining a state variable participation factor that forms a mode are proved to be incorrect for complex eigenvalue of characteristic matrix of linear deferential equations system. For solving this problem we presented new expression to calculate participation factor state variables in mode which gave correct results in cases real and complex eigenvalues. The participation factor must not depend on the initial conditional. It has to be defined a power system configuration, generators and regulators types, generator relations, etc. The state–in–mode participation factor shows which part of excitation will participate in the total oscillations under current frequency. To validate the proposed approach we considered a two–mass mechanics system which proved the correctness of our participation factor theory.

Modern software applications designed to analyze power system stability deploy old approaches that provide incorrect results of modal analysis and pose risks to the operation of real power systems.

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