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ANALYTICAL METHOD FOR DETERMINING ANTROPOMETRIC PARAMETERS IN BODY MASS REDUCTION PROCESS IN THE ASPECT OF HEALTH SAFETY

Abstract. In this study, for the first time we developed a mathematical model showing that the reduction in parameters such as total body mass, total fat mass, visceral fat mass, waist circumference, hip circumference and body mass index (BMI) can be described using exponential equations. Constants k_t, k_f, k_{fv}, k_{BMI}, k_p, k_b that occur in particular equations characterize individual features related to these anthropometric parameters. The presented model was verified using experimental data obtained by analysis of body composition. These individuals were overweight in the initial moment of the experiment and eliminated or limited easily digestible products containing sugar from their diet. We showed that there is a critical time, when each individual achieved normal values of body mass index, fat mass, visceral fat mass, waist circumference, hip circumference, and waist-hip ratio. This critical time ranged between 35 and 46 weeks.
Keywords: Overweight reduction, mathematical model, bioelectric impendance method, overweight reduction coefficient, low-sugar diet, health security.

ANALITYCZNA METODA OKREŚLANIA PARAMETRÓW ANTROPOMETRYCZNYCH W PROCESIE REDUKCJI MASY CIAŁA W ASPEKCIE BEZPIECZEŃSTWA ZDROWIA

Streszczenie: W tym badaniu jako pierwsi opracowaliśmy model matematyczny pokazujący, że zmniejszenie parametrów, takich jak całkowita masa ciała, całkowita masa tłuszczu, trzewna masa tłuszczowa, obwód talii, obwód bioder i wskaźnik masy ciała (BMI), można opisać za pomocą równania wykładniczego. Stałe k_t, k_f, k_{fv}, k_{BMI}, k_p, k_b występujące w poszczególnych równaniach charakteryzują poszczególne cechy związane z tymi parametrami antropometrycznymi. Przedstawiony model zweryfikowano za pomocą danych eksperymentalnych uzyskanych poprzez analizę składu ciała. Osoby badane miały nadwagę w początkowym momencie eksperymentu i wyeliminowały lub ograniczyły z diety łatwo przyswajalne produkty zawierające cukier. Pokazaliśmy krytyczny czas, kiedy każdy człowiek osiąga normalne wartości wskaźnika masy ciała, masy tłuszczu, masy trzewnej, obwodu talii, obwodu bioder i stosunku talii do bioder. Ten krytyczny czas waha się od 35 do 46 tygodni.
Słowa kluczowe: redukcja nadwagi, model matematyczny, metoda impedancji bioelektrycznej, współczynniki redukcji nadwagi, dieta niskocukrowa, bezpieczeństwo zdrowotne.

Abbreviations: m_t, total body mass; m_f, total fat mass; m_{fv}, visceral fat mass; *BMI*, body mass index; l_p, waist circumference; l_b, hips circumference; *WHR*, wist-hip ratio.

Introduction

Nourishment is the source of the major driving force of every living organism that enables whole organism as well as its elements to do the useful work such as daily physical and intellectual activity [16]. Overeating leads to weight gain and obesity that significantly increases risk of developing life-threatening conditions such as type II diabetes, chronic cardiovascular disease, stroke, certain types of cancer, gall stones, calcification osteoarthritis and gout [15, 6, 7, 11]. According to World Health Organization, 2014 39% of the world's adult population were overweight in and 13% were obese [20]. This phenomenon is expanding and is considered one of the most important health problems of 21-st century in all age groups of the human population [17]. Moreover, obesity is considered to be the mother of all diseases and dysfunctions [8]. Therefore, body mass reduction is an important activity from the point of view of health safety.

Mechanism of obesity can be described by several concepts that include caloric imbalance, metabolic-hormonal imbalance and genetic disorder. According to the caloric imbalance concept, obesity is the result of an distortion in the energy balance between energy intake, accumulation and expenditure [19].

The concept of the metabolic-hormonal imbalance imply that excessive accumulation of fat in adipocytes is caused by stimulation of insulin secretion as a results of diet rich in high-processed carbohydrates [5, 18]. Genetic contribution to obesity include multiple gene alterations rather than one specific gene mutation [5, 12]. Thus, both the environmental and genetic factors contribute to obesity.

To analyze epidemiological studies of eating disorders, it is important to determine basic anthropometric parameters describing composition of the human body [3, 9]. This studies can be made at the following levels: global (total body mass, height, body surface area, volume, waist and hips circumference), organ and tissue (muscle mass, fat and bone mass), cellular (cellular mass, extracellular space), molecular (fat, water, protein, minerals) and atomic (10 most important elements: oxygen, carbon, hydrogen, nitrogen, calcium, phosphorus, potassium, sulfur, sodium, chlorine) [2, 1]. To evaluate overweight and obesity at the global level in epidemiological studies indicators are often used such as body mass index (BMI) or waist-hip ratio (WHR) [9, 10]. For a normal body weight BMI is comprised between 18.5 and 24.9 kg/m^2, for overweight between 25 and 29.9 kg/m^2,and obesity occurs when BMI $\geq$ 30 kg/m^2. WHR represents one of the five indicators of metabolic syndrome and it is used as an index of adipose tissue. The value of the WHR index for women of normal weight is $(WHR)_w < 0.8$, and for men $(WHR)_m < 1$. Body composition on the organ and tissue level can be analyzed using bioelectrical impedance method [10, 14].

The overweight and light obesity reduction proccess may be described by mathematical models showing time dependence of quantitative reduction of well established anthropometric parameters. To our knowledge there is no mathematical model applied to body mass reduction described in the literature. In this paper we present a mathematical model showing that reduction process of basic anthropometric parameters such as total body mass (m_t), fat mass (m_f), visceral fat mass (m_{fv}), waist (l_p) and hips (l_b) circumference can be described by exponent equations. We also confirm the model with experimental data. The study aim to show how elimination or limitation of carbohydrate-rich products affect the value of the anthropometric parameters. Additionally, the obtained model is compared to Michaelis-Menten model that is often applied in biochemical studies.

Materials and Methods

Mathematical model

Each of the anthropometric indicators related to the state of the human body at a given timepoint and various physiological conditions (age, obesity, etc.) can be described in the general form:

$$X = X_a + X_e \quad (1)$$

where: X – anthropometric indicator related to the state of the human body at a given timepoint, X_a – anthropometric indicator related to an acceptable state of the human body, X_e – excess of the anthropometric indicator. These indicators may be: total body mass (m_t), fat mass (m_f), BMI, visceral fat mass (m_{fv}), water mass (m_w), waist (l_p) and hips (l_b) circumference, WHR, etc.

The human body consists of water, fat, muscle, bone and viscera and that a quantitative measure of these components is their mass. Therefore, the total body mass can be expressed by the equation:

$$m_t = m_f + m_w + m_m + m_o + m_v + m_s \quad (2)$$

where m_f – fat mass , m_w – water mass, m_m – muscle mass, m_o – bone mass, and m_v – organ mass (heart, liver, brain, intestines, stomach etc.), m_s – fascia mass. Fat mass is the main component that changes during weight gain.

For normal body mass (m_a), BMI fulfil condition 18,5 kg/m^2 < BMI < 24.9 kg/m^2, therefore the most appropriate body mass index would be in the middle of this range, BMI = 21.7 kg/m^2. It should be noted that a fraction of irreducible fat in adipose tissue is necessary as it plays important physiological role. For this study, we assumed that the normal values of anthropometric parameters (m_a, l_{pa}, l_{ba}) was obtained when BMI ≤ 24.9 kg/m^2. Total fat mass in the human body (m_f) is the sum of the normal fat mass (m_{fa}) and excessive fat mass (m_{fr}). Both normal (m_{fa}) and excessive (m_{fr}) fat masses include visceral part (m_{fav}), (m_{fap}) and remaining (m_{frv}) and (m_{frp}). This can be written in the form

$$m_f = m_{fa} + m_{fr} = m_{fav} + m_{fap} + m_{frv} + m_{frp} \quad (3)$$

where $m_{fa} = m_{fav} + m_{fap}$ and $m_{fr} = m_{frv} + m_{frp}$.

This means that the normal body mass (m_a) can be presented by the formula

$$m_a = m_{fa} + C = m_{fav} + m_{fap} + C \quad (4)$$

where $C = m_m + m_o + m_v + m_s$.

Therefore, the Equation (2) can be written in the following form

$$m_t = m_a + m_{fr} \quad (5)$$

For overweight, waist circumference (l_p) is the sum of normal waist circumference (l_{pa}) and its excess that is caused by accumulation of fat (l_{pf})

$$l_p = l_{pa} + l_{pf} \quad (6)$$

Similarly, hip circumference (l_b) is the sum of normal hip circumference (l_{ba}) and its excess that is caused by accumulation of fat (l_{bf})

$$l_b = l_{ba} + l_{bf} \quad (7)$$

Macroscopic manifestation of excessive accumulation of fat in adipocytes is an increased fat mass and thus total body weight, whereas macroscopic manifestation of removal of fat from the adipocytes is reduced fat mass and thus total body weight. Considering the Eq. (2), the mass reduction process can be analyzed by analysis of the reduction of fat in adipose tissue. It is impossible to predict which adipocytes will remove fat in the process of weight loss, yet adipocytes, reduction speed of a generalized anthropometric parameter (X_j) can be written as

$$\frac{dX_e}{dt} = -k_e X_e \tag{8}$$

where (dX_e/dt) – reduction speed of anthropometric parameter X_e, k_e – reduction constant of anthropometric parameter characteristic for each individual (1/s). X_e can be substituted with the total body weight (m_c), fat mass (m_f), visceral fat mass (m_{fv}), waist circumference (l_p) or hip circumference (l_b).

The solution to this differential equation is the expression

$$X_e = X_{e0} \exp[-k_e(t - t_0)] \tag{9}$$

where X_{e0} – value of an anthropometric parameter at the initial moment i.e. at overweight condition (t_0), X_e – value of an anthropometric parameter after a time (t) different than the initial moment (in condition of lack of overweight or obesity).

Differentiating the Eq. (9) after time, we obtain

$$\frac{dX_e}{dt} = -k_e X_{e0} \exp[-k_e(t - t_0)] \tag{10}$$

This equation can be written in the alternative to the Eq. (10) form as

$$R_e = R_{e0} \exp[-k_e(t - t_0)] \tag{11}$$

where: $R_e = dX_e/dt$ – reduction speed of anthropometric parameter, $R_{e0}=k_eX_{e0}$ – reduction speed of anthropometric anthropometric parameter at the initial moment.

The Eq. (11) shows that the constant of anthropometric parameter reduction k_e determines reduction rate of anthropometric parameter R_e that is decreasing with time t. By logarithm of both sides of Eq. (11) and applying the appropriate algebraic transformations we obtain

$$k_e = \frac{1}{t - t_0} \ln \frac{R_{e0}}{R_e} \tag{12}$$

Measurement procedure

The experimental study involved 4 healthy individuals, 2 men and 2 women and started in September 2014. Participants eliminated from their diet products containing corn syrup, sweets, alcohol, refined carbohydrates (white flour), sucrose, fruits rich in fructose (plums, pears, grapes), and reduced to a minimum starch-rich vegetables (potatoes, beans). They followed a regime of having 4-5 meals a day between 7 a.m. and 6 p.m. Participation in the study was voluntary. Participants decided to change diet mainly for aesthetic and not medical reasons.

Sex, age, height (h), total body mass (m_t), BMI, waist circumference (l_p) hip circumference (l_b) and WHR of individuals marked 1, 2, 3, 4 at the start of experiment are presented in Table 1.

Tab. 1. Anthropomorphic indicators at the initial timepoint ($t_{0)}$ sex, age, height (h) and total body mass (m_t), BMI (body mass index), waist circumference (l_p), hips circumference (l_b), WHR (wist-hip ratio).

Individual	**Sex**	**Age (years)**	***h* (m)**	**m_t (kg)**	**BMI (kg/m^2)**	**l_p (m)**	**l_b (m)**	**WHR**
1	M	62	1.85	113.0	33.02	1.06	1.15	0.92
2	M	41	1.82	104.8	31.64	1.02	1.12	0.91
3	W	41	1.65	89.6	32.91	0.98	1.14	0.86
4	W	36	1.66	81.8	29.68	0.99	1.14	0.87

BMI was calculated based on formula BMI = m_t/h^2 kg/m^2 and WHR based on the formula WHR = l_p/l_b. As shown in the Table 1, the subjects 1−3 show light obesity, and the subject 4 overweight. All participants were equipped with ergonomic circumference measuring tape SECA 201 (Seca gmbh, Hamburg, Germany), Microtoise 2.20 (Girodmedical, Nantes, France) and body composition analyzer TANITA BC 420 SMA (Tanita Corporation, Tokyo, Japan) and were instructed in the measuring procedures. Waist circumference (l_p) and hips circumference (l_b) were measured to the nearest 1 mm using ergonomic circumference measuring tape SECA 201. The standing body height (stature) was measured to the nearest 1 mm using a Microtoise 2.20. Each measurement was done in triplicate and arithmetic average of these measurements was used. Body composition was analyzed by measuring bioelectrical impedance using TANITA BC 420 SMA analyzer [10, 14]. The analyzer cooperate with GMON programme (Tanita Corporation, Tokyo, Japan) and provides data such as total body mass (m_t), fat mass (m_f), total body water or water mass (m_w), muscle mass (m_m), bone mass (m_o), and visceral fat mass (m_{fv}). Total body mass and muscle mass were measured to the nearest 100 g, fat mass and water mass to the nearest 0.1%. Measurements were performed by participant every Saturday, i.e., at week intervals for 54 weeks at their place of residence. All measurements were performed in the morning (~7^{00}), in the vertical body position, without clothes and jewelry, 30 minutes after urination, before breakfast. Intense physical activity was avoided at least 12 hours before the measurement.

Curve fitting

For measurements of m_t, m_f, m_{fv}, BMI, l_p and l_b, a first-order exponential decay curve was fitted into data points using Origin Software 6.1 (OriginLab Co, Northampton, MA, USA). The equation obtained for the fitted curves can be writted in a general form:

$$X = X_a + A_{e0}\exp(-k_e t) \qquad (13)$$

where: X_a – normal value of the generalized anthropometric parameter, $A_{e0} = X_0 - X_a$ – amplitude of reduction of generalized anthropometric parameter, k_e – constant of reduction of generalized anthropometric parameter, t – time. The constances X_a, A_{e0} and k_e are parameters for the fitted curves.

The fitted curves calculated based on Eq. (13) are shown in graphs as full lines for reduction of m_t, m_f, m_{fv}, BMI, l_p, l_b parameters. Eq. (13) is presented in a corresponding form for each analyzed parameter in the figure legends.

Constant of reduction of generalized anthropometric parameter (k_e) was calculated separately for m_t, m_f, m_{fv}, BMI, l_p, l_b parameters using equations obtained from curves fitted to set of data points for each individual. This curve fitting resulted in the certain constances k_t, k_f, k_{fv}, k_{BMI}, k_p and k_b for measured parameters.

Results and Discussion

This paper presents the measurement results of the total body mass (m_t), fat mass (m_f), visceral fat mass (m_{fv}), waist circumference (l_p), hip circumference (l_b) for an individual 1 (□), 2 (○), 3 (◊) and 4 (△) obtained during 54 weeks. In addition, we calculated BMI and WHR for each individual and timepoint. The average results of measurements of water mass (m_w), muscle mass (m_m), bone mass (m_o) are listed in Table 2. The measurements shown in Table 2 indicate that the total water mass (m_w), the total muscle mass (m_m) and the total bone mass (m_o) fluctuate around the average values. It can therefore be assumed that the total water mass (m_w), total muscle mass (m_m) and total bone mass (m_o) are constant within the measurement uncertainty. Given that we can simplify Eq. (2) to the form

$$m_t = m_f + C \qquad (14)$$

where $C = m_m + m_o + m_v + m_s = \text{const}$. A constant C is different for each individual (Table 2). C is similar for the two men ($C_1 \approx C_2$) and the two women ($C_3 \approx C_4$) and $C_1 \approx C_2 > C_3 \approx C_4$.

Tab. 2. Mean values of the water mass (m_w), muscle mass (m_m), bone mass (m_o) determined by bioelectrical impedance for four individuals

Individual	mass (kg) of			C (kg)
	water (m_w)	muscles (m_m)	bones (m_o)	
1	51.24±0.92	19.75±0.30	3.68±0.10	74.67±0.44
2	50.84±0.74	19.76±0.19	3.62±0.06	74.22±0.33
3	35.76±0.77	12.23±0.19	2.56±0.07	50.55±0.34
4	33.44±0.36	11.22±0.17	2.50±0.02	47.16±0.18

Reduction of total body mass was shown in Fig. 1 as nonlinear dependence $m_t = f(t)$. For the individual 1 (□), there was a 28.6 kg decrease in the total body mass that constituted 25.3% of initial m_t, for the individual 2 (○) there was 23.6 kg decrease that was 23.6% of initial m_t, for the individual 3 (◇) there was a 24.8 kg decrease that was 27.7% of initial m_t, while for the individual 4 (△), there was a 14.7 kg decrease that was 18.0% of initial m_t in 54 weeks (t_k). In the beginning the rate of body mass reduction was substantially faster than in the end of the experiment.

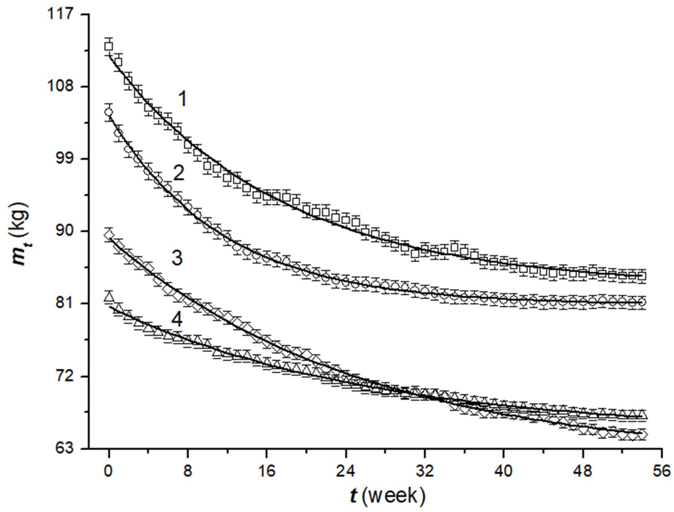

Fig. 1. Dependence $m_t = f(t)$ for the individual 1 (□), 2 (○), 3 (◇) and 4 (△). Solid lines 1-4 illustrate the results of the calculations of the total body mass (m_t) based on the equation $m_t = m_{ta} + A_{t0}$ exp $(-k_t t)$. For the individual 1 m_{ta} = 83.14 ± 0.27 kg, A_{t0} = 28.68 ± 0.29 kg, k_t = (5.70 ± 0.02) × 10^{-2} $(\text{week})^{-1}$ (χ^2 = 0.329, R^2 = 0.995). For the individual 2 m_{ta} = 80.88 ± 0.06 kg, A_{t0} = 23.57 ± 0.12 kg, k_t = (8.64 ± 0.01) × 10^{-2} $(\text{week})^{-1}$ (χ^2 = 0.047, R^2 = 0.999). For the individual 3 m_{ta} = 61.44 ± 0.29 kg, A_{t0} = 27.70 ± 0.24 kg, k_t = (3.87±0.01) × 10^{-2} $(\text{week})^{-1}$ (χ^2 = 0.107, R^2 = 0.998). For the individual 4 m_{ta} = 64.91 ± 0.24 kg, A_{t0} = 15.75 ± 0.20 kg, k_t = (3.77 ± 0.01) × 10^{-2} $(\text{week})^{-1}$ (χ^2 = 0.066, R^2 = 0.996).

At the starting point of the experiment, the BMI was in the obesity range for individuals 1-3 and in overweight range for individual 4. Reduction of BMI shown in Fig. 2 as nonlinear dependence BMI = $f(t)$ was calculated using total body mass measurement shown in Figure 1 and height measurements listed in Table 1. From these curves it results that a decline in BMI was 8.36 kg/m^2 for the individual 1 (□), 7.12 kg/m^2 for the individual 2 (○), 9.11 kg/m^2 for the individual 3 (⋄) and 5.31 kg/m^2 for the individual 4 (△). All 4 individuals reached normal body mass with BMI < 24.9 kg/m^2 by the end of experiment.

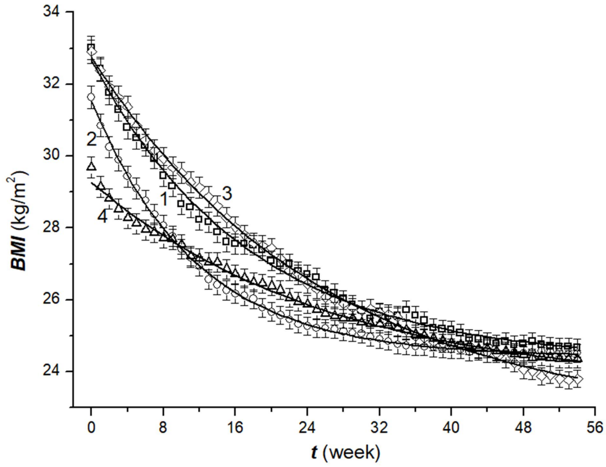

Fig. 2. Dependence $BMI = f(t)$ for the individual 1 (□), 2 (○), 3 (⋄) and 4 (△) calculated based on formula $BMI = m_t/h^2$ (kg/m^2) and the test results shown in Fig. 1 and listed in Table 1. Solid lines 1-4 illustrate the results of the BMI calculations based on equation $BMI = (BMI)_a + A_{BMI0} \exp(-k_{BMI}t)$. For the individual 1 $(BMI)_a = 24.29 \pm 0.08$ kg/m^2, $A_{BMI0} = 8.38 \pm 0.09$ kg/m^2, $k_{BMI} = k_t = (5.70 \pm 0.02) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.329$, $R^2 = 0.995$). For the individual 2 $(BMI)_a = 24.42 \pm 0.02$ kg/m^2, $A_{BMI0} = 7.11 \pm 0.03$ kg/m^2, $k_{BMI} = k_t = (8.64 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.047$, $R^2 = 0.999$). For the individual 3 $(BMI)_a = 22.58 \pm 0.11$ (kg/m^2), $A_{BMI0} = 10.17 \pm 0.09$ kg/m^2, $k_{BMI} = k_t = (3.87 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.107$, $R^2 = 0.998$). For the individual 4 $(BMI)_a = 23.55 \pm 0.09$ kg/m^2, $A_{BMI0} = 5.72 \pm 0.07$ kg/m^2, $k_{BMI} = k_t = (3{,}77 \pm 0.01)$ (week)$^{-1}$ ($\chi^2 = 0.066$, $R^2 = 0.996$).

Reduction of total fat mass was shown as a nonlinear dependence $m_f = f(t)$ in Fig. 3. The dependence $m_f = f(t)$ indicates that the total fat mass dropped 26.6 kg (71.5% of initial m_f) for the individual 1 (□), 19.0 kg (72.2% of initial m_f) for the individual 2 (○), 21.8 kg (60.2% of initial m_f) for the individual 3 (⋄) and 12.8 kg (38.9% of initial m_f) for the individual 4. Next, we calculated reduction of total visceral fat mass shown in Figure 4 as a nonlinear dependence $m_{fv} = f(t)$. The dependence $m_{fv} = f(t)$ indicates that the total visceral fat decreased 11.2 kg (71.3% of initial m_{fv}) for the individual 1 (□), 7.8 kg (70.9% of initial m_{fv}) for the individual 2 (○), 4.8 kg (60.0% of initial m_{fv}) for the individual 3 (⋄) and 2.7 kg (38.6% of initial m_{fv}) for the individual 4 (△). For each individual, the percentage of reduction for total fat mass and viscelar fat mass was similar.

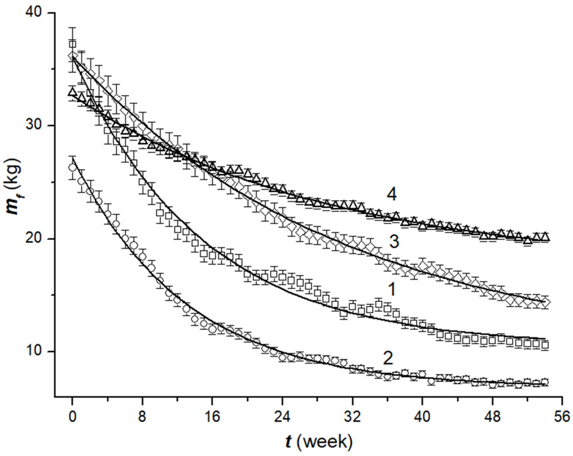

Fig. 3. Dependence $m_f = f(t)$ for the individual 1 (□), 2 (○), 3 (⋄) and 4 (△). Solid lines 1-4 illustrate the results of the calculations of total fat mass (m_f) based on the equation $m_f = m_{fa} + A_{f0}$ exp ($-k_f t$). For the person 1 $m_{fa} = 10.50 \pm 0.25$ kg, $A_{f0} = 25.62 \pm 0.35$ kg, $k_f = (7.14 \pm 0.03) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.457$, $R^2 = 0.991$). For the individual 2 $m_{fa} = 6.81 \pm 0.11$ kg, $A_{f0} = 20.33 \pm 0.19$ kg, $k_f = (7.75 \pm 0.02) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.126$, $R^2 = 0.996$). For the individual 3 $m_{fa} = 9.64 \pm 0.38$ kg, $A_{f0} = 26.50 \pm 0.32$ kg, $k_f = (3.17 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.085$, $R^2 = 0.998$). For the individual 4 $m_{fa} = 17.43 \pm 0.27$ kg, $A_{f0} = 15.27 \pm 0.22$ kg, $k_f = (3.40 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.056$, $R^2 = 0.996$).

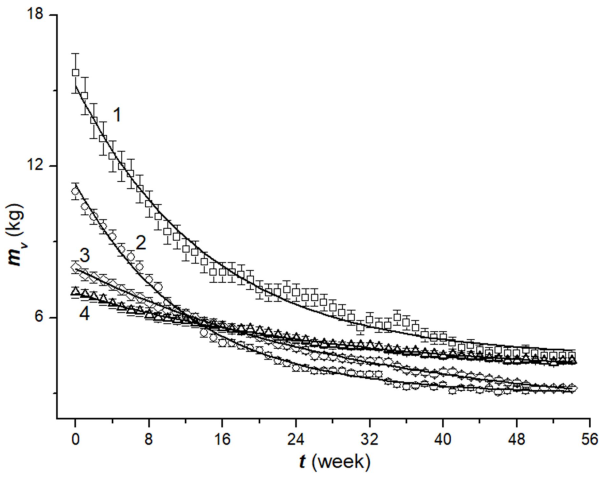

Fig. 4. Dependence $m_{fv} = f(t)$ for the individual 1 (□), 2 (○), 3 (⋄) and 4 (△). Solid lines 1-4 illustrate the results of the calculations of visceral fat mass (m_{fv}) based on the equation $m_{fv} = m_{fva} + A_{fv0}$ exp ($-k_{fv}t$). For the individual 1 $m_{fva} = 4.43 \pm 0.11$ kg, $A_{fv0} = 10.74 \pm 0.15$ kg, $k_{fv} = (6.80 \pm 0.02) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.086$, $R^2 = 0.990$). For the individual 2 $m_{fva} = 2.96 \pm 0.04$ kg, $A_{fv0} = 8.31 \pm 0.07$ kg, $k_{fv} = (8.01 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.020$, $R^2 = 0.996$). For the individual 3 $m_{fva} = 2.15 \pm 0.09$ kg, $A_{fv0} = 5.80 \pm 0.07$ kg, $k_{fv} = (3.21 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.005$, $R^2 = 0.998$). For the individual 4 $m_{fva} = 3.71 \pm 0.06$ kg, $A_{fv0} = 3.25 \pm 0.05$ kg, $k_{fv} = (3.41 \pm 0.01) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.003$, $R^2 = 0.996$).

Fig. 5 shows that time dependence of waist circumference (l_p) is nonlinear and can be presented as $l_p = f(t)$. The difference in waist circumference between the start and the end of experiment was 0.14 m for the individual 1 (□), 0.15 m for the individual 2 (○), 0.115 m for the individual 3 (⋄) and 0.08 m for the individual 4 (△). The reduction of initial waist circumference ranged from 14.7% for individual 2 to 8.0% for individual 4. Next, the results of time dependence of hips circumference (l_b) presented in Fig. 6 show that that dependence $l_b = f(t)$ is nonlinear. The decrease in hips circumference for each individual is as follows: 0.15 m for the individual 1 (□), 0.09 m for the individual 2 (○), 0.1 m for the individual 3 (⋄) and 0.08 m for the individual 4 (△). Thus, reduction of initial hips circumference ranged from 13% for individual 1 to 7% for individual 4. A comparison of the test results shown in Figure 5 and 6 indicated that stabilization value of the l_b occured earlier than the stabilization of the l_p for all individuals. For the individual 1, 3 and 4, normal l_p appeared after $t_{c1} = 48$ weeks and for the individual; 2 for $t_{c2} \geq 40$ weeks. For all four individuals, normal l_b appeared at similar timepoint of $t_{c1} = t_{c2} = t_{c3} = t_{c4} \geq 16$ weeks.

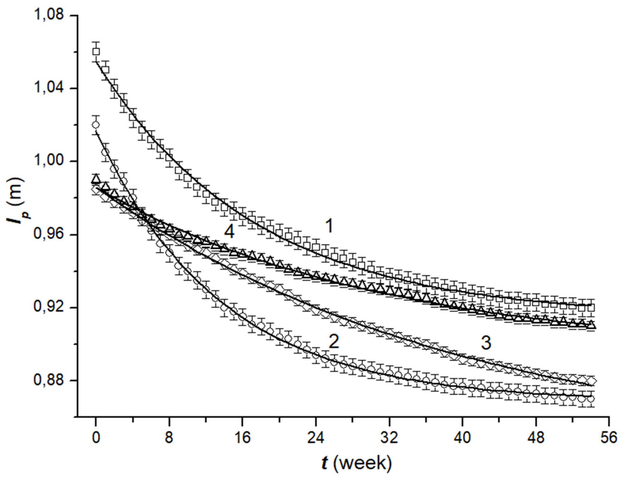

Fig. 5. Dependence $l_p = f(t)$ for the individual 1 (□), 2 (○), 3 (⋄) and 4 (△). Solid lines 1-4 illustrate the results of the calculations of waist circumference (l_p) based on the equation $l_p = l_{pa} + A_{p0}$ exp ($-k_p t$). For the individual 1 $l_{pa} = 0.920 \pm 0.004$ m, $A_{p0} = 0.140 \pm 0.001$ m, $k_p = (5.757 \pm 0.001) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.044$, $R^2 = 0.998$). For the individual 2 $l_{pa} = (0.870 \pm 0.001)$ m, $A_{p0} = (0.150 \pm 0.001)$ m, $k_p = (7.303 \pm 0.002) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.01$, $R^2 = 0.999$). For the individual 3 $l_{pa} = (0.880 \pm 0.001)$ m, $A_{p0} = (0.105 \pm 0.001)$ m, $k_p = (2.473 \pm 0.001) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.001$, $R^2 = 0.999$). For the individual 4 $l_{pa} = (0.911 \pm 0.002)$ m, $A_{p0} = (0.080 \pm 0.001)$ m, $k_p = (3.119 \pm 0.001) \times 10^{-2}$ (week)$^{-1}$ ($\chi^2 = 0.001$, $R^2 = 0.996$).

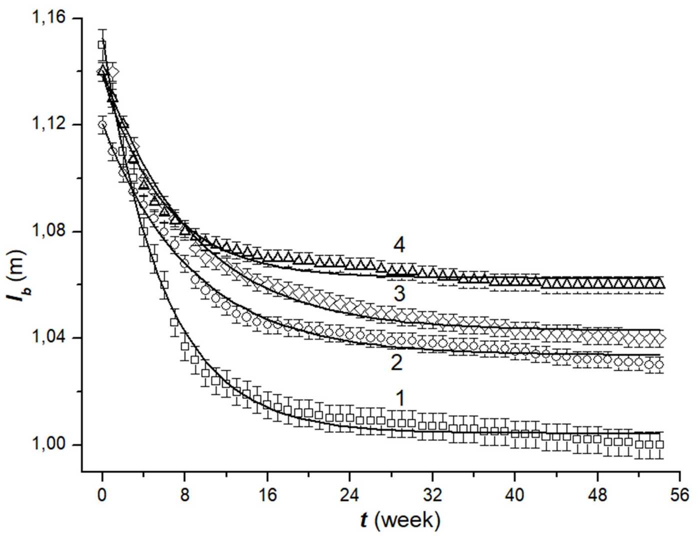

Fig. 6. Dependence $l_b = f(t)$ for the individual 1 (□), 2 (○), 3 (◊) and 4 (△). Solid lines 1-4 illustrates the results of the calculations of hips circumference (l_b) based on the equation $l_b = l_{ba} + A_{b0}$ exp $(-k_b t)$. For the individual 1 l_{ba} = 1.000 ± 0.001 m, A_{b0} = 0.1483 ± 0.0019 m, k_b = (16.990 ± 0.001) × 10^{-2} (week)$^{-1}$ (χ^2 = 0.001, R^2 = 0.994). For the individual 2 l_{ba} = 1.030 ± 0.001 m, A_{b0} = 0.0868 ± 0.0013 m, k_b = (157.602 ± 0.29) × 10^{-2} (week)$^{-1}$ (χ^2 = 0.001, R^2 = 0.991). For the individual 3 l_{ba} = 1.0401 ± 0.0001 m, A_{b0} = 0.0977 ± 0.0016 m, k_b = (11.349 ± 0.003) × 10^{-2} (week)$^{-1}$ (χ^2 = 0.001, R^2 = 0.989). For the individual 4 l_{ba} = 1.061 ± 0.001 m, A_{b0} = 0.0076 ± 0.0018 m, k_b = (16.802 ± 0.001) × 10^{-2} (week)$^{-1}$ (χ^2 = 0.001, R^2 = 0.979).

Fig. 7 shows dependence WHR = $f(t)$ for the person 1 (□), 2 (○), 3 (◊) and 4 (△), calculated based on the results of $l_p = f(t)$ and $l_b = f(t)$ shown in Figs. 5 and 6. Curves 1, 3 and 4 in this figure demonstrate that the WHR initially increases, and for t_{10}=10 weeks, reaches the maximal value WHR = 0.97 (curve 1), WHR = 0.89 (curves 3 and 4), and then decrease linearly to a fixed value WHR = 0.92 (curve 1), WHR = 0.84 (curve 3) and WHR = 0.85 (curve 4). Exponential decrease in WHR is observed only for curve 2, where WHR decreases from WHR = 0.91 (for t_0=0) to WHR = 0.84 (for t_k=54 weeks). The Lorenz curves to curves 1, 3 and 4 were fitted. From Fig. 7 it results that a clear stabilization of WHR occured after t_{c1}=t_{c2} ≥28 weeks for individuals 1 and 2. For individuals 3 and 4 the WHR values tend to decrease in the direction of stability, but did not reach an normal value within 54 weeks. In addition, a comparison of the results shown in Figs. 5 and 7 indicates that stabilization of WHR is coupled to the stabilization of the l_p for all individuals.

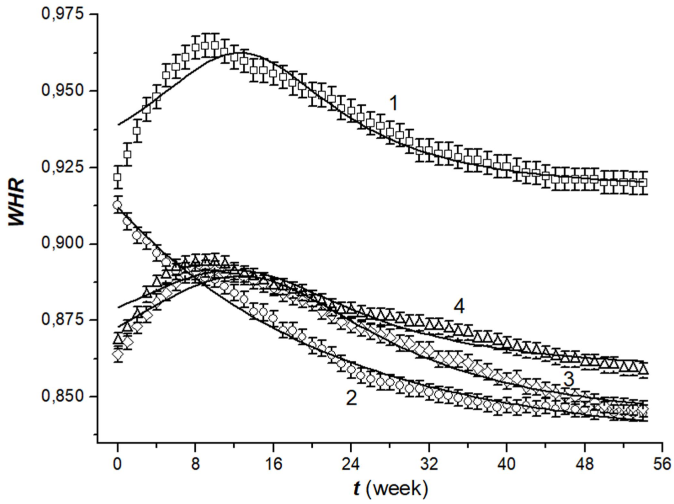

Fig. 7. Dependence $WHR = f(t)$ for the individual 1 (□), 2 (○), 3 (◊) and 4 (△) calculated based on the test results of $l_p = f(t)$ and $l_b = f(t)$ listed in Fig. 5 and 6 and equation WHR = $l_p \times l_b^{-1}$. Solid lines 1-4 illustrate the results of the curve fitting: the Lorenz curve (curve 1) (area – 1.761, center – 12.57, width – 24.4, offset – 0.92 and height – 0.046, χ^2 = 0.728, R^2 = 0.934), the exponent (curve 2) $WHR = (WHR)_a + A_{WHR0} \exp(-k_{WHR}t)$, $(WHR)_a$ = 0.836 ± 0.001 m, A_{WHR0} = 0.0764 ± 0.0013 m, k_{WHR} = (0.0457 ± 0.0029) × 10^{-2} (week)$^{-1}$ (χ^2 = 0.0001, R^2 = 0.988), the Lorenz curve (curve 3) (area – 2.71, center – 12.55, width – 35.06, offset – 0.84 and height – 0.049, χ^2 = 0.0001, R^2 = 0.971), the Lorenz curve (curve 4) (area – 1.67, center – 11.82, width – 31.31, offset – 0.86 and height – 0.034, χ^2 = 0.0001, R^2 = 0.925).

The course of the curves that are demonstrated in Figs. 1-7 shows that there is a critical time (t_{ci}, i =1, 2, 3, 4), different for the individual curves, after which normal values of body weight (m_{ta}) (Fig. 1), BMI (BMI_a) (Fig. 2), fat mass (m_{fa}) (Fig. 3), visceral fat mass (m_{fva}) (Fig. 4) waist circumference (l_{pa}) (Fig. 5) hips circumference (l_{ba}) (Fig. 6) and WHR (WHR_a) (Fig. 7) were achieved. For the curves 1 and 4, which are presented in Figs. 1-4, this critical time is $t_{c1} = t_{c4}$ = 44 weeks, for the curve 2 – t_{c2} = 35 weeks, and for the curve 3 – t_{c3} = 46 weeks.

Results shown in Figs. 1-6 demonstrate that constants k_t, k_f, k_{fv}, k_{BMI}, k_p and k_b characterized individual features that corresponded to the rate of the reduction of macroscopic parameters, such as total body mass (m_t), fat mass (m_f), visceral fat mass (m_{fv}), BMI, waist circumference (l_p) and hips circumference (l_b), respectively. Table 3 demonstrate that constants k_t, k_{BMI}, k_f, k_{fv} and k_{pi} can be determined for each individual and these constants fulfil the following condition (within the uncertainty of measurement)

$$k_t = k_{BMI} = k_f = k_{fv} = k_p = k_i \quad (15)$$

where i = 1, …, 4.

Tab. 3. Values of reduction constants of total body mass (k_t), BMI (k_{BMI}), fat mass (k_f), visceral fat mass (k_{fv}), waist circumference (k_p), hips circumference (k_b), k_i – the arithmetic mean of the k_t, k_f, k_{fv} annd k_{pi} for individual 1, 2, 3 and 4.

Individual i	$k_t = k_{BMI}$ **×10²** **((week)⁻¹)**	k_f**×10²** **((week)⁻¹)**	k_{fv} **×10²** **((week)⁻¹)**	k_{pi} **×10²** **((week)⁻¹)**	k_i **×10²** **((week)⁻¹)**	k_{bi} **×10²** **((week)⁻¹)**
1	5.70	7.14	6.82	5.76	6.15±0.33	16.99
2	8.64	7.75	8.05	7.30	7.72±0.27	157.60
3	3.87	3.17	3.20	2.47	2.94±0.27	11.35
4	3.77	3.40	3.39	3.20	3.36±0.11	16.80

The condition presented above is the result of coupling the reduction processes of total body mass with reduction of BMI, total fat mass, visceral fat mass and waist circumference. Furthermore, $k_2 > k_1 > k_4 > k_3$, wherein k_2 is insignificantly greater than k_1 and k_4 is insignificantly greater than $k_{3,}$ whereas k_2 and k_1 are 2 fold greater than k_4 and k_3. Therefore, this condition can be written as $k_2 \approx k_1 > k_4 \approx k_3$. Individuals 1 and 2 are men, whereas 3 and 4 are women, therefore k constantsvalue is strongly linked to a sex of the individuals. In contrast, various relationships between the coefficients k_i and k_{bi} did not depend on a gender of the subjects. It can merely be written that $k_i < k_{bi}$ (i = 1, …, 4). This relationship indicates that there was no coupling between the processes of reduction of total body mass with reduction of BMI, total fat mass, total visceral fat mass and waist circumference with the process of reduction of hips circumference.

The results presented in this paper show that described nutrition regime resulted in reduction of fat mass (total and visceral), which is revealed as reduction of total body mass, as well as waist and hips circumference. Fat mass decreased exponentially. It seems that the coefficient k_e which occurs in the Eq. (13) is a measure of inter-individual variability and its value is ~2 fold higher for men than for women (Table 3). In addition, it can be noted that the k_p coefficient calculated based on the measurements of waist circumference (l_p) has a similar value (within the measurement uncertainty) as coefficients k_f, k_m and k_{fv}. Therefore, the measurements of waist circumference correspond well with the measurements of total body mass, fat mass and visceral fat mass.

To describe the process of removing substrate of the human body Michaelis-Menten equation is often used [13, 4]. For processes of reduction anthropometric parameters considered in the paper the equation we will write in the form

$$\frac{dX_e}{dt} = -\frac{v_{\max} X_e}{K_m + X_e} \quad (16)$$

where $v_{max} = (dX_e/dt)_{max}$ – maximal reduction speed of anthropomorphic parameter X_e, K_m – Michaelis-Menten constant, which quantifies the anthropometric parameter value at which the rate of the reduction process of this parameter is equal to half of the v_{max}.

To discuss the usefulness of the Eq. (16) to describe the process of reducing X_e, is enough to compare the Eqs. (9) and (16) and check whether there is a relationship

$$k_e = \frac{v_{\max}}{K_m + X_e} \quad (17)$$

The results described in paper indicate that a process of reduction anthropometric parameters is coupled to the reduction of fat mass (m_f), contained in a non-uniformly flattened adipose tissue. Accordingly, the analysis of Eq. (17) we can be limited to calculation of k_f, assuming $v_{max} = v_{fmax} = (dm_f/dt)_{max}$, $K_m = K_{mf}$ oraz $X_e = m_{fa}$. In order to determine v_{fmax} and K_{mf} we use the experimental results shown in Fig. 3 and the method of Linewaeaver-Burk [Moody et al. 1974; Han and Lavenspiel 1988]. Linewaeaver-Burk method is based on the linear curve fitting in the experimental dependence described with $\Delta t/(X_{e0} - X_e) = f(1/X_e)$ equation to aquire values of $1/K_m$ and $1/v_{max}$ from the fitted curve. To obtain k_e the values of K_m and v_{max} are considered in Eq. (17).

The results of the calculations of v_{fmax}, K_{mf} and k_{ec} are summarized in Table 4. The table shows that for the individual 1 $k_{fc} = 0.49k_f$, for the individual 2 $k_{fc} = 0.65k_f$, for the individual 3 $k_{fc} = 0.94k_f$ and for the individual 4 $k_{fc} = 0.2k_f$. Hence it follows that only for the individual 3 it can be assumed that $k_{fc} \approx k_f$. For the rest of the participants $k_{fc} < k_f$, wherein for the individual 4 $k_{fc} << k_f$. It means that using Michaelis-Menten type equation to describe the process of reducing the fat content in the human fat tissue is problematic.

Tab. 4. Values of maximal reduction speed of fat mass (v_{fmax}), Michaelis-Menten constant (v_{fmax}), normal fat mass (m_{fa}), reduction constant of fat mass (k_{fc}) calculated based on Eq. (17), reduction constant of fat mass (k_f) for individual 1, 2, 3 and 4.

Individual *i*	**v_{fmax} (kg (week)$^{-1}$)**	**K_{mf} (kg)**	**m_{fa} (kg)**	**$k_{fc} \times 10^2$ ((week)$^{-1}$)**	**$k_f \times 10^2$ ((week)$^{-1}$)**	**k_{fc}/k_f**
1	2.92	73.14	10.50	3.49	7.14	0.49
2	8.44	160.0	6.81	5.06	7.75	0.65
3	3.20	98.07	9.64	2.97	3.17	0.94
4	0.52	59.03	17.43	0.68	3.40	0.20

Low-sugar diet leads to reduction of anthropometric parameters such as total body mass, total fat mass, visceral fat mass, waist and hip circumference. Applied diet and a proper eating regime satisfied the hunger. Importantly, the reduction of anthropometric parameters is long-lasting. In this study, the normal body mass was defined as body mass, for which BMI is lower than 24.9 kg/m^2. However, achieving BMI < 24.9 kg/m^2 does not nessesary mean that the persons achieved body mass accepted by the individual and/or a doctor. The mass reduction may be continued for both aesthetic and the health purposes. Our results show that body mass and other anthropometric parameters decline substantially slower when they reach normal values.

Conclussions

1. Anthropometric parameters decline according to the equation of exponent. There is a critical time (t_{ci}), when normal body mass with BMI < 24.9 kg/m^2 is achieved. It ranges between 35 weeks $\leq t_c \leq$ 46 weeks for the four individuals.
2. The reduction of anthropometric parameters can be described by constants k_t, k_f, k_{fv}, k_{BMI}, k_p and k_b, that are present in the particular exponent equations that correspond to the rate of the reduction of macroscopic parameters, such as total body mass (m_t), fat mass (m_f), visceral fat mass (m_{fv}), BMI, waist circumference (l_p) and hips circumference (l_b), respectively. The values of constants k_t, k_{BMI}, k_f, k_{fv} and k_p are equal and characteristic for each of the four individuals.

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References

[1] Duren D.L., Sherwood R.J., Czerwinski S.A., Lee M., Choh A.C., Siervogel R.M., Chumlea W.C., Body composition methods: comparisons and interpretation, [in.] Journal of Diabetes Science and Technology, Vol. 2, 2008, p. 1139–1146,
DOI: http://dx.doi.org/10.1177/193229680800200623

[2] Ellis K.J., Human body composition: in vivo methods, [in.] Physiological Reviews, Vol. 80, 2000, p. 649–680,
DOI: http://dx.doi.org/10.1152/physrev.2000.80.2.649

[3] Flegal K.M., Carroll M.D., Kit B.K., Ogden C.L., Prevalence of obesity and trends in the distribution of body mass index among US adults, 1999–2010, [in.] JAMA, Vol. 307, 2012, p. 491–497, DOI: http://dx.doi.org/10.1001/jama.2012.39

[4] Han K., Lavenspiel O., Extended monod kinetics for substrat, product and cell inhibition, [in.] Biotechogy and Bioengineering, Vol. 32, 1988, p. 430–437, DOI: http://dx.doi.org/10.1002/bit.260320404

[5] Heindel J.J., Newbold R., Schug T.T., Endocrine disruptors and obesity, [in.] Nature Reviews Endocrinology, Vol. 11, 2015, p. 653–661; DOI: http://dx.doi.org/10.1038/nrendo.2015.163

[6] Johnson R.J., Segal M.S., Sautin Y., Nakagawa T., Feig D.I., Kang D-H., et al., Potential role of sugar (fructose) in the epidemic of hypertension, obesity and the metabolic syndrome, diabetes, kidney disease, and cardiovascular disease, [in.] The American Journal of Clinical Nutrition, Vol. 86, 2007, p. 899–906.

[7] Johnson R.J., Stenvinkel P., Martin S.L., Jani A., Sánchez-Lozada L.G., Hill J.O., Lanaspa M.A., Redefining metabolic syndrome as a fat storage condition based on studies of comparative physiology, [in.] Obesity, Vol. 21, 2013, p. 659–664, DOI: http://dx.doi.org/10.1002/oby.20026

[8] Kishore B.K., Obesity: the mother of all diseases, http://www.atlantadunia.com/Dunia/health/Articledetail.aspx?q=4 [accessed 26.01.18]

[9] Krakauer N.Y., Krakauer J.C., A new body shape index predicts mortality hazard independently of body mass index, [in.] PLoS One, Vol. 7, 2012, p. e39504, DOI: http://dx.doi.org/10.1371/journal.pone.0039504

[10] Kyle U.G., Bosaeus I., De Lorenzo A.D., Deurenberg P., Elia M., Gómez J.M., Heitmann B.L., et al. (2004): Bioelectrical impedance analysis-part I: review of principles and methods. Clin. Nutr. 23, 1226–1243, DOI: http://dx.doi.org/10.1016/j.clnu.2004.06.004

[11] Liu H., Huang D., McArthur D.L., Boros L.G.., Nissen N., Heaney A.P., Fructose induces transketolase flux to promote pancreatic cancer growth, [in.] Cancer Research, Vol. 70, 2010, p. 6368–6376.

[12] Liu Y.J., Araujo S., Recker R.R., Deng H.W., Molecular and genetic mechanisms of obesity: implications for future management, Current Molecular Medicine, Vol. 3, 2003, 325–340, DOI: http://dx.doi.org/10.2174/1566524033479735

[13] Moody F.G., Rikkers L.F. Aldrete J.S., Estimation of the funtianol reverse of human liver, [in.] Annales of Surgery, Vol. 180, 1974, 592–598, DOI: http://dx.doi.org/10.1097/00000658-197410000-00024

[14] Ritchie J.D., Miller C.K., Smiciklas-Wright H., Tanita foot-to-foot bioelectrical impedance analysis system validated in older adults, [in.] Journal of American. Dietetic Association, Vol. 5, 2005, p. 1617–1619, DOI: http://dx.doi.org/10.1016/j.jada.2005.07.011
[15] Sánchez-Lozada L.G., Tapia E, Jiménez A., Bautista P., Cristóbal M., Nepomuceno T., et al., Fructose-induced metabolic syndrome is associated with glomerular hypertension and renal microvascular damage in rats, [in.] Am. J. Physiol. Renal. Physiol. Vol. 292, 2007, p. 423–429, DOI: http://dx.doi.org/10.1152/ajprenal.00124.2006
[16] Shermer M., How science can inform ethics and champion sentient beings, [in.] Scientific American, Vol. 312, 2015, p. 16.
[17] Swinburn B.A., Sacks G., Hall K.D., McPherson K., Finegood D.T., Moodie M.L., Gortmaker S.L., The global obesity pandemic: shaped by global drivers and local environments, Lancet, Vol. 378, 2011, p. 804–814, DOI: http://dx.doi.org/10.1016/S0140-6736(11)60813-1
[18] Taubes G., Which one will make you fat? [in.] Scientific American, Vol. 309, 2013, p. 60–65, DOI: http://dx.doi.org/10.1038/scientificamerican0913-60
[19] Vandevijvere S., Carson C.C., Hall K.D., Umali E., Swinburn B.A., Increased food energy supply as a major driver of the obesity epidemic: a global analysis, [in.] Bulletin of the World Health Organisation, Vol. 93, 2015, p. 446–456, DOI: http://dx.doi.org/10.2471/BLT.14.150565
[20] World Health Organisation. Fact sheets: Obesity and overweight, http://www.who.int/mediacentre/factsheets/fs311/en [accessed 26.01.19].