TRANSACTIONS OF THE INSTITUTE OF FLUID-FLOW MACHINERY

No. 133, 2016, 109–115

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Electric field influence on a height of rise of a dielectric liquid between two parallel electrodes

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Abstract

A differential equation for temporal dependence of the liquid height of rise in a microchannel consisted of two parallel plates has been modified to account for a dielectrophoretic force, which is a result of an electric field occurring between the plates. The equation takes into account viscous, surface tension, gravitational forces, capillary entrance pressure and dielectrophoretic force. Numerical calculations have been performed for 2-propanol as a liquid. Time dependence of the height of rise and the velocity of liquid front has been obtained for two cases of the initial liquid height and several values of a voltage applied to the plates.

Keywords: Dielectrophoretic force; Microchannel flow controller

Nomenclature

B	–	half of the electrode distance, m
E	–	electric field, V/m
g	–	gravitational accelerationy, m/s^2
h	–	fluid height of rise, m
p	–	pressure, Pa
t	–	time, s
V	–	voltage, V
W	–	half of the electrode length, m

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Greek symbols

σ – surface tension, N/m
ϵ_0 – electric permittivity of vacuum, F/m
ϵ_r – relative electric permittivity
θ – dynamic contact angle, rad
ρ – density, kg/m^3
μ – dynamic viscosity, Pa s

1 Introduction

Understanding and controlling the fluid-flow at microscale is of great importance due to its growing range of applications nowadays, among others in medicine, bio-engineering, and chemistry. Microfluidic devices are also used for heat transfer in microelectronic systems as well as in refrigeration, cooling, and organic Rankine cycle (ORC) systems. To enhance a capillary-force driven flow that is characteristic for microfluidic devices, the electric field is often employed to induce electrowetting, electrocapillarity or dielectrophoretic (DEP) effects [1].

A microchannel flow controller based on DEP effect has been proposed lately [2]. The pressure-driven flow of a liquid in a channel with rectangular cross-section is enhanced by capillary and DEP forces in this device. It was shown that by changing the electric voltage applied to the electrodes placed on vertical walls of the channel, the liquid flow rate can be effectively regulated. However, a optimization of this device, particularly with the goal to control its time-response properties, is required because the controller is meant as a part of a feed-back system for temperature regulation. To this end, knowledge of the influence of the forces on the liquid height of rise is necessary, because its value is correlated with the liquid flow rate in the controller [2–4].

A time evolution of the height of rise for capillary driven flow was analyzed theoretically first for tubes of circular cross-sections [5]. Classical Lucas-Washburn equation takes into account viscous, surface tension, and gravitational forces. More advanced models include also inertial force and entrance pressure loss effects [6,7]. Newer papers take into consideration parallel-plate and rectangular capillaries [8,9] as well. A generalized theoretical analysis [8] gives results which agree very well with experimental results for different geometries, dimensions and liquids.

Despite the fact that the effect of the capillary rise seems a very well understood issue, to our knowledge, there is no an analysis on the temporal evolution of the height of rise for the case when DEP forces exist together with capillary forces. Papers that deals with the electric field effects on the capillary dynamics

are devoted to electrowetting [10] or electrocapillarity [11] effects.

The goal of the presented paper is to perform theoretical and numerical analysis of the influence of the DEP forces on the temporal dependence of the liquid height of rise in capillary system consists of two parallel plates constituting two electrodes.

2 Matematical model for capillary flow

The presented model is based on that described in [9], where the temporal dependence of the height of rise, called there the penetration depth, $h(t)$, for two geometries of the capillary, namely parallel plates and circular tube, is considered. The model [9] starts from the integral momentum equation for homogeneous, incompressible, Newtonian fluid and takes into account the following forces acting on the control deformable fluid volume: the capillary force, viscous force, the gravitational force, and the pressure force at the inlet to the channel. The latter force – not taken into account in early analyses [5,6] – is obtained based on [12], where it is assumed that an infinite reservoir represented by a hemispherical control volume is placed at the capillary entrance. From the momentum balance in the control volume, the fluid pressure at the entrance can be found. Other shapes of the control volume have been also considered, but their influence is not significant for the case when channel length is much greater than the width [7].

In the present study, when the capillary with a dielectric fluid is placed in the electric field, the DEP force [13] is added to the momentum balance equation. Taking into account this force leads to a modification of the differential equation for a time dependence of the height-of-rise presented in [9]. For a channel consisted of two rectangular electrodes separated by the distance $2B$ and of the length $2W$ this equation takes the form

$$(h+1.11\sqrt{BW})\frac{d^2h}{dt^2}+0.958\left(\frac{dh}{dt}\right)^2+\left(\frac{3}{B^2}h+\frac{1.772}{\sqrt{BW}}\right)\frac{\mu}{\rho}\left(\frac{dh}{dt}\right) +gh-\frac{\sigma}{\rho}\left(\frac{\cos\theta}{B}-\frac{1}{W}\right)-\frac{p_{\mathrm{DEP}}}{\rho}=0\,, \qquad (1)$$

where μ and ρ are the dynamic viscosity and the density of the liquid, respectively, σ is the surface tension of the liquid in contact with air, θ is the dynamic contact angle of the air-liquid-electrode interface, and g is the gravitational acceleration. The pressure p_{DEP} follows from the DEP force acting on the liquid surface and

according to [13] can be expressed as

$$p_{\mathrm{DEP}} = \frac{\varepsilon_0(\varepsilon_{\mathrm{r}} - 1)V^2}{8B^2}\,, \tag{2}$$

where ε_0 is the electric permittivity of vacuum, ε_r is the relative permittivity of the liquid, and V is the electric potential between the electrodes, related to the electric field E by the relation $E = V/(2B)$. It is worth to note that the numerical coefficients in Eq. (1) are the result of theoretical analysis and are not taken from experiments.

3 Results

Equation (1), which is a nonlinear nonhomogeneous second order equation, was solved using commercial COMSOL ver. 5.1 software [14] employing its Global ODEs and DAEs interface. To validate the calculation method, the results were compared with that from [9] for different liquids, where no electric field was applied. The results agreed perfectly for all cases.

The calculations were performed for the same liquid that was used in the microchannel described in [2,3], namely 2-propanol. Its physical properties were taken assuming temperature 25 °C and are as follows [15]: the density 786 kg/m^3, surface tension 20.9 mN/m, dynamic viscosity 2.34 mPa s, relative electric permittivity 19.9 and the contact angle 0 rad. The channel dimensions were: $2B = 3 \times 10^{-4}$m, $2W = 5 \times 10^{-3}$ m.

Two cases were analyzed: (i) the initial level of liquid is the same as in the external reservoir, $h(0) = 0$, and (ii) the initial level corresponds to the level for the stationary equilibrium of all forces except of the DEP force, $h(0) = h_{cap}$. This value can be found from

$$h_{cap} = \frac{\sigma}{\rho g}\left(\frac{\cos\theta}{B} - \frac{1}{W}\right) \tag{3}$$

and for the selected parameters is equal to about 1.71×10^{-2} m.

Figure 1 presents how the height of rise changes with the time for different applied voltages. It is seen that the height of rise increases with the applied voltage and after about 5 s reaches the equilibrium level of rise. This time is smaller for lower voltage. The velocity of the liquid front is shown in Fig. 2. In case (i), the velocity reaches almost 0.5 m/s for $V = 500$ V in the initial stage of rising because both forces capillary and DEP act on the liquid surface. In case (ii) the velocity is about ten times smaller for the same case because only DEP force causes the liquid flow.

The presented results show that changing the voltage applied to the electrodes it is possible to effectively modify the liquid level height of rise. The voltage above 100 V should be applied to achieve a noticeable effect whereas for voltage 300 V the level rises about 50%.

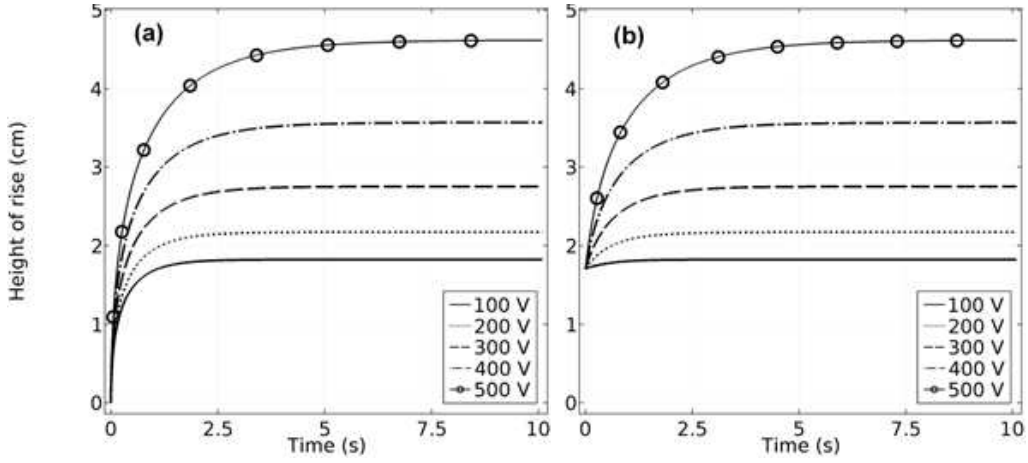

Figure 1: Temporal dependence of the height of rise for different voltage applied to the electrodes (a) $h(0) = 0$ and (b) $h(0) = h_{cap}$.

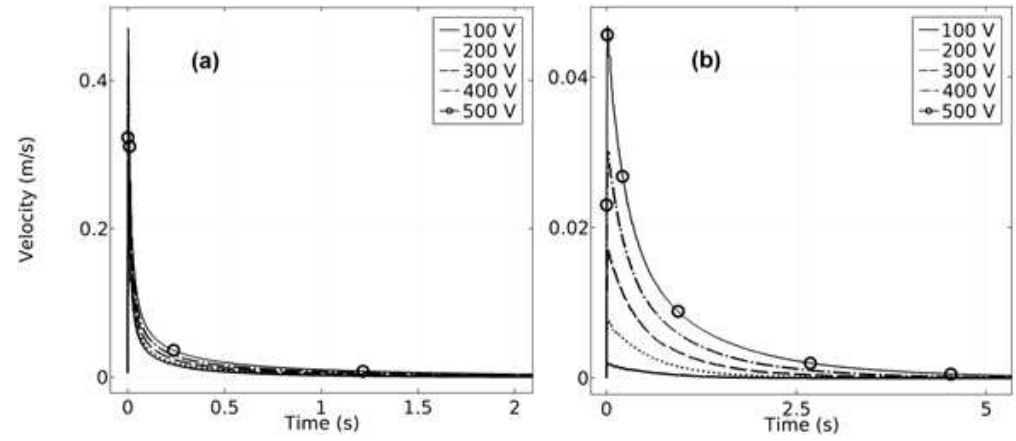

Figure 2: Temporal dependence of the velocity of liquid front for different voltage applied to the electrodes (a) $h(0) = 0$ and (b) $h(0) = h_{cap}$.

4 Conclusion

Applying the electric voltage to the electrodes with a dielectric liquid between them causes a flow of the liquid due to presence of DEP force. The temporal dependence of the height of rise of the liquid when the voltage is applied can be obtained by a term describing the DEP pressure to the equation derived for temporal dependence of height of rise in ordinary capillary systems. By changing the electric voltage applied to the electrodes it is possible to control the liquid level and the velocity of the liquid flow.

Acknowledgements This research has been supported by National Science Centre within the Project No. 2012/05/B/ST8/02742

Received in July 2016

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