

Design of inverse kinematics algorithms: extended Jacobian approximation of the dynamically consistent Jacobian inverse

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The paper presents the approximation problem of the inverse kinematics algorithms for the redundant manipulators. We introduce the approximation of the dynamically consistent Jacobian by the extended Jacobian. In order to do that, we formulate the approximation problem and suitably defined approximation error. By the minimization of this error over a certain region we can design an extended Jacobian inverse which will be close to the dynamically consistent Jacobian inverse. To solve the approximation problem we use the Cholesky decomposition and the Ritz method. The computational example illustrates the theory.

Key words: redundant manipulator, inverse kinematics, Jacobian, approximation.

1. Introduction

The inverse kinematics problem consists in finding a manipulator's configuration corresponding to a prescribed location (position and orientation) in task space of the end-effector. For the redundant manipulator, whose the number of Degrees of Freedom is greater than the task space dimension, there is an infinite number of inverse kinematics problem solutions. In order to find a unique solution, we shall define some additional constraints, e.g. minimization of the joints velocities during the motion, obstacle avoidance, etc. The state of the art in the resolving of the inverse kinematics problem for the redundant manipulators has been reviewed in [4] and references therein.

Typically, the inverse kinematic problem is solved numerically, using the Jacobian inverse kinematics algorithms, e.g. the extended Jacobian algorithm. The extended Jacobian is constructed by extension with the derivative of an augmenting kinematics function [2] and as a result the Jacobian is a square matrix. By an appropriate choice of the augmenting kinematics functions, we may form the algorithm dynamic. For that reason,

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The work of the author was supported by a statutory grant No. S40142W4/K7 provided by the Wrocław University of Technology.

Received 17.10.2014. Revised 26.02.2015.

this algorithm can solve the inverse kinematics problem and simultaneously keep an objective function at its minimum [12]. By design, the extended Jacobian algorithm has the property of repeatability [16], which guarantees that the closed paths in the task space are converted into the closed paths in the joint space. This property is significant for planning the cyclic tasks in the industry applications.

The idea of shaping the algorithm performance by combining the advantages of two algorithms has been introduced by Roberts and Maciejewski in [13, 14, 15]. Also, in a series of papers [7, 8, 9] the authors showed that based on the properties of the extended Jacobian algorithm, one can design the repeatable inverse kinematics algorithm which approximates the Jacobian pseudo inverse algorithm.

Another important task for the redundant manipulators is force control. This task consists in imposing the predetermined force by the end-effector. In this case, the manipulator's redundancy can be used to fulfill some additional tasks. As an example, beyond exerting a constant force by the effector, the end-effector will be moving along a prescribed curve. In order to solve that kind of problem, we enroll the dynamically consistent Jacobian inverse [10], which does not guarantee the repeatability. In this paper, our aim will be to define the extended Jacobian inverse which will behave similar to the dynamically consistent Jacobian inverse in its behavior as well as preserving the property of repeatability.

Summarizing, the main contribution of this article is the new definition of the approximation problem of which solution allows us to obtain new Jacobian algorithm which has a repeatability property and is similar to the dynamically consistent Jacobian algorithm. Namely, the new algorithm combines the advantages of its two constituent algorithms. Additionally, in order to solve the approximation problem we enroll the Cholesky decomposition and the Ritz method.

This paper is organized in the following way. Section 2 introduces the basic concepts. The dynamically consistent Jacobian inverse idea is presented in section 3. Main results concerned with the approximation problem and its solution are provided in section 4. Section 5 is devoted to computational example. Section 6 concludes the paper.

2. Preliminaries

We shall study a coordinate representation kinematics of a stationary redundant manipulator

$$k: \mathbb{R}^n \rightarrow \mathbb{R}^m, \quad y = k(q) \quad (1)$$

with n degrees of freedom and m -dimensional task space. For the redundant manipulators $n > m$ and the number $s = n - m$ stands for the degree of redundancy of the kinematics. Let $J(q) = \frac{\partial k(q)}{\partial q}$ denote the manipulator's analytic Jacobian. Given the kinematics (1) and desirable point y_d in task space we shall define the inverse kinematic problem as: find the joint position q_d such that $k(q_d) = y_d$. The solution of inverse kinematic problem is usually delivered by a Jacobian inverse kinematics algorithm. A derivation of the

Jacobian algorithms is based on the continuation method [5]. For an initial configuration q_0 , we define a curve $q(t)$ in joint space which passes through q_0 such that the corresponding task space error along this curve $e(t) = k(q(t)) - y_d$ decreases exponentially with decay rate $\gamma > 0$, so that

$$\frac{de(t)}{dt} = -\gamma e(t).$$

By substitution of the error in the above formula we obtain the Wazewski–Davidenko equation

$$J(q(t)) \frac{dq(t)}{dt} = -\gamma(k(q(t)) - y_d).$$

If $J^\#(q)$ is a right inverse of the Jacobian ($J(q)J^\#(q) = I_m$) we get a dynamic system

$$\dot{q}(t) = -\gamma J^\#(q)e(t),$$

whose trajectory approaches to a solution of the inverse kinematic problem with a limit $q_d = \lim_{t \rightarrow +\infty} q(t)$.

Usually, the Jacobian inverse kinematics algorithms employ the Jacobian pseudo inverse at regular joint positions of the manipulator

$$J^{P\#}(q) = J^T(q)(J(q)J^T(q))^{-1}$$

or the extended Jacobian inverse. The extended Jacobian inverse can be introduced in following way. Given the kinematics (1) we choose an augmenting kinematics map

$$h: \mathbb{R}^n \rightarrow \mathbb{R}^s, \quad \tilde{y} = h(q), \quad s = n - m. \quad (2)$$

Then using the map (2) we define the extended kinematics

$$l = (k, h): \mathbb{R}^n \rightarrow \mathbb{R}^n, \quad \bar{y} = l(q)$$

which transforms the joint space into itself, and the extended Jacobian $\bar{J}(q) = \frac{\partial l(q)}{\partial q}$. Finally, the extended Jacobian inverse takes form

$$J^{E\#}(q) = \bar{J}^{-1}(q) \Big|_{m \text{ first columns}}. \quad (3)$$

By design, the extended Jacobian inverse is a right inverse of the Jacobian

$$J(q)J^{E\#}(q) = I_m$$

and has the annihilation property

$$\frac{\partial h(q)}{\partial q} J^{E\#}(q) = 0.$$

It is well known that the extended Jacobian inverse kinematics algorithm is repeatable.

An alternative to both Jacobian inverses can be dynamically consistent Jacobian inverse derived by Oussama Khatib in [11].

3. Dynamically consistent Jacobian inverse

The joint space equation of motion can be written

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D(q) = \Gamma, \quad (4)$$

where $M(q) = M^T(q)$ is an inertia matrix, vector $C(q, \dot{q})\dot{q}$ describes influence of the centrifugal and Coriolis forces, $D(q)$ is the gravity force vector and Γ is the vector of generalized joint forces. For non-redundant manipulators the basic relationship between end-effector forces F and joint torques Γ is as follows

$$\Gamma = J^T(q)F.$$

This relationship is obtained using the identity between the virtual works associated with joint space and task space. However, this equality becomes incomplete for the redundant manipulators that are in motion. It is known that when the redundant manipulator is not at static equilibrium, there exist an infinity of joint torques vectors that could be applied without producing the resulting forces at the end-effector [10]. Those joint torques are acting within the null space. Taking into consideration the above reasoning, the relationship between end-effector forces and joint torques takes the form

$$\Gamma = J^T(q)F + (I_n - J^T(q)J^{T\#}(q))\Gamma_0, \quad (5)$$

where Γ_0 is an arbitrary generalized joint torque vector and $J^{T\#}$ is a generalized inverse of J^T . As it can be seen, the equation (5) depends on $J^{T\#}$. Further we will see that only one of generalized inverses is consistent with the system dynamics [11]. Let us begin with applying a joint torque vector in the general form (5) to the manipulator's dynamics (4)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D(q) = J^T(q)F + (I_n - J^T(q)J^{T\#}(q))\Gamma_0. \quad (6)$$

To determine the relation between operational acceleration and the operational force we shall multiply equation (6) by the matrix $J(q)M^{-1}(q)$. Next, using the differential $\ddot{y} = \dot{J}(q)\dot{q} + J(q)\ddot{q}$ and some simple computations the resulting equation can be written as

$$\begin{aligned} \ddot{y} + (J(q)M^{-1}(q)C(q, \dot{q})\dot{q} - \dot{J}(q)\dot{q}) + J(q)M^{-1}(q)D(q) = \\ (J(q)M^{-1}(q)J^T(q))F + J(q)M^{-1}(q)(I - J^T(q)J^{T\#}(q))\Gamma_0. \end{aligned} \quad (7)$$

Equation (7) expresses the relationship between the operational accelerations \ddot{y} and the operational force F . It can be noticed that as long as the term $J(q)M^{-1}(q)(I - J^T(q)J^{T\#}(q))\Gamma_0$ is non-zero the operational point is affected by Γ_0 . In order for the joint torques associated with the null space in (5) to not produce any acceleration at end-effector, it is necessary that

$$J(q)M^{-1}(q)(I - J^T(q)J^{T\#}(q))\Gamma_0 = 0. \quad (8)$$

So,

$$\begin{aligned} J(q)M^{-1}(q) &= J(q)M^{-1}(q)T^T(q)J^{\#\Gamma}(q), \\ M^{-1}(q)J^T(q) &= J^{\#}(q) (J(q)M^{-1}(q)J^T(q))^T, \\ J^{\#}(q) &= M^{-1}(q)J^T(q) (J(q)M^{-1}(q)J^T(q))^{-1}. \end{aligned}$$

Equation (8) is satisfied by generalized inverse of Jacobian which is called dynamically consistent Jacobian inverse

$$J^{DC\#}(q) = M^{-1}(q)J^T(q) (J(q)M^{-1}(q)J^T(q))^{-1}.$$

4. The approximation problem

In order to obtain the new extended Jacobian which will be similar to the dynamically consistent Jacobian first we shall state and solve the following approximation problem: find an extended Jacobian inverse $J^{E\#}(q)$ that approximates in an optimal way the given dynamically consistent inverse $J^{DC\#}(q)$. Invoking the approach presented in [17], we introduce a pair of the matrices

$$E_1(q) = \begin{bmatrix} J(q) \\ Dh(q) \end{bmatrix}^{-1} = \begin{bmatrix} J^{E\#}(q) & Q(q) \end{bmatrix} \quad (9)$$

and

$$E_2(q) = \begin{bmatrix} J(q) \\ K^T(q) \end{bmatrix}^{-1} = \begin{bmatrix} J^{DC\#}(q) & M^{-1}(q)K(q) \end{bmatrix}, \quad (10)$$

where $h(q)$ is the augmenting kinematics map, $Dh(q) = \frac{\partial h(q)}{\partial q}$ and matrix $K(q)$ has the following properties

$$J(q)M^{-1}(q)K(q) = 0 \quad (11)$$

and

$$K^T(q)M^{-1}(q)K(q) = I_s. \quad (12)$$

Let $Q \in \mathbb{R}^n$ stand for a set of regular configurations, then using (9) i (10) we define the approximation error as the Frobenius norm of a measure of the difference between $J^{DC\#}(q)$ and $J^{E\#}(q)$

$$\mathcal{E}(h) = \int_Q \|E_1^{-1}(q)E_2(q) - I_n\|_F^2 m(q) dq, \quad (13)$$

where $m(q) dq$ denotes the volume form with $m(q) = \det(J(q)M^{-1}(q)J^T(q))$ [17].

After substituting (9) and (10) into (13) the approximation error functional takes form

$$\mathcal{E}(h) = \int_Q \text{tr} \left(Dh(q) P^{DC}(q) (Dh(q))^T - 2Dh(q) M^{-1}(q) K(q) + I_s \right) m(q) dq, \quad (14)$$

where

$$P^{DC}(q) = J^{DC\#}(q) (J^{DC\#}(q))^T + M^{-1}(q) K(q) K^T(q) M^{-1}(q).$$

The augmenting kinematics function minimizing the functional error (14) allows us to introduce the new extended Jacobian.

4.1. The Cholesky decomposition

To define the approximation error functional we need to find the matrix $K(q)$ which fulfills (11) and (12). In order to do that we enroll the Cholesky decomposition. This being so, let us assume that $W(q) = M(q)^{-1} K(q)$ so $K(q) = M(q) W(q)$. In that case, the equations (11) and (12) take form

$$J(q) W(q) = 0 \quad (15)$$

and

$$K^T(q) W(q) = W^T(q) M(q) W(q) = I_s. \quad (16)$$

It is known that the inertia matrix $M(q)$ is symmetric and positively definite, $M(q) = M^T(q) > 0$. We apply the Cholesky decomposition [3]

$$M(q) = T^T(q) T(q)$$

and based on that, from (16) we receive

$$W^T(q) T^T(q) T(q) W(q) = (T(q) W(q))^T (T(q) W(q)) = I_s. \quad (17)$$

Let $Z(q) = T(q) W(q)$, then the equation (15)

$$J(q) W(q) = J(q) T^{-1}(q) Z(q) = 0 \quad (18)$$

and from (17) we get

$$Z^T(q) Z(q) = I_s. \quad (19)$$

Finally, after substituting $J(q) T^{-1}(q) = S(q)$ the constraints (11), (12) are as follows

$$S(q) Z(q) = 0$$

and

$$Z^T(q) Z(q) = I_s.$$

The matrix $Z(q)$ is a matrix with orthonormal columns spanning the $S(q)$ kernel. For obtained $T(q)$ and $Z(q)$ we compute $W(q) = T^{-1}(q) Z(q) = M^{-1}(q) K(q)$. Please notice that to compute the approximation error we need only the $W(q)$ matrix instead of the term $M^{-1}(q) K(q)$.

4.2. The Ritz method

The approximation problem becomes the calculus of variations problem because the error functional (14) should be minimized with respect to augmenting kinematics functions $h(q)$. That is why the optimal augmenting maps can be found from Euler–Lagrange equations that takes form of linear elliptic partial differential equations. For realistic redundant manipulators to calculate the optimal maps $h(q)$ we need to use direct methods of calculus of variations, e.g. the Ritz method [6]. To do so, we assume that

$$h(q) = c^T \phi(q), \quad (20)$$

where $\phi(q) = (\phi_1(q), \phi_2(q), \dots, \phi_p(q))$ represents the vector composed of basic functions in the joint space and $c \in \mathbb{R}^p$ is a vector of parameters. Thanks to the parametrization of $h(q)$ the approximation error takes the quadratic form

$$\mathcal{E}(c) = c^T Q c - 2c^T R + D, \quad (21)$$

where

$$Q = \int_{\mathcal{Q}} \frac{\partial \phi(q)}{\partial q} P^{DC}(q) \left(\frac{\partial \phi(q)}{\partial q} \right)^T m(q) dq,$$

$$R = \int_{\mathcal{Q}} \frac{\partial \phi(q)}{\partial q} M^{-1}(q) K(q) m(q) dq,$$

$$D = \int_{\mathcal{Q}} m(q) I_s dq.$$

After differentiation of the equation (21) with respect to c and equate to zero we receive

$$\frac{\partial \mathcal{E}(c)}{\partial c} = 2Qc - 2R = 0.$$

Under assumption that the matrix Q is non-singular, the optimal augmenting kinematics map $h(q) = c^{*T} \phi(q)$ is obtained when $c^* = Q^{-1}R$.

5. Example

5.1. Manipulators

As a testbed for the performance evaluation of the newly proposed approximation algorithm we have chosen the planar 3R manipulator shown in Fig. 1 and a 4 DOF sub-manipulator of the PUMA 560 manipulator, Fig. 2.

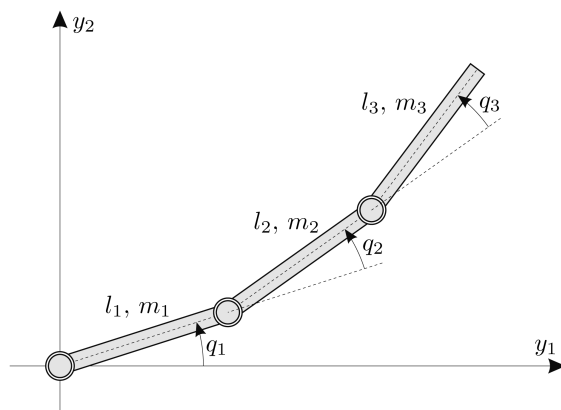


Figure 1: 3R planar manipulator.

Planar 3R manipulator Using the Cartesian position coordinates, the kinematic of the planar 3R manipulator can be represented as

$$k(q) = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) + l_3 \cos(q_1 + q_2 + q_3) \\ l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) + l_3 \sin(q_1 + q_2 + q_3) \end{bmatrix}$$

where l_i is the length of i -th arm. The dynamics equation of the manipulator is as follows

$$\underbrace{\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix}}_{M(q)} \ddot{q} + C(q, \dot{q})\dot{q} + D(q) = \tau,$$

where

$$M_{11} = \frac{1}{3}(l_3^2 m_3 + l_2^2(m_2 + 3m_3) + l_1^2(m_1 + 3(m_2 + m_3))) + l_1 l_2(m_2 + 2m_3) \cos(q_2) + l_2 l_3 m_3 \cos(q_3) + l_1 l_3 m_3 \cos(q_2 + q_3),$$

$$M_{12} = \frac{1}{6}(2(l_3^2 m_3 + l_2^2(m_2 + 3m_3)) + 3l_1 l_2(m_2 + 2m_3) \cos(q_2) + 3l_3 m_3(2l_2 \cos(q_3) + l_1 \cos(q_2 + q_3))),$$

$$M_{13} = \frac{1}{6}l_3 m_3(2l_3 + 3l_2 \cos(q_3) + 3l_1 \cos(q_2 + q_3)),$$

$$M_{22} = \frac{1}{3}(l_3^2 m_3 + l_2^2(m_2 + 3m_3)) + l_2 l_3 m_3 \cos(q_3),$$

$$M_{33} = \frac{l_3^2 m_3}{3},$$

τ_i is control force/torque and m_i stands for the mass of i -th arm.

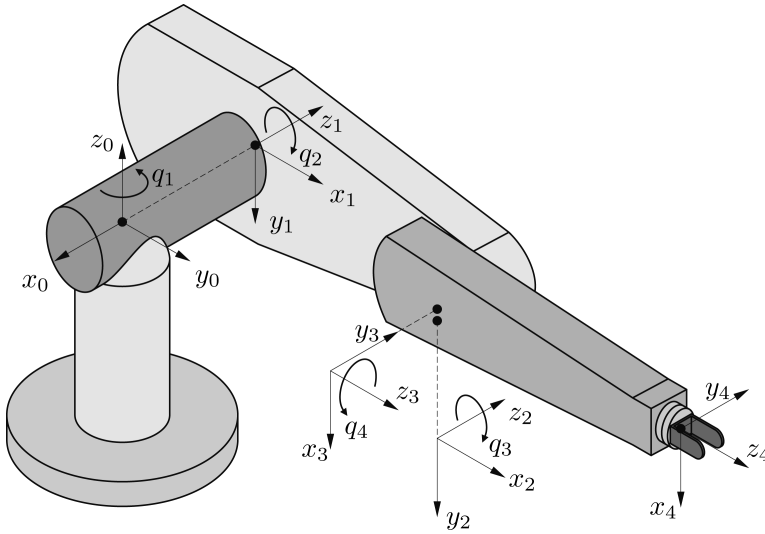


Figure 2: Puma 560.

Puma 560 manipulator The kinematics of the manipulator (Fig. 2) is defined by the XY position of the end-effector and the rotation around the z_4 axis, that is

$$k(q) = \begin{bmatrix} -0.1501s_1 + c_1(s_2(0.4331c_3 + 0.0203s_3) + c_2(0.4318 - 0.0203c_3 + 0.4331s_3)) \\ 0.1501c_1 + s_1(s_2(0.4331c_3 + 0.0203s_3) + c_2(0.4318 - 0.0203c_3 + 0.4331s_3)) \\ q_4 \end{bmatrix},$$

where s_i, c_i denote respectively $\sin(q_i)$ and $\cos(q_i)$.

For given manipulator's dynamics $M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D(q) = \tau$, the elements of the inertia matrix are as follows [1]

$$\begin{aligned} M_{11} &= 2.57 + 1.38 \cos^2(q_2) + 0.3 \sin^2(q_2 + q_3) + 0.744 \cos(q_2) \sin(q_2 + q_3), \\ M_{12} &= 0.69 \sin(q_2) - 0.134 \cos(q_2 + q_3) + 0.0238 \cos(q_2), \\ M_{13} &= -0.134 \cos(q_2 + q_3) - 0.00397 \sin(q_2 + q_3), \\ M_{14} &= M_{24} = M_{34} = 0, \\ M_{22} &= 6.79 + 0.744 \sin(q_3), \\ M_{23} &= 0.333 + 0.372 \sin(q_3) - 0.011 \cos(q_3), \\ M_{33} &= 1.16, \\ M_{44} &= 0.2. \end{aligned}$$

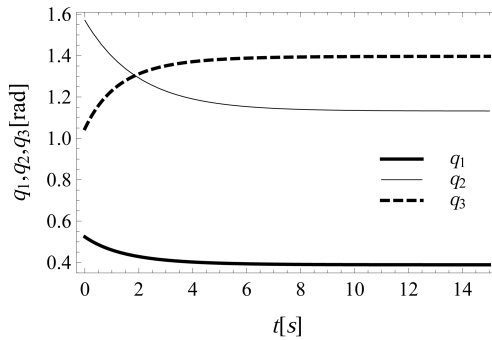


Figure 3: 3R planar manipulator: convergence of q_1-q_3 for extended Jacobian algorithm.

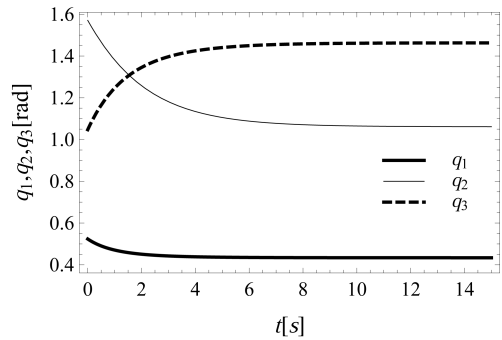


Figure 4: 3R planar manipulator: convergence of q_1-q_3 for dynamically consistent Jacobian inverse algorithm.

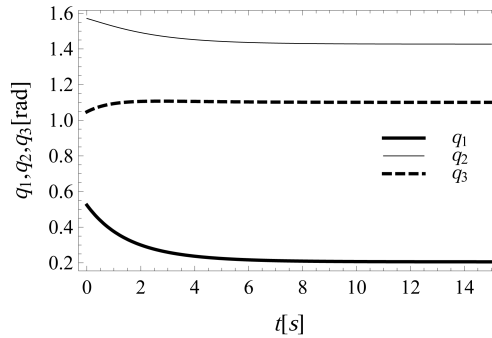


Figure 5: 3R planar manipulator: convergence of q_1-q_3 for Jacobian pseudo inverse algorithm.

5.2. Inverse kinematics problem

Planar 3R manipulator Since the degree of redundancy of the 3R manipulator equals 1, we shall find only one augmenting kinematics function $h(q)$ which minimizes the approximation error (14). For the sake of simplicity, we have chosen a linear function $h(q) = c^T q$, $c \in \mathbb{R}^3$. The approximation will be performed over a region

$$Q = \left\{ q \in \mathbb{R}^3 \mid 0 < q_1 < \frac{\pi}{2}, 0 < q_2 < \frac{\pi}{2}, 0 < q_3 < \frac{\pi}{2} \right\}.$$

After a computations we obtained the following augmenting kinematics function

$$h(q) = 0.485q_1 - 0.064q_2 + 0.107q_3. \quad (22)$$

Function (22) determines the extended Jacobian and the extended Jacobian inverse (3) which has been applied in order to solve inverse kinematic problem for the manipulator.

The following data are assumed: the initial joint space configuration $q(0) = (\frac{\pi}{6}, \frac{\pi}{2}, \frac{\pi}{3})$, the desired tasks space position $y_d = (0, 1.6)$, the convergence rate $\gamma = 0.5$ and simulation time interval $T = [0, 15][s]$.

The results have been displayed in Figs. 3–5. The solution of the inverse kinematics has been compared for the optimal extended Jacobian, the dynamically consistent Jacobian inverse and the Jacobian pseudo inverse.

To evaluate the performance of the extended Jacobian algorithm we defined the quality measure of approximation

$$\delta(q(t)) = \|J(q(t))M^{-1}(q(t)) (I_n - J^T(q(t))J^{\#T}(q(t)))\|_F^2.$$

For the solution obtained by the dynamically consistent Jacobian inverse the quality measure is equal zero during the trajectory. We expect the measure to be as close as possible to zero for the extended Jacobian inverse. It is worth to notice that the approximation over the certain region is optimal in a mean way. In Figs. 6 and 7 one can see that the criterion value is much lower for the approximate Jacobian than the pseudo inverse Jacobian.

It follows that the approximation problem is solved in a satisfactory way in a sense that the solution stays acceptably close to the solution provided by the dynamically consistent Jacobian inverse algorithm.

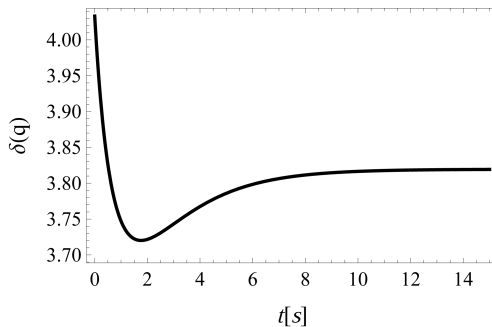


Figure 6: 3R planar manipulator: quality measure of approximation for extended Jacobian algorithm.

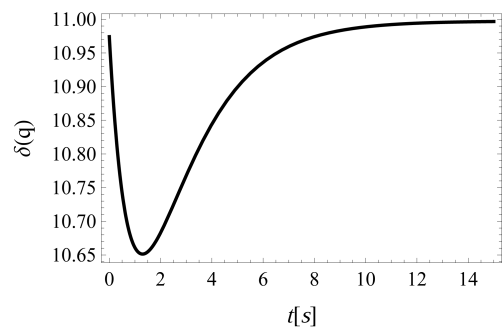


Figure 7: 3R planar manipulator: quality measure of approximation for Jacobian pseudo inverse algorithm.

Puma 560 As in previous example we shall find the augmenting kinematics function which will minimize the approximation error (14) over a prescribed joint space region

$$Q = \left\{ q \in \mathbb{R}^3 \mid 0 < q_1 < \frac{\pi}{2}, 0 < q_2 < \frac{\pi}{2}, 0 < q_3 < \frac{\pi}{2}, 0 < q_4 < \frac{\pi}{2} \right\}. \quad (23)$$

In this case the augmenting kinematics function is computed as

$$h(q) = -0.0274q_1 - 0.1862q_2 + 0.04414q_3 + 0q_4. \quad (24)$$

The inverse kinematic problem is solved under the conditions: $q(0) = (\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, 0)$, $y_d = (-0.13, 0.2, \frac{\pi}{3})$, $\gamma = 0.5$ and $T = [0, 15][s]$. Figures 8–10 illustrate the solution of the inverse kinematics problem. The criterion value during the motion for the extended Jacobian algorithm and the Jacobian pseudo inverse algorithm are shown respectively in Fig. 11 and Fig. 12.

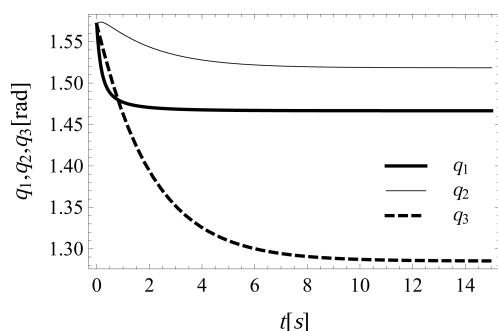


Figure 8: PUMA: trajectories of q_1 – q_3 for extended Jacobian algorithm.

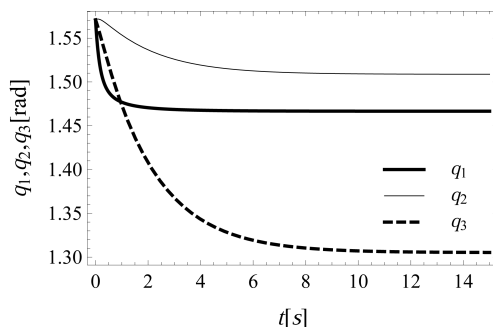


Figure 9: PUMA: trajectories of q_1 – q_3 for dynamically consistent Jacobian inverse algorithm.

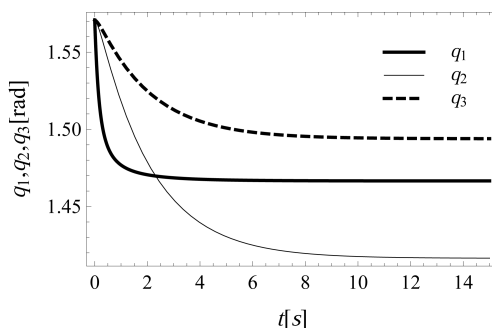


Figure 10: PUMA: trajectories of q_1 – q_3 for Jacobian pseudo inverse algorithm.

5.3. Crucial properties

To investigate the crucial features of the new algorithm we propose two kind of numerical tests for the 3R planar manipulator. Firstly, we may consider the repeatability property. For this purpose we will make a series of movements between the three points $A = k(\frac{\pi}{2}, \frac{\pi}{4}, 0)$, $B = k(\frac{\pi}{4}, \frac{\pi}{2}, \frac{\pi}{3})$, $C = k(\frac{\pi}{3}, \frac{\pi}{4}, \frac{\pi}{4})$ which compose the closed path in task space. The results are shown in Figs. 13 and 14. It can be seen that for closed path in tasks space the dynamically consistent Jacobian algorithm does not return the close path in joint space unlike the extended Jacobian algorithm.

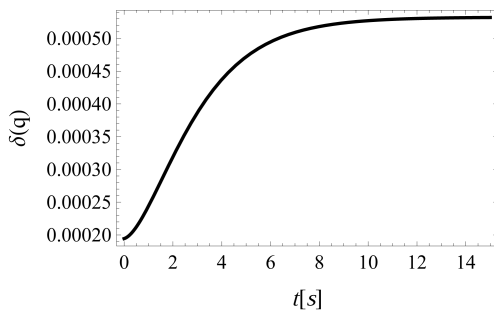


Figure 11: PUMA: quality measure of approximation for extended Jacobian algorithm.

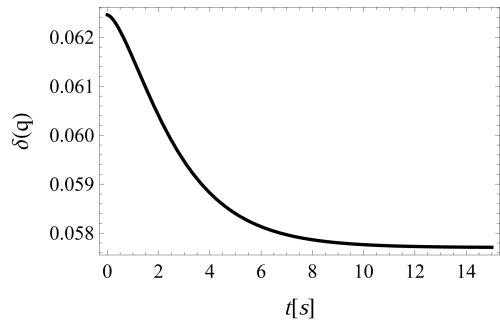


Figure 12: PUMA: quality measure of approximation for Jacobian pseudo inverse algorithm.

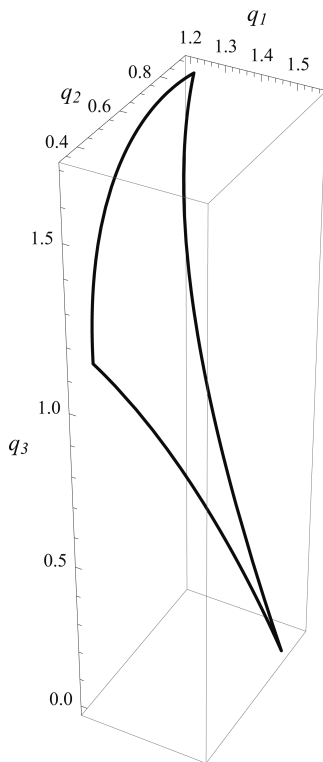


Figure 13: Trajectories in joint space for extended Jacobian algorithm.

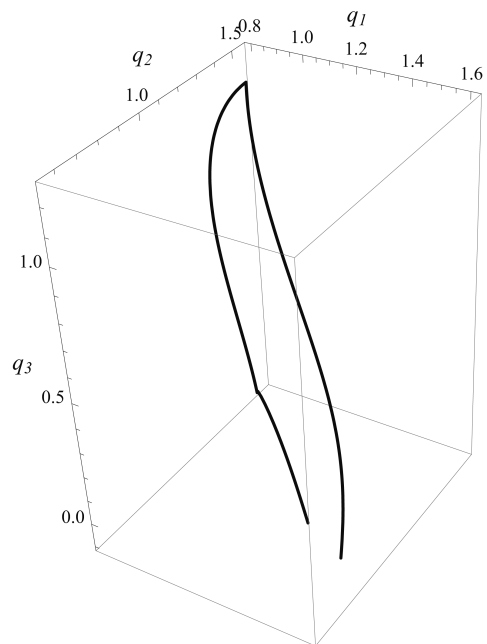


Figure 14: Trajectories in joint space for dynamically consistent Jacobian algorithm.

To check the dynamic consistency, let us consider the following example. The goal is to maintain the end-effector at static equilibrium by application of the operational force which compensates the gravity, centrifugal and Coriolis forces reflected at the end-

effector. All internal motions should be constrained to the null space. Let us select $\Gamma_0 = (2(q_1 - \frac{\pi}{2}), 2(q_2 + \frac{\pi}{6}), 2(q_3 - \frac{\pi}{2}))$. In case of dynamically consistent Jacobian the end-effector remains at the same place while the joints are moving, Fig. 15. After apply the optimal extended Jacobian it can be seen in Fig. 16 that the end-effector is slightly moving, however this motion is smaller than in case of Jacobian pseudo inverse, Fig. 17.

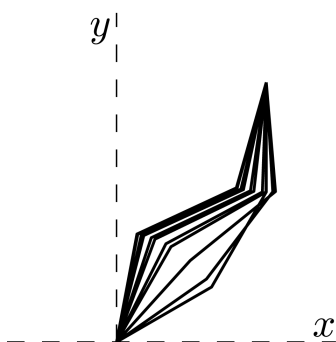


Figure 15: Gravity compensation – dynamically consistent Jacobian.

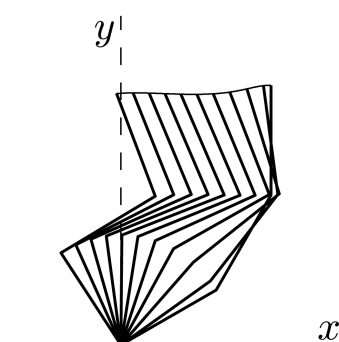


Figure 16: Gravity compensation – extended Jacobian.

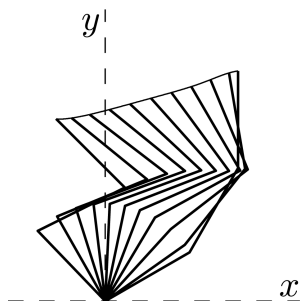


Figure 17: Gravity compensation – Jacobian pseudo inverse.

6. Conclusion

We have proposed the new optimal extended Jacobian inverse kinematics algorithm. We have approximated the dynamically consistent Jacobian inverse by the extended Jacobian. The extended Jacobian obtained from the solution of the approximation problem combines the advantages of both the extended Jacobian algorithm and the dynamically consistent Jacobian algorithm, which has been shown by the numerical tests. The presented results confirm that the approximation problem has been solved successfully.

The computations have been done using the WOLFRAM MATHEMATICA. The simulations show that the quality of approximation is dependent on the choice of the basis functions employed in the Ritz method. On the other hand it has been checked that the linear augmenting kinematics function is a good compromise between the quality of simulations and the computational effort. For example, in case of 3R planar manipulator for the linear augmenting kinematics function the time required to obtain the solution is 5 seconds (PC equipped with 2.4[GHz] processor). Obviously, for more complicated manipulators (with higher degree of redundancy, much more complicated inertia matrix or more Degrees of Freedom) the optimization could be more time-consuming. Nevertheless, the obtained extended Jacobian can be used to solve many different tasks.

Presented results underlie base for the future work in which the possibility of the application the approximated extended Jacobian in the force control algorithms for the redundant manipulator should be investigated.

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