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## PREVENTIVE MAINTENANCE MODELS – HIGHER OPERATIONAL RELIABILITY

### MODELE KONSERWACJI ZAPOBIEGAWCZEJ A WYŻSZA NIEZAWODNOŚĆ EKSPLOATACYJNA

*The authors present a method for determining the optimal interval for preventive periodical maintenance and an optimal diagnostic parameter for predictive maintenance/replacement. Additionally, the authors raise the question: how does preventive maintenance influence the probability of failure and the operational reliability of system elements that have undergone preventive periodical maintenance? They answer the question using analytical and simulation computing approaches. The results are in quantitative form, giving relationships between preventive maintenance intervals and reliability functions. Examples demonstrate suitability of the method for typical engineering objects using a three parameters Weibull distribution. Application of the method is of substantial benefit to both the manufacturer and the user of technical equipment.*

**Keywords:** preventive maintenance, predictive maintenance, maintenance interval optimization, reliability improvement.

*Autorzy przedstawiają metodę określania optymalnego czasu przerwy na okresową konserwację zapobiegawczą oraz optymalnego parametru diagnostycznego dla konserwacji predykcyjnej/wymiany. Dodatkowo, autorzy zadają pytanie, jaki jest wpływ konserwacji zapobiegawczej na prawdopodobieństwo wystąpienia uszkodzenia oraz na niezawodność eksploatacyjną elementów systemu, w stosunku do których zastosowano okresową konserwację zapobiegawczą. Odpowiedzi na te pytania, autorzy poszukują posługując się metodami analizy i symulacji komputerowej. Wyniki podane w formie ilościowej, informują o związkach między przerwami na konserwację predykcyjną a funkcjami niezawodnościowymi. Podane przykłady pokazują, z wykorzystaniem trójparametrowego rozkładu Weibulla, że proponowana metoda może być stosowana w przypadku typowych obiektów inżynierskich. Zastosowanie omawianej metody przynosi znaczące korzyści zarówno wytwórcom jak i użytkownikom sprzętu technicznego.*

**Słowa kluczowe:** konserwacja zapobiegawcza, konserwacja predykcyjna, optymalizacja przerw konserwacyjnych, doskonalenie niezawodności.

#### 1. Introduction and literature survey

Many maintenance strategies, policies and methods have been developed, which are aimed at making maintenance cheaper and more effective. Such programs have the minimization of costs, downtime and losses due to failure of critical objects of the equipment as their main objective. Cost minimization improves the effectiveness and profitability of the organization [1, 2, 9, 12, 13, 20].

For creation of the maintenance policies, well described data mining input is very important. [4].

In recent years, useful models of preventive and predictive maintenance optimization with different complexity and applicability have been further developed.

In the paper, [5] the authors proposed a quasi-periodic imperfect preventive maintenance policy. Finally, a real case study of preventive maintenance on Chinese diesel locomotives is examined to illustrate the proposed maintenance policy.

The paper [6] proposes an approach in which preventive and failure replacement costs as well as inspection cost are taken into account to determine the optimal replacement policy and an age-based inspection scheme, such that the total average costs of replacements and inspections is minimized.

Determination of the preventive effect of optimal replacement policies in the paper [8] is based on aging intensity and the cost ratio of failure and preventive replacements. One of its conclusions is that

not every preventive maintenance is fully effective and a policy of, “run to failure” can be more effective (note: in some cases).

The proposed model in the paper [10] takes into consideration the stochastic nature of equipment failures. The output from the model is a cost distribution against the time from which the minimum cost may be found for a particular period and this period is defined as the optimum lifespan of the machine part.

The paper [11] considers periodic preventive maintenance policies for a deteriorating repairable system. On each failure, the system is repaired and, at the planned times, it is periodically maintained to improve its performance reliability. Most periodic preventive maintenance (PM) models for repairable systems have been studied assuming that the failure process between two PMs follows the nonhomogeneous Poisson process (NHPP), implying the minimal repair on each failure.

The paper [14] regarding warranty policy considering three maintenance options for products with multiple failure modes also showed the broad usability of the Weibull distribution. This fact supports the decision of the authors to also use the Weibull function.

The paper [15] presents a new mathematical function to model an improvement based on the ratio of maintenance and repair costs, and demonstrate how it outperforms fixed improvement factor models by analyzing the effectiveness in terms of cost and reliability of a system.

It determines the optimal preventive maintenance and replacement schedule of the system.

The paper [16] takes into account degradation modeling and maintenance policy for a two-stage degradation system, which degradation process is nonlinear and degradation rate is change over time in both stages. Influence analysis of different model parameter and maintenance policy is studied in numerical examples with results that the proposed optimal maintenance policy can help to reduce the mean cost rate.

In the paper, [17] the authors proposed a hybrid imperfect maintenance model with random adjustment-reduction parameters and a maintenance policy. Furthermore, a numerical example and an example of the fuel injection pump of diesel engines are carried out and presented to illustrate the proposed method.

A mathematic model of optimization of maintenance intervals having regard to the risk is presented in the paper [18]. Precise calculations were made for steam turbines that operate in power units.

Maintenance can represent a significant portion of the cost in asset intensive organizations, as breakdowns have an impact on the capacity, quality and cost of operation [21]. However, the formulation of a maintenance strategy depends on a number of factors, including the cost of down time, reliability characteristics and redundancy of assets. Consequently, the balance between preventive maintenance (PM) and corrective maintenance (CM) for minimizing costs varies between organizations and assets. Nevertheless, there are some rules of thumb on the balance between PM and CM, such as the 80/20 rule.

In the paper [22], an approach is presented, which allows evaluation of various possible maintenance scenarios with respect to both reliability and economic criteria. Authors included three deterioration states ( $D1 \div D3$ ) and three repairs: minor (index = 1), medium (2) and major (3), but in real machine operation it is difficult to define these general states and repairs exactly.

In the paper [23] a double-fold Weibull competing risk model using the real failure data from railway operation, was developed for the engine system of a diesel locomotive and its current maintenance. Results show that the maintenance period varies widely between winter and summer, and that optimized maintenance can increase the availability and decrease cost more than the existing policy.

The paper [7] is a very large review on machinery diagnostics and prognostics implementing condition-based maintenance using 271 references and other reviews in the paper [19] using 104 references which point to future perspectives on maintenance optimization. These two references [7, 19] fully support the authors method, from data collection through data processing to optimal maintenance decision making.

These references proposed interesting models regarding concrete application on particular technical systems with different structures and as well a general solution. The authors did not find in the review, a simple model of predictive maintenance optimization for industrial practice and no idea that preventive maintenance improves reliability including utilization of a three parameters Weibull distribution. According to authors' experiences from different fields of industry, maintenance managers need simple and general methods for design of maintenance programs and policies optimization. Therefore, the objective of this paper is to contribute to the optimization of predictive maintenance with a new simple semi-stochastic model. A further objective was to give maintenance staff evidence that preventive maintenance improves operational reliability based on a mathematical theory of reliability [1, 20] and authors works [9, 12, 13]. Finally, all models are demonstrated using numerical simulation with a three parameters Weibull distribution supported by table processor Excel.

## 2. Optimization of predictive maintenance

In discussing machine maintenance strategy, it is customary to distinguish between the following methods (policies) [3]:

- a) **corrective maintenance** - maintenance carried out after fault recognition and intended to put an item into a state in which it can perform a required function,
- b) **preventive maintenance** - maintenance carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning of an item; following policies c), d) and e) are also preventive maintenance,
- c) **predetermined maintenance**, preventive maintenance carried out in accordance with established intervals of time or number of units of use but without previous condition investigation,
- d) **condition based maintenance** - preventive maintenance which includes a combination of condition monitoring and/or inspection and/or testing, analysis and the ensuing maintenance actions; the condition monitoring and/or inspection and/or testing may be scheduled, on request or continuous,
- e) **predictive maintenance** - condition based maintenance carried out following a forecast derived from repeated analysis or known characteristics and evaluation of the significant parameters of the degradation of the item.

The proposed model of predictive maintenance optimization is based on minimization of unit maintenance, diagnostics and failure risk cost  $c(S_p)$  of a component [1]:

$$c(S_p) = \frac{C_{pr} + L_f \cdot F(S_p)}{\bar{t}(S_p)} + c_d \quad (1)$$

where  $S_p$  is a diagnostic signal for predictive maintenance; diagnostic signal is allowed to be a random variable,  $C_{pr}$  is cost of preventive maintenance,  $L_f$  is loss due to failure risk ( $L_f \cdot F(S_p)$ ); loss due to failure risk can be calculated as a difference between cost of corrective maintenance and cost of preventive maintenance, it means  $L_f = C_{cm} - C_{pr}$ ,  $F(S_p)$  is probability of failure depending on diagnostic signal  $S_p$ ,  $c_d$  denotes unit costs of condition monitoring to obtain diagnostic signal  $S_p$ ,  $\bar{t}(S_p)$  is the mean time corresponding to diagnostic signal  $S_p$ , which can be determined from operational data using the formula:

$$\bar{t}(S_p) = \frac{1}{n} \left[ \sum_{i=1}^{m(S_p)} t_i(S_p) + \sum_{j=1}^{n-m(S_p)} t_j(S_p) \right] \quad (2)$$

where  $t_i(S_p)$  denotes the operating time of the  $i$ th object surviving at the level  $S_p$ ,  $t_j(S_p)$  denotes the time to failure of the  $j$ th object which failed before reaching the state  $S_p$ ,  $m(S_p)$  is the number of objects reaching state  $S_p$  without failure and  $n$  is the total number of objects in the investigated population. To obtain these data it is necessary to carry out an operational observation – life test of objects population including on-line diagnostic measurement till failure or at least diagnostic signals  $S_{pf}$  closely before failure occurs. In the first case, it is easy to apply equation (2) and probability of failure (distribution function)  $F(S_p)$  can be obtained by means of diagnostic signals  $S_p$  shortly before failure. If there are only recognized diagnostic signals (technical states) closely before failures, it is necessary to calculate operating time related to selected diagnostic signal  $S_p$  which is used as an indicator for predictive maintenance of an object.

For calculation of mean operating time, versus diagnostic signal for predictive maintenance  $\bar{t}(S_p)$ , authors use a simplified model in which the technical state degradation (a change of diagnostic signal)

runs along a straight line from start state  $S_{pz}$  to limit value of technical state (to failure)  $S_{pfi}$  i-th object. The accuracy of this approximation from point of technical solution is sufficient. Calculation of the  $t_i(S_p)$  is carried out in a case of the  $S_{pi} < S_{pfi}$  according to equation (3):

$$t_i(S_p) = t_i(S_{pfi}) \frac{S_{pi} - S_{pz}}{S_{pfi} - S_{pz}}, \quad (3)$$

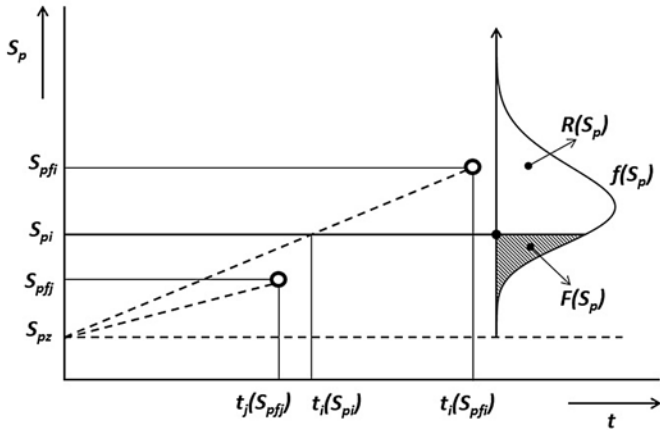


Fig. 1. Principle of input data determination for calculation of mean operating time versus diagnostic signal for predictive maintenance  $\bar{t}(S_p)$

If  $S_{pj} \geq S_{pfi}$ , the operating time to failure of the  $j$ th object which failed before reaching the diagnostic signal  $S_p$ , we can read directly from the database of operating time to failure of the  $j$ th object which failed before reaching the diagnostic signal  $t_j(S_p)$ . Interpretation of these input data is clear from Fig. 1.

Unit costs of preventive maintenance and failure risk versus diagnostic signal for predictive maintenance and optimal diagnostic signal for predictive maintenance  $S_{po}$  (for  $c(S_p) = \text{minimum}$ ) we can calculate, using equation (4):

$$c(S_p) = \frac{C_{pr} + L_f \cdot F(S_p)}{\bar{t}(S_p)} + c_d = \frac{C_{pr} + L_f \cdot F(S_p)}{\frac{1}{n} \left[ \sum_{i=1}^{m(S_p)} t_i(S_p) + \sum_{j=1}^{n-m(S_p)} t_j(S_p) \right]} + c_d \quad (4)$$

For a proposed model of predictive maintenance optimization (4) it is necessary to obtain or calculate input data as follows:

- cost of preventive maintenance  $C_{pr}$
- losses due to failure risk  $L_f$
- probability of failure versus diagnostic signal for predictive maintenance  $F(S_p)$
- mean operating time versus diagnostic signal for predictive maintenance  $\bar{t}(S_p)$
- unit cost of diagnostics (condition monitoring)  $c_d$
- diagnostic signal for predictive maintenance  $S_p$

Optimal predictive dispositional operating time  $t_d(S_{po})$  from actual operating time  $t(S)$  in decision making state to optimal operating time  $t(S_{po})$  for predictive maintenance (restoration, replacement) is calculated from equation

$$t_d(S_{po}) = t(S_{po}) - t(S) \quad (5)$$

### 3. Calculation of mean life and reliability functions of preventive predetermined maintained objects

If we should prove that preventive predetermined maintenance increases operational reliability, we must calculate reliability function of object predetermined maintained in operating time  $t_p$  and its mean life  $ET$  of preventively predetermined maintained objects in time  $t_p$  comparing with corrective maintenance of the same object.

Let us monitor a series of objects that underwent preventive predetermined maintenance (were replaced) after time interval  $t_p$  using a new object with the same reliability properties. Also, let us suppose that its durability is characterized by a random variable  $X$  with a continuous density function  $f$  and distribution function  $F$ .

Object reliability can be improved during operation by preventive predetermined replacement at time  $t_p$ . Durability of  $k$ -th component is also described by a random variable  $X_k$  with the same density function  $f$  and distribution function  $F$ . We suppose that random variables  $X_1, X_2, \dots$  are independent.

Let us denote by  $T$  a random variable which describes the life of preventively predetermined replaced objects. Further, we derive the formula of the density function  $f_T$  and the distribution function  $F_T$  for the random variable  $T$ . We are particularly interested in the mean value  $ET$ .

Let us denote  $p = P[X_k < t_p]$ ,  $q = P[X_k \geq t_p] = 1 - p$  and  $I = \int_0^{t_p} xf(x)dx$ . We express the random variable  $T$  using  $X_k$  in the following way:

$$T = \begin{cases} X_1 & \text{for } X_1 < t_p \\ t_p + X_1 & \text{for } X_1 \geq t_p, X_2 < t_p \\ 2t_p + X_1 & \text{for } X_1 \geq t_p, X_2 \geq t_p, X_3 < t_p \\ \dots & \dots \\ kt_p + X_{k+1} & \text{for } X_1 \geq t_p, X_2 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p. \end{cases}$$

With respect to independence  $X_1, X_2, \dots$ , from the total probability theorem it holds for arbitrary  $x \in \langle 0; \infty \rangle$ :

$$\begin{aligned} F_T(x) &= P[T < x] = \\ &= P[T < x / X_1 < t_p] \times P[X_1 < t_p] + \\ &+ P[T < x / X_1 \geq t_p, X_2 < t_p] \times P[X_1 \geq t_p, X_2 < t_p] + \dots + \\ &+ P[T < x / X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p] \times \\ &\cdot P[X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p] + \dots = \\ &= P[T < x / X_1 < t_p] \times p + P[T < x / X_1 \geq t_p, X_2 < t_p] \times pq + \dots + \\ &+ P[T < x / X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p] \times pq^k + \dots \end{aligned} \quad (6)$$

Further, we calculate according to the definition of conditional probability with respect to the independence of  $X_1, X_2, \dots$

$$\begin{aligned} P[T < x / X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p] &= \\ &= P[kt_p + X_{k+1} < x / X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p] = \\ &= \frac{P[X_{k+1} < x - kt_p, X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p]}{P[X_1 \geq t_p, \dots, X_k \geq t_p, X_{k+1} < t_p]} = \\ &= \frac{P[X_{k+1} < \min(x - kt_p, t_p)] \cdot q^k}{q^k \cdot p}. \end{aligned} \quad (7)$$

After substitution (7) into equation (6) we obtain:

$$F_T(x) = P[T < x] = \sum_{k=0}^{\infty} P[X_{k+1} < \min(x - kt_p, t_p)] \cdot q^k = \sum_{k=0}^{\infty} F(\min(x - kt_p, t_p)) \cdot q^k$$

It is possible to itemize the distribution function  $F_T$  around the following intervals:

$$F_T(x) = \begin{cases} F(x) & \text{on } (0; t_p) \\ F(t_p) + qF(x - t_p) & \text{on } (t_p; 2t_p) \\ F(t_p) + qF(x - t_p) + q^2F(x - 2t_p) & \text{on } (2t_p; 3t_p) \\ \dots & \dots \end{cases} \quad (8)$$

Last, using a modified equation (8), we calculate the failure probability  $F_T(t)$  and the reliability function  $R_T(t)$  for the components that underwent preventive predetermined maintenance:

$$F_T(t) = F(t_{po}) + R(t_{po})F(t - t_{po}) + R^2(t_{po})F(t - 2t_{po}) + R^3(t_{po})F(t - 3t_{po}) + \dots \quad (9)$$

and

$$R_T(t) = 1 - F_T(t) \quad (10)$$

We obtain density function  $f_T$  by differentiation of  $F_T$ :

$$f_T(x) = \begin{cases} f(x) & \text{on } (0; t_p) \\ qf(x - t_p) & \text{on } (t_p; 2t_p) \\ q^2f(x - 2t_p) & \text{on } (2t_p; 3t_p) \\ \dots & \dots \end{cases} \quad (11)$$

Finally, we calculate the mean value of life (sum of particular operating time) for objects that underwent preventive predetermined replacement:

$$\begin{aligned} ET &= \int_0^{\infty} xf_T(x)dx = \sum_{k=0}^{\infty} \int_{kt_p}^{(k+1)t_p} q^k xf(x - kt_p)dx = \\ &= \sum_{k=0}^{\infty} q^k \int_0^{t_p} (x + kt_p)f(x)dx = \left( \int_0^{t_p} xf(x)dx \right) \sum_{k=0}^{\infty} q^k + qt_p \left( \int_0^{t_p} f(x)dx \right) \sum_{k=0}^{\infty} kq^{k-1} = \\ &= I \frac{1}{1-q} + pqt_p \frac{1}{(1-q)^2} = \\ &= \frac{I + qt_p}{p}, \end{aligned} \quad (12)$$

we have used the formula for the sum of the geometrical series  $1 + q + q^2 + \dots + q^k + \dots = 1 / (1 - q)$  and from this formula through differentiation we have obtained the derived formula

$$1 + 2q + 3q^2 + \dots + kq^{k-1} + \dots = 1 / (1 - q)^2.$$

Integral  $I$  can be modified using integration by parts:

$$\begin{aligned} I &= \int_0^{t_p} xf(x)dx = t_p F(t_p) - \int_0^{t_p} F(x)dx = \\ &= t_p p - t_p + \int_0^{t_p} R(x)dx = -qt_p + \int_0^{t_p} R(x)dx, \end{aligned} \quad (13)$$

where  $R(x) = 1 - F(x)$  is reliability function.

For mean value  $ET$  of the life of preventively predetermined maintained objects at time  $t_p$  we obtain the following equation:

$$ET = \frac{1}{F(t_p)} \int_0^{t_p} R(x)dx = \frac{\int_0^{t_p} R(x)dx}{1 - R(t_p)} \quad (14)$$

From equation (14) it is clear that mean life of preventive predetermined maintained objects  $ET > \int_0^{\infty} R(x)dx$  (mean life of corrective maintained objects) for  $t_p < \infty$ . This fact proves that preventive predetermined maintenance increases operational reliability of objects comparing with corrective maintenance.

Optimal value of operating time to predetermined maintenance  $t_{po}$  [1] it is possible to calculate from equation (15) which is analogical to equation (1) and (4) and using three parameters Weibull distribution function, we obtain:

$$c(t_p) = \frac{C_{pr} + L_f \cdot F(t_p)}{\bar{t}(t_p)} = \frac{C_{pr} + L_f \cdot (1 - \exp(-(\frac{t_p - \gamma_t}{\beta_t})^{\alpha_t}))}{\int_0^{t_p} \exp(-(\frac{t - \gamma_t}{\beta_t})^{\alpha_t}) dt} \quad (15)$$

For solution of equation (14) and (15) it is possible to use a numerical method, e.g. to use MS Excel.

#### 4. Numerical solution

We have simulated the life  $t_f$  of 44 objects to failure and their technical state (diagnostic signals)  $S_f$  shortly before failure, including costs and losses. Value of diagnostic signal may represent the ratio of a two values of variable, therefore, the value presented by diagnostic signal is a dimensionless number. There were obtained input data – see Table 1.

Using input reliability and economic data regarding life time  $t_f$  from Table 1 and software [http://wessa.net/rwasp\\_fitdistrweibull.wasp](http://wessa.net/rwasp_fitdistrweibull.wasp) we obtained mean operating time to failure ( $MOTTF$ ), standard deviation ( $SD$ ) and parameters  $\alpha_t$ ,  $\beta_t$  and  $\gamma_t$  of the Weibull distribution function – see Table 2.

Using input reliability data regarding technical state (diagnostic signal)  $S_f$  from Table 1 and software [http://wessa.net/rwasp\\_fitdistrweibull.wasp](http://wessa.net/rwasp_fitdistrweibull.wasp), we obtained average diagnostic signal (technical state)  $\bar{S}_f$ , standard deviation ( $SD_S$ ) and parameters  $\alpha_S$ ,  $\beta_S$  and  $\gamma_S$  of the Weibull distribution function – see Table 3.

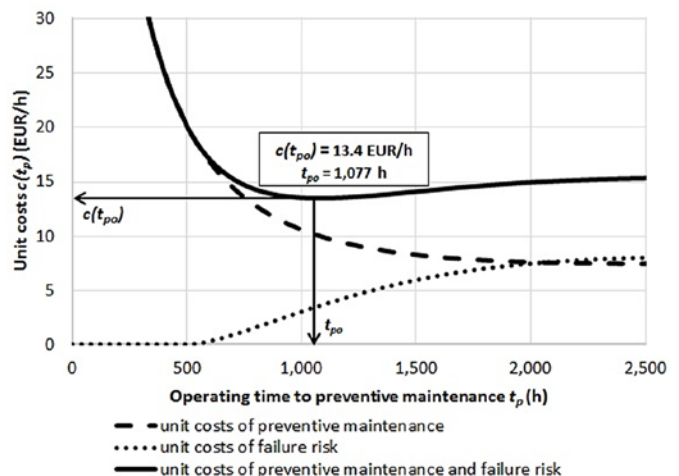


Fig. 2. Dependency of unit costs of preventive maintenance and unit costs of failure risk  $c(t_p)$  versus operating time to preventive predetermined maintenance  $t_p$

Table 1. Simulated input data – life  $t_f$  and diagnostic signal  $S_f$  closely before failure including costs

Object Nr.	1	2	3	4	5	6	7	8	9	10	11
$t_f$ (h)	501	635	727	753	799	941	988	995	1012	1087	1111
$S_f$	3.01	3.07	3.09	3.10	3.13	3.16	3.18	3.19	3.20	3.21	3.22
Object Nr.	12	13	14	15	16	17	18	19	20	21	22
$t_f$ (h)	1125	1163	1194	1199	1205	1210	1223	1238	1245	1256	1277
$S_f$	3.23	3.24	3.25	3.26	3.27	3.28	3.28	3.29	3.29	3.40	3.40
Object Nr.	23	24	25	26	27	28	29	30	31	32	33
$t_f$ (h)	1298	1356	1375	1399	1410	1447	1492	1512	1544	1588	1625
$S_f$	3.41	3.43	3.46	3.47	3.48	3.49	3.49	3.50	3.52	3.53	3.55
Object Nr.	34	35	36	37	38	39	40	41	42	43	44
$t_f$ (h)	1678	1739	1749	1763	1799	1832	1979	2030	2213	2375	2700
$S_f$	3.57	3.58	3.59	3.60	3.62	3.65	3.69	3.72	3.79	3.85	3.97
Costs of preventive maintenance $C_{pr}$ (EUR)											10,000
Costs of corrective maintenance $C_{cm}$ (EUR)											21,000
Production losses due to failure and following down time $L_f$ (EUR)											11,000
Unit costs of condition monitoring $c_d$ (EUR/h)											1.2

Table 2. Parameters of the Weibull distribution function – MOTTF,  $SD_t$ ,  $\alpha_t$ ,  $\beta_t$  and  $\gamma_t$

MOTTF (h)	Standard deviation $SD_t$ (h)	Shape parameter $\alpha_t$	Scale parameter $\beta_t$	Location parameter $\gamma_t$
$MTTF = \beta_t \cdot \Gamma(\frac{1}{\alpha_t} + 1) + \gamma_t$ $\Gamma$ is Gama function	$SD_t = \beta_t \sqrt{\Gamma(\frac{2}{\alpha_t} + 1) - \Gamma(\frac{1}{\alpha_t} + 1)^2}$ $\Gamma$ is Gama function	1.823	971.465	500
1363.39	490.56			

Table 3. Parameters of the Weibull distribution function –  $\bar{S}_f$ ,  $SD_S$ ,  $\alpha_S$ ,  $\beta_S$  and  $\gamma_S$

Mean value $\bar{S}_f$	Standard deviation $SD_S$	Shape parameter $\alpha_S$	Scale parameter $\beta_S$	Location parameter $\gamma_S$
$\bar{S}_f = \beta_S \cdot \Gamma(\frac{1}{\alpha_S} + 1) + \gamma_S$ $\Gamma$ is Gama function	$SD_S = \beta_S \sqrt{\Gamma(\frac{2}{\alpha_S} + 1) - \Gamma(\frac{1}{\alpha_S} + 1)^2}$ $\Gamma$ is Gama function	1.825	0.450	3.00
3.4	0.227			

Table 4. Unit costs of preventive maintenance and failure risk versus operating time (period) of preventive predetermined maintenance (optimal data are formatted bold)

$t_p$ (h)	1,000	1,020	1,040	1,060	<b>1,077</b>	1,100	1,120	3,500
$c(t_p)$ (EUR/h)	13.486	13.465	13.451	13.444	<b>13.442</b>	13.445	13.452	15.400

Table 5. Unit costs of predictive maintenance, diagnostics and failure risk versus diagnostic signal for predictive maintenance (optimal data are formatted bold)

$S_p$ (-)	3.1	3.2	3.3	<b>3.4</b>	3.5	3.6	3.7	3.8
$\bar{t}(S_p)$ (h)	732.0	930.1	1,075.2	<b>1,186.3</b>	1,275.3	1,328.8	1,355.7	1,370.8
$c(S_p)$ (EUR/h)	15.182	12.753	11.861	<b>11.592</b>	11.613	11.937	12.445	12.999

Unit costs of preventive maintenance and failure risk  $L_f \cdot (1 - \exp(-(\frac{t_p - \gamma_t}{\beta_t})^{\alpha_t}))$  versus operating time (period) of preventive predetermined maintenance we can calculate by means of equation (15) – see some results in Table 4 and on Fig. 2.

$$c(S_p) = \frac{C_{pr} + L_f \cdot (1 - \exp(-(\frac{S_p - \gamma_S}{\beta_S})^{\alpha_S}))}{\frac{1}{n} \left[ \sum_{i=1}^{m(S_p)} t_i(S_p) + \sum_{j=1}^{n-m(S_p)} t_j(S_p) \right]} + c_d \quad (16)$$

Now we can calculate optimal diagnostic signal (technical state) for restoration  $S_{po}$  using equation (4) substituting Weibull distribution function to equation (16) and by application MS Excel, we can calculate unit costs  $c(S_p)$  of preventive maintenance, diagnostics and failure risk versus diagnostic signal for predictive maintenance, optimal diagnostic signal  $S_{po}$  (technical state) for restoration and mean operating time versus diagnostic

$$\bar{t}(S_p) = \frac{1}{n} \left[ \sum_{i=1}^{m(S_p)} t_i(S_p) + \sum_{j=1}^{n-m(S_p)} t_j(S_p) \right] - \text{see Table 5 and Fig. 3.}$$

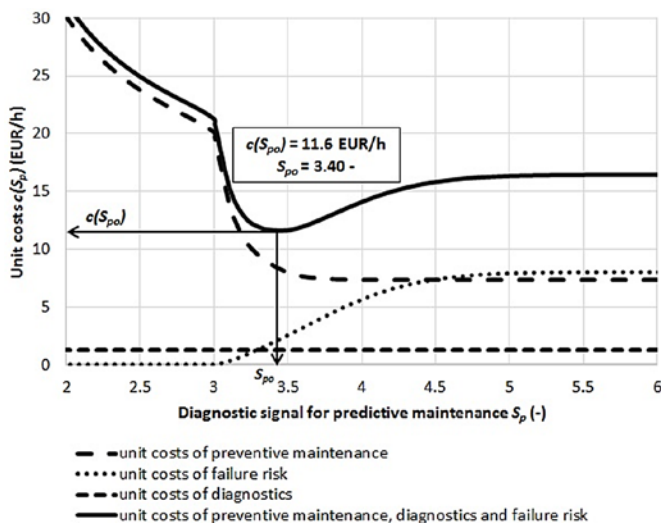


Fig. 3. Dependency of unit costs of preventive maintenance, diagnostics and failure risk versus diagnostic signal of predictive maintenance  $S_p$

Knowledge of optimal diagnostic signal  $S_{po}$  (see Fig. 3) is very important for the design of predictive maintenance. We can very easily indicate dispositional operating time  $t_d(S_p)$  according to equation (5) to be able to plan the maintenance of an object.

Now we use MS Excel to compute mean life  $ET$  of the objects that have undergone preventive maintenance after the optimal interval  $t_{po} = 1,076.7$  hours (according to the equation (15)) by the Weibull distribution function with parameters  $\alpha_t$ ,  $\beta_t$  and  $\gamma_t$  using numerical method of  $R(t)$  integration.

$$ET = \frac{\int_0^{t_{po}} R(x) dx}{1 - R(t_{po})} = \frac{\gamma_t + \int_{\gamma_t}^{t_{po}} \exp(-(\frac{t_p - \gamma_t}{\beta_t})^{\alpha_t}) dt}{1 - \exp(-(\frac{t_{po} - \gamma_t}{\beta_t})^{\alpha_t})} = \frac{1,076.72}{1 - 0.6795} = 3,359.4 \text{ hours} \quad (17)$$

Numerical calculation (for the Weibull distribution function with parameters  $\alpha_t = 1.823$ ,  $\beta_t = 971.466$  and  $\gamma_t = 500$ ) of  $R_T(t)$  is done according to equations (9) and (10) and of  $R(t)$  is done according to equation (18) – see Fig. 4.

$$R(t) = \exp(-(\frac{t - \gamma_t}{\beta_t})^{\alpha_t}) \quad (18)$$

From this figure it is clear that the object with predetermined maintenance has a much better reliability function  $R_T(t)$  than the same object maintained after failure (reliability function  $R(t)$ ).

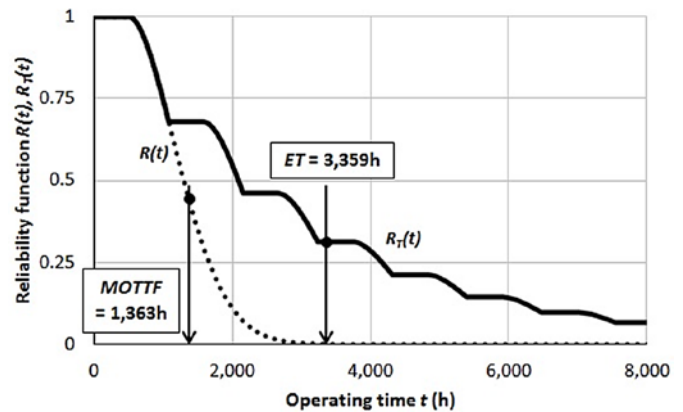


Fig. 4. Reliability functions  $R(t)$  (object is running to failure without preventive maintenance) and  $R_T(t)$  (preventive predetermined maintained object) versus operating time

### 5. Conclusion

Authors offer a tool for maintenance managers which represents general methods of calculating the optimal interval for predetermined maintenance and the optimal diagnostic signal for predictive maintenance/replacement – equations (1) and (15). Further, the authors deduced equations for mean life and probability reliability function of predetermined maintained machine objects and equations for predictive maintenance optimization – equations (10), (12) and (14). Authors proof that preventive maintenance improves reliability. From equation (14) it is clear that mean life of preventive predetermined maintained objects  $ET > \int_0^{\infty} R(x) dx$  (mean life of corrective main-

tained objects) for  $t_p \ll \infty$ . Numerical solution presented graphically on Fig. 4 also shows that reliability of preventive predetermined maintained objects decreases more slowly than the reliability of objects which are running to failure.

The example shows an application of the proposed mathematical model on a virtual machine object. When we replace the component after failure, the  $MOTTF = 1,363$  hours and production losses due to the failure risk  $L_f = 11,000$  EUR and unit costs of preventive maintenance and failure risk, then  $c(MOTTF) = 15.4$  EUR/hour. When we introduce predetermined maintenance (for  $t_{po} = 1,077$  hours) of the object, the  $MOTTF$  increases to  $ET = 3,360$  hours and unit costs of preventive maintenance and failure risk decrease to 13.4 EUR/hour – see Fig. 2 and Fig. 4.

When we introduce predictive maintenance on the same object using the derived equation (16), we obtained optimal diagnostic signal  $S_{po} = 3.4$  and unit costs  $c(S_{po}) = 11.6$  EUR/hour. If we compare these unit costs with unit costs of periodic maintenance ( $c(t_{po}) = 13.4$  EUR/hour), we see that this predictive maintenance strategy brings economical effect of 1.8 EUR/hour. We can see the comparison of all results of the example of chosen maintenance policies from Table 6.

Table 6. Comparison of all results of maintenance policies from the example

Maintenance policy	Diagnostic signal (h, -)	Unit costs (EUR/h)
Corrective maintenance	$t_p \rightarrow \infty$	$c(t_p \rightarrow \infty) = 15.4$
Predetermined maintenance	$t_{po} = 1.077$	$c(t_{po}) = 13.4$
Predictive maintenance	$S_{do} = 3.4$	$c(S_p) = 11.6$

The best maintenance policy from point of unit costs for this example is a predictive maintenance.

The benefit of the proposed mathematical models is not only the ability to compute the optimal interval of predetermined maintenance and optimal diagnostic signal for predictive maintenance, but also to

provide quantitative proof that preventive predetermined maintenance increases operational reliability of machine objects. The decision lies with maintenance specialists, whether or not they adopt and apply these models and methods for improving maintenance effectiveness of industry production equipment.

#### Acknowledgement

The paper was prepared with support from project IGA CULS Prague 2016: 31190/1312/3117 Effect of mixtures of biofuels on production of nitrogen oxide and smoke of combustion engine during NRSC.

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