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SOLUTION OF THE MODIFIED TIME FRACTIONAL COUPLED BURGERS EQUATIONS USING LAPLACE ADOMIAN DECOMPOSTION METHOD

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Abstract: In this work, a coupled system of time-fractional modified Burgers' equations is considered. Three different fractional operators: Caputo, Caputo-Fabrizio and Atangana-Baleanu operators are implemented for the equations. Also, two different scenarios are examined for each fractional operator: when the initial conditions are $u(x, y, 0) = \sin(xy)$, $v(x, y, 0) = \sin(xy)$, and when they are $u(x, y, 0) = e^{\{-kxy\}}$, $v(x, y, 0) = e^{\{-kxy\}}$, where k, α are some positive constants. With the aid of computable Adomian polynomials, the solutions are obtained using Laplace Adomian decomposition method (LADM). The method does not need linearization, weak nonlinearity assumptions or perturbation theory. Simulations are also presented to support theoretical results, and the behaviour of the solutions under the three different fractional operators compared.

Key words: Burgers equations, Fractional derivatives, Laplace Adomian decomposition method, Semi-analytic solutions, Simulations

1. INTRODUCTION

Fractional differential equations (FDEs) are beginning to enjoy widespread application in many real life modelling problems. Fractional operators involving power-law kernel were first proposed by Riemann-Liouville and Caputo [1]. Although, these kernels are singular and constitute serious setbacks to their usage, more recent and improved operators such as Caputo-Fabrizio (CF) [2] and Atangana-Baleanu (AB) [3] operators have emerged.

The time-fractional Burgers equation is a kind of sub-diffusion convection equation. It is widely used to describe many physical problems such as unidirectional propagation of weakly nonlinear acoustic waves, shock waves in a viscous medium, flow systems, electromagnetic waves, compressible turbulence and weak shock propagation, etc [4]. Within the literature, lots of methods have been used to solve different versions of the Burgers equations, both integer order and time-fractional forms [5-10].

Agheli used the new homotopic perturbation method (NHPM) to solve a system of time fractional Burgers' equations [5]. Kaya [6] considered an explicit solution of the coupled viscous Burgers equation with the aid of the decomposition method. Majeed et al. [7] considered the solution of a one-dimensional time fractional Burgers and Fishers equations numerically with the help of the cubic B-spline approximation method. Singh et al. [8] analyzed a one-dimensional time-fractional model for damped Burgers equation involving the Caputo-Fabrizio fractional derivative. Also, the authors [9] considered the approximate analytic solution of the time-fractional damped Burgers and Cahn-Allen equations involving the Riemann-Liouville derivative using Homotopy analysis method (HAM). The existence of solutions for a coupled system of time-fractional partial differential equations (FPDEs) including continuous functions and the Caputo-Fabrizio fractional derivative was examined by Alsaedi et al. [10].

Also, several other methods have been proposed to solve non-linear fractional partial differential equations. The authors [11] numerically solved space-time fractional Burgers equations with the help of a new semi-analytical method. Safari and Sun [12] solved a fractional Rayleigh-Stokes using an improved singular boundary and dual reciprocity methods. Safari and Chen [13] solved a multi-term time-fractional mixed diffusion-wave equations with coupling of the improved singular boundary and dual reciprocity methods. In [14], Safari et al. used a meshless method to solve a variable-order fractional diffusion problems with fourth-order derivative term.

Among the available methods, the Laplace-Adomian decomposition method (LADM) has proven to be one of the most effective and straight forward method for solving non-linear FDES. This method combines both the Adomian decomposition method and Laplace transform. Also, it does not involve any predefined size declaration, discretization or linearization [15].

In this work, a modified two-dimensional system of time-fractional Burgers' equations is considered, with the help of three different fractional derivatives: Caputo, Caputo-Fabrizio and Atangana-Baleanu. The system is solved by applying LADM and the obtained results are compared. We hope this work will open up new research questions for further studies in this regard.

1.1 Preliminaries

Definition 1.1 [16] The Caputo fractional (CF) derivative of a function f of order $\theta \in R^+$ is defined by

$$ {}_0^C D_t^\theta f(t) = \frac{1}{\Gamma(1-\theta)} \int_0^t (t-\zeta)^{-\theta} f'(\zeta) d\zeta \quad (1)$$

Definition 1.2 [16] The Caputo Fractional integral of a function f of order $\theta \in R^{+}$ is defined by

$$ {}_{0}^{C}I_{t}^{\theta} f(t) = \frac{1}{\Gamma(\theta)} \int_{0}^{t} (t-\zeta)^{\theta-1} f(\zeta) d\zeta \qquad t>0 \tag{2}$$

If $f(t)=1$, the Caputo fractional integral is defined as

$$ {}_{0}^{C}I_{t}^{\theta}(1) = \frac{1}{\Gamma(\theta)} \int_{0}^{t} (t-\zeta)^{\theta-1}(1)(\zeta) d\zeta = \frac{t^{\theta}}{\Gamma(\theta+1)} \tag{3}$$

Definition 1.3 [16] For the Caputo derivative, the Laplace transform is defined by:

$$ L\{{}_{0}^{C}D_{t}^{\theta} f(t)\} = s^{\theta} L\{f(s)\} - s^{\theta-1} f(0), \quad 0<\theta<1 \tag{4}$$

Definition 1.4 [2] Let $f \in H^{1}(a_1,a_2),\ a_2>a_1, \theta \in (0,1)$.The Caputo-Fabrizio fractional (CF) derivative [2] of a function f of order $\theta \in R^{+}$ is defined by

$$ {}_{0}^{CF}D_{t}^{\theta} f(t) = \frac{G(\theta)}{(1-\theta)} \int_{0}^{t} \exp\left[-\frac{\theta}{1-\theta}(t-\tau)\right] f'(\zeta) d\zeta \tag{5}$$

where $G(\theta)=(1-\theta)+\frac{\theta}{\Gamma(\theta)}$, denotes a normalization function satisfying $G(0)=G(1)=1$.

However, if $f \notin H^{1}(a_1,a_2)$, then the Caputo-Fabrizio derivative is defined by

$$ {}_{0}^{CF}D_{t}^{\theta} f(t) = \frac{\theta G(\theta)}{(1-\theta)} \int_{0}^{t} \exp\left[-\frac{\theta}{1-\theta}(t-\tau)\right] (f(t)-f(\zeta)) d\zeta \tag{6}$$

Definition 1.5 [2] The Laplace transform of the Caputo-Fabrizio derivative is given by:

$$ L\{{}_{0}^{CF}D_{t}^{\theta} f(t)\} = G(\theta) \frac{sL\{f(t)\}-f(0)}{[s+\theta(1-s)]} \tag{7}$$

Definition 1.6 [3] Let $f \in H^{1}(a_1,a_2),\ a_2>a_1, \theta \in (0,1)$.The Atangana-Baleanu derivative of a function f of order $\theta \in R^{+}$ in Caputo sense is defined by

$$ {}_{0}^{ABC}D_{t}^{\theta} f(t) = \frac{G(\theta)}{(1-\theta)} \int_{0}^{t} E_{\theta}\left[-\frac{\theta}{1-\theta}(t-\tau)\right] f'(\zeta) d\zeta \tag{8}$$

where, $E_{\theta}(.)$ is the Mittag-Leffler function defined by

$$ E_{\theta}(t) = \sum_{k=0}^{\infty} \frac{t^{\theta}}{\Gamma(\theta k+1)}, \quad \theta>0. \tag{9}$$

Definition 1.7 [3] The Atangana-Baleanu (AB) fractional integral in Caputo for a given function f of order $\theta \in R^{+}$ is defined by

$$ {}_{0}^{AB}I_{t}^{\theta} f(t) = \frac{1-\theta}{G(\theta)} f(t) + \frac{\theta}{G(\theta)\Gamma(\theta)} \int_{0}^{t} (t-\tau)^{\theta-1} f(\zeta) d\zeta \tag{10}$$

Definition 1.8 [3] For the AB derivative, the Laplace transform is defined as:

$$ L\{{}_{0}^{ABC}D_{t}^{\theta} f(t)\} = G(\theta) \frac{s^{\theta}L\{f(t)\}-s^{\theta-1}f(0)}{[\theta+(1-\theta)s^{\theta}]} \tag{11}$$

2. APPLICATIONS

Various forms of the time fractional Burgers equations have been considered by different authors. For instance, Mohammed [17] used the Conformable double Sumudu transform in solving a scalar time-fractional coupled Burgers equation. Also, the authors [18] solved a nonlinear one-dimensional fractional Burgers' equations with the aid of the Elzaki transform and homotopy perturbation method. In [19], the authors developed a modified variational iteration Laplace transform method and compared with Laplace Adomian decomposition method in solving a one-dimensional time-fractional Burgers Equations.

To the best of our knowledge, no authors have considered the coupled system of time fractional Burgers' equation using the three different fractional derivatives. In this paper, this is now solved with the aid of the Laplace Adomian Decomposition method (LADM).

2.1. The Atangana-Baleanu fractional operator

Example 2.1 Let us consider the system of modified time-fractional two-dimensional Burgers' equations

$$\begin{aligned} &\frac{\partial^{\theta}}{\partial t^{\theta}} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \\ &\frac{\partial^{\theta} v}{\partial t^{\theta}} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \alpha\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \end{aligned} \tag{12}$$

Subject to the initial conditions:

Case 2.1 $u(x,y,0)=\sin(xy)\,,\ v(x,,0)=\sin(xy)$

By applying the Laplace transform of the AB derivative to the equation (12), we obtain

$$ L\left[\frac{\partial^{\theta}}{\partial t^{\theta}}\right] = L\left[\alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y}\right]$$

$$ L\left[\frac{\partial^{\theta} v}{\partial t^{\theta}}\right] = L\left[\alpha\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y}\right],$$

which can be re-written as

$$\begin{aligned} L[u(x,y,t)] &= \frac{u(x,y,0)}{s} + \frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y}\right] \\ L[v(x,y,t)] &= \frac{v(x,y,0)}{s} + \frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y}\right]. \end{aligned} \tag{13}$$

Taking inverse Laplace transform of both sides, we obtain

$$\begin{aligned} u(x,y,t) &= L^{-1}\left[\frac{u(x,y,0)}{s} + \frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]\right] \\ v(x,y,t) &= L^{-1}\left[\frac{v(x,y,0)}{s} + \frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]\right] \end{aligned} \tag{14}$$

which is equivalent to

$$\begin{aligned} u(x,y,t) &= \sin(xy) + L^{-1}\left[\frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y}\right]\right] \\ v(x,y,t) &= \sin(xy) + L^{-1}\left[\frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) - u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y}\right]\right] \end{aligned} \tag{15}$$

Using the ADM, we obtain

$$ \sum_{j=0}^{\infty} u_j(x,y,t) = \sin(xy) + L^{-1}\left[\frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) - \sum_{j=0}^{\infty} A_j - \sum_{j=0}^{\infty} B_j\right]\right]$$

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$$\sum_{j=0}^{\infty} v_j(x,y,t) = \sin(xy) + L^{-1}\left[\frac{s^{\theta}(1-\theta)+\theta}{s^{\theta}G(\theta)} L\left[\alpha\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \sum_{j=0}^{\infty} C_j - \sum_{j=0}^{\infty} D_j\right]\right] \quad (16)$$

where, the Adomian polynomial components A_j, B_j, C_j and D_j are given as:

$$A_0 = u_0\frac{\partial u_0}{\partial x},\ \ A_1 = u_0\frac{\partial u_1}{\partial x} + u_1\frac{\partial u_0}{\partial x},$$
$$A_2 = u_0\frac{\partial u_2}{\partial x} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_0}{\partial x},$$
$$B_0 = v_0\frac{\partial u_0}{\partial y},\quad B_1 = v_0\frac{\partial u_1}{\partial y} + v_1\frac{\partial u_0}{\partial y},$$
$$B_2 = v_0\frac{\partial u_2}{\partial y} + v_1\frac{\partial u_2}{\partial y} + v_2\frac{\partial u_0}{\partial y},$$
$$C_0 = u_0\frac{\partial v_0}{\partial x},\ \ C_1 = u_0\frac{\partial v_1}{\partial x} + u_1\frac{\partial v_0}{\partial x},$$
$$C_2 = u_0\frac{\partial v_2}{\partial x} + u_1\frac{\partial v_1}{\partial x} + u_2\frac{\partial v_0}{\partial x},$$
$$D_0 = v_0\frac{\partial v_0}{\partial y},\quad D_1 = v_0\frac{\partial v_1}{\partial y} + v_1\frac{\partial v_0}{\partial y},$$
$$B_2 = v_0\frac{\partial v_2}{\partial y} + v_1\frac{\partial v_2}{\partial y} + v_2\frac{\partial v_0}{\partial y}, \quad (17)$$

For $j = 0,1,2,$

$$u_1(x,y,t) = \sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2}\right) - u_0\frac{\partial u_0}{\partial x} - v_0\frac{\partial u_0}{\partial y}\right] = \sin(\mathrm{xy}) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-\alpha(x^2+y^2)\sin(xy) - (x+y)\sin(xy)\cos(xy)\right]$$

$$v_1(x,y,t) = \sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial y^2}\right) - u_0\frac{\partial v_0}{\partial x} - v_0\frac{\partial v_0}{\partial y}\right] = \sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-\alpha(x^2+y^2)\sin(xy) - (x+y)\sin(xy)\cos(xy)\right]$$

$$u_2(x,y,t) = \sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u_0}{\partial y^2}\right) - u_0\frac{\partial u_1}{\partial x} - u_1\frac{\partial u_0}{\partial x} - v_0\frac{\partial u_1}{\partial y} - v_1\frac{\partial u_0}{\partial y}\right] = \sin(\mathrm{xy}) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) - \sin(xy)\left(y\cos(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-2\alpha x\sin(xy) - \alpha y(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)\right]\right) - \left(\sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-\alpha(x^2+y^2)\sin(xy) - (x+y)\sin(xy)\cos(xy)\right]\right) y\cos(xy) - \sin(xy)\left(x\cos(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-2\alpha y\sin(xy) - \alpha x(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)\right]\right) - \left(\sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-\alpha(x^2+y^2)\sin(xy) - (x+y)\sin(xy)\cos(xy)\right]\right) x\cos(xy)\right]$$

$$v_2(x,y,t) = \sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial y^2}\right) - u_0\frac{\partial v_1}{\partial x} - u_1\frac{\partial v_0}{\partial x} - v_0\frac{\partial v_1}{\partial y} - v_1\frac{\partial v_0}{\partial y}\right] = \sin(\mathrm{xy}) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}\right) - \sin(xy)\left(y\cos(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-2\alpha x\sin(xy) - \alpha y(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)\right]\right) - \left(\sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-\alpha(x^2+y^2)\sin(xy) - (x+y)\sin(xy)\cos(xy)\right]\right) y\cos(xy) - \sin(xy)\left(x\cos(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-2\alpha y\sin(xy) - \alpha x(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)\right]\right) - \left(\sin(xy) + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[-\alpha(x^2+y^2)\sin(xy) - (x+y)\sin(xy)\cos(xy)\right]\right) x\cos(xy)\right]$$

Case 2.2 $u(x,y,0) = \sin(xy)\,,\ v(x,,0) = \sin(xy)$

$$u_1(x,y,t) = e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}\right]$$

$$v_1(x,y,t) = e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}\right]$$

$$u_2(x,y,t) = e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(k^2y^2e^{-kxy} + k^2x^2e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(k^4y^4e^{-kxy} + k^4x^4e^{-kxy} + 4k^2e^{-kxy})\right]\right) - e^{-kxy}\left(kye^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(-k^3y^3e^{-kxy} + 2k^2xe^{-kxy} - k^3yx^2e^{-kxy}) + 2k^2y^2e^{-2kxy} - ke^{-4kxy} + 4k^2xye^{-4kxy}\right]\right) - \left(e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}\right]\right) kye^{-2kxy}\left(-kxe^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(2k^2ye^{-kxy} - k^3xy^2e^{-kxy} - k^3x^3e^{-kxy}) - k^2e^{-2kxy} + 2k^2xye^{-2kxy} - 4k^2x^2ye^{-4kxy}\right]\right) - \left(e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}\right]\right)(-kxe^{-kx})\right]$$

$$v_2(x,y,t) = e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha\left(k^2y^2e^{-kxy} + k^2x^2e^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(k^4y^4e^{-kxy} + k^4x^4e^{-kxy} + 4k^2e^{-kxy})\right]\right) - e^{-kxy}\left(kye^{-kxy} + \left[\frac{1}{G(\theta)}\left(1-\theta+\frac{\theta t^{\theta}}{\Gamma(\theta)}\right)\right]\left[\alpha(-k^3y^3e^{-kxy} + 2k^2xe^{-kxy} - k^3yx^2e^{-kxy}) + 2k^2y^2e^{-2kxy} - ke^{-4kxy} + \right.\right.\right.$$

$4k^2xye^{-4kxy}]\Big) - \Big(e^{-kxy} + \Big[\frac{1}{G(\theta)}\Big(1-\theta+\frac{\theta t^\theta}{\Gamma(\theta)}\Big)\Big][\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}]\Big)kye^{-2kxy}\Big(-kxe^{-kxy} + \Big[\frac{1}{G(\theta)}\Big(1-\theta+\frac{\theta t^\theta}{\Gamma(\theta)}\Big)\Big][\alpha(2k^2ye^{-kxy} - k^3xy^2e^{-kxy} - k^3x^3e^{-kxy}) - k^2e^{-2kxy} + 2k^2xye^{-2kxy} - 4k^2x^2ye^{-4kxy}]\Big) - \Big(e^{-kxy} + \Big[\frac{1}{G(\theta)}\Big(1-\theta+\frac{\theta t^\theta}{\Gamma(\theta)}\Big)\Big][\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}]\Big)(-kxe^{-kx})\Big]$

2.2. The Caputo fractional operator

Applying the Laplace transform to system (12) and solving via the Caputo fractional derivative, we have using the initial conditions:

Case 2.3 $u(x,y,0) = \sin(xy)$, $v(x,y,0) = \sin(xy)$

$u_2(x,y,t) = \sin(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big]\Big[\alpha\Big(\frac{\partial^2u_0}{\partial x^2} + \frac{\partial^2u_0}{\partial y^2}\Big) - u_0\frac{\partial u_1}{\partial x} - u_1\frac{\partial u_0}{\partial x} - v_0\frac{\partial u_1}{\partial y} - v_1\frac{\partial u_0}{\partial y}\Big] = \sin(\mathrm{xy}) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big]\Big[\alpha\Big(\frac{\partial^2u_1}{\partial x^2} + \frac{\partial^2u_1}{\partial y^2}\Big) - \sin(xy)\Big(ycos(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-2\alpha xsin(xy) - \alpha y(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)]\Big) - \Big(\sin(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-\alpha(x^2+y^2)sin(xy) - (x+y)\sin(xy)\cos(xy)]\Big)ycos(xy) - \sin(xy)\Big(xcos(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-2\alpha ysin(xy) - \alpha x(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)]\Big) - \Big(\sin(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-\alpha(x^2+y^2)\sin(xy) - (x+y)sin(xy)\cos(xy)]\Big)xcos(xy)\Big]$

$v_2(x,y,t) = \sin(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big]\Big[\alpha\Big(\frac{\partial^2v_0}{\partial x^2} + \frac{\partial^2v_0}{\partial y^2}\Big) - u_0\frac{\partial v_1}{\partial x} - u_1\frac{\partial v_0}{\partial x} - v_0\frac{\partial v_1}{\partial y} - v_1\frac{\partial v_0}{\partial y}\Big] = \sin(\mathrm{xy}) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big]\Big[\alpha\Big(\frac{\partial^2u_1}{\partial x^2} + \frac{\partial^2u_1}{\partial y^2}\Big) - \sin(xy)\Big(ycos(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-2\alpha xsin(xy) - \alpha y(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)]\Big) - \Big(\sin(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-\alpha(x^2+y^2)sin(xy) - (x+y)\sin(xy)\cos(xy)]\Big)ycos(xy) - \sin(xy)\Big(xcos(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-2\alpha ysin(xy) - \alpha x(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)]\Big) - \Big(\sin(xy) + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][-\alpha(x^2+y^2)\sin(xy) - (x+y)sin(xy)\cos(xy)]\Big)xcos(xy)\Big]$

Case 2.4 $u(x,y,0) = e^{-kxy}$, $v(x,,0) = e^{-kxy}$

$u_2(x,y,t) = e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big]\Big[\alpha\Big(k^2y^2e^{-kxy} + k^2x^2e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(k^4y^4e^{-kxy} + k^4x^4e^{-kxy} + 4k^2e^{-kxy})]\Big) - e^{-kxy}\Big(kye^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(-k^3y^3e^{-kxy} + 2k^2xe^{-kxy} - k^3yx^2e^{-kxy}) + 2k^2y^2e^{-2kxy} - ke^{-4kxy} + 4k^2xye^{-4kxy}]\Big) - \Big(e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}]\Big)kye^{-2kxy}\Big(-kxe^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(2k^2ye^{-kxy} - k^3xy^2e^{-kxy} - k^3x^3e^{-kxy}) - k^2e^{-2kxy} + 2k^2xye^{-2kxy} - 4k^2x^2ye^{-4kxy}]\Big) - \Big(e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}]\Big)(-kxe^{-kx})\Big]$

$v_2(x,y,t) = e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big]\Big[\alpha\Big(k^2y^2e^{-kxy} + k^2x^2e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(k^4y^4e^{-kxy} + k^4x^4e^{-kxy} + 4k^2e^{-kxy})]\Big) - e^{-kxy}\Big(kye^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(-k^3y^3e^{-kxy} + 2k^2xe^{-kxy} - k^3yx^2e^{-kxy}) + 2k^2y^2e^{-2kxy} - ke^{-4kxy} + 4k^2xye^{-4kxy}]\Big) - \Big(e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}]\Big)kye^{-2kxy}\Big(-kxe^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(2k^2ye^{-kxy} - k^3xy^2e^{-kxy} - k^3x^3e^{-kxy}) - k^2e^{-2kxy} + 2k^2xye^{-2kxy} - 4k^2x^2ye^{-4kxy}]\Big) - \Big(e^{-kxy} + \Big[\frac{t^\theta}{\Gamma(\theta+1)}\Big][\alpha(k^2y^2e^{-kxy} + k^2x^2e^{-kxy}) + kye^{-2kxy} + kxe^{-4kxy}]\Big)(-kxe^{-kx})\Big]$

2.3. The Caputo-Fabrizio fractional operator

Applying the Laplace transform to system (12) and solving via the Caputo-Fabrizio fractional derivative, we have, using the initial conditions:

Case 2.5 $u(x,y,0) = e^{-kxy}$, $v(x,,0) = e^{-kxy}$

$u_2(x,y,t) = \sin(xy) + [1+\theta(t-1)]\Big[\alpha\Big(\frac{\partial^2u_0}{\partial x^2} + \frac{\partial^2u_0}{\partial y^2}\Big) - u_0\frac{\partial u_1}{\partial x} - u_1\frac{\partial u_0}{\partial x} - v_0\frac{\partial u_1}{\partial y} - v_1\frac{\partial u_0}{\partial y}\Big] = \sin(\mathrm{xy}) + [1+\theta(t-1)]\Big[\alpha\Big(\frac{\partial^2u_1}{\partial x^2} + \frac{\partial^2u_1}{\partial y^2}\Big) - \sin(xy)(ycos(xy) + [1+\theta(t-1)][-2\alpha xsin(xy) - \alpha y(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)]) - (\sin(xy) + [1+\theta(t-1)][-\alpha(x^2+y^2)sin(xy) - (x+y)\sin(xy)\cos(xy)])ycos(xy) - \sin(xy)(xcos(xy) + [1+\theta(t-1)][-2\alpha ysin(xy) - \alpha x(x^2+y^2)\cos(xy) - y(x+y)[\cos^2(xy) - \sin^2(xy)] - \sin(xy)\cos(xy)]) - (\sin(xy) + [1+\theta(t-1)][-\alpha(x^2+y^2)\sin(xy) - (x+y)sin(xy)\cos(xy)])xcos(xy)\Big]$

$v_2(x,y,t) = \sin(xy) + [1+\theta(t-1)]\Big[\alpha\Big(\frac{\partial^2v_0}{\partial x^2} + \frac{\partial^2v_0}{\partial y^2}\Big) - u_0\frac{\partial v_1}{\partial x} - u_1\frac{\partial v_0}{\partial x} - v_0\frac{\partial v_1}{\partial y} - v_1\frac{\partial v_0}{\partial y}\Big] = \sin(\mathrm{xy}) + [1+\theta(t-$

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$$1)]\left[\alpha\left(\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}\right)-\sin(xy)(y\cos(xy)+[1+\theta(t-1)][-2\alpha x\sin(xy)-\alpha y(x^2+y^2)\cos(xy)-y(x+y)[\cos^2(xy)-\sin^2(xy)]-\sin(xy)\cos(xy)])-(\sin(xy)+[1+\theta(t-1)][-\alpha(x^2+y^2)\sin(xy)-(x+y)\sin(xy)\cos(xy)])y\cos(xy)-\sin(xy)\,(x\cos(xy)+[1+\theta(t-1)][-2\alpha y\sin(xy)-\alpha x(x^2+y^2)\cos(xy)-y(x+y)[\cos^2(xy)-\sin^2(xy)]-\sin(xy)\cos(xy)])-(\sin(xy)+[1+\theta(t-1)][-\alpha(x^2+y^2)\sin(xy)-(x+y)\sin(xy)\cos(xy)])x\cos(xy)\right]$$

Case 2.6 $u(x,y,0)=e^{-kxy},\ v(x,,0)=e^{-kxy}$

$$u_2(x,y,t)=e^{-kxy}+\left[\frac{t^\theta}{\Gamma(\theta+1)}\right]\left[\alpha\left(k^2y^2e^{-kxy}+k^2x^2e^{-kxy}+\left[\frac{t^\theta}{\Gamma(\theta+1)}\right][\alpha(k^4y^4e^{-kxy}+k^4x^4e^{-kxy}+4k^2e^{-kxy})]\right)-e^{-kxy}\left(kye^{-kxy}+\left[\frac{t^\theta}{\Gamma(\theta+1)}\right][\alpha(-k^3y^3e^{-kxy}+2k^2xe^{-kxy}-k^3yx^2e^{-kxy})+2k^2y^2e^{-2kxy}-ke^{-4kxy}+4k^2xye^{-4kxy}]\right)-\left(e^{-kxy}+\left[\frac{t^\theta}{\Gamma(\theta+1)}\right][\alpha(k^2y^2e^{-kxy}+k^2x^2e^{-kxy})+kye^{-2kxy}+kxe^{-4kxy}]\right)kye^{-2kxy}(-kxe^{-kxy}+1+\theta(t-1)[\alpha(2k^2ye^{-kxy}-k^3xy^2e^{-kxy}-k^3x^3e^{-kxy})-k^2e^{-2kxy}+2k^2xye^{-2kxy}-4k^2x^2ye^{-4kxy}])-(e^{-kxy}+[1+\theta(t-1)][\alpha(k^2y^2e^{-kxy}+k^2x^2e^{-kxy})+kye^{-2kxy}+kxe^{-4kxy}])(-kxe^{-kx})\right]$$

$$v_2(x,y,t)=e^{-kxy}+[1+\theta(t-1)][\alpha(k^2y^2e^{-kxy}+k^2x^26+1+\theta(t-1)[\alpha(k^4y^4e^{-kxy}+k^4x^4e^{-kxy}+4k^2e^{-kxy})])-e^{-kxy}(kye^{-kxy}+[1+\theta(t-1)][\alpha(-k^3y^3e^{-kxy}+2k^2xe^{-kxy}-k^3yx^2e^{-kxy})+2k^2y^2e^{-2kxy}-ke^{-4kxy}+4k^2xye^{-4kxy}])-(e^{-kxy}+[1+\theta(t-1)][\alpha(k^2y^2e^{-kxy}+k^2x^2e^{-kxy})+kye^{-2kxy}+kxe^{-4kxy}])kye^{-2kxy}(-kxe^{-kxy}+[1+\theta(t-1)][\alpha(2k^2ye^{-kxy}-k^3xy^2e^{-kxy}-k^3x^3e^{-kxy})-k^2e^{-2kxy}+2k^2xye^{-2kxy}-4k^2x^2ye^{-4kxy}])-(e^{-kxy}+[1+\theta(t-1)][\alpha(k^2y^2e^{-kxy}+k^2x^2e^{-kxy})+kye^{-2kxy}+kxe^{-4kxy}])(-kxe^{-kx})]$$

2.4. Numerical simulations

In this section, we present some numerical results to justify the theoretical analysis and computations. It is imperative to state that the MATLAB R2020a version was used to run all the simulations in this section. The solution profiles for u at initial time $t = 0$ when the fluid viscosity is 1.0, using the three different fractional operators are presented in Figs.1(a)-(b).

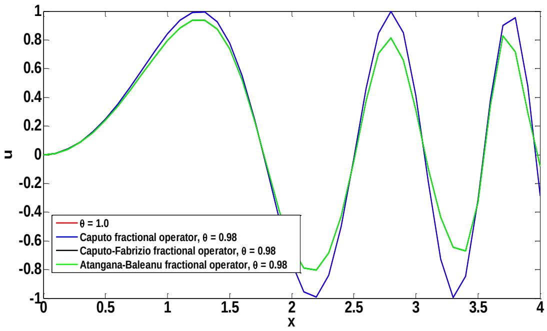

Fig. 1(a). Case 1: Solution profile for u when $t = 0, \alpha = 1.0$

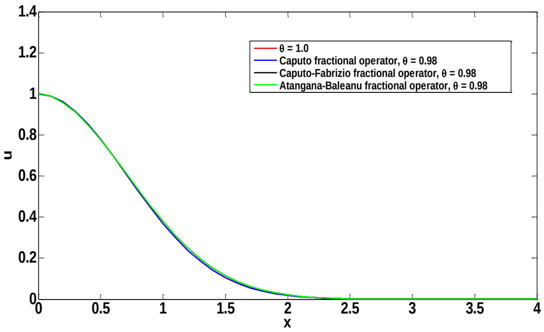

Fig. 1(b). Case 2: Solution profile for u when $t = 0, \alpha = 1.0$

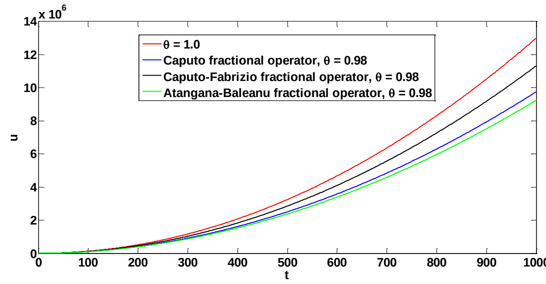

Fig. 2(a). Case 2: Solution profile for u when $x = y = 1, \alpha = 1.0$

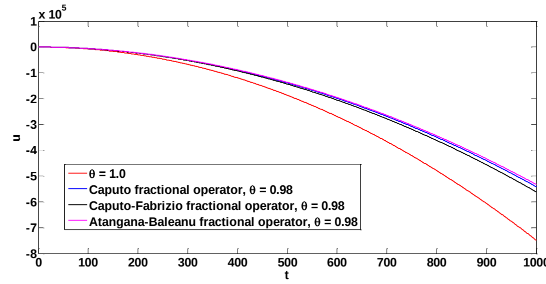

Fig. 2(b). Case 2: Solution profile for u when $x = y = 1, \alpha = 1.0$

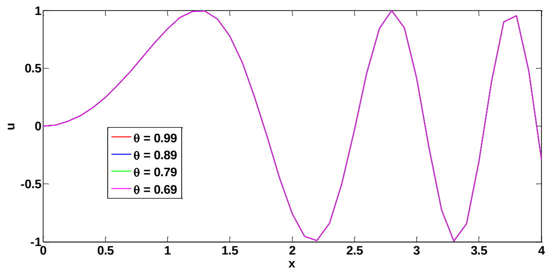

Fig. 3(a). Case 1: Solution profile for u when $t = 0, \alpha = 1.0$

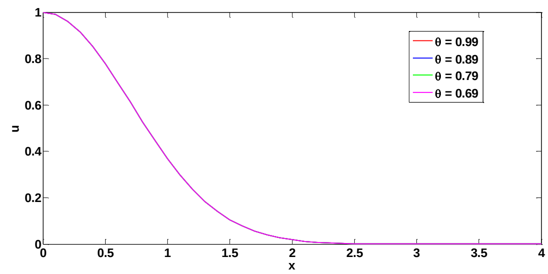

Fig. 3(b). Case 2: Solution profile for u when $t = 0, \alpha = 1.0$

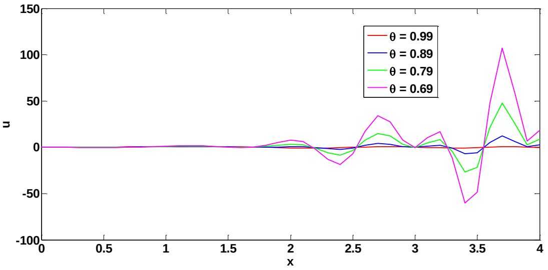

Fig. 4(a). Case 1: Solution profile for u when $t = 0, \alpha = 1.0$

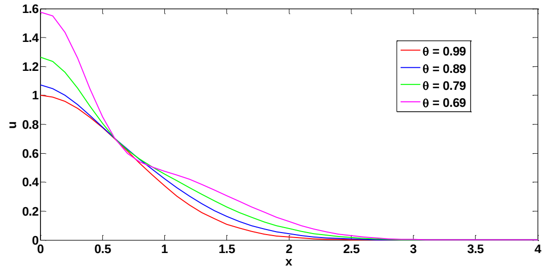

Fig. 4(b). Case 2: Solution profile for u when $t = 0, \alpha = 1.0$

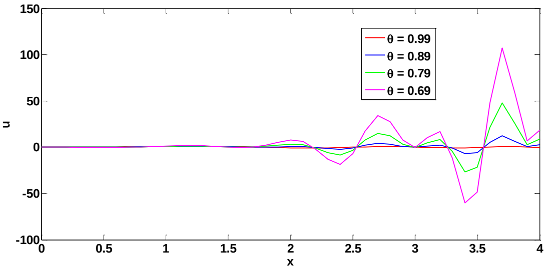

Fig. 5(a). Case 1: Solution profile for u when $t = 0, \alpha = 1.0$

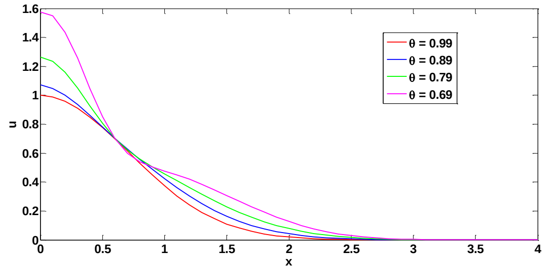

Fig. 5(b). Case 2: Solution profile for u when $t = 0, \alpha = 1.0$

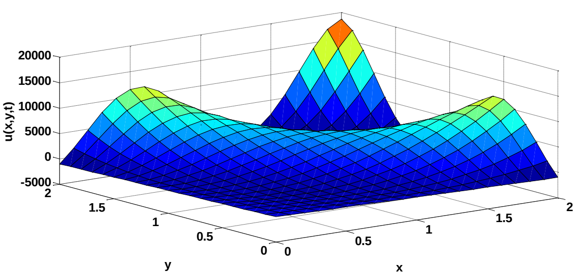

Fig. 6(a). Case 1: Solution profile for u when $\alpha = 1.0,\ \theta = 0.95, t = 20$

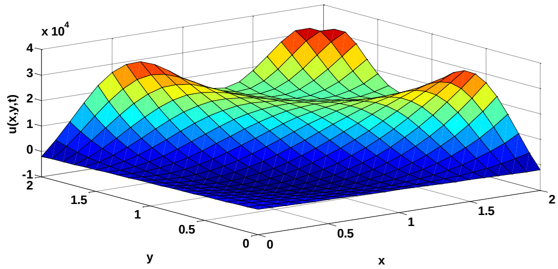

Fig. 6(b). Case 2: Solution profile for u when $\alpha = 2.0,\ \theta = 0.95, t = 20$

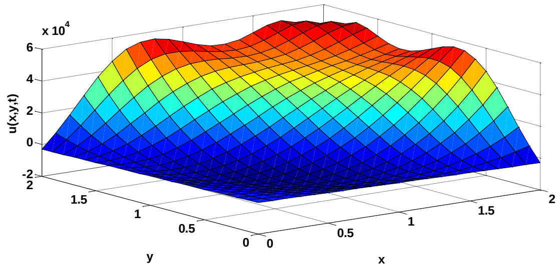

Fig. 6(c). Case 1: Solution profile for u when $\alpha = 3.0,\ \theta = 0.95, t = 20$

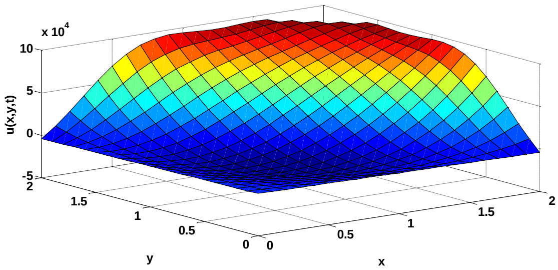

Fig. 6(d). Case 1: Solution profile for u when $\alpha = 4.0,\ \theta = 0.95, t = 20$

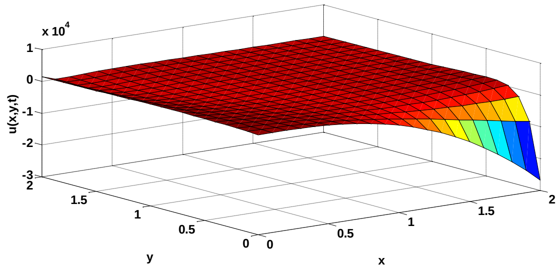

Fig. 7(a). Case 2: Solution profile for u when $\alpha = 1.0,\ \theta = 0.95, t = 20$

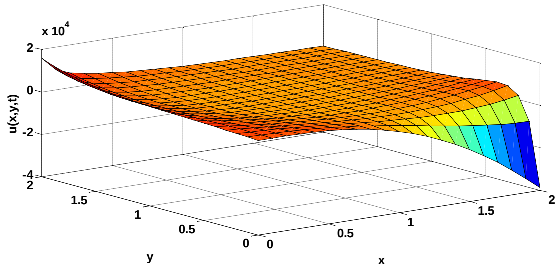

Fig. 7(b). Case 2: Solution profile for u when $\alpha = 2.0,\ \theta = 0.95, t = 20$

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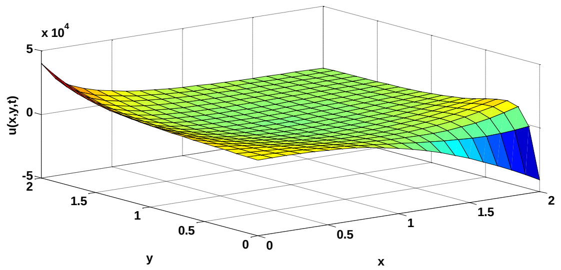

Fig. 7(c). Case 2: Solution profile for u when $\alpha = 3.0,\ \theta = 0.95, t = 20$

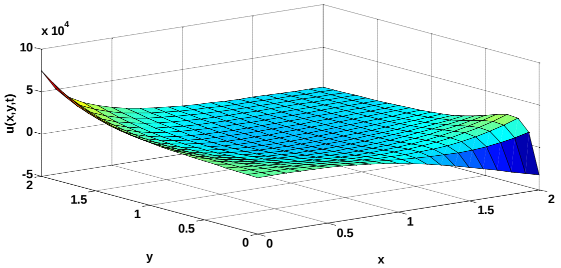

Fig. 7(d). Case 2: Solution profile for u when $\alpha = 4.0,\ \theta = 0.95, t = 20$

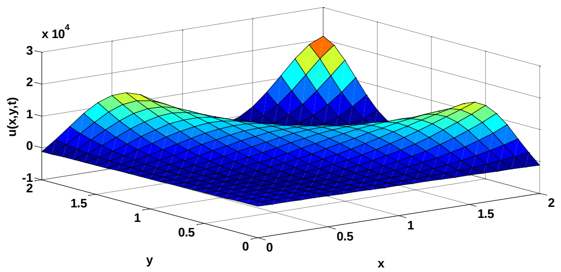

Fig. 8(a). Case 1: Solution profile for u when $\alpha = 1.0,\ \theta = 0.95, t = 20$

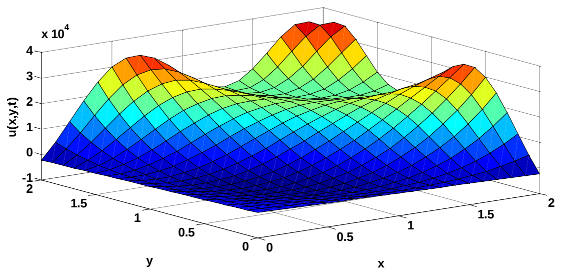

Fig. 8(b). Case 1: Solution profile for u when $\alpha = 2.0,\ \theta = 0.95, t = 20$

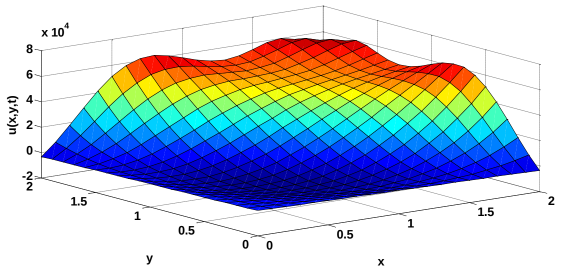

Fig. 8(c). Case 1: Solution profile for u when $\alpha = 3.0,\ \theta = 0.95, t = 20$

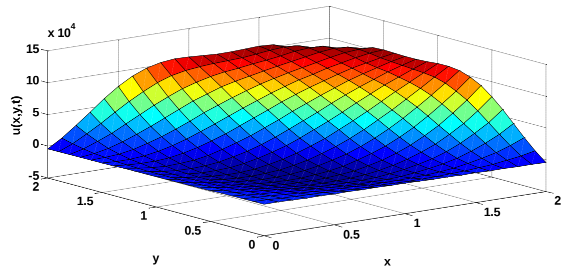

Fig. 8(d). Case 1: Solution profile for u when $\alpha = 4.0,\ \theta = 0.95, t = 20$

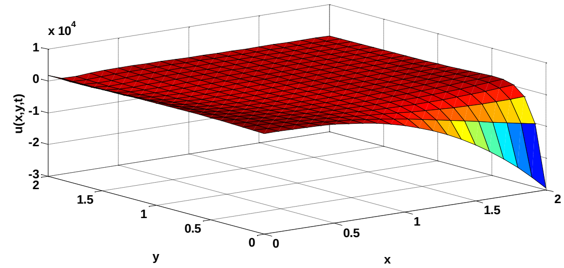

Fig. 9(a). Case 2: Solution profile for u when $\alpha = 1.0,\ \theta = 0.95, t = 20$

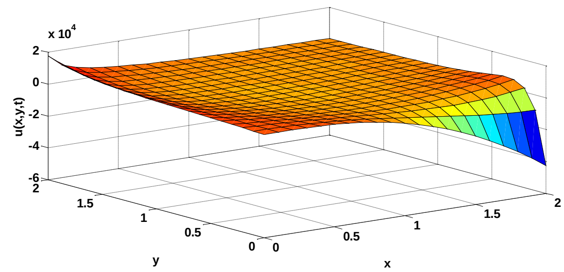

Fig. 9(b). Case 2: Solution profile for u when $\alpha = 2.0,\ \theta = 0.95, t = 20$

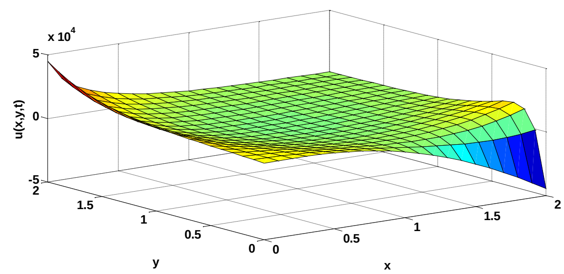

Fig. 9(c). Case 2: Solution profile for u when $\alpha = 3.0,\ \theta = 0.95, t = 20$

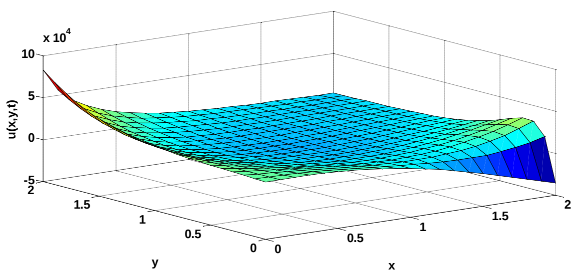

Fig. 9(d). Case 2: Solution profile for u when $\alpha = 4.0,\ \theta = 0.95, t = 20$

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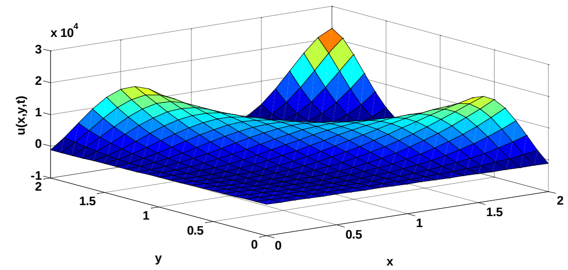

Fig. 10(a). Case 1: Solution profile for u when $\alpha = 1.0, \ \theta = 0.95, t = 20$

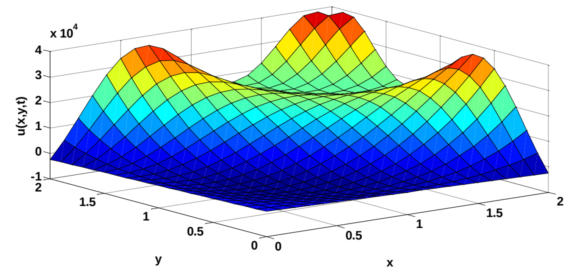

Fig. 10(b). Case 1: Solution profile for u when $\alpha = 2.0, \ \theta = 0.95, t = 20$

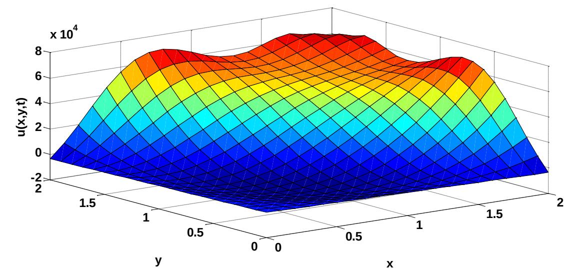

Fig. 10(c). Case 1: Solution profile for u when $\alpha = 3.0, \ \theta = 0.95, t = 20$

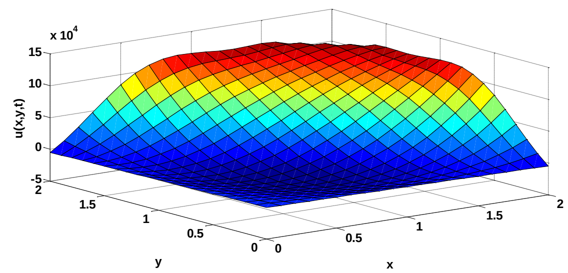

Fig. 10(d). Case 1: Solution profile for u when $\alpha = 4.0, \ \theta = 0.95, t = 20$

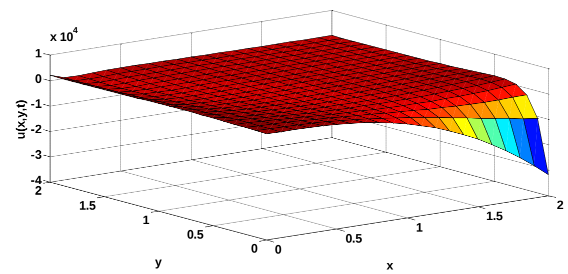

Fig. 11(a). Case 2: Solution profile for u when $\alpha = 1.0, \ \theta = 0.95, t = 20$

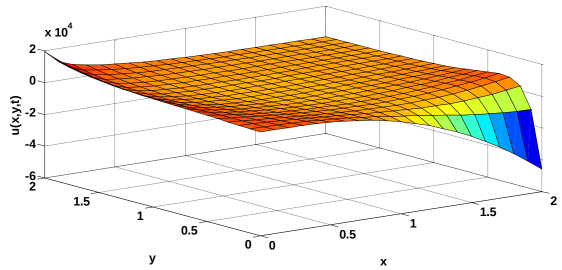

Fig. 11(b). Case 2: Solution profile for u when $\alpha = 2.0, \ \theta = 0.95, t = 20$

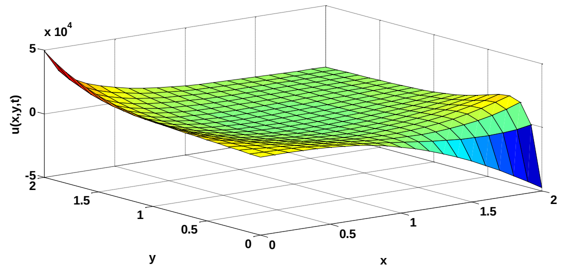

Fig. 11(c). Case 2: Solution profile for u when $\alpha = 3.0, \ \theta = 0.95, t = 20$

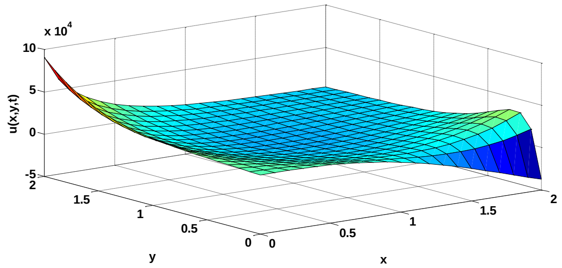

Fig. 11(d). Case 2: Solution profile for u when $\alpha = 4.0, \ \theta = 0.95, t = 20$

The solution profiles for u over time when $\mathrm{x} = \mathrm{y} = 1$ and when the fluid viscosity is 1.0, using the three different fractional operators are presented in Figs.2(a)-(b).The solution profiles for u at initial time $\mathrm{t} = 0$ when the fluid viscosity is 1.0, using the Caputo fractional operator are presented in Figs.3(a)-(b). The solution profiles for u at initial time $\mathrm{t} = 0$ when the fluid viscosity is 1.0, using the Caputo-Fabrizio fractional operator are presented in Figs.4(a)-(b).The solution profiles for u at initial time $\mathrm{t} = 0$ when the fluid viscosity is 1.0, using the Atangana-Baleanu fractional operator are presented in Figs.5(a)-(b).The solution profiles for u at initial conditions $\mathrm{u(x, y, 0)} = \sin(\mathrm{xy}), \mathrm{v(x, y, 0)} = \sin(\mathrm{xy})$, over time, when the fluid viscosity α is varied from 1.0 to 4.0, and the fractional order $\theta = 0.95$, using the Atangana-Baleanu fractional operator are presented in Figs. 6(a)-(d). It is observed that as the viscosity is increased, the fluid velocity is stabilized.The solution profiles for u at initial conditions $\mathrm{u(x, y, 0)} = \mathrm{e}^{-\mathrm{kxy}}$, $\mathrm{v(x, y, 0)} = \mathrm{e}^{-\mathrm{kxy}}$, over time, when the fluid viscosity α is varied from 1.0 to 4.0, and the fractional order $\theta = 0.95$, using the Atangana-Baleanu fractional operator are presented in Fig.ure 7(a)-(d).The solution profiles for u at initial conditions $\mathrm{u(x, y, 0)} = \sin(\mathrm{xy})$, $\mathrm{v(x, y, 0)} = \sin(\mathrm{xy})$, over time, when the fluid viscosity α is varied from 1.0 to 4.0, and the fractional order $\theta = 0.95$, using the Caputo fractional operator are presented in Figs. 8(a)-(d).The solution profiles for$\mathrm{u}$ at initial conditions $\mathrm{u(x, y, 0)} = \mathrm{e}^{-\mathrm{kxy}}$, $\mathrm{v(x, y, 0)} = \mathrm{e}^{-\mathrm{kxy}}$, over time,

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when the fluid viscosity α is varied from 1.0 to 4.0, and the fractional order $\theta = 0.95$, using the Caputo fractional operator are presented in Figs. 9(a)-(d).The solution profiles for u at initial conditions $u(x, y, 0) = \sin(xy)$, $v(x, y, 0) = \sin(xy)$, over time, when the fluid viscosity α is varied from 1.0 to 4.0, and the fractional order $\theta = 0.95$, using the Caputo-Fabrizio fractional operator are presented in Fig.ure 10(a)-(d).The solution profiles for u at initial conditions $u(x, y, 0) = e^{-kxy}$, $v(x, y, 0) = e^{-kxy}$, over time, when the fluid viscosity α is varied from 1.0 to 4.0, and the fractional order $\theta = 0.95$, using the Caputo-Fabrizio fractional operator are presented in Figs. 11(a)-(d). It is observed from the figures that as the viscosity is increased, the fluid velocity is stabilized.

It is also worth stating that the CPU time using the Caputo fractional operator was 0.863006 seconds. Using the Caputo-Fabrizio operator, the CPU time was 0.954948 seconds while with the AB fractional operator the CPU time was 0.860035 seconds.

3. CONCLUSION

In this work, a coupled system of time-fractional modified Burgers' equations with appropriate initial values is solved using the Laplace Adomian decomposition method. Three different fractional operators: Caputo, Caputo-Fabrizio and Atangana-Baleanu operators are considered for the equations. Also, two different scenarios are examined for each fractional operator: when the initial conditions are $u(x, y, 0) = \sin(xy)$, $v(x, y, 0) = \sin(xy)$, and when they are $u(x, y, 0) = e^{-kxy}$, $v(x, y, 0) = e^{-kxy}$, where k, α are some positive constants. With the aid of computable Adomian polynomials, the solutions are obtained. The method does not need linearization, weak nonlinearity assumptions or perturbation theory. Simulations are also presented to support theoretical results, and the behaviour of the solutions under the three different fractional operators compared.

Future work shall consider other numerical schemes such as singular boundary and dual reciprocity methods on the current coupled system. We shall also consider modified Burgers equations with higher order dissipation term.

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