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Template Matching Using Improved Rotations Fourier Transform Method

Marvin Chandra Wijaya

***Abstract*—Template matching is a process to identify and localize a template image on an original image. Several methods are commonly used for template matching, one of which uses the Fourier transform. This study proposes a modification of the method by adding an improved rotation to the Fourier transform. Improved rotation in this study uses increment rotation and three shear methods for the template image rotation process. The three shear rotation method has the advantage of precise and noise-free rotation results, making the template matching process even more accurate. Based on the experimental results, the use of 10°angle increments has increased template matching accuracy. In addition, the use of three shear rotations can improve the accuracy of template matching by 13% without prolonging the processing time.**

***Keywords*—template matching; Fourier transform; three shear rotations; image rotation**

I. Introduction

TEMPLATE matching identifies one part of the image from a complete or larger image. Template matching is beneficial in the real world, and many applications require the template matching process [1]. The need for image search, image identification, and image location search are important aspects of computer vision systems [2]. Template matching is widely used in the real world, such as facial recognition, traffic light detection, fingerprint recognition and others [3]. Simple template matching is to see the normalized correlation between a template or image slice (t) and a whole image (f) which is calculated as in Equation (1):

$$tm(u,v) = \frac{\sum_x \sum_y f(u+x, v+y)t(x,y)}{\sqrt{\sum_x \sum_y f^2(u+x)(v+y)}} \quad (1)$$

If a high value of tm(u,v) is produced, it is indicated that there is a suitable image piece at the coordinate location (u,v). In the development of the template matching method, there are two approaches to developing the template matching method. There are two approaches in the template matching process: the feature-based approach and the template-based approach. The feature-based approach uses feature extraction in the target image, such as shape, color, and texture [4]. The features used are converted into vectors that will be processed using a deep learning classifier or Neural Network [5], [6]. The feature-based classification approach effectively recognizes objects in the search process [7]. However, this method has a weakness in the relatively long processing time compared to the template-based approach. The template-based approach does not use feature extraction but uses image similarity searches in searching for image searches [8]. This approach has the advantage of speeding up the template matching process, so it is widely used when large image sizes or many images need to be identified [9]. This approach also allows a reduction in image resolution, which makes the image file size smaller. The template matching process will be faster with smaller image size. In the template-based approach, image identification accuracy depends on the method used, and it is not very accurate compared to the feature-based approach.

M. C. Wijaya is with Computer Engineering Departement, Maranatha Christian University, Bandung, Indonesia (e-mail: marvin.cw@eng.maranatha.edu).

In this study, a method that can overcome this problem is proposed, a method that has a relatively fast processing time and has better image identification accuracy. The use of templates that can have various scales and rotations will increase the accuracy of image identification. In addition, accuracy can be achieved by combining a template-based approach and a feature-based approach. Still, this combination requires several requirements, such as the template image having features that support feature matching.

II. Related Works

A. Fourier Transform

The Fourier transform, developed by Joseph Fourier, is an integral transformation that transforms a function into a sinusoidal basis function [10]. The sinusoidal function is a sum or integral multiplied by several coefficients. The coefficient serves to be the amplitude of the function [11]. Fourier transform converts a signal in the time region into a frequency region, as shown in Figure 1. The signal in the digital image representation is the bitmap value for each pixel. An example of the bitmap value of each pixel is the grayscale value which is 0 - 255.

The discretization and reduction of computational processes that a computer can process are called fast Fourier transform (FFT). FFT will reduce the computational volume from $O(N^2)$ to O(N log N), which causes the transformation process to be faster.

B. Template Matching Application

Template matching is widely applied to various fields, such as in the field of architecture or buildings. In 2013 Tinghua Ai conducted a study entitled “A shape analysis

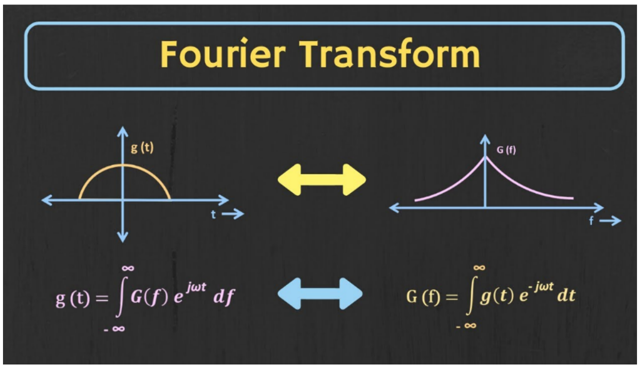

Fig. 1. Fourier Transform

and template matching of building features by the Fourier transform method" to perform shape analysis on building features [12]. A building shape drawn vertically from a height needs to be investigated for its forms to see the geographical phenomenon. The study uses various building templates to recognize a building plan (on the top) and various building templates to recognize a building plan (on the top). The Fourier transform method is used to identify the similarity of the shape of the building to be sought. Due to the similarity of the shape, normalization cannot be done for all building form templates, so the membership function is used for each template.

In the electronics field, template matching can be used to check for defects in the production of electronic goods. In 2019, Du Ming Tsai conducted a study entitled "Defect Detection in Electronic Surfaces Using Template-Based Fourier Image Reconstruction" aiming to detect electronic surfaces [13]. An electronic device such as an integrated circuit (IC) and a printed circuit board (PCB) may have defects that affect the quality and performance of the electronic circuit. This study checked the defects using template matching with the Fourier transform method with modified global Fourier image reconstruction. This method can detect and locate defects in non-periodic pattern images. In addition, it is investigated how to reconstruct the pattern using the inverse Fourier transform to return it to its original condition.

In the biomedical field, template matching using Fourier transform can detect heart sounds. In 2014 Anandarup Mukherjeea conducted a study entitled "A Fourier series based Template Matching Approach to Detect the Splitting of Second Heart Sound" which aims to detect heart sounds. This method is capable of splitting the S2 heart sound in humans [14]. The S2 heart sound can be used to detect congenital heart diseases, which is very useful in helping doctors detect heart abnormalities. The experiment was successfully carried out using an i3 processor using 4GB of RAM, which can detect heart sounds in 0.6856 seconds. Template matching can be applied to identify retinal [15]. In 2012, Masoud Sabaghi conducted a study entitled "Retinal Identification System Based on the Combination of Fourier and Wavelet Transform" which aims to identify the eye [16]. The methods used are Fourier transform, Wavelet transforms, and both transform. Table I shows that template matching succeeded in identifying retinal with an average accuracy of more than 95

TABLE I
COMPARISON BETWEEN RESULTS OF DIFFERENT METHODS.

Method	Average Identification Rate
Fourier transform	95.4%
Wavelet transform	97.2%
Fourier and Wavelet transform	99.1%

C. Improved The Accuracy And Speed Of The Template Matching Process

Two considerations of using the Fourier transform as a method for template matching are the long processing time, especially if the amount of data sampling is large or massive and efforts to improve its accuracy.

Increasing the processing time on the Fourier transform can be done by modifying the Fourier transform. In 1997, M. Uenohara conducted a study entitled "Use of Fourier and Karhunen-Loeve decomposition for fast pattern matching with a large set of templates" to process a large set of template matching [17]. Because this study uses a lot of template matching data, the speed of the process is an essential parameter. Therefore, the combined method of Fourier transform and Karhunen-Loeve transform is used. This study uses a vector subspace as the desired pattern recognition medium. The combination of these methods succeeded in processing a large set of template matching with significantly increased processing speed.

In 2009, Guangjun Zhang conducted a study entitled "Novel template matching method with sub-pixel accuracy based on correlation and Fourier-Mellin transform" which aims to determine the reference location of the image being analyzed [18]. This study uses sub-pixel modification based on Fourier-Mellin transformation correlation to improve template matching accuracy. The experimental results show that the method is robust on noise (Gaussian) and produces high accuracy template matching. The flow chart for the Fourier-Mellin transform method is shown in Figure 2.

Using a graphic processing unit (GPU) to calculate Fourier transforms is faster than using a central processing unit (CPU). In 2011, Yunhui Liu conducted a study entitled "GPU Accelerated Fourier Cross-Correlation Computation and Its Application in Template Matching" to calculate GPU speed in calculating Fourier transform [19]. The Fourier transform method is modified into Fourier cross-correlation, which can speed up the computation time. This study uses the NVIDIA GeForce 9400 graphics card as the GPU. Experimental results show that the processing time is ten times faster than using the CPU.

III. METHOD

This study proposes a template matching method with adapted rotation on the Fourier transform. The weakness of the Fourier transform method is the problem of rotation of the template image. Even though the image being searched

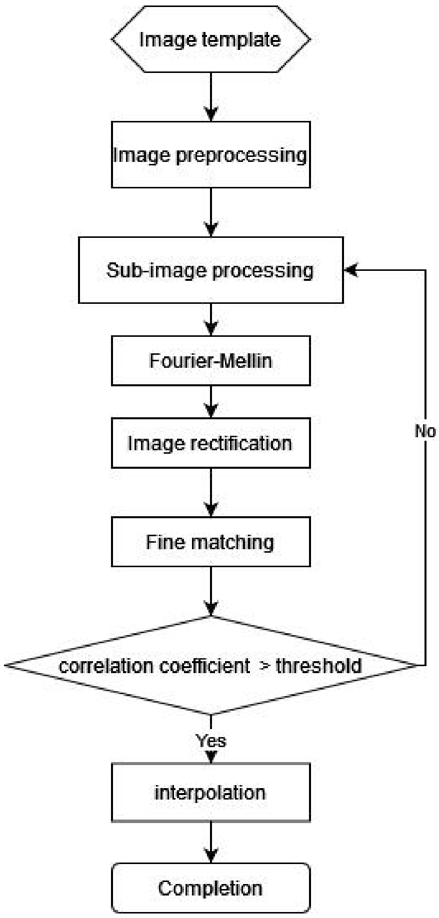

Fig. 2. Fourier-Mellin transform template matching

for is the same, the difference in the axis often results in wrong identification. The use of rotation can help to increase the accuracy of image identification but will prolong the identification process. Therefore, the rotation process must adapt the number of rounds so that the processing time does not increase drastically but still increases the accuracy of identifying and locating template images. There are several steps in identifying and determining the location of the image template: grayscaling process, the adapted rotation process, Fourier transform, matrix multiplication, inverse Fourier transform and the image threshold, as shown in Figure 3. Images and image templates in the form of RGB files are converted to grayscale to be processed faster using the Fourier transform. There are several formulas for converting RGB values to grayscale values [20]. Equation (2) is the formula to convert RGB to a grayscale intensity.

$$I = 0.299 \times R + 0.587 \times G + 0.114 \times B \qquad (2)$$

Where:
I = Grayscale intensity
R = Red intensity
G = Green intensity
B = Blue intensity

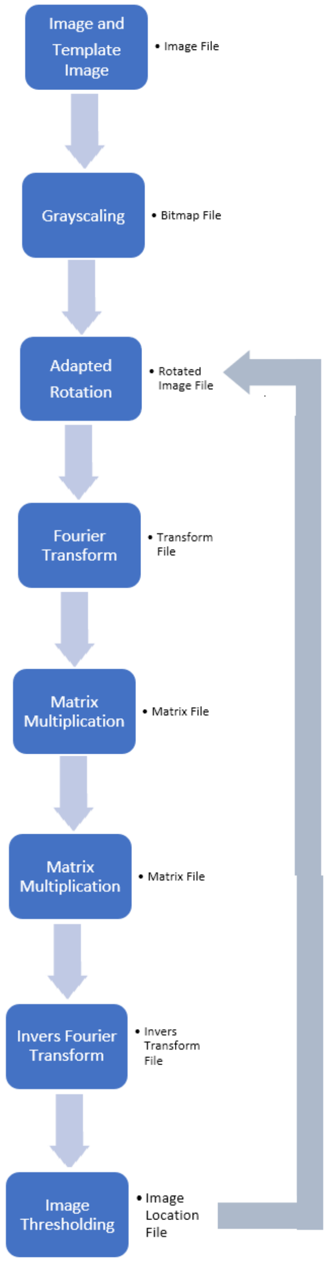

Fig. 3. Steps in identifying and determining the location of the image template

Image rotation is an image manipulation to be seen from various angles or perspectives. In template matching, image rotation is required because not all template images have the same orientation as the large image [21]. The value of θ (rotation angle) and the center of rotation are required in the rotation routine. If a point in the image with coordinates (x1,y1) is rotated with an angle θ and the center of rotation (x0,y0), then the rotation coordinates (x2,y2) will be generated with Equation (3) and Equation (4).

$$x2 = (x1 - x0)cos(\theta) + (y1 - y0)sin(\theta) \qquad (3)$$

$$y2 = -(x1 - x0)sin(\theta) + (y1 - y0)cos(\theta) \qquad (4)$$

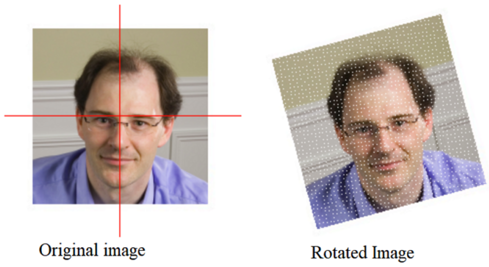

Fig. 4. Rotating image

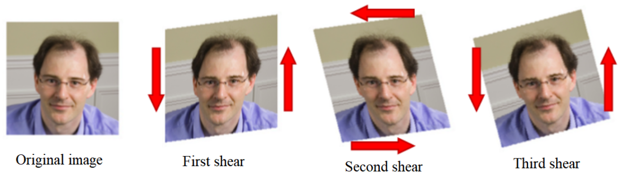

Fig. 5. Rotating image

The center point of rotation for template matching is (0,0) so the rotation formula becomes Equation (5) and Equation (6). Equation (7) is the matrix form of Equations (5) and Equation (6).

$$x2 = (x1)cos(\theta) + (y1)sin(\theta) \tag{5}$$

$$y2 = -(x1)sin(\theta) + (y1)cos(\theta) \tag{6}$$

$$\begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{7}$$

Rotation using these formulas or matrices will result in an image that is rotated by a θ angle. The weakness of using this formula is that the rotated image has aliasing, causing the rotated image has a lot of noise or white spots, as shown in Figure 4.

Several methods can overcome the aliasing problem. The first method is to over-sample the original image. This method divides every single pixel in the original image into several pixels then calculates the rotation process for each pixel. The second method is to do area mapping. This method uses the inverse problem, each pixel in the rotated image is searched for 4 pixels from the original image. The colour in the rotated image is taken from the 4 pixels colour average of the original image colour. The third method to solve this problem is the three shear rotation method, by expanding the rotation matrix into three different matrices. The three matrices have similarities in the first and last matrices and each matrix has a determinant of 1.0. The three shear method's rotation process will produce a rotated image with good image quality, as shown in Figure 5. Equation 8 is the three shear rotation formula.

$$\begin{bmatrix} x1 \\ y1 \end{bmatrix} = \begin{bmatrix} 1 & -tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sin\theta & 1 \end{bmatrix} \begin{bmatrix} 1 & -tan(\theta/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \tag{8}$$

The Fourier transform is used to break the image into components of the sine function and its cosine function. The Fourier transform transforms the input image in the form of a spatial domain into an image representation in the frequency domain. The digital Fourier transform output does not cover all the image frequencies in the spatial domain but a set of frequencies that represent the entire contents of the input image. The number of frequencies is called the sampling frequency which is related to the size or number of pixels in the image. Fourier transform on an image using 2D discrete Fourier transform with Equation 9.

$$F(x,y) = \frac{1}{WH} \sum_{w=0}^{W-1} \sum_{h=0}^{H-1} f(w,h) e^{-2j\pi(\frac{x}{W}w + \frac{y}{H}h)} \tag{9}$$

The computational process in the 2D discrete Fourier transform has a weakness in the long computation time when an image gets bigger because it needs to be accelerated using the fast Fourier transform algorithm. The Cooley-Tukey FFT algorithm is the most widely used in accelerating the computation time of the Fourier transform [22]. The Cooley-Tukey algorithm recursively breaks the problem into sub-problems of the same type. The solutions to those sub-problems will be recombined to provide a solution to the original problem [23]. The Fourier inversion theorem is a function that allows recovering a function from the Fourier transform [24]. The inverse Fourier transform converts the signal from the frequency domain to the time domain. In this study, the inverse Fourier transform will reconstruct a re-image from combining two images processed by matrix multiplication in the frequency domain. Equation 10 is the inverse Fourier transform formula for a 2D matrix.

$$f(x,y) = \sum_{w=0}^{W-1} \sum_{h=0}^{H-1} F(w,h) e^{-2j\pi(\frac{x}{W}w + \frac{y}{H}h)} \tag{10}$$

The inverse Fourier transform can be computed much faster using the inverse fast Fourier transform (IFFT). Inverse FFT is processed by reversing the order in the function and then calculated into the FFT function.

Image thresholding is a simple way to segment an image [25]. In this study, the image segmentation resulting from the inverse Fourier transformation is carried out into two parts. The image resulting from the inverse Fourier transform in the form of grayscale becomes a binary image (black and white). The white image indicates the location of the image being searched for or identified. Equation 11 is the formula for thresholding used in this study.

$$f(x,y) = \begin{cases} 255 & ,iff(x,y) > 100 \\ 0 & ,ifelse \end{cases} \tag{11}$$

The template matching algorithm with improved rotation is adjusted to the template matching implementation steps, as discussed in Figure 3. Algorithm 1 is the algorithm for template matching using improved rotations Fourier transform method.

Algorithm 1: Template matching algorithm

```
Input image property;
Input image file;
Input template property;
Input template file;
Input increment value;
angle ← 0;
max ← 0;
rotate ← true;
Execute grayscaling;
while rotate do
    Execute three shear rotation;
    Execute image Fourier transform;
    Execute template Fourier transform;
    Execute matrix multipication;
    Execute inverse fast Fourier transform;
    temp ← 0;
    for i=0 to Result.W do
        for j=0 to Result.H do
            if temp ≤ Result.f(i,j) then
                temp ← Result.f(i,j)
            end
        end
    end
    if temp ≥ max then
        max ← temp
    else
        rotate ←false
    end
end
Execute thresholding;
for x=0 to Result.W do
    for y=0 to Result.H do
        if Result(x,y)=255 then
            Location.x ← x;
            Location.y ← y;
        end
    end
end
```

Algorithm 2: Graysaling Algorithm

```
for x=0 to image.W do
    for y=0 to image.H do
        image.F(x,y) ← 0.299×image.R(x,y) +
          0.587×image.G(x,y) + 0.114×image.B(x,y);
    end
end
for x=0 to template.W do
    for y=0 to template.H do
        template.F(x,y) ← 0.299×template.R(x,y) +
          0.587×template.G(x,y) +
          0.114×template.B(x,y);
    end
end
```

Algorithm 3: Three shear rotation algorithm

```
angle ← angle + increment;
a ← -tan(angle/2);
b ← sin(angle);
for x=0 to template.W do
    for y=0 to template.H do
        x1 ← x + (abx) + (2ay) + (a^2 by);
        y1 ← (bx) + (aby) + 1;
        template.F(x1,y1) ← Oldtemplate.F(x,y);
    end
end
```

Algorithm 4: image Fourier transform algorithm

```
Divide the coefficient vector of the polynomial into
  two vectors;
for for image.x=0 to image.W do
    for for image.y=0 to image.H do
        for image.w=0 to image.W-1 do
            for image.h=0 to image.H-1 do
                Calculate image.F(x,y);
            end
        end
    end
end
```

Algorithm 5: Template Fourier Transform algorithm

```
Divide the coefficient vector of the polynomial into
  two vectors;
for template.x=0 to template.W do
    for template.y=0 to template.H do
        for template.w=0 to template.W-1 do
            for template.h=0 to template.H-1 do
                Calculate template.F(x,y);
            end
        end
    end
end
```

Algorithm 6: Matrix multiplication algorithm

```
for i=1 to W do
    for j=1 to H do
        sum ← 0;
        for k=1 to m do
            sum ← image.F(i,k) × template(k,j) + sum;
        end
        Result(i,j) = sum;
    end
end
```

Algorithm 7: Inverse fast Fourier transform algorithm

```
Divide the coefficient vector of the polynomial into
  two vectors;
for template.x=0 to template.W do
    for Result.y=0 to Result.H do
        for Result.w=0 to Result.W-1 do
            for Result.h=0 to Result.H-1 do
                Calculate Result.f(x,y);
            end
        end
    end
end
```

Algorithm 8: Thresholding algorithm

```
temp ← 0;
for i=1 to Result.W do
    for j=1 to Result.H do
        if Result(i,j) ≥ threshold.value then
            Result(i,j) ← 255
        else
            Result(i,j) ← 0
        end
    end
end
```

IV. Results

Experiments in this study were piloted using several images. Each template image will be rotated to measure the accuracy and time required for the template matching process. Figure 6 is an example of an image and a template identified using improved rotation on the Fourier transform.

The rotation of the template is given in different angle (θ) increments, and each experiment is carried out ten times. This experiment aims to determine the relationship between accuracy and time required for the template matching process. Figure 7 shows an example of the template matching process, and a template image is about to look for similarities to the full image. The template image is not rotated and is processed immediately along with the full image. The results of the Fourier transform of the two images will be multiplied, and the inverse Fourier transform will be performed. The results of the inverse Fourier transform (before the thresholding process) will produce a black and white image with a slightly gray circle or oval where the template image is located. The gray image does not mean that the template image has been found. It is necessary to rotate the template image to strengthen the certainty that the template image has been found.

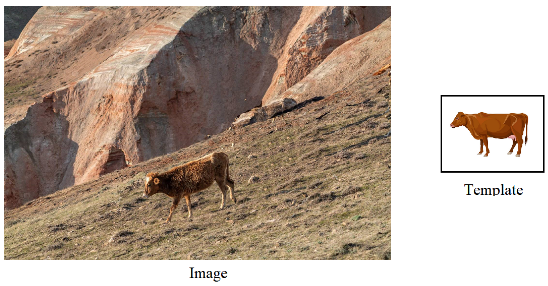

Fig. 6. An image and a template example

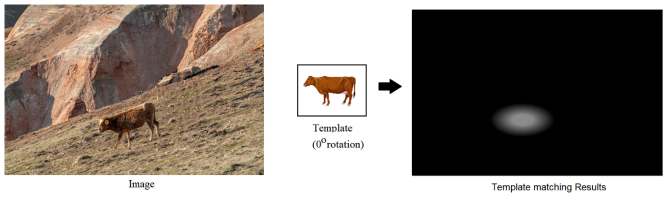

Fig. 7. Template matching result with 0°angle (before thresholding)

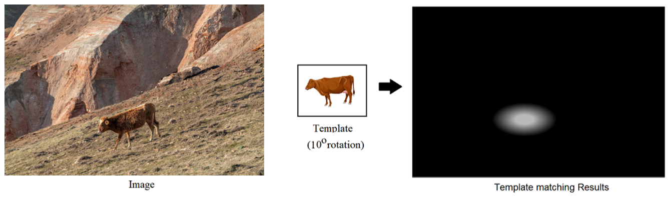

Fig. 8. Template matching result with 10°angle (before thresholding)

By using the three shear rotation process, the template image will be rotated by 10°angle. Figure 8 shows the results of the template matching process with a rotation of 10°angle. Using a rotation of 10°angle on the template image will result in a circle or oval that is a lighter gray color.

According to the algorithm proposed by this study, the rotation of the template image will continue in increments of 10 degrees. Figure 9 shows the results of the template matching process with a rotation of 20°angle which produces a circle or oval that is almost white.

The template matching results in the next rotation (30°angle) will produce a darker circle or oval, as shown in Figure 10 . The template matching algorithm will stop executing because the results obtained are whiter in the previous rotation.

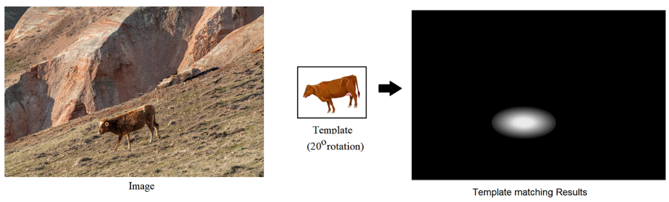

Fig. 9. Template matching result with 20°angle (before thresholding)

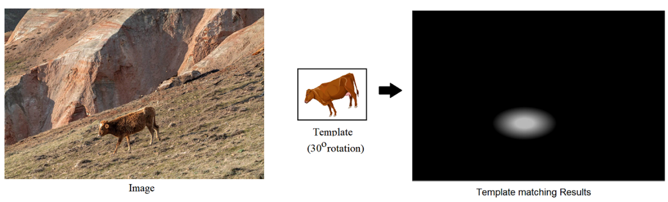

Fig. 10. Template matching result with 30°angle (before thresholding)

TABLE II
DIFFERENT ANGLE INCREMENTS EXPERIMENTS

Angle (θ) increments	Accuracy (%)	Time required (ms)
5	100	2100
10	100	1305
15	98	815
20	95	597
25	92	421
30	88	355
35	84	301
40	80	265
45	75	229

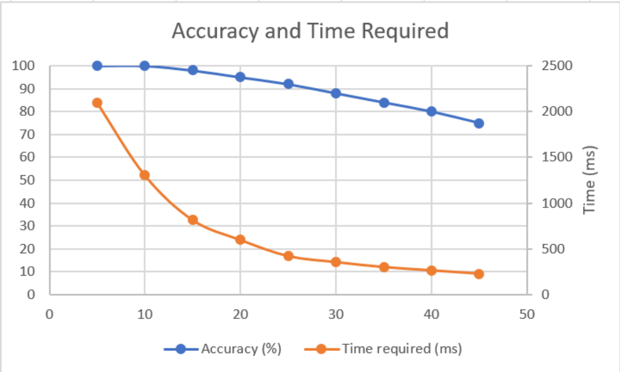

Fig. 11. Accuracy and Time Required

Table II and Figure 11 show that if the rotation angle increment is smaller, a higher level of accuracy will be produced, but the time required for the template matching process is longer. Processing time was measured using an Intel Pentium 4417U 2.30GHz processor and 4GB RAM.

The next experiment is to measure the accuracy of the template matching process using three shear rotations on the Fourier transform. Table III and Figure 12 show that using three shear rotations in the Fourier transform increases the accuracy of the template matching process. Image rotation without using the three shear method will produce a lot of noise in the image. Moreover, the larger the rotation required, the larger the noise size in the rotated image. Using three shear rotations will increase the average accuracy by 13% compared to without using three shear rotations.

TABLE III
DIFFERENT ANGLE INCREMENTS EXPERIMENTS

Angle (θ) increments	Accuracy with three shear rotation (%)	Accuracy without three shear rotation (%)
5	100	97
10	100	95
15	98	92
20	95	88
25	92	83
30	88	78
35	84	71
40	80	64
45	75	58

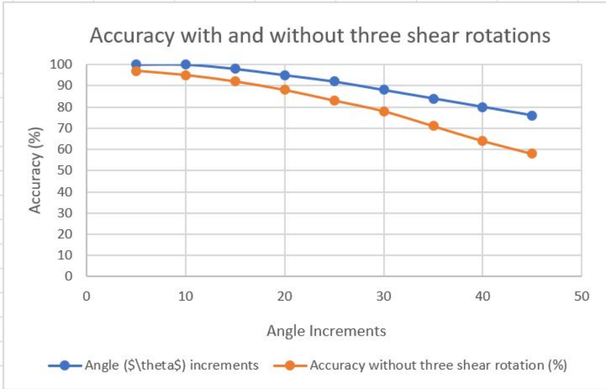

Fig. 12. Accuracy with and without three shear rotations

V. CONCLUSIONS

Template matching is a process whose accuracy and processing time are influenced by several parameters. This study uses the method of improved rotation Fourier transform. Rotation can affect the level of accuracy and time required in template matching. Based on data from experimental results, the use of rotation with an angle increment of 10 degrees will increase accuracy and process time significantly faster with an angle increment of 5 degrees. Using three shear rotations in the improved rotation proposed in this study significantly increases template matching accuracy by 13% without longer processing time.

REFERENCES

[1] R. Susik, S. Grabowski, K. Fredriksson, "Revisiting Multiple Pattern Matching", Computing and Informatics, 2019, vol. 38, no. 4, pp. 937–962, https://doi.org/10.31577/cai20194937.

[2] H. Schweitzer, J. W. Bell, F. Wu, "Very fast template matching", Lecture Notes Computer Science, vol. 2353, no. 009741, 2002, pp. 358–372, 2002, https://doi.org/10.1007/3-540-47979-1_24.

[3] M. Che, M. Che, Z. Chao, X. Cao, "Traffic Light Recognition for Real Scenes Based on Image Processing and Deep Learning", Computing and Informatics, 2020, vol. 39, no. 3, pp. 439–463, 2020, https://doi.org/10.31577/cai_2020_3_439.

[4] N. Roshanbin, J. Miller, "A comparative study of the performance of local feature-based pattern recognition algorithms", Pattern Analysis and Applications, 2017, vol. 20, no. 4, pp. 1145–1156, https://doi.org/10.1007/s10044-016-0554-y.

[5] W. Q. Huang, M. Z. Huang, Y. M. Wang, "Detecting objects using Rolling Convolution and Recurrent Neural Network," International Journal Electronics and Telecommunication, vol. 65, no. 2, pp. 293–301, 2019, https://doi.org/10.24425/ijet.2019.126313.

[6] K. S. Yadav, K. A. Monsley, R. H. Laskar, S. Misra, M. K. Bhuyan, T. Khan, "A selective region-based detection and tracking approach towards the recognition of dynamic bare hand gesture using deep neural network", Multimedia Systems, 2022, in press, https://doi.org/10.1007/s00530-022-00890-1.

[7] C. Vázquez, M. Ghazal, A. Amer, "Feature-based detection and correction of occlusions and split of video objects", Signal, Image and Video Processing, 2009, vol. 3, pp. 13–25, https://doi.org/10.1007/s11760-008-0055-6.

[8] P. H. Chien, G. C. Lee, "A template-based method for identifying input regions in survey forms", Pattern Recognition and Image Analysis volume, 2011, vol. 21, pp. 469–472, https://doi.org/10.1134/S1054661811020210.

[9] G. Du, M. Zhou, C. Yin, Z. Wu, W. Shui, "Classifying fragments of terracotta warriors using template-based partial matching", Multimedia Tools and Applications, 2018, vol. 77, no. 15, pp. 19171–19191, https://doi.org/10.1007/s11042-017-5396-0.

[10] H. Weber, "Numerical computation of the fourier transform using Laguerre functions and the Fast Fourier Transform", Numerische Mathematik, 1980, vol. 36, pp. 197–209, https://doi.org/10.1007/BF01396758.

[11] E. Liflyand, "Fourier transform versus Hilbert transform", Journal of Mathematical Sciences, 2012, vol. 187, pp. 49–56, https://doi.org/10.1007/s10958-012-1048-0.

[12] T. Ai, X. Cheng, P. Liu, M. Yang, "A shape analysis and template matching of building features by the Fourier transform method", Computers, Environment and Urban Systems, 2013, vol. 41, pp. 219–233, https://doi.org/10.1016/j.compenvurbsys.2013.07.002.

[13] D. M. Tsai, C. K. Huang, "Defect Detection in Electronic Surfaces Using Template-Based Fourier Image Reconstruction", IEEE Transactions on Components, Packaging and Manufacturing Technology, 2019, vol. 9, no. 1, pp. 163–172, https://doi.org/10.1109/TCPMT.2018.2873744.

[14] A. Mukherjeea, A. Khanb, "A Fourier series based Template Matching Approach to Detect the Splitting of Second Heart Sound", IOSR Journal of VLSI and Signal Processing, 2014, vol. 4, no. 4, pp. 9–13, https://doi.org/10.9790/4200-04430913.

[15] P. Elangovan, M. K. Nath, "A Review: Person Identification using Retinal Fundus Images," International Journal Electronics and Telecommunication, vol. 65, no. 4, pp. 585–596, 2019, https://doi.org/10.24425/ijet.2019.129817.

[16] M. Sabaghi, S. R. Hadianamrei, M. Fattahi, M. R. Kouchaki, A. Zahedi, "Retinal Identification System Based on the Combination of Fourier and Wavelet Transform", Journal of Signal and Information Processing, 2012, vol. 3, no. 1, pp. 35–38, https://doi.org/10.4236/jsip.2012.31005.

[17] M. Uenohara, T. Kanade, "Use of Fourier and Karhunen-Loeve decomposition for fast pattern matching with a large set of templates", IEEE Transactions on Pattern Analysis and Machine Intelligence, 1997, vol. 19, no. 8, pp. 891–898, https://doi.org/10.1109/34.608291.

[18] G. Zhang, M. Lei, X. Liu, "Novel template matching method with sub-pixel accuracy based on correlation and Fourier-Mellin transform", Optical Engineering, 2009, vol. 48, no. 4, p. 055001, https://doi.org/10.1117/1.3125425.

[19] Y. Liu, Q. Zou, S. Luo, "GPU Accelerated Fourier Cross Correlation Computation and Its Application in Template Matching", Communications in Computer and Information Science (ICHCC 2011), 2011, p. 163, https://doi.org/10.1007/978-3-642-25002-6_68.

[20] A. M. Grigoryan, A. John, S. S. Agaian, "A Novel Color Image Enhancement Method by the Transfor- mation of Color Images to 2-D Grayscale Images", International Journal of Signal Processing and Analysis, 2017, vol. 2, no. 2, pp. 1–18, https://doi.org/10.35840/2631-5114/3502.

[21] T. Ahonen, J. Matas, C. He, M. Pietikäinen, "Rotation Invariant Image Description with Local Binary Pattern Histogram Fourier Features", in Scandinavian Conference on Image Analysis, 2009, pp. 61–70, https://doi.org/10.1007/978-3-642-02230-2_7.

[22] M. V. Noskov, V. S. Tutatchikov, "Modification of a two-dimensional fast Fourier transform algorithm with an analog of the Cooley–Tukey algorithm for image processing", Pattern Recognition and Image Analysis, 2017, vol. 27, no. 1, pp. 110–113, https://doi.org/10.1134/S1054661817010096.

[23] Y. Mei, M. N. Omidvar, X. Li, X. Yao, "A Competitive Divide-and-Conquer Algorithm for Unconstrained Large-Scale Black-Box Optimization", ACM Transactions on Mathematical Software, 2016, vol. 42, no. 2, pp. 1–24, https://doi.org/10.1145/2791291.

[24] V. Sukhoy, A. Stoytchev, "Generalizing the inverse FFT off the unit circle", 2019, Scientific reports, vol. 4, no. 14443, pp. 1–12, https://doi.org/10.1038/s41598-019-50234-9

[25] T. Y. Goh, S. N. Basah, H. Yazid, M. J. A. Safar, F. S. A. Saad, "Performance analysis of image thresholding: Otsu technique", Measurement: Journal of the International Measurement Confederation, 2018, vol. 114, no. 1, pp. 298–307, https://doi.org/10.1016/j.measurement.2017.09.052