Jarosław KACZOR*

THE INFLUENCE OF THE SHAPE OF RACES FOR THE BALL BEARINGS ON THE LIFE OF BEARINGS

WPŁYW KSZTAŁTU BIEŻNI ŁOŻYSKA KULKOWEGO NA TRWAŁOŚĆ ŁOŻYSK

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Summary

The bearings of the three bearing shaft are usually regarded as ideally stiff articulated supports. In technical literature, the shaft elasticity is the only factor taken into account when determining the reaction and bending moments of three bearing shafts. The elasticity of the bearings is not taken into consideration. In fact, the bearings experience deformation, which causes additional shaft deflection and has an impact on contact stresses in the bearings.

* Institute of Environmental Engineering and Building Installations, Technical University of Lodz, Lodz, Poland, al. Politechniki 6, 90-924 Lodz, Poland, Tel: 48 42 6313514, e-mail: jaroslaw.kaczor@p.lodz.pl

These deformations depend on, among others, the osculation. The objective of this paper is to determine the effect of the osculation on the contact stress of the three bearing shaft, as well as on its durability.

INTRODUCTION

The durability of the plain ball bearings depends on the design, and technological and wearing factors. Among the design factors (features), the osculation is one of the most crucial ones, defined as follows:

$$k = \frac{r_b}{r_k} \tag{1}$$

where: (see **Figure 1**)
r_b – the radius of the race in axial plane,
r_k – the radius of the ball.

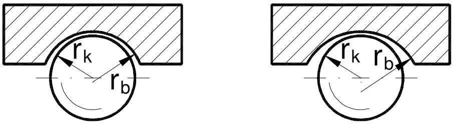

Fig. 1. Two cross sections of ball bearings with different osculation
Rys. 1. Dwa przekroje łożysk kulkowych o różnych współczynnikach przylegania

This coefficient determines the race profile in relation to the ball profile (the bigger it is, the smaller the race concavity); therefore, it can be regarded as the coefficient of the shape of the race.

Catalogue calculation of the fatigue durability of a roller bearing **[L. 1]** is reduced to taking into account the dynamic bearing capacity, its actual equivalent load, and the average rotating speed. Calculations that are more precise also involve the expected probability of achieving the calculated durability, the material of the bearing, and the friction conditions; whereas, the distribution of external load on different bearing elements is omitted. The distribution depends on the following:

– The displacement within the bearing (the displacement in three directions and angles in two planes),
– The clearance in the bearing, and
– The osculation, which is the ratio of the race radius r_b against the ball radius r_k (see **Figure 1**).

In commonly used bearings, the manufacturers apply established values of the osculation. However, recently, special bearings have been developed, which are tailored to specific applications. This trend is justified in pursuit of better developments, more suited to their functions, and at the same time, more economical. Within this group of bearings, osculation plays a significant role as a parameter in the adjustment of the bearing. In the field of miniature bearings, manufacturers even offer bearings with the osculation tailored to the desired consumer specifications.

The previous bearing durability calculations are based on the Lundberg and Palmgren model **[L. 1]**. These calculations have been introduced based on the assumption of the radial load of the bearing and the so-called ‘normal’ clearance in the bearing. Applying this model does not give the possibility of taking the current clearance into account, which has an obvious impact on the distribution of the load in the bearing. The load distribution is also affected by the osculation, because the increase of the osculation (the increase of the race radius) in the bearing under the axial load has the same impact as the increase of clearance.

The impact of the osculation on the durability of the single bearing carrying the radial load is known. The authors have set themselves a task to examine the above impact in the case of three bearing shaft, in which there are mutual interactions between the bearings, and the specific complications are caused by the presence of the axial forces acting on the shaft.

Quantifying the impact of the osculation on the bearings’ durability is not easy. The solution involves maximum orthogonal shear stresses τ_0, which are difficult to calculate because of the impossibility to directly determine certain components having an impact on the stresses τ_0 (e.g. the depth of occurrence).

SUBSURFACE STRESSES

In a static contact, the stresses occur under the contact area of a rolling element with the race.

The stress values are set at the maximum normal stress in the direction of the main axis of the contact ellipse and the normal axis against the surface, and constitute the main stresses, σ_x, σ_y, σ_z (**Fig. 2a**).

The maximum value is reached by shear stresses τ_{yz} located at some depth along the z-axis, defined by the following formula:

$$\tau_{yz} = \frac{1}{2} \cdot \left(\sigma_z - \sigma_y\right) \tag{2}$$

where:

σ_z – the main stress in the direction of the z-axis,
σ_y – the main stress in the direction of the y-axis.

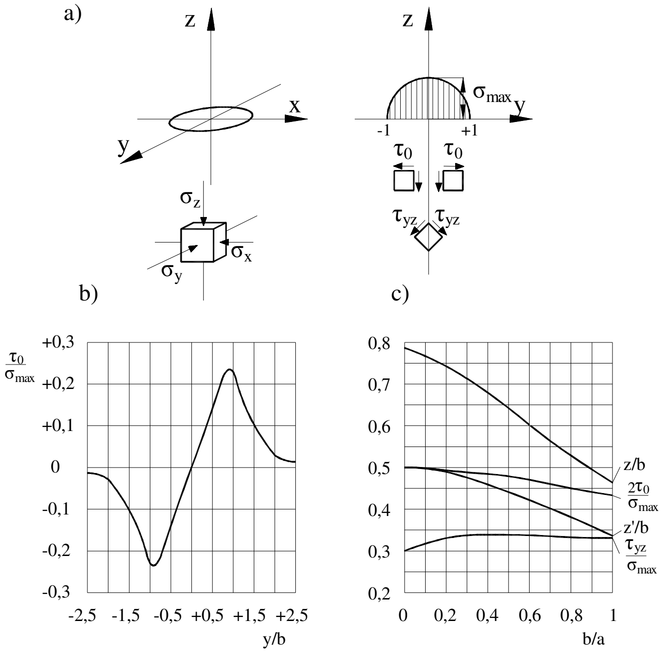

Fig. 2. The subsurface stresses [L. 2]
Rys. 2. Naprężenia podpowierzchniowe **[L. 2]**

Figure 2c shows the ratio of the maximum orthogonal shear stresses τ_0 against the maximum normal stresses τ_{yz}/σ_{max} and their relative depth of occurrence z/b depending on the ratio of the semi-axis of the contact of the ellipse b/a (**Figure 3**). For a circular contact area, the depth of the location point of the maximum orthogonal shear stresses is 0.467 for point contact and for 0.786 for linear contact.

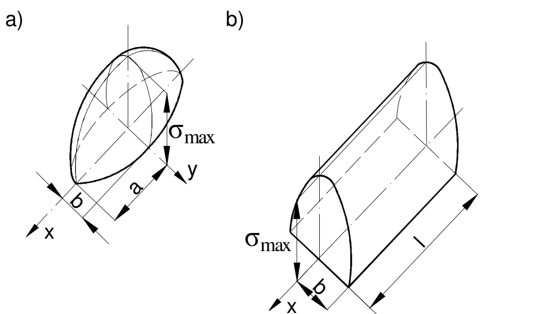

Fig. 3. The model of contact stresses distribution: a) point, b) linear [L. 2]
Rys. 3. Model rozkładu naprężeń kontaktowych: a) punktowy, b) liniowy **[L. 2]**

When a rolling element rolls up the race, the orthogonal shear stresses occur in the planes parallel to the surface, whose turns in the front and rear end of the contact area are opposite.

The maximum orthogonal stresses τ_0 show the course of such variability, as it is illustrated for the linear contact point in **Figure 2b**. As a result, the evidence of stresses in the material change after a moving point of contact passes the discussed race point. The amplitude of changes in these stresses is $2\tau_0$, and their relative value in relation to the maximum normal stresses is σ_{max} o, and the depth of occurrence z'/b may be determined on the basis of the ratio and the auxiliary parameter t according to the following equations **[L. 2**, **3]**:

– The amplitude of the relative value of orthogonal shear stresses:

$$\frac{\tau_0}{\sigma_{max}} = \frac{(2 \cdot t - 1)^{1/2}}{2 \cdot t \cdot (t + 1)} \tag{3}$$

– The depth of occurrence:

$$\frac{z'}{b} = \frac{1}{(t + 1) \cdot (2 \cdot t - 1)^{1/2}} \tag{4}$$

– An auxiliary parameter t (depending on the ratio b/a):

$$\frac{b}{a} = \left[\left(t^2 - 1\right) \cdot (2 \cdot t - 1)\right]^{1/2} \tag{5}$$

The relative stress value $2\tau_0/\sigma_{max}$ and the relative depth of occurrence z'/b, depending on the ratio b/a, is also depicted in **Fig. 2c**.

These stresses are considered the main cause of fatigue cracks occurring beneath the surface subjected to continuous rolling. The values of these stresses depend, among others, on the size of the contact area between the rolling element and the race, and on the maximum normal stresses σ_{max}, the size of which is affected by the osculation k.

Therefore, it has been assumed that the maximum orthogonal stresses τ_0 change in the same way as the maximum normal stresses σ_{max} (the contact pressure p_{max}). The bearing durability changes depending on the orthogonal normal stresses according to the formula below [4]:

$$L_{10} \sim \frac{1}{\tau_0^{c/e}} = \frac{1}{\tau_0^{9,3}} \tag{6}$$

This allows specifying the approximate change of durability depending on the osculation.

THE PHYSICAL MODEL

The machine shaft of the model, made of steel, is supported on three roller bearings. It can have a variable diameter. It is elastic, and the dependence of the load and deflection results from a well-known equation of the deflection line of the shaft. The shaft bearings are typical ball bearings (single row). The bearings are elastic, i.e., they show radial, axial, and flexural elasticity. The axial shaft load is taken over by the bearing marked as 'the left one' in the adopted scheme. Thus, this is the retainer bearing. The simplifying assumptions are as follows:

– The problem is considered statically, i.e., the forces and the changes in them caused by the rotation of the shaft and the inside element of the bearings are omitted.
– The load of the shaft occurs in one axial plane and is represented by component forces in the radial direction x and y-axis.
– There are no errors in the shape of the balls or the bearing rings.
– The elastic deformations of the bearings occur only in contact points with the bearing rings, and free surfaces of the rings retain a cylindrical shape.
– The location of the bearings is geometrically accurate, and the external bearing rings axes lie on one straight line.
– The clearances associated with the fitting of the bearings have been omitted, and the values of the working clearances depend only on the values of the internal bearing clearances.

THE MATHEMATICAL APPROACH

A structure consisting of a shaft and three bearings is a statically indeterminate structure. The value of the bearings load is affected by the stiffness of the shaft and the bearings and the external loads. One should take into account the fact that the supporting points of the conventional shaft line of contact (determined by the inner bearing rings) move because of:

a) clearances in the bearings, and
b) the elastic displacements in the bearings.

The way to solve the statically indeterminate structure, which considers the above factors, and the way to determine the maximum normal stresses σ_{max} (p_{max}) in the bearings are presented precisely in the works **[L. 5]**.

The contact pressure in the contact point with the ball against the race is defined by the Formula (7), according to **[L. 2]**:

$$p_{max} = \frac{858}{a^* b^*}\left[Q_{max} \cdot (\Sigma\rho)^2\right]^{\frac{1}{3}} \tag{7}$$

where

Q_{max} – the load of the most loaded ball,
$\Sigma\rho$ – the sum of curvatures,
a*, b* – the coefficients of the semi-axis ellipse of contact.

The coefficients of the contact ellipse semi-axis, similarly to the sum of the curvatures of the bodies in contact (the ball and the ring), depend on the ball radius and the race cross section in two planes, which is described among others in **[L. 2]**.

The value of the sum of the curvatures is illustrated in the Formulas (8) and (9):

a) For the race of the inner ring:

$$\Sigma\rho_i = \frac{1}{D_t}\left(4 - \frac{2}{k_i} + \frac{2\kappa}{1-\kappa}\right) \tag{8}$$

b) For the race of the external ring:

$$\Sigma\rho_o = \frac{1}{D_t}\left(4 - \frac{2}{k_o} - \frac{2\kappa}{1+\kappa}\right) \tag{9}$$

In these formulas, respectively:

$$\kappa = \frac{2 \cdot r_k}{d_m} \tag{10}$$

$$k_i = k_o = k \tag{11}$$

where

D_t – the diameter of the rolling element (of the ball),
d_m – the pitch diameter of the bearing,
k_i – the osculation for the ball contact point with the inner ring race,
k_o – the osculation for the ball contact point with the external ring race.

The ellipse parameters a* and b* can be read from the graphs given in **[L. 2]**. In this way, the variable osculation is included in the calculation of stresses. To make the example of determining of normal stress σ_{max} and the approximate durability of the three-bearing system, the arrangement was used as shown in **Figure 4**.

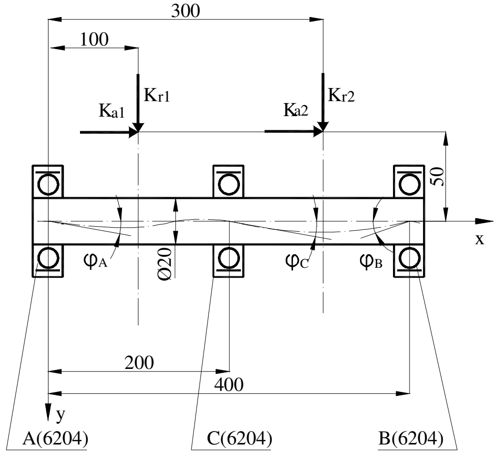

Fig. 4. The example of the bearing adopted for calculation
Rys. 4. Przykład łożyskowania przyjętego do obliczeń

The bearings from the clearance group "N" with have the radial clearance equal to Δr = 0.014 mm applied for calculations. The shaft load consists of forces K_{r1} = 2000 N, K_{r2} = 3000 N and K_{a1} = 0, 500, 1500, 2000 N, K_{a2} = 0, 500, 1500, 2000 N.

Figure 5 shows the effect of the osculation to the maximum normal stress σ_{max}, and **Figure 6** illustrates the impact of this factor on the bearing durability. Additionally, the influence of this coefficient on the bearings reaction moments is shown.

A clear dependence of the stresses σ_{max} on the osculation may be noted. This is due to the changes in the size of contact of the race and the balls. In addition, the increase of the coefficient causes the increase of the bearing reaction moment, because of which, there are additional loads on the balls, and the contact stresses are on the rise.

Based on changes in the maximum normal stresses, one can determine the approximate durability of the bearings. For example, for the bearings with the osculation of k = 1.03 (used by a local manufacturer of the bearings) with a load of K_{r1} = 2000 N, K_{r2} = 3000 N and K_{a1} = 500 N, K_{a2} = 500 N, the durability of the bearings are the following, respectively:

– For Bearing A, the cataloguing durability is $L_{10} = 2.71 \cdot 10^2$ million rotations. Based on the authors' calculations, $L_{10} = 5.27 \cdot 10^2$ million rotations.
– For Bearing B, the cataloguing durability of the bearing is $L_{10} = 6.46 \cdot 10^2$ million rotations. Based on authors calculations, $L_{10} = 7.79 \cdot 10^2$ million rotations.

- For Bearing C, The cataloguing durability of the bearing is $L_{10} = 2.46 \cdot 10^1$ million rotations; and, on the basis of authors' calculations, $L_{10} = 2,69 \cdot 10^1$ million rotations.

The durability has been determined for the maximum stresses given as follows, respectively:

- For Bearing A: $\sigma_{max} = p_{max} \approx 2100$ MPa.
- For Bearing B: $\sigma_{max} = p_{max} \approx 2100$ MPa.
- For Bearing C: $\sigma_{max} = p_{max} \approx 2900$ MPa.

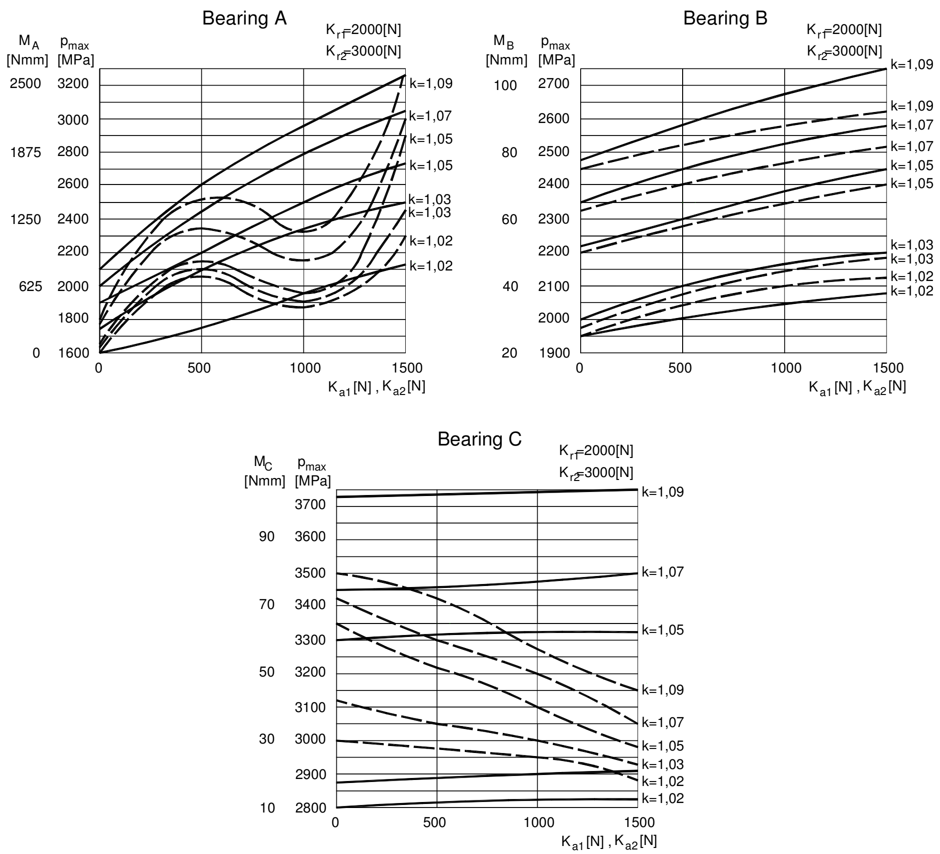

Fig. 5. The values for the reaction bearing moments (dashed lines) and maximum bearing contact stresses (solid lines)

Rys. 5. Wartości momentów reakcyjnych (linie kreskowe) i maksymalnych naprężeń kontaktowych w łożyskach (linie ciągłe)

The characteristics illustrating the results of calculations (**Fig. 5**) show that the equivalent bearing loads are slightly dependent on the osculation, while the stresses are still clearly dependent on the given subject. This is due to changes in the size of the contact of the race and the balls. Moreover, the increase of this ratio results in the increase of the bearing reaction moment. Consequently, there are additional loads on the balls; therefore, the contact stresses increase more. Thus, it would seem beneficial to reduce the osculation. However, it is common knowledge that, in the choice process of the coefficient, both the impact on stresses and the resistance moment of the bearing are taken into account. Reducing this factor by increasing the contact point of the ball and race results in the decrease of the contact stresses but, on the other hand, causes an increase of the resistance due to higher macro-sliding activity in the bearings. In conclusion, it must be emphasized that the exact durability calculation should not neglect the importance of the osculation. This is also valid in three-bearing shafts in which the deflection of the shaft is reduced, but the bearing reaction moments play an important role.

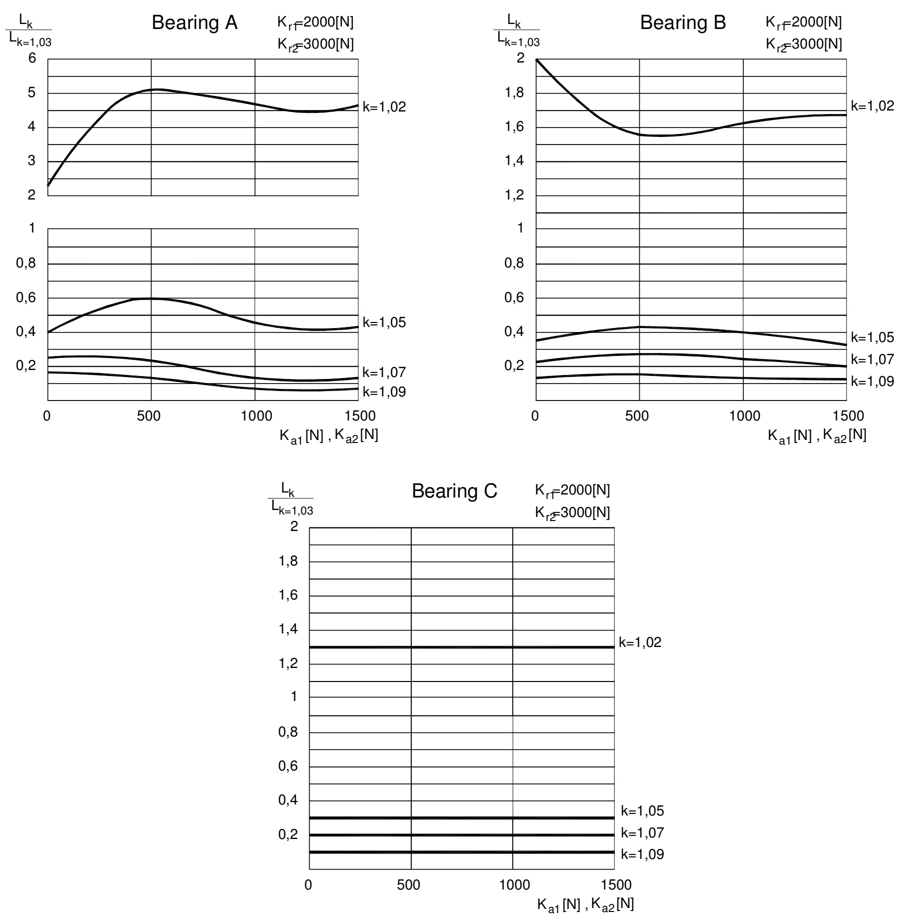

Fig. 6. The influence of osculation on the bearing durability
Rys. 6. Wpływ współczynnika przylegania na trwałość łożyskowania

It should be noted that the presented results of calculations are appropriate only for a specific case of the structure and load, and they serve to illustrate the possibilities of using the calculation program held by the authors.

For the osculation of k = 1.07, the obtained σ_{max} maximum normal stresses is greater than the stress for k = 1.03, as follows:

– For Bearing A, by about 16.6%,
– For Bearing B, by about 16.6%, and
– For Bearing C, by about 19.7%.

The result is that the durability ratio is (as noted in 6) as follows:

– For Bearing A: $\frac{L_{k=1.07}}{L_{k=1.03}} = 0.238$,

– For Bearing B: $\frac{L_{k=1.07}}{L_{k=1.03}} = 0.238$,

– For Bearing C: $\frac{L_{k=1.07}}{L_{k=1.03}} = 0.182$.

For the osculation of k = 1.09, the increase in the maximum normal stresses in relation to the stress for k = 1.03 is as follows:

– For Bearing A, about 23.8%,
– For Bearing B, approximately 22.8%,
– For Bearing C, approximately 29.4% C,

which means that the durability ratio is

– For Bearing A: $\frac{L_{k=1.09}}{L_{k=1.03}} = 0.134$,

– For Bearing B: $\frac{L_{k=1.09}}{L_{k=1.03}} = 0.142$,

– For Bearing C: $\frac{L_{k=1.09}}{L_{k=1.03}} = 0.091$.

Reducing the osculation (k = 1.02), or reducing the radius of the bearing ring race cross section, reduces stress σ_{max} in relation to the stresses obtained for k=1.03 as follows:

– For Bearing A, about 16.19%,
– For Bearing B, approximately 4.76%, and
– For Bearing C of about 2.76%.

The decrease the maximal normal stresses σ_{max} causes the durability ratio to be

– For Bearing A: $\frac{L_{k=1.02}}{L_{k=1.03}} = 5.16$,

– For Bearing B: $\frac{L_{k=1.02}}{L_{k=1.03}} = 1.57$,

– For Bearing C: $\frac{L_{k=1.02}}{L_{k=1.03}} = 1.3$.

As shown by the calculations, by reducing the osculation, the increase of bearing durability can be achieved in the case of bearing mountings of a three-bearing shaft.

However, it is worth noting that this effect is dependent on the value of the axial force loading the shaft.

An especially large increase of durability can be achieved by this method at moderate axial forces (0.2–0.3 of a radial force).

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Streszczenie

Łożyska wału trzypodporowego traktuje się zazwyczaj jako podpory doskonale sztywne o charakterze przegubowym. Książkowe wyznaczanie reakcji i momentów gnących wałów trzypodporowych odbywa się przy uwzględnieniu tylko sprężystości wału. Nie uwzględnia się sprężystości podpór. W rzeczywistości w łożyskach również występują odkształcenia, które powodują dodatkowe ugięcie wału i wpływają na naprężenia kontaktowe łożyskach. Odkształcenia te są zależne m.in. od współczynnika przylegania. Celem tej pracy jest stwierdzenie, jak wpływa współczynnik przylegania na naprężenia kontaktowe łożysk wału trzypodporowego oraz na trwałość łożyskowania.

Trwałość łożysk kulkowych zwykłych zależy od czynników konstrukcyjnych, technologicznych i eksploatacyjnych. Wśród czynników (cech) konstrukcyjnych jednym z najistotniejszych jest współczynnik przylegania.