AERODYNAMIC MODEL IDENTIFICATION OF TURBOPROP AIRCRAFT BASED ON FLIGHT TEST DATA

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Abstract

The aerodynamic model of an aircraft can be considered as a mathematical representation of the forces and moments acting on the aircraft. These forces and moments are usually approximated by polynomials as function of aircraft's state variables. The coefficients in the polynomials are known as the aerodynamic parameters. These aerodynamic parameters are of importance in the evaluation of aircraft performance and stability-control characteristics of an aircraft. These parameters also can be used in the design of, for example, automatic flight control systems and mathematical model of flight simulator.

This paper will discuss the principles of the aerodynamic model identification of the aircraft based on flight test data using parameter identification techniques. The paper starts with discussion of a mathematical model of an aircraft. Next, parameter identification techniques will be described. Two techniques often used for estimation of the aerodynamic parameters will be discussed. Results of the modelling and estimation of the aerodynamic parameters, state trajectory of the aircraft, bias errors in the instrumentation system, and calibration factors of the vane-angle of attack from flight test data of the N-250 turboprop aircraft will be presented in this paper. Original achievement of that paper is the mathematical model of a turboprop aircraft.

Keywords: *turboprop aircraft, flight testing, parameter identification*

1. Introduction

In principle, the aerodynamic model of an aircraft can be obtained from theoretical prediction, wind tunnel test or flight test. Determination of the aerodynamic parameters from flight test is usually carried out from stationary flight measurements. However, measurements in stationary flight conditions are time consuming and require highly trained and competent test pilots to obtain realistic results. Furthermore, the resulting accuracies are limited due to unavoidable deviation from the nominal stationary flight conditions [1, 2].

Dynamic or nonstationary flight test measurements have advantages that it will yield accurate results and an important reduction in flight test time compared to stationary measurements. These measurements become more attractive in combination with accurate flight test instrumentation system and sophisticated flight test data analysis software such as parameter identification techniques [1, 3-5].

Parameter identification techniques give the opportunity of including in the analysis the prior knowledge of aircraft parameters such as the aerodynamic parameters, obtained from theoretical prediction, wind tunnel measurements and/or previous flight measurements. It also provides a tool for obtaining more accurate flight test data by reconstructing some output variables that are difficult to be measured.

This paper is organised as follows. Section 2 will describe the mathematical model of turboprop aircraft in which the forces and moment are expressed as function of aircraft state variables. Next, parameter identification principles will be briefly explained ii section 3. Estimation of aircraft state variable and aerodynamic parameter based on flight data will be shown in section 4. Some concluding remarks will be given in section 5.

2. Mathematical model of turboprop aircraft

Longitudinal state equation

The longitudinal state equation of the tuboprop aircraft is a set of differential equations relating the forces and moments acting on the aircraft. This model can be derived from equation of motion of the aircraft. In the body-fixed reference frame, this equation can be written as follows.

$$\begin{aligned} \dot{u} &= -qw - g\sin\theta + \frac{F_x}{m}, \\ \dot{w} &= qu + g\cos\theta + \frac{F_z}{m}, \\ \dot{\theta} &= q, \\ \dot{q} &= \frac{M}{I_y}, \\ \dot{h} &= u\sin\theta - w\cos\theta. \end{aligned} \tag{1}$$

In equation (1), u and w indicate the velocity components of the helicopter along the X and Z axes respectively, q is the pitch rate, θ denotes the angle of pitch, h is the altitude, g is the acceleration due to gravity, F_x and F_z are the total forces along the X and Z axes respectively, M is the total aerodynamic-propulsion pitching moment about the Y axis, m is the mass of the aircraft and I_y is the inertia about the Y axis.

Aerodynamic-propulsion model

The total aerodynamic-propulsion force and moment in equation (1) can be expressed as follows:

$$\begin{aligned} F_x &= \tfrac{1}{2}\rho V^2 S C_X, \\ F_z &= \tfrac{1}{2}\rho V^2 S C_Z, \\ M &= \tfrac{1}{2}\rho V^2 S \bar{c} C_m, \end{aligned} \tag{2}$$

where C_x, C_z, and C_m denote the coefficients of the aerodynamic-propulsion forces along the X and Z axes, and pitching moment about Y axis respectively, V is the forward speed, ρ is the air density, S is wing reference area and $\bar{c}$ is the mean aerodynamic chord of the wing.

The aerodynamic forces and moment coefficients in equation (2) can be expressed in terms of several state and control variables of the aircraft, for example the angle of attack α, pitch rate **q**, elevator control surface deflection δ_e, and the thrust coefficient T_c. The dependency of those coefficients on these variables can be expressed as follows:

$$\{C_X; C_Z; C_m\} = f(\alpha, q, \delta_e, T_c). \tag{3}$$

If terms up to the first order are included, except the α^2 term in the C_X equation, these coefficients can be expressed as follows [ ]:

$$
\begin{aligned}
C_X &= C_{X_0} + C_{X_\alpha}\alpha + C_{X_{\alpha^2}}\alpha^2 + C_{X_q} q + C_{X_{\delta_e}}\delta_e + C_{X_{T_c}} T_c,\\
C_Z &= C_{Z_0} + C_{Z_\alpha}\alpha + \qquad\quad + C_{Z_q} q + C_{Z_{\delta_e}}\delta_e + C_{Z_{T_c}} T_c,\\
C_m &= C_{m_0} + C_{m_\alpha}\alpha + \qquad\quad + C_{m_q} q + C_{m_{\delta_e}}\delta_e + C_{m_{T_c}} T_c.
\end{aligned} \tag{4}
$$

In equation (4), $C_{X_0}, C_{X_\alpha}, C_{X_{\alpha^2}}, C_{X_q}, C_{Z_0}, C_{Z_\alpha}, C_{Z_q}, C_{m_0}, C_{m_\alpha}$ and C_{m_q} are the aerodynamic parameters, $C_{X_{\delta_e}}$, $C_{Z_{\delta_e}}$ and $C_{m_{\delta_e}}$ are the control parameters, and $C_{X_{T_c}}, C_{Z_{T_c}}, C_{m_{T_c}}$ are the thrust parameters expressing the effect of the thrust on the total aerodynamic forces and moment. These parameters can be determined from either theoretical prediction or experimental. In this paper, these parameters will be estimated from flight data.

By substituting equation (4) into (2) and the result is inserting into equation (1) yields the longitudinal equation of motion of the aircraft as follows:

$$
\begin{aligned}
\dot{u} &= -qw - g\sin\theta + \frac{\frac{1}{2}\rho V^2 S}{m}\left(C_{X_0} + C_{X_\alpha}\alpha + C_{X_{\alpha^2}}\alpha^2 + C_{X_q} q + C_{X_{\delta_e}}\delta_e + C_{X_{T_c}} T_c\right)\\
\dot{w} &= \ qu + g\cos\theta + \frac{\frac{1}{2}\rho V^2 S}{m}\left(C_{Z_0} + C_{Z_\alpha}\alpha + C_{Z_q} q + C_{Z_{\delta_e}}\delta_e + C_{Z_{T_c}} T_c\right),\\
\dot{\theta} &= q,\\
\dot{q} &= \tfrac{M}{I_y} = \frac{\frac{1}{2}\rho V^2 S\bar{c} C_m}{I_y}\left(C_{m_0} + C_{m_\alpha}\alpha + C_{m_q} q + C_{m_{\delta_e}}\delta_e + C_{m_{T_c}} T_c\right),\\
\dot{h} &= u\sin\theta - w\cos\theta.
\end{aligned} \tag{5}
$$

In principle, equation (5) can be integrated numerically to obtain the aircraft state variables. In this case, the elevator deflection and the thrust coefficient are the input variables.

Observation equation

The observation equation relates the measured aircraft variables (using sensors) with the state variables. If airspeed V, angle of attack α, and altitude h are measured, then the observation equations are given as follows:

$$
\begin{aligned}
V &= \sqrt{u^2 + w^2} + n_V,\\
\alpha &= a_0 + a_1 \tan^{-1}\left(\tfrac{w}{u}\right) + n_\alpha,\\
h &= h_0 + \Delta h + n_h.
\end{aligned} \tag{6}
$$

In equation (6), measurement noise is added in the model.

3. Parameter identification

The mathematical model of the aircraft discussed in the previous section, see equations (5) and (6) can be written in non linear stochastic differential equations as follows [-] :

$$
\begin{aligned}
\dot{x}(t) &= f[x(t), u(t), \theta] + w(t); \quad x(0) = x_0,\\
y(t) &= g[x(t), u(t), \theta]
\end{aligned} \tag{7}
$$

and the discrete form of the observations as:

$$
z(i) = y(i) + v(i); \quad i = 1,2,3,\ldots N, \tag{8}
$$

where x(t), u(t), y(t) and z(i) are the state, input, observations and measurement vectors respectively, θ represents the vector of the unknown parameters, i.e. the aerodynamic parameters,

x_0 is the vector of the (unknown) initial conditions of the state, w(t) and v(i) are the process and measurement noise vectors respectively, and N is the number of data points.

In general, the problem of estimating the state trajectory and the aerodynamic parameters of the aircraft can be solved using the so-called maximum likelihood method. In this method, both the state and the parameters are estimated simultaneously by maximization of the so-called likelihood function. The likelihood function can be defined in terms of the difference between the measured and the simulated value from the sensor model.

If certain conditions concerning the accuracy and type of the variables measured in flight are met, for example the total force in X, Y and Z directions are measured using accurate accelerometers and the rotational rate are measured using rate gyros, then the maximum likelihood estimation problem can be decomposed into two separate estimation problems, i.e. a state trajectory estimation problem followed by a parameter estimation problem. In the case of longitudinal motion, the state equation (1) can be re-written as follow:

$$\begin{aligned} \dot{u} &= -wq - g\sin\theta + a_x, \\ \dot{w} &= uq + g\cos\theta + a_z, \\ \dot{\theta} &= q, \\ \Delta\dot{h} &= u\sin\theta - w\cos\theta, \end{aligned} \tag{9}$$

where accelerations a_x and a_z and pitch rate q serve as input in state equation (9). These two separate problems are much easier to be solved than the original estimation problem.

State estimation

The state trajectory estimation is a recursive prediction of the aircraft state and its covariance matrix from a non-linear mathematical model, see equation (5) or (6), using a state estimation technique such as the Extended Kalman Filter (EKF) []. In principle, EKF consisted of three steps as follows:

- prediction of the state, observation and covariance matrix at time (i),
- calculation of the Kalman gain,
- updating the state and covariance matrix using the observed data.

Parameter estimation

Assuming that the airspeed V, altitude h (consequently the air density ρ can be estimated) and the angle of attack α are estimated using EKF, the coefficients of the total forces and moment in equation (4) can be expressed as follows:

$$\begin{aligned} C_x &= \frac{F_X}{0.5\rho V^2 S} = \frac{m a_x}{0.5\rho V^2 S} = C_{x_0} + C_{x_\alpha}\alpha + C_{x_{\alpha^2}}\alpha^2 + C_{x_q}q + C_{x_{\delta_e}}\delta_e + C_{x_{T_c}}T_c + \varepsilon_x, \\ C_z &= \frac{F_Z}{0.5\rho V^2 S} = \frac{m a_z}{0.5\rho V^2 S} = C_{z_0} + C_{z_\alpha}\alpha + C_{z_q}q + C_{z_{\delta_e}}\delta_e + C_{z_{T_c}}T_c + \varepsilon_z, \\ C_m &= \frac{M}{0.5\rho V^2 Sc} = \frac{I_y \dot{q}}{0.5\rho V^2 Sc} = C_{m_0} + C_{m_\alpha}\alpha + C_{m_q}q + C_{m_{\delta_e}}\delta_e + C_{m_{T_c}}T_c + \varepsilon_m. \end{aligned} \tag{10}$$

The force coefficients C_X and C_Z in equation (10) can be obtained from measurement of longitudinal and vertical accelerations. The moment coefficient C_m is obtained from numerical differentiation of q_m by taking the 4 points Lagrange time derivative [-].

From the flight data, the value of angular rate *q,* the elevator deflection δ_e and the thrust coefficient T_c are obtained. The speed and the angle of attack are estimated using EKF.
In a regression equation form, equation (10) can be expressed as

$$y(i) = \theta_0 + \theta_1 x_1(i) + \theta_2 x_2(i) + ... + \theta_p x_p(i) + \varepsilon(i), \tag{11}$$

where y(i) is the dependent variable, i.e. the aerodynamic force and moment coefficients, $x_p(i)$

denote the independent variables, i.e. the (estimated) state and control variables, θ_p is the vector of the aerodynamic parameters, and $\varepsilon(i)$ is the stochastic equation error, accounting for measurement error on the dependent variable.

In the matrix form, equation (11) can be written as:

$$Y_m = X\theta + \varepsilon. \tag{12}$$

The matrix of independent variables X is assumed known exactly from the state estimation, i.e. using EKF, or from direct measurement of state variable using instrumentation system with highest accuracy.

Using the Least Squares (LS) technique, the parameter vector θ can be estimated from:

$$\hat{\theta} = [X^T X]^{-1}[X^T Y]. \tag{13}$$

Following the estimated parameter $\hat{\theta}$, the estimated dependent variable and the estimated error model can be calculated from:

$$\begin{aligned} \hat{Y} &= X\hat{\theta}, \\ \hat{\varepsilon} &= Y - \hat{Y} = Y - X\hat{\theta}. \end{aligned} \tag{14}$$

Finally, the goodness of fit the model can be expressed in terms of a correlation coefficient between Y and $\hat{Y}$. This coefficient is usually referred to as the multiple correlation coefficient whose square is :

$$R^2 = \frac{[\hat{Y} - \overline{Y}]^T[\hat{Y} - \overline{Y}]}{[Y - \overline{Y}]^T[Y - \overline{Y}]}; \qquad \overline{Y} = \frac{1}{N}\sum_{i=1}^{N}\hat{Y}(i). \tag{15}$$

The value of the multiple correlation coefficient R can be as high as one when the model fit is perfect, or zero when no correlation exists between the estimated and the measured dependent variable Y.

4. Estimation of state and aerodynamic parameter

In this section estimation of aircraft state using EKF and aerodynamic parameters using LS method based on flight data will be given. The flight data are generated from simulation of the aircraft. As mentioned above, the flight data will be analyzed by using the EKF estimation technique followed by the Least Square algorithm.

Simulated flight data

To demonstrate the estimation techniques using EKF as well as LS method, the simulated flight data of the N-250 turboprop aircraft during longitudinal maneuverer will be used. Fig. 1 shows the prototype of the N-250 aircraft. This aircraft is powered by 2 engines and 6 blades of propeller [1]. The geometrical data and mass properties of this aircraft is given in Tab. 1.

Fig. 1. Prototype of N250 turboprop aircraft [1]

Tab. 1. Data of the N250 turboprop aircraft

Parameter	Symbol	Value
Wing area	S	65 m^2
Wing span	b	26 m
Wing m.a.c.	c	2.471 m
Mass	m	21500 kg
Inertia about Y-axis	Iy	600000 kg-m^2

Figure 2 shows the simulated flight data of the N250 turboprop aircraft during dynamic manoeuvre of elevator input followed by the thrust input. The observed airspeed V, the angle of attack α, altitude h, the accelerations along the longitudinal and vertical axes *(a_x, a_z)*, pitch rate q are also presented in Fig. 2.

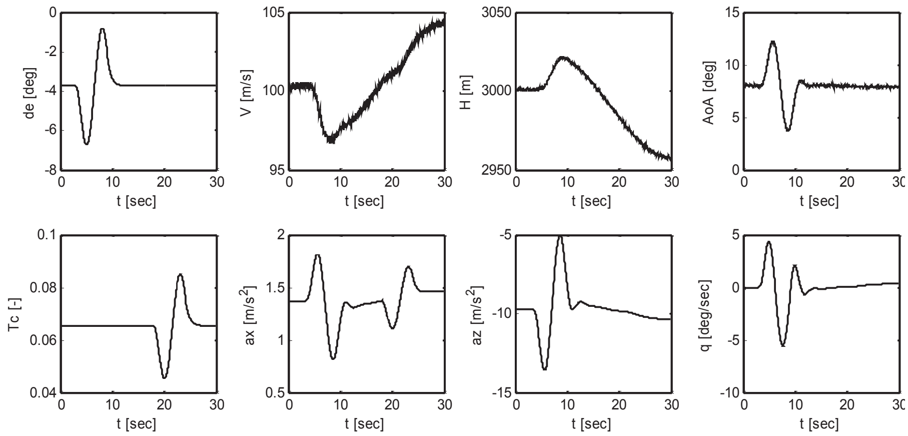

Fig. 2. The simulated flight data of the N250 aircraft

Estimated state and aerodynamic parameters

Based on the simulated flight data, the estimation of the aircraft state trajectory using EKF and the estimation of the aerodynamic parameters using LS will be performed.

Figure 3 shows an example of time history of the estimated aircraft state trajectory using EKF algorithm. The flight data was obtained from simulated flight maneuverer of the elevator followed by thrust at altitude of 10000 ft and nominal airspeed 100 m/sec. The filtered estimates of the airspeed component along the longitudinal axis $\hat{u}$, the airspeed component along the vertical axis $\hat{w}$, the pitch angle $\hat{\theta}$, and the altitude variation $\Delta\hat{h}$, see equation (9) are presented in this figure.

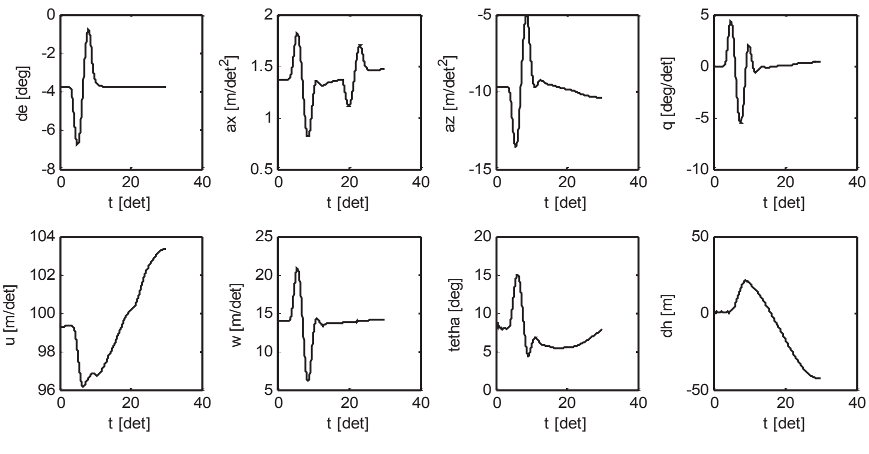

Fig. 3. The aircraft inputs (elevator deflection, longitudinal acceleration, vertical acceleration, pitch rate) and the estimated aircraft state (u, w, θ, Δh) using EKF

Figure 4 shows the EKF estimate of airspeed V, altitude variation Δh, and angle of attack α (red, dotted line). The measured airspeed, altitude variation, and angle of attack are also given in this figure (black, solid line). The difference between the estimated and the measured (residual) are also presented in the bottom of this figure. It is shown in the figure that the estimated airspeed, altitude variation and angle of attack are in a good agreement with the measured value.

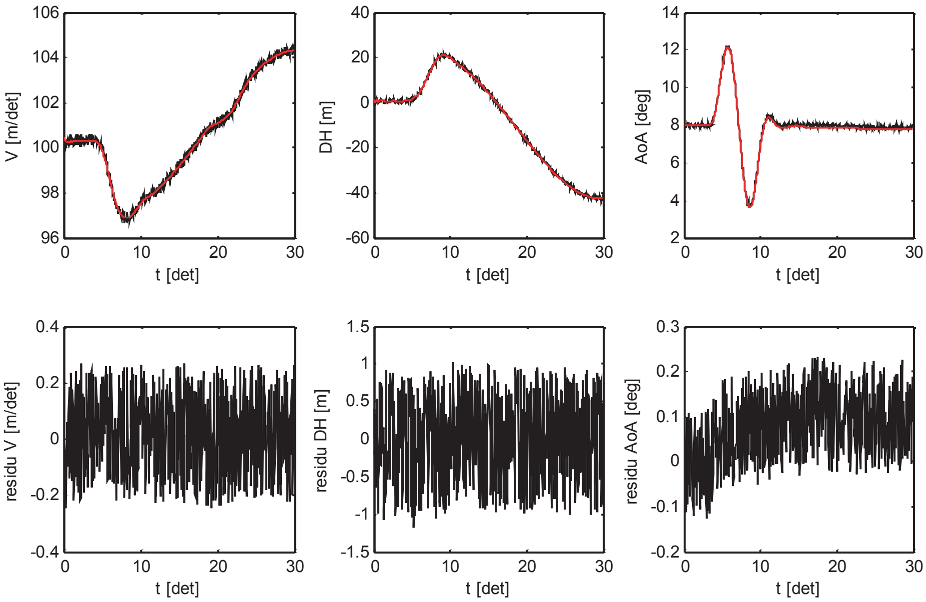

Fig. 4. The measured (solid line) and the estimated (dotted line) of the airspeed, altitude variation and angle of attack

The estimation of the aerodynamic parameters is performed with Least Squares method. Fig. 5 shows the measured (solid line) and estimated (dotted line) values of the longitudinal force coefficient C_x, the vertical force C_z, and the pitching moment coefficient C_m.

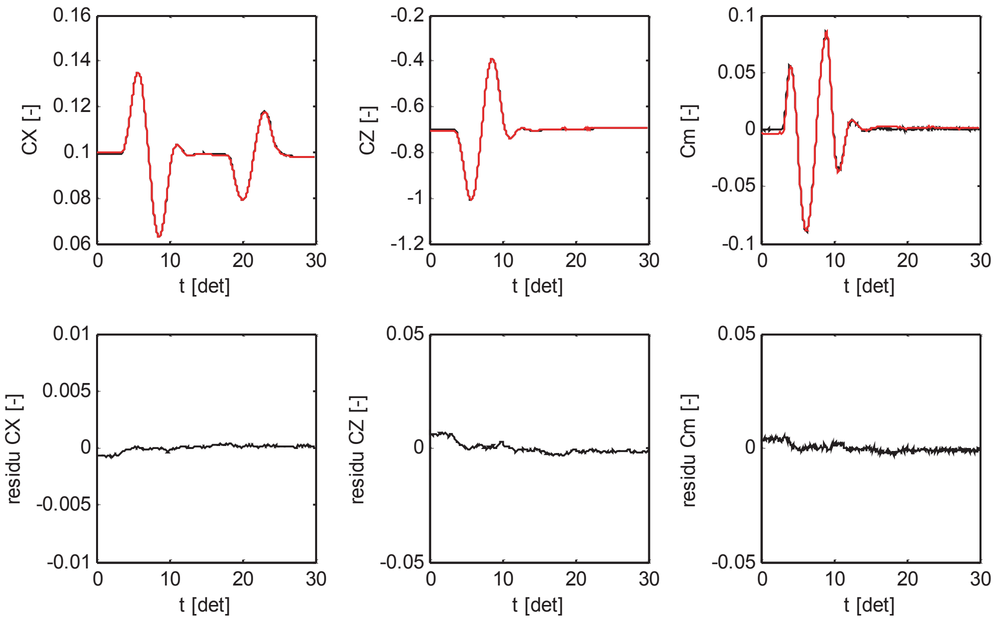

Fig. 5. The measured (solid line) and estimated (dotted line) of the longitudinal force cofficient C_X, vetical force coefficient C_Z and the pitching moment coefficient C_m

The difference between the measured C_X, C_Z, C_m and the estimated C_X, C_Z, C_m are also presented in this figure (the bottom side). In general, the estimated values of C_X, C_Z and C_m fit well with the measured values. The total correlation coefficient R^2, see equation (15), of the forces and moment coefficients are 0.999, 0.998 and 0.993 respectively.

5. Conclusions

The mathematical model of a turboprop aircraft has been derived in this paper. The aerodynamic parameter in the model has been estimated based on flight data using Extended Kalman Filter followed by Least Squares method. The parameter identification results show in a good agreement between the estimated and measured value.

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