

## MULTIPLE SOFT FAULT DIAGNOSIS OF BJT CIRCUITS

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### Abstract

This paper deals with multiple soft fault diagnosis of nonlinear analog circuits comprising bipolar transistors characterized by the Ebers-Moll model. Resistances of the circuit and beta forward factor of a transistor are considered as potentially faulty parameters. The proposed diagnostic method exploits a strongly nonlinear set of algebraic type equations, which may possess multiple solutions, and is capable of finding different sets of the parameters values which meet the diagnostic test. The equations are written on the basis of node analysis and include DC voltages measured at accessible nodes, as well as some measured currents. The unknown variables are node voltages and the parameters which are considered as potentially faulty. The number of these parameters is larger than the number of the accessible nodes. To solve the set of equations the block relaxation method is used with different assignments of the variables to the blocks. Next, the solutions are corrected using the Newton-Raphson algorithm. As a result, one or more sets of the parameters values which satisfy the diagnostic test are obtained. The proposed approach is illustrated with a numerical example.

Keywords: analog circuits, BJT circuits, fault diagnosis, multiple faults.

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### 1. Introduction

The fault diagnosis of analog circuits is an important and open problem for design validation [1]. A fault is named soft (parametric) if a parameter drifts from its tolerance range, but does not lead to any topological changes. Otherwise a fault is named catastrophic. The entire fault diagnosis includes detection of a fault, location faulty elements, and determination of their parameters. If most of circuit simulations take place before any testing, the diagnostic method is called the simulation-before-test (SBT) approach, otherwise it is called the simulation-after-test (SAT) approach. Unlike the catastrophic fault diagnosis, where the faulty parameters tend to infinity or zero, the soft fault diagnosis deals with the parameters that may take arbitrary values within some ranges. Therefore, the SBT approach, which requires building a fault dictionary, is difficult to arrange and the SAT approach is preferred.

Most of the previous research work in the area of a soft fault diagnosis addresses only the case where just one element is faulty, e.g. [2–9]. Less work deals with a multiple fault diagnosis, e.g. [10–16]. The SAT methods employ test equations which involve measured voltages and/or currents and unknown parameters to be determined. Frequently, these equations have an algebraic form and are nonlinear. If the parameters slightly drift from their nominal values, the equations can be linearized which simplifies the diagnosis. Unfortunately, such an approach is not allowed if the parameters deviate considerably from their nominal values and the equations are strongly nonlinear. In such a case several sets of the parameters values can satisfy the test, because the set of nonlinear equations can actually have multiple solutions. Most algorithms allow finding only one solution (one set of the parameters values), even if the set of nonlinear equations has several solutions. However, finding just one specific solution which is not necessarily the actual one is rarely of interest and not sufficient for a reliable diagnosis of the circuit. The Newton-Raphson algorithm which is the best known and

the most frequently used for solving nonlinear equations is useless as a general purpose tool in this area. In many cases this algorithm applied to the system of nonlinear equations fails, even if the system of equations has a unique solution. To find different sets of the parameters which meet the diagnostic test, the parametric homotopy method and the homotopy–simplicial algorithm have been recently proposed [15–16]. They are rather complex, but capable of solving the test equations which are not given in an explicit analytical form.

This paper is devoted to the multiple soft fault diagnosis of nonlinear circuits, containing bipolar transistors characterized by the Ebers-Moll model [17] and offers a method for localizing faulty elements and evaluating their parameters, without any linearization of the test equations. Resistances of the circuits and the beta forward factor of a transistor are considered as potentially faulty parameters. The proposed method is based on measurement of voltages at accessible nodes and some currents, the node approach, and the block relaxation concept. It allows finding different sets of values of all the elements considered as potentially faulty which meet the diagnostic test.

Localization of faulty parameters and determination their values are useful at the preproduction stage, where corrections of the technological process are possible. In such a case the time consumed by the diagnostic procedure is off–line and not crucial.

## 2. Fault diagnosis of BJT circuits

Let us consider a circuit consisting of  $n_t$  bipolar transistors,  $n_d$  diodes,  $r$  resistors considered as potentially faulty,  $h$  resistors considered as fault–free, and  $k$  DC voltage sources. Moreover, the beta forward factor ( $\beta_F$ ) of one transistor is considered as potentially faulty. We assume that one terminal of each voltage source is grounded. The transistors are characterized by the Ebers-Moll model composed of two diode-controlled source combinations and emitter, base, and collector resistors [17] (see Fig. 1 for npn transistors).

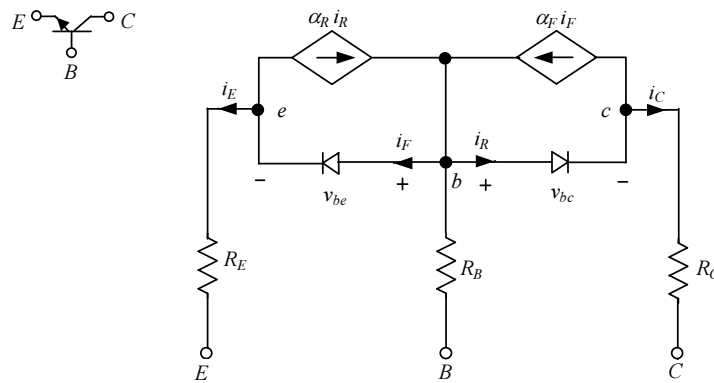


Fig. 1. The Ebers-Moll model of an npn transistor.

The emitter and collector diodes are described by the equations

$$i_F = g_F(v_{be}) = I_{ES} \left( \exp\left(\frac{v_{be}}{n_F v_T}\right) - 1 \right), \quad (1)$$

$$i_R = g_R(v_{bc}) = I_{CS} \left( \exp\left(\frac{v_{bc}}{n_R v_T}\right) - 1 \right). \quad (2)$$

The diodes acting alone are specified by the equation

$$i_D = g_D(v_D) = I_D \left( \exp\left(\frac{v_D}{n_D v_T}\right) - 1 \right). \quad (3)$$

The emitter and collector currents of the transistors and the currents of all the diodes acting alone will be considered as elements of the vector  $i_3$ , whereas the corresponding voltages as elements of the vector  $v_3$ . Then we can write

$$i_3 = Tg(v_3), \quad (4)$$

where  $g$  is a vector function consisting of the functions  $g_F, g_R, g_D$  of all the transistors and diodes,  $T$  is a block diagonal matrix of the form

$$T = \begin{bmatrix} 1 & -\alpha_R^{(1)} & & & & \\ -\alpha_F^{(1)} & 1 & & & & \\ & & \ddots & & & \\ & & & 1 & -\alpha_R^{(n_t)} & \\ & & & -\alpha_F^{(n_t)} & 1 & \\ & & & & & \mathbf{1} \end{bmatrix}, \quad (5)$$

where  $\mathbf{1}$  is  $n_d \times n_d$  identity matrix. In the discussed circuit we wish to find the actual values of  $r$  resistors and  $\alpha_F^{(i)} = \beta_F^{(i)} / (\beta_F^{(i)} + 1)$  factor ( $i \in \{1, \dots, n_t\}$ ), considered as potentially faulty. For this purpose the diagnostic test is arranged as follows. We apply a DC voltage source to the input node and - for different values of this voltage - measure the voltages  $\hat{v}_1, \dots, \hat{v}_m$  at the nodes accessible for measurement and the currents flowing through all independent voltage sources acting in the circuit (see Fig. 2).

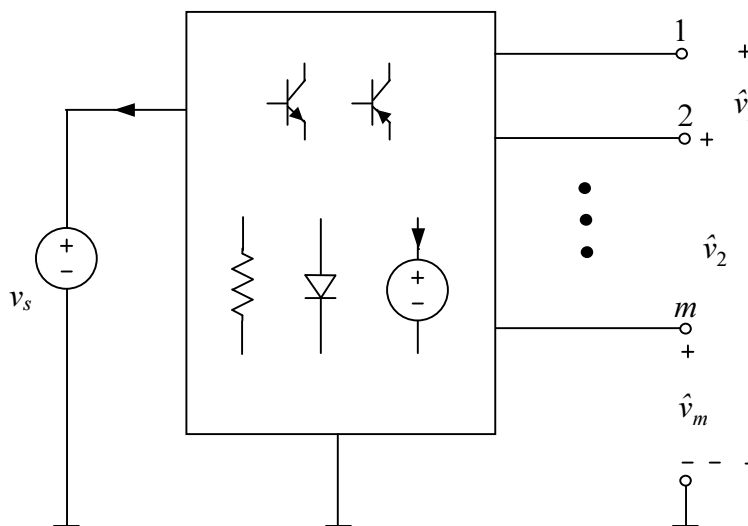


Fig. 2. Arrangement of a diagnostic test.

To describe this circuit we use the node approach. For this purpose we choose a grounded node as the datum node and introduce the node-to-datum voltages  $e_1, \dots, e_n$ , where  $n$  is the number of the remaining nodes. Next we create the incidence matrix [17], i.e. a rectangular

matrix of  $n$  rows and  $b = r + h + 2n_t + n_d + k$  columns whose  $(i, j)$ -th element  $a_{i,j}$  is defined by

$$a_{i,j} = \begin{cases} 1 & \text{if branch } j \text{ leaves node } i, \\ -1 & \text{if branch } j \text{ enters node } i, \\ 0 & \text{if branch } j \text{ is not incident with node } i. \end{cases}$$

The columns correspond, in succession, to the resistors considered as potentially faulty, resistors considered as fault-free, the branches  $be$  and  $bc$  of the transistors as well as the branches of the diodes acting alone, and the DC voltage sources. The nodes are numbered in such a way that at the end the measurement nodes and the nodes corresponding to the non-grounded terminals of the voltage sources are drawn up. As a result, an incidence matrix can be presented in the form

$$A = [A_1 \ A_2 \ A_3 \ A_4], \quad (6)$$

and the equation

$$Ai = \mathbf{0}, \quad (7)$$

holds, where  $i = [i_1^T \ i_2^T \ i_3^T \ i_4^T]^T$ . The sub-vectors  $i_1, i_2, i_3, i_4$  of the vector  $i$  correspond to the columns of the sub-matrices  $A_1, A_2, A_3, A_4$ . Hence, it holds

$$v_1 = A_1^T e, \quad (8)$$

$$v_2 = A_2^T e, \quad (9)$$

$$v_3 = A_3^T e, \quad (10)$$

where  $v_1, v_2, v_3$  are the vectors of the branch voltages corresponding to the vectors of the branch currents  $i_1, i_2, i_3$ . The vector  $e = [e_1 \ \dots \ e_n]^T$  consists of the node voltages. The branch equations are as follows

$$i_1 = G_1 v_1 = G_1 A_1^T e, \quad (11)$$

where  $G_1 = \text{diag}(x_1, \dots, x_r)$ , where  $x_1, \dots, x_r$  are the conductances of the resistors considered as potentially faulty,

$$i_2 = G_2 v_2 = G_2 A_2^T e, \quad (12)$$

where  $G_2 = \text{diag}(G_1, \dots, G_h)$  consists of the conductances of the resistors considered as fault-free,

$$i_3 = T(x_{r+1}) g(A_3^T e), \quad (13)$$

where  $T(x_{r+1})$  is given by (5) with  $i$ -th coefficient  $\alpha_F^{(i)} = x_{r+1}$ ,  $i \in \{1, \dots, n_t\}$ , considered as potentially faulty. Equations (7) – (13) lead to the matrix node equation

$$A_1 \text{diag}(x_1, \dots, x_r) A_1^T e + A_2 G_2 A_2^T e + A_3 T(x_{r+1}) g(A_3^T e) + A_4 i_4 = \mathbf{0}, \quad (14)$$

which describes the circuit. The node voltage vector  $e$  can be decomposed as follows

$$e = \begin{bmatrix} y \\ \hat{v} \end{bmatrix}, \quad (15)$$

where  $\hat{\mathbf{v}}$  is an  $(m+k)$  - vector consisting of  $m$  measured voltages and  $k$  voltages of DC voltage sources. The  $m+k$  elements of the vector  $\hat{\mathbf{v}}$  are known. The remaining node voltages being elements of the vector  $\mathbf{y} = [y_1 \dots y_{\hat{n}}]^T$ , where  $\hat{n} = n - (m+k)$ , are unknown variables.

The equation (14) consists of  $n$  individual equations with  $n+r+1-(m+k)$  unknown variables  $y_1, \dots, y_{\hat{n}}, x_1, \dots, x_{r+1}$ . Let  $r+1 > m+k$ , which means that the number the unknown variables is larger than the number of equations. For example, if  $n=15$ ,  $m+k=4$  and  $r+1=8$ , then the number of equations is 15, whereas the number of unknown variables is 19, including 11 node voltages and 8 unknown parameters (7 conductances of the resistors and one  $\beta_F$  coefficient  $x_{r+1} = x_8$ ). In such a case we need 2 equations of the form (14) corresponding to 2 different voltages of  $v_s = v_s^{(j)}$ ,  $j=1, 2$ . As a result, we obtain a set of 30 individual equations with 30 unknown variables, including 22 node voltages, and 8 unknown parameters.

Let  $r+1 = 2(m+k)$ , then we create 2 equations of the form (14), which can be presented in the form

$$\begin{aligned} \mathbf{f}^{(1)}(\mathbf{y}^{(1)}, \mathbf{x}) &= \mathbf{0}, \\ \mathbf{f}^{(2)}(\mathbf{y}^{(2)}, \mathbf{x}) &= \mathbf{0}, \end{aligned} \tag{16}$$

where  $\mathbf{y}^{(j)} = [y_1^{(j)} \dots y_{\hat{n}}^{(j)}]^T$ ,  $j=1, 2$ , is the vector of the node voltages in the circuit with the input voltage  $v_s = v_s^{(j)}$ ,  $\mathbf{x} = [x_1 \dots x_{r+1}]^T$ , and  $\mathbf{f}^{(j)}$  ( $j=1, 2$ ) is the symbol of the function on the left-hand side of the equation (14). Each of the equations describes the circuit driven by a different value of  $v_s$ , the  $j$ -th equation ( $j=1, 2$ ) of the system (16) is as follows

$$\begin{aligned} \mathbf{f}^{(j)}(\mathbf{y}^{(j)}, \mathbf{x}) &= \\ &= \mathbf{A}_1 \text{diag}(x_1, \dots, x_r) \mathbf{A}_1^T \begin{bmatrix} \mathbf{y}^{(j)} \\ \hat{\mathbf{v}}^{(j)} \end{bmatrix} + \mathbf{A}_2 \mathbf{G}_2 \mathbf{A}_2^T \begin{bmatrix} \mathbf{y}^{(j)} \\ \hat{\mathbf{v}}^{(j)} \end{bmatrix} + \mathbf{A}_3 \mathbf{T}(x_{r+1}) \mathbf{g} \left( \mathbf{A}_3^T \begin{bmatrix} \mathbf{y}^{(j)} \\ \hat{\mathbf{v}}^{(j)} \end{bmatrix} \right) + \mathbf{A}_4 \mathbf{i}_4^{(j)} = \mathbf{0}, \end{aligned}$$

where  $\hat{\mathbf{v}}^{(j)}$  and  $\mathbf{i}_4^{(j)}$  are given or known from measurement. The number of individual equations of the system (16) is  $2n$ . Since any vector  $\mathbf{y}^{(j)}$  ( $j=1, 2$ ) contains  $n-(m+k)$  unknown node voltages, the number of unknown variables of the system (16) is  $2(n-(m+k))+r+1=2n$  and equals to the number of the individual equations. To solve the system, the Newton-Raphson algorithm can be directly applied. Unfortunately, numerical experiments show that in such a case the Newton-Raphson algorithm usually fails, as it is shown in Section 3. This is why we propose a more efficient approach based on the idea of the block relaxation with the Newton-Raphson algorithm used to solve the block equations and to correct the final solution, as described in the sequel.

The unknown variables  $x_1 \dots x_{r+1}$ , where  $x_{r+1} = \alpha_F^{(l)}$ ,  $l \in \{1, \dots, n_l\}$ ,  $r+1 = 2(m+k)$ , will be divided into 2 sets, each consisting of  $(m+k)$  elements and assigned to 2 sub-vectors  $\mathbf{x}^{(1)}$  and  $\mathbf{x}^{(2)}$ . Then, the system (16) becomes

$$\begin{aligned} \mathbf{f}^{(1)}(\mathbf{y}^{(1)}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}) &= \mathbf{0}, \\ \mathbf{f}^{(2)}(\mathbf{y}^{(2)}, \mathbf{x}^{(1)}, \mathbf{x}^{(2)}) &= \mathbf{0}. \end{aligned} \tag{17}$$

Let  $\hat{\mathbf{x}}_1^{(1)}$  and  $\hat{\mathbf{x}}_1^{(2)}$  be the vectors  $\mathbf{x}^{(1)}$  and  $\mathbf{x}^{(2)}$ , respectively, consisting of the nominal values of the corresponding parameters, whereas  $\hat{\mathbf{y}}_1^{(1)}$  and  $\hat{\mathbf{y}}_1^{(2)}$  are the vectors  $\mathbf{y}^{(1)}$  and  $\mathbf{y}^{(2)}$  which satisfy the circuit equation in this case. We set  $\mathbf{x}^{(2)} = \hat{\mathbf{x}}_1^{(2)}$  into the first equation of the system (17)

$$\mathbf{f}^{(1)}(\mathbf{y}^{(1)}, \mathbf{x}^{(1)}, \hat{\mathbf{x}}_1^{(2)}) = \mathbf{0} . \quad (18)$$

The equation (18) consists of  $n$  individual equations with  $n - (m + k)$  unknown node voltages (elements of  $\mathbf{y}^{(1)}$ ) and  $m + k$  unknown parameters (elements of  $\mathbf{x}^{(1)}$ ). We solve this equation for  $\mathbf{y}^{(1)}$  and  $\mathbf{x}^{(1)}$  using the Newton-Raphson algorithm, obtaining  $\tilde{\mathbf{y}}_1^{(1)}$  and  $\tilde{\mathbf{x}}_1^{(1)}$ . Then we proceed to the second equation of the system (17), substituting  $\mathbf{x}^{(1)} = \tilde{\mathbf{x}}_1^{(1)}$ ,

$$\mathbf{f}^{(2)}(\mathbf{y}^{(2)}, \tilde{\mathbf{x}}_1^{(1)}, \mathbf{x}^{(2)}) = \mathbf{0} \quad (19)$$

and solve the equation (19) for  $\mathbf{y}^{(2)}$ ,  $\mathbf{x}^{(2)}$ , obtaining  $\tilde{\mathbf{y}}_1^{(2)}$ ,  $\tilde{\mathbf{x}}_1^{(2)}$ . Next, we find

$$\hat{\mathbf{x}}_1^{(i)} := \hat{\mathbf{x}}_1^{(i)} + \frac{1}{2}(\tilde{\mathbf{x}}_1^{(i)} - \hat{\mathbf{x}}_1^{(i)}), \quad i = 1, 2 , \quad (20)$$

$$\hat{\mathbf{y}}_1^{(i)} := \hat{\mathbf{y}}_1^{(i)} + \frac{1}{2}(\tilde{\mathbf{y}}_1^{(i)} - \hat{\mathbf{y}}_1^{(i)}), \quad i = 1, 2 , \quad (21)$$

and repeat the described above procedure. As a result, the sequences

$$\begin{aligned} & \hat{\mathbf{y}}_1^{(1)}, \hat{\mathbf{y}}_2^{(1)}, \hat{\mathbf{y}}_3^{(1)}, \dots \\ & \hat{\mathbf{y}}_1^{(2)}, \hat{\mathbf{y}}_2^{(2)}, \hat{\mathbf{y}}_3^{(2)}, \dots, \\ & \hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2, \hat{\mathbf{x}}_3, \dots \end{aligned} \quad (22)$$

are generated, where  $\hat{\mathbf{x}}_j = \left[ (\hat{\mathbf{x}}_j^{(1)})^T, (\hat{\mathbf{x}}_j^{(2)})^T \right]^T$ ,  $j = 1, 2, 3, \dots$ . The procedure is terminated if  $\|\hat{\mathbf{y}}_{j+1}^{(i)} - \hat{\mathbf{y}}_j^{(i)}\| < \varepsilon_1$ ,  $i = 1, 2$ , and  $\|\hat{\mathbf{x}}_{j+1} - \hat{\mathbf{x}}_j\| < \varepsilon_2$ , where  $\varepsilon_1$  and  $\varepsilon_2$  are tolerances which define the required accuracy of the obtained solution. To improve the efficiency of the method we do not assume restrictive values of  $\varepsilon_1$  and  $\varepsilon_2$ , obtaining an approximate solution. This is considered as initial guess of the Newton-Raphson algorithm applied to the system (16) which usually rapidly converges to the accurate solution. As a result, we find  $R_i = 1/G_i = 1/x_i$

( $i = 1, \dots, r$ ) and  $\beta_F^{(l)} = \frac{x_{r+1}}{1 - x_{r+1}}$  of the  $l$ -th transistor.

**Note**

If  $m + k < r + 1 < 2(m + k)$  the last equation of the system (16) is reduced. E.g., if  $m + k = 4$  and  $r + 1 = 7$ , then one individual equation of  $\mathbf{f}^{(2)}(\mathbf{y}^{(2)}, \mathbf{x}) = \mathbf{0}$  is discarded.

Convergence of the block relaxation method strongly depends on the way in which the elements of vector  $\mathbf{x}$  are assigned to subvectors  $\mathbf{x}^{(1)}$  and  $\mathbf{x}^{(2)}$ . To make possible finding multiple solutions of the system of equations (16) we consider all possible assignments of the elements  $x_1, \dots, x_{r+1}$  to  $\mathbf{x}^{(1)}$  and  $\mathbf{x}^{(2)}$ , and for each of them solve the system (17) using the described above approach based on the block relaxation method. Usually, for some of the systems this method is divergent and they are discarded. The solutions of the others may be

identical or different. In the last case there are several solutions of the system (16), what means that several sets of the parameters values meet the diagnostic test. For instance, if  $r + 1 = 8$  and  $m + k = 4$ , eight elements of the vector  $\mathbf{x}$  are assigned to  $\mathbf{x}^{(1)}$  or  $\mathbf{x}^{(2)}$ . There are 70 combinations of the assignments, leading to the same number of systems (17) which have to be solved using the proposed method. Usually, the time of solving all these systems is very short (in the example given in Section 3 it is less than 1s).

Two variants of the described method are discussed in this paper. The first one is devoted to the diagnosis of multiple faults of resistors only. In this case the transistors are considered as fault-free and  $x_{r+1}$  is a fixed parameter. The method gives one or more sets of the parameters values  $\{x_1, \dots, x_r\}$  which meet the diagnostic test. The second variant of the method deals with the fault diagnosis of a set of resistors  $\{R_1, \dots, R_r\}$  and one of the transistors. In such a case we perform  $n_t$  diagnoses, each for the same set of parameters  $\{x_1, \dots, x_r\}$  corresponding to the conductances of resistors  $R_1, \dots, R_r$  and one parameter  $x_{r+1}^{(i)}$   $i \in \{1, \dots, n_t\}$  corresponding to the  $i$ -th transistor. If only one of the diagnoses gives an accepted solution, e.g. for  $i = p$ ,  $\{x_1, \dots, x_r, x_{r+1}^{(p)}\}$ , and the others fail, just this set of parameters meets the test. As a result, we find values of the resistances of all the resistors considered as potentially faulty and  $\beta_F$  of  $p$ -th transistor, which together meet the diagnostic test. Otherwise we exploit another test, repeat the procedure and find the common set. The details are explained in Section 3.

### 3. Numerical example

The proposed method was implemented in Delphi and tested using several circuits. The calculations were executed on a computer with the processor INTEL(R) Core(TM) i7-2600. One of the exemplary circuits is discussed below.

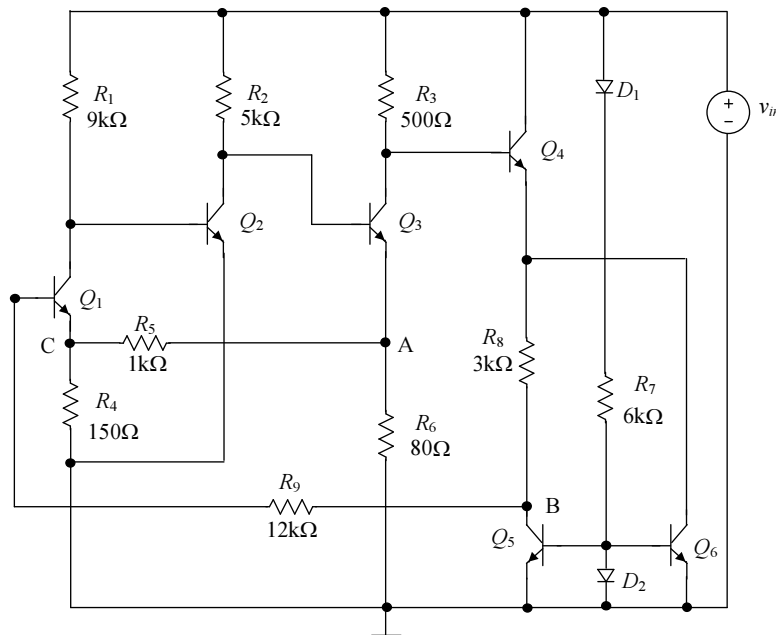


Fig. 3. An example of a circuit.

Let us consider the diode-transistor circuit shown in Fig. 3. The nominal values of all the resistances are indicated in this figure and their tolerances are 5%. The parameters of the

Ebers-Moll model for all the transistors are as follows:  $\alpha_F = 0.9901$  ( $\beta_F = 100$ ),  $\alpha_R = 0.5$ ,  $I_{ES} = 10.302 \text{ fA}$ ,  $I_{CS} = 20.400 \text{ fA}$ ,  $v_T = 25.8642 \text{ mV}$ ,  $n_F = 1$ ,  $n_R = 1$ ,  $R_C = R_B = R_E = 0$ . The parameters of both diodes are:  $I_D = 10.200 \text{ fA}$ ,  $n_D = 1$ ,  $v_T = 25.8642 \text{ mV}$ . The exponential characteristics of the diodes included in the transistor model and acting alone have been linearized above  $0.8 \text{ V}$ . To perform the diagnostic test we measure node voltages at nodes A, B, C ( $m = 3$ ) for two values of the input voltage  $v_{in}^{(1)} = 15 \text{ V}$ ,  $v_{in}^{(2)} = 5 \text{ V}$  and the currents flowing through the voltage source  $v_m$  in both cases. The measurement accuracy of the voltages and the currents is  $0.1 \mu\text{V}$  and  $0.1 \mu\text{A}$ , respectively. The number of the nodal equations is  $n = 11$ . Hence, the number of equations of the representation (16) is 22. We assume the following values of tolerances:  $\varepsilon_1 = 0.01$  and  $\varepsilon_2 = 0.0001$ , whereas the tolerance of the Newton-Raphson method used to correct the solution of the system (16) is  $10^{-6}$ .

**Variant 1**

Let the set of resistors  $\{R_1, \dots, R_8\}$  be considered as potentially faulty and the other circuit parameters have their nominal values. In this case all the transistors are considered as fault-free,  $x_{r+1}$  is fixed,  $r = 8$ ,  $m = 3$ ,  $k = 1$ , hence,  $r = 2(m + k)$ . The representation (16) consists of 22 equations with  $2(n - (m + k)) = 14$  unknown node voltages and  $r = 8$  unknown parameters. We examined 25 different sets of the resistor values every time performing the diagnostic procedure described in Section 2. Two of the cases are discussed below.

**Case 1**

For the actual values of parameters  $\{R_1, R_2, \dots, R_8\}$  indicated in column 2 of Table 1 the proposed method gives two sets of the resistances which satisfy the test, shown in columns 3 and 4. To find the correct set of the resistances we apply another test assuming  $v_{in}^{(1)} = 12 \text{ V}$ ,  $v_{in}^{(2)} = 7 \text{ V}$ . As a result, the method gives two sets indicated in columns 5 and 6. The common part of the sets provided by both tests is the actual one.

Table 1. Results of Variant 1, Case 1.

Parameters	Actual values	Results (test 1)		Results (test 2)	
		Set 1	Set 2	Set 1	Set 2
$R_1$ [k $\Omega$ ]	12 (+33.3%)	12.00	12.00	12.00	12.00
$R_2$ [k $\Omega$ ]	4 (-20%)	4.00	3.89	4.00	3.89
$R_3$ [k $\Omega$ ]	1 (+100%)	1.00	0.60	1.00	0.60
$R_4$ [ $\Omega$ ]	220 (+46.7%)	220.00	220.00	220.00	220.00
$R_5$ [ $\Omega$ ]	700 (-30.0%)	700.00	700.00	700.00	700.00
$R_6$ [ $\Omega$ ]	70 (-12.5%)	70.13	35.76	70.38	35.37
$R_7$ [k $\Omega$ ]	5.7 (-5.0%)	5.69	57.15	5.67	71.38
$R_8$ [k $\Omega$ ]	2.7 (-10.0%)	2.70	22.04	2.70	26.79

**Case 2**

The resistances are as follows:  $\{R_1 = 9.27 \text{ k}\Omega (+3.0\%), R_2 = 4.92 \text{ k}\Omega (-1.6\%), R_3 = 488 \Omega (-2.4\%), R_4 = 147 \Omega (-2.0\%), R_5 = 1020 \Omega (+2.0\%), R_6 = 81 \Omega (+1.2\%), R_7 = 5.83 \text{ k}\Omega (-2.8\%), R_8 = 3.9 \text{ k}\Omega (+30\%)\}$ . The proposed method gives one (correct) set of



resistances which satisfy the test:  $\{R_1 = 9.27\text{k}\Omega, R_2 = 4.92\text{k}\Omega, R_3 = 488\Omega, R_4 = 147.00\Omega, R_5 = 1020\Omega, R_6 = 80.91\Omega, R_7 = 5.84\text{k}\Omega, R_8 = 3.90\text{k}\Omega\}$ .

In both cases the Newton-Raphson algorithm fails, when applied directly to the system (16) consisting of all 22 equations.

The results of 25 diagnoses, performed using the proposed approach, are as follows. The percentage of the correct results equals to 92%; in 32% we obtain only one correct set, whereas in 60% - the correct set and one or two virtual ones. In 8% the method diverges. The average time of one diagnosis is 0.8s. The Newton-Raphson algorithm applied directly to the system consisting of all 22 equations fails in 23 out of 25 cases.

**Variant 2**

Let the set of seven resistors  $\{R_1, R_2, R_3, R_4, R_5, R_6, R_8\}$ , and  $\beta_F$  of one transistor be considered as potentially faulty. In this case  $r + 1 = 8, m = 3, k = 1$ . Hence,  $r + 1 = 2(m + k)$ . The representation (16) consists of 22 equations with 14 unknown node voltages and  $r + 1 = 8$  parameters.

**Case 1**

The actual values of parameters are shown in column 2 of Table 2. For the same test as in Variant 1 the procedure described in Section 2 gives two sets of parameters that meet the test, indicated in column 3 and 4. Using another test with  $v_{in}^{(1)} = 12\text{V}, v_{in}^{(2)} = 7\text{V}$  and choosing the common set we find the actual set of parameters.

Table 2. Results of Variant 2, Case 1.

Parameters	Actual values	Results	
		Set 1	Set 2
$R_1$ [kΩ]	9.36 (+4.0%)	9.36	2.95
$R_2$ [kΩ]	4.75 (-5.0%)	4.75	529.42
$R_3$ [Ω]	650 (+30%)	649.97	484.81
$R_4$ [Ω]	157 (+4.7%)	157.00	50.39
$R_5$ [kΩ]	1.03 (+3.0%)	1.03	234.00
$R_6$ [Ω]	100 (+20.0%)	100.00	125.85
$R_8$ [kΩ]	4.0 (+33.3%)	4.00	4.32
$\beta_F^{(1)}$	10 (-90.0%)	10.00	35.52

For the sets:  $\{R_1, R_2, R_3, R_4, R_5, R_6, R_8, \beta_F^{(i)}\}, i = 2, 3, 4, 5, 6$ , the method either is divergent or leads to an incorrect solution (e.g. containing negative values of some parameters). Thus, we obtain a unique (correct) set of the parameters which meet the test.

**Case 2**

The actual values of parameters are indicated in column 2 of Table 3. Applying the procedure described in Section 2 we obtain three sets of parameters which meet the test having equal rights; they are shown in columns 3, 4, and 5.

Table 3. Results of Variant 2, Case 2.

Parameters	Actual values	Results		
		Set 1	Set 2	Set 3
$R_1$ [k $\Omega$ ]	8.1 (-10.0%)	8.10	9.43	9.68
$R_2$ [k $\Omega$ ]	4.7 (-6.0%)	4.70	4.32	4.38
$R_3$ [k $\Omega$ ]	520 (+4.0%)	520.00	519.22	527.96
$R_4$ [ $\Omega$ ]	140 (-6.7%)	140.00	139.73	138.30
$R_5$ [k $\Omega$ ]	1.2 (+20.0%)	1.20	1.18	1.14
$R_6$ [ $\Omega$ ]	85 (+6.2%)	85.00	85.09	66.51
$R_8$ [k $\Omega$ ]	3.3 (+10.0%)	3.30	3.30	3.24
$\beta_F$	$\beta_F^{(2)} = 10$ (-90.0%)	$\beta_F^{(2)} = 10.00$	$\beta_F^{(1)} = 99.59$	$\beta_F^{(3)} = 3.67$

In all the cases the Newton-Raphson algorithm fails, when applied directly to the system (16) consisting of all 22 equations.

We considered 36 different sets of resistances and beta forward factors and every time applied the proposed diagnostic method. The results are as follows. The percentage of the correct results equals to 80% ; in 20% we obtain only one correct set, whereas in 60% - the correct set and one or more (up to five) virtual ones. In 20% the method diverges. The average time of one diagnosis, comprising 6 sets of parameters (including the same resistors and different  $\beta_F$ ), is 2.5s.

If more than one set of the parameters values are obtained, an additional test should be performed and the common set selected.

#### 4. Conclusion

The paper deals with the multiple soft fault diagnosis of analog circuits containing bipolar transistors. This class of circuits is difficult to diagnosis due to strong nonlinearities of BJT model. A set of resistors and one transistor are considered as potentially faulty. The proposed block relaxation method for solving nonlinear equations describing the circuit with different assignments of the unknown variables to the blocks, combined with the Newton-Raphson algorithm, is efficient. It is capable of finding one or several sets of the parameters values which meet the diagnostic test. The method is effective, fast and does not require much computing power. A desirable attribute of the proposed approach is obtaining values of all elements considered as possible faulty, in the circuits with a rather small number of nodes accessible for measurement. The drawbacks of the proposed method are as follows. Only single transistor can be considered as potentially faulty in one diagnostic procedure. The method cannot be directly extended to CMOS circuits.

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