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# Port oil terminal reliability optimization

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**Abstract**

A method for reliability optimization, which is relevant for critical infrastructure activity governed by operational processes, is presented and applied to a port oil terminal. The optimal values for the reliability and resilience indicators related to the operation, are determined for this critical type of infrastructure. Simple suggestions regarding optimized infrastructural operation strategies are formulated and implemented towards reorganizing the port oil terminal processes in order to maximize its lifetime in defined reliability states.

## Introduction

A critical infrastructure operating in a fixed area may be vulnerable to damage and degradation caused by external threats, and similarly, it may impose threats to other critical infrastructures (Lauge, Hernantes & Sarriegi, 2015). Therefore, it is valuable for the overall industrial practices to improve the reliability indicators related to such critical infrastructure. In order to reach this goal, various tools are required to determine the critical infrastructure's reliability and resilience indicators, as well as their optimal forms. In this way, procedures that promote positive changes to the system operation processes can be proposed. Comparing the optimized values of the critical infrastructure's indicators with their values before the process optimization provides a measure of the enhancement in reliability (Klabjan & Adelman, 2006; Tang, Yin & Xi, 2007; Kołowrocki & Soszyńska-Budny, 2011; 2015). This report presents and describes the critical infrastructure procedural optimization, whereby we determine the optimal reliability and risk functions relevant for a port oil terminal. Other significant reliability and resilience indicators determined for this type of critical infrastructure are (i) its mean lifetime up to and exceeding a critical reliability state, (ii) the moment when its risk function value exceeds the acceptable reliability level, (iii) the intensity of changes due to ageing / degradation of the critical infrastructure, (iv) the coefficient of the operation process's impact on infrastructural ageing, and (v) the coefficient of resilience related to impacts on the operational process. The results are compared with the reliability indicator values determined for the port oil terminal before the optimization, and a new operational strategy is proposed.

## Methodology

### Optimization of operation and reliability

We consider the reliability function, shown below, defining the critical infrastructure impacted by operation process coordinates (Kołowrocki & Soszyńska-Budny, 2011; 2015).

$$\boldsymbol{R}(t,\cdot) = [1, \boldsymbol{R}(t,1),\ldots, \boldsymbol{R}(t,z)],\ \ t \geq 0$$

$$\boldsymbol{R}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{R}(t,u)]^{(b)}$$

$$t \geq 0,\ \ u = 1, 2,\ldots, z \qquad (1)$$

In this equation, $p_b$ ($b = 1, 2,\ldots, v$) represents the limit transient probabilities of the critical infrastructure operation process at the operation states, $z_b$ ($b = 1, 2,\ldots, v$). Additionally, $[\boldsymbol{R}(u)]^{(b)}$ ($b = 1, 2,\ldots, v$, and $u = 1, 2,\ldots, z$) at these operation states represent the conditional reliability functions of the critical infrastructure conditional lifetimes in the safety state subsets, $\{u, u+1,\ldots, z\}$, where $u = 1, 2,\ldots, z$. It is natural to assume that the critical infrastructure operation process has a significant influence on the resulting safety of that infrastructure. This relationship is also clearly expressed in equation (2), below (Kołowrocki & Soszyńska-Budny, 2011; 2015), for the mean values of the critical infrastructure's unconditional lifetimes in the safety state subsets.

$$\boldsymbol{\mu}(u) = \sum_{b=1}^{v} p_b [\boldsymbol{\mu}(u)]^{(b)} \tag{2}$$

From the linear equation (2), it is clear that the mean value of the critical infrastructure's unconditional lifetime, $\boldsymbol{\mu}(u)$ ($u = 1, 2,\ldots, z$), is determined by the limit values of transient probabilities, $p_b$ ($b = 1, 2,\ldots, v$), of the critical infrastructure operation process at the operation states, $z_b$ ($b = 1, 2,\ldots, v$), as well as the mean values, $[\boldsymbol{\mu}(u)]^{(b)}$ ($b = 1, 2,\ldots, v$, and $u = 1, 2,\ldots, z$), of the critical infrastructure conditional lifetimes in the safety state subsets, $\{u, u+1,\ldots, z\}$ ($u = 1, 2,\ldots, z$), of these operation states. Therefore, an infrastructure lifetime optimization approach based on linear programming can be proposed (Klabjan & Adelman, 2006; Kołowrocki & Soszyńska-Budny, 2011; 2015). In order to do so, we must determine the corresponding optimal values, $\dot{p}_b$ ($b = 1, 2,\ldots, v$), of the transient probabilities, $p_b$ ($b = 1, 2,\ldots, v$), related to the critical infrastructure operation process at each operation state in order to maximize the mean value, $\boldsymbol{\mu}(u)$, of the unconditional critical infrastructure lifetime in the safety state subsets, $\{u, u+1,\ldots, z\}$ ($u = 1, 2,\ldots, z$). This is carried out under the assumption that the mean values, $[\boldsymbol{\mu}(u)]^{(b)}$ ($b = 1, 2,\ldots, v$, and $u = 1, 2,\ldots, z$), of the critical infrastructure conditional lifetimes in the safety state subsets are fixed. One special case arises if $r$ ($r = 1, 2,\ldots, z$) is considered a critical infrastructure safety state, and we want to find the optimal values, $\dot{p}_b$ ($b = 1, 2,\ldots, v$), of the transient probabilities of the critical infrastructure operation process at the various operation states. In such a case, to maximize the mean value, now denoted as $\boldsymbol{\mu}(r)$, of the unconditional critical infrastructure lifetime in the safety state subset, $\{r, r+1,\ldots, z\}$ ($r = 1, 2,\ldots, z$), it must be assumed that the mean values, $[\boldsymbol{\mu}(r)]^{(b)}$ ($b = 1, 2,\ldots, v$, and $r = 1, 2,\ldots, z$), of the critical infrastructure conditional lifetimes in this safety state subset are fixed. Essentially, we formulate the optimization problem as a linear programming model, with the objective function of the following form,

$$\boldsymbol{\mu}(r) = \sum_{b=1}^{v} p_b [\boldsymbol{\mu}(r)]^{(b)} \tag{3}$$

for a fixed $r \in \{1, 2,\ldots, z\}$, and with the following bound constraints:

$$\breve{p}_b \le p_b \le \hat{p}_b, \;\; b = 1, 2,\ldots, v \tag{4}$$

$$\sum_{b=1}^{v} p_b = 1 \tag{5}$$

Therefore, $[\boldsymbol{\mu}(r)]^{(b)}$, $[\boldsymbol{\mu}(r)]^{(b)} \ge 0$ ($b = 1, 2,\ldots, v$), are fixed mean values of the infrastructure conditional lifetimes in the safety state subset, $\{r, r+1, \ldots, z\}$, and

$$\breve{p}_b,\; 0 \le \breve{p}_b \le 1 \text{ and } \hat{p}_b,\; 0 \le \hat{p}_b \le 1,\; \breve{p}_b \le \hat{p}_b$$
$$b = 1, 2, \ldots, v \tag{6}$$

are the lower and upper bounds of the unknown transient probabilities, $p_b$ ($b = 1, 2,\ldots, v$), respectively. The optimal solution of the formulas represented by (3)–(6) can be obtained using linear programming, i.e. we can determine the optimal values $\dot{p}_b$ of the transient probabilities, $p_b$ ($b = 1, 2,\ldots, v$), which maximize the objective function given by (3).

First, we arrange the critical infrastructure conditional lifetime mean values, $[\boldsymbol{\mu}(r)]^{(b)}$ ($b = 1, 2,\ldots, v$), in decreasing order

$$[\boldsymbol{\mu}(r)]^{(b_1)} \ge [\boldsymbol{\mu}(r)]^{(b_2)} \ge \cdots \ge [\boldsymbol{\mu}(r)]^{(b_v)},$$

where $b_i \in \{1, 2,..., v\}$ ($i = 1, 2,\ldots, v$).

Next, we substitute

$$x_i = p_{b_i},\; \breve{x}_i = \breve{p}_{b_i},\; \hat{x}_i = \hat{p}_{b_i},\; i = 1, 2,\ldots, v \tag{7}$$

and maximize the linear form of equation (3) with respect to $x_i$ ($i = 1, 2,\ldots, v$), which takes the following form after the transformation:

$$\boldsymbol{\mu}(r) = \sum_{i=1}^{v} x_i [\boldsymbol{\mu}(r)]^{(b_i)} \tag{8}$$

for fixed $r \in \{1, 2,\ldots, z\}$, with bound constraints, $\breve{x}_i \le x_i \le \hat{x}_i$ ($i = 1, 2,\ldots, v$), such that

$$\sum_{i=1}^{v} x_i = 1.$$

Here, $[\boldsymbol{\mu}(r)]^{(b)}$ and $[\boldsymbol{\mu}(r)]^{(b)} \ge 0$ ($i = 1, 2,\ldots, v$) represent the fixed mean values of the critical infrastructure conditional lifetimes in the safety state subset, $\{r, r+1, \ldots, z\}$, arranged in decreasing order, and

$$\breve{x}_i, 0 \le \breve{x}_i \le 1, \quad \hat{x}_i, 0 \le \hat{x}_i \le 1, \quad \breve{x}_i \le \hat{x}_i$$
$$b = 1, 2, \ldots, v \tag{9}$$

are the lower and upper bounds of the unknown probabilities, $x_i$ ($i = 1, 2, \ldots, v$), respectively.

To find the optimal values of $x_i$ ($i = 1, 2, \ldots, v$), we define

$$\breve{x}_i = \sum_{i=1}^{v} \breve{x}_i, \quad \hat{y} = 1 - \breve{x} \tag{10}$$

and

$$\breve{x}_i^0 = 0, \ \hat{x}_i^0 = 0 \ \text{and} \ \breve{x}^I = \sum_{i=1}^{I} \breve{x}_i, \quad \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i$$
$$I = 1, 2, \ldots, v \tag{11}$$

Next, we determine the largest value, $I \in \{0,1,\ldots,v\}$, such that

$$\breve{x}^I - \hat{x}^I < \hat{y} \tag{12}$$

and fix the optimal solution in order to maximize equation (8) in the following way:
i) If $I = 0$, the optimal solution is:

$$\dot{x}_1 = \hat{y} + \breve{x}_1 \ \text{and} \ \dot{x}_1 = \hat{x}_i, \quad i = 1,2,\ldots,v \tag{13}$$

ii) If $0 < I < v$, the optimal solution is:

$$\dot{x}_1 = \hat{x}_i, \quad i = 1,2,\ldots,I,$$
$$\text{or} \ \ \dot{x}_{I+1} = \hat{y} - \hat{x}_1^I + \breve{x}_1^I + \breve{x}_{I+1}^I$$
$$\text{and} \ \dot{x}_1 = \breve{x}_i, \quad i = I+1, I+2, \ldots, v \tag{14}$$

iii) If $I = v$, the optimal solution is:

$$\dot{x}_1 = \hat{x}_i, \quad i = 1,2,\ldots,v \tag{15}$$

Finally, after conducting the inverse substitution in (7), we obtain the optimal limit transient probabilities,

$$\dot{p}_{b_i} = \dot{x}_i, \ i = 1, 2, \ldots, v \tag{16}$$

that maximize the critical infrastructure mean lifetime in the safety state subset, $\{r, r+1, \ldots, z\}$, which is defined by the linear form shown in equation (3). Thus, its maximum value takes the form,

$$\dot{\boldsymbol{\mu}}(r) = \sum_{b=1}^{v} \dot{p}_b [\boldsymbol{\mu}(r)]^{(b)} \tag{17}$$

for a fixed $r \in \{1, 2, \ldots, z\}$.

#### Optimal reliability characteristics

From equation (17), which expresses the maximum mean value, $\dot{\boldsymbol{\mu}}(r)$, of the critical infrastructure's unconditional lifetime in the safety state subset, $\{r, r+1, \ldots, z\}$, we can replace the critical safety state, $r$, with the safety state, $u$ ($u = 1, 2, \ldots, z$) to obtain the corresponding optimal solutions for the mean values of the critical infrastructure unconditional lifetimes in the safety state subsets, $\{u, u+1, \ldots, z\}$, as shown in the equation,

$$\dot{\boldsymbol{\mu}}(u) = \sum_{b=1}^{v} \dot{p}_b [\boldsymbol{\mu}(u)]^{(b)}, \ u = 1, 2, \ldots, z \tag{18}$$

Further, according to equation (1), the corresponding optimal unconditional reliability function of the critical infrastructure is the vector,

$$\dot{\boldsymbol{R}}(t,\cdot) = \left[1, \dot{\boldsymbol{R}}(t,1), \ldots, \dot{\boldsymbol{R}}(t,z)\right], \ t \ge 0 \tag{19}$$

with the coordinates given by

$$\dot{\boldsymbol{R}}(t,u) = \sum_{b=1}^{v} \dot{p}_b \left[\boldsymbol{R}(t,u)\right]^{(b)}, \ t \ge 0, \ u = 1, 2, \ldots, z \tag{20}$$

The optimal values of the variances of the critical infrastructure unconditional lifetimes in the corresponding safety state subsets are

$$\dot{\boldsymbol{\sigma}}(u) = 2\int_0^\infty t\dot{\boldsymbol{R}}(t,u)dt - [\dot{\boldsymbol{\mu}}(u)]^{(2)}, \ u = 1, 2, \ldots, z \tag{21}$$

where $\dot{\boldsymbol{\mu}}(r)$ is calculated from equation (18), and $\dot{\boldsymbol{R}}(t,u)$ is given by equation (19). The optimal solutions for the mean values of the critical infrastructure unconditional lifetimes in the particular safety states are:

$$\dot{\bar{\boldsymbol{\mu}}}(u) = \dot{\boldsymbol{\mu}}(u) - \dot{\boldsymbol{\mu}}(u+1), \ u = 1,2,\ldots z-1,$$
$$\dot{\bar{\boldsymbol{\mu}}}(z) = \dot{\boldsymbol{\mu}}(z) \tag{22}$$

The corresponding optimal critical infrastructure risk function and the optimal moment when the risk exceeds a permitted level, $\delta$, are given by the following two equations (Kołowrocki & Soszyńska-Budny, 2011; 2015):

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,r), \ t \ge 0 \tag{23}$$

and

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \tag{24}$$

where $\dot{\boldsymbol{R}}(t,r)$ is defined by equation (20), for $u = r$, and $\dot{\boldsymbol{r}}^{-1}(t)$, represents the inverse function of the optimal risk function, $\dot{\boldsymbol{r}}(t)$, if it exists.

#### Optimal operation strategy

Some useful and easily applicable tools that can help in designing and planning more reliable and safe operation processes for critical infrastructures are the optimal mean values of the total operational sojourn times, $\hat{\theta}_b$, at each operation state, $z_b$ ($b = 1, 2, \ldots, v$), during the fixed operation time, $\theta$. These values can be obtained by replacing the transient probabilities, $p_b$, at the operation states, $z_b$, in the following formula (Kołowrocki & Soszyńska-Budny, 2011; 2015)

$$\hat{M}_b = E\left[\hat{\theta}_b\right] = p_b\theta,\ b = 1, 2, \ldots, v \tag{25}$$

with their optimal values, $\dot{p}_b$ (Kołowrocki & Soszyńska-Budny, 2011), which results in the following expression,

$$\dot{\hat{M}}_b = \dot{E}\left[\hat{\theta}_b\right] = \dot{p}_b\theta,\ b = 1, 2, \ldots, v \tag{26}$$

Knowing the optimal mean values, $\dot{\hat{M}}_b$, of the total sojourn times at each particular operation state during a fixed operation time represents the basis for changing the relevant operation procedures in order to ensure more reliable and safe operations for critical infrastructures.

## Application

### Port oil terminal; critical infrastructure

In this report, we specifically consider a port oil terminal as the critical infrastructure impacted by its operational process. This port oil terminal is located at the Baltic seaside, and is designated for receiving oil products from ships, storing these materials, and sending them off via carriages or trucks. The terminal considered in this work is composed of three regions, A, B and C, which are linked by the piping transportation system within the pier, as illustrated in Figure 1.

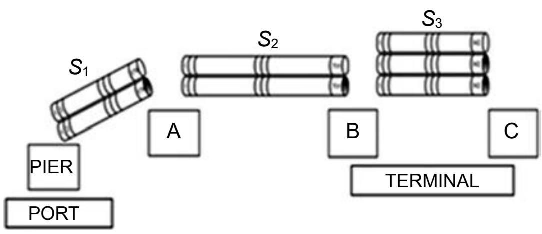


**Figure 1. Scheme of the port oil terminal and the associated piping transportation systems**

The main technical assets of the port oil terminal's critical infrastructure are:
- $A_1$ – port oil piping transportation system,
- $A_2$ – internal pipeline technological system,
- $A_3$ – supporting pump station,
- $A_4$ – internal pump system,
- $A_5$ – port oil tanker shipment terminal,
- $A_6$ – loading railway carriage station,
- $A_7$ – loading road carriage station,
- $A_8$ – unloading railway carriage station,
- $A_9$ – oil storage reservoir system.

The asset $A_1$ (i.e., the port oil piping transportation system) operating at the site consists of three subsystems (see Figure 1):
- $S_1$ is composed of two pipelines, each comprising 176 pipe segments and 2 valves;
- $S_2$ is composed of two pipelines, each containing 717 pipe segments and 2 valves;
- $S_3$ is composed of three pipelines, each with 360 pipe segments and 2 valves.

The port oil transportation system is set up as a series composed of two parallel subsystems, namely $S_1$ and $S_2$, each of which contains two pipelines (assets), as well as one series ("2 out of 3") subsystem, $S_3$, containing 3 pipelines (assets). The operation of the asset, $A_1$, represents the main activity of the port oil terminal, therefore its functioning dictates the operational processes involving the remaining assets, $A_2 – A_9$.

Based on statistical data and expert opinions, it is possible to fix the following parameters of the operation process of the oil terminal critical infrastructure:
- the number of operation process states = 7;

The operation process states ($z_{1–7}$) are therefore defined as follows:
- $z_1$ – transport of one type of medium from terminal area B to area C using two out of three pipelines of the subsystem, $S_3$, of asset, $A_1$, and assets $A_2$, $A_4$, $A_6$, $A_7$, $A_9$;
- $z_2$ – transport of one type of medium from terminal area C to area B using one out of three pipelines of the subsystem, $S_3$, of asset, $A_1$, and assets $A_2$, $A_4$, $A_8$, $A_9$;
- $z_3$ – transport of one type of medium from terminal area B, through area A, to the pier using one out of two pipelines of the subsystem, $S_1$, and one out of two pipelines of the subsystem, $S_2$, of the asset, $A_1$, and assets $A_2$, $A_4$, $A_5$, $A_9$;
- $z_4$ – transport of one type of medium from the pier, through areas A and B, to area C using one out of two pipelines of the subsystem, $S_1$, one out of two pipelines in subsystem, $S_2$, and two out of three pipelines of the subsystem, $S_3$, of the asset, $A_1$, and assets $A_2$, $A_3$, $A_4$, $A_5$, $A_6$, $A_7$, $A_9$;
- $z_5$ – transport of one type of medium from the pier, through area A, to area B using one out of two pipelines of the subsystem, $S_1$, and one out of two pipelines of the subsystem, $S_2$, of the asset, $A_1$, and assets $A_2$, $A_3$, $A_4$, $A_5$, $A_9$;
- $z_6$ – transport of one type of medium from terminal area B to area C using two out of three pipelines

of the subsystem, $S_3$, and simultaneously transporting one type of medium from the pier, through area A, to area B, using one out of two pipelines of the subsystem, $S_1$, and one out of two pipelines of the subsystem, $S_2$, of the asset, $A_1$, and assets $A_2$, $A_3$, $A_4$, $A_5$, $A_6$, $A_7$, $A_9$;

- $z_7$ – transport of one type of medium from terminal area B to area C using one pipeline, and simultaneously transporting a second type of medium from terminal area C to area B using another one of the three pipelines of subsystem, $S_3$, of the asset, $A_1$, and assets $A_2$, $A_4$, $A_6$, $A_7$, $A_8$, $A_9$.

The main characteristics of the port oil terminal's critical infrastructure operation process, $Z(t)$, are the limit values of the transient probabilities of the operation process, $Z(t)$, at the particular operation states, $z_b$ ($b = 1, 2,\ldots, 7$) (Magryta, 2019):

$$p_1 = 0.395,\ p_2 = 0.060,\ p_3 = 0.003,\ p_4 = 0.002,\ p_5 = 0.20,\ p_6 = 0.058,\ p_7 = 0.282 \quad (27)$$

#### Optimization of the operation process

Assuming that the port oil terminal critical state is $r = 1$, and considering the conditional mean values determined by Magryta (2019):

$$[\boldsymbol{\mu}(1)]^{(1)} = [\boldsymbol{\mu}(1)]^{(2)} = [\boldsymbol{\mu}(1)]^{(7)} \cong 7.85 \text{ years},$$
$$[\boldsymbol{\mu}(1)]^{(3)} = [\boldsymbol{\mu}(1)]^{(5)} \cong 7.19 \text{ years},$$
$$[\boldsymbol{\mu}(1)]^{(4)} = [\boldsymbol{\mu}(1)]^{(6)} \cong 6.64 \text{ years},$$

the objective function defined by equation (3) takes the form,

$$\boldsymbol{\mu}(1) = 7.85\cdot p_1 + 7.85\cdot p_2 + 7.19\cdot p_3 + 6.64\cdot p_4 + 7.19\cdot p_5 + 6.64\cdot p_6 + 7.85\cdot p_7 \quad (28)$$

where the transient probabilities, $p_b$ ($b = 1, 2,\ldots, 7$), are given by (27).

The approximate values of the lower ($\breve{p}_b$) and upper ($\hat{p}_b$) bounds of the unknown transient probabilities, $p_b$ ($b = 1, 2,\ldots, 7$), based on expert opinions are:

$$\breve{p}_1 = 0.31,\ \breve{p}_2 = 0.04,\ \breve{p}_3 = 0.002,\ \breve{p}_4 = 0.001,\ \breve{p}_5 = 0.15,\ \breve{p}_6 = 0.04,\ \breve{p}_7 = 0.25,$$
$$\hat{p}_1 = 0.46,\ \hat{p}_2 = 0.08,\ \hat{p}_3 = 0.006,\ \hat{p}_4 = 0.004,\ \hat{p}_5 = 0.26,\ \hat{p}_6 = 0.08,\ \hat{p}_7 = 0.40 \quad (29)$$

Thus, we assume the following bound constraints:

$$0.31 \le p_1 \le 0.46$$
$$0.04 \le p_2 \le 0.08$$
$$0.002 \le p_3 \le 0.006$$
$$0.001 \le p_4 \le 0.004$$
$$0.15 \le p_5 \le 0.26$$
$$0.04 \le p_6 \le 0.08$$
$$0.25 \le p_7 \le 0.40$$
$$\sum_{i=1}^{7} p_b = 1 \quad (30)$$

Before determining the optimal values, $\dot{p}_b$, of the transient probabilities, $p_b$ ($b = 1, 2,\ldots, 7$), which maximize the objective function, it is necessary to arrange the system conditional lifetime mean values, $[\boldsymbol{\mu}(1)]^{(b)}$ ($b = 1, 2,\ldots, 7$), in decreasing order, which, in this case is:

$$[\boldsymbol{\mu}(1)]^{(1)} \ge [\boldsymbol{\mu}(1)]^{(2)} \ge [\boldsymbol{\mu}(1)]^{(7)} \ge [\boldsymbol{\mu}(1)]^{(3)} \ge [\boldsymbol{\mu}(1)]^{(5)} \ge [\boldsymbol{\mu}(1)]^{(4)} \ge [\boldsymbol{\mu}(1)]^{(6)}$$

Then, according to equation (7), we substitute the values,

$$x_1 = p_1,\quad x_2 = p_2,\quad x_3 = p_7,\quad x_4 = p_3,\quad x_5 = p_5,\quad x_6 = p_4,\quad x_7 = p_6 \quad (31)$$

and

$$\breve{x}_1 = \breve{p}_1 = 0.31,\ \breve{x}_2 = \breve{p}_2 = 0.04,\ \breve{x}_3 = \breve{p}_7 = 0.25,\ \breve{x}_4 = \breve{p}_3 = 0.002,\ \breve{x}_5 = \breve{p}_5 = 0.15,\ \breve{x}_6 = \breve{p}_4 = 0.001,\ \breve{x}_7 = \breve{p}_6 = 0.04$$
$$\hat{x}_1 = \hat{p}_1 = 0.46,\ \hat{x}_2 = \hat{p}_2 = 0.08,\ \hat{x}_3 = \hat{p}_7 = 0.40,\ \hat{x}_4 = \hat{p}_3 = 0.006,\ \hat{x}_5 = \hat{p}_5 = 0.26,\ \hat{x}_6 = \hat{p}_4 = 0.004,\ \hat{x}_7 = \hat{p}_6 = 0.08 \quad (32)$$

and maximize the linear form of equation (28) with respect to $x_i$ ($i = 1, 2,\ldots, 7$). According to the values in (31)–(32), the resulting expression takes the form,

$$\boldsymbol{\mu}(1) = 7.85\cdot x_1 + 7.85\cdot x_2 + 7.19\cdot x_3 + 6.64\cdot x_4 + 7.19\cdot x_5 + 6.64\cdot x_6 + 7.85\cdot x_7 \quad (33)$$

with the following bound constraints:

$$0.31 \le x_1 \le 0.46$$
$$0.04 \le x_2 \le 0.08$$
$$0.25 \le x_3\ 0.40$$
$$0.002 \le x_4 \le 0.006$$
$$0.15 \le x_5 \le 0.26$$
$$0.001 \le x_6 \le 0.004$$
$$0.04 \le x_7 \le 0.08$$
$$\sum_{i=1}^{7} x_i = 1 \quad (34)$$

According to equation (10), we calculate

$$\breve{x} = \sum_{i=1}^{7} \breve{x}_i = 0.793$$

$$\hat{y} = 1 - \breve{x} = 1 - 0.793 = 0.207 \quad (35)$$

and employing equation (11), we further find

$$\breve{x}^0 = 0,\ \widehat{x}^0 = 0,\ \widehat{x}^0 - \breve{x}^0 = 0$$
$$\breve{x}^1 = 0.31,\ \widehat{x}^1 = 0.46,\ \widehat{x}^1 - \breve{x}^1 = 0.15$$
$$\breve{x}^2 = 0.35,\ \widehat{x}^2 = 0.54,\ \widehat{x}^2 - \breve{x}^2 = 0.19$$
$$\breve{x}^3 = 0.60,\ \widehat{x}^3 = 0.94,\ \widehat{x}^3 - \breve{x}^3 = 0.34$$
$$\breve{x}^4 = 0.602,\ \widehat{x}^4 = 0.946,\ \widehat{x}^4 - \breve{x}^4 = 0.3444$$
$$\breve{x}^5 = 0.752,\ \widehat{x}^5 = 1.206,\ \widehat{x}^5 - \breve{x}^5 = 0.454$$
$$\breve{x}^6 = 0.753,\ \widehat{x}^6 = 1.21,\ \widehat{x}^6 - \breve{x}^6 = 0.457$$
$$\breve{x}^7 = 0.793,\ \widehat{x}^7 = 1.29,\ \widehat{x}^7 - \breve{x}^7 = 0.497 \quad (36)$$

Based on the above calculations, expression (12) takes the form,

$$\breve{x}^I - \widehat{x}^I < 0.207 \quad (37)$$

so it is determined that the largest value, $I \in \{0, 2, \ldots, 7\}$, such that this inequality holds true, is $I = 2$.

Therefore, we fix the optimal solution to maximize the linear function of equation (33) according to the rule expressed in (14) and we obtain,

$$\dot{x}_1 = \widehat{x}_1 = 0.46,\ \dot{x}_2 = \widehat{x}_2 = 0.08,$$
$$\dot{x}_3 = \hat{y} - \widehat{x}^2 + \breve{x}^2 + \breve{x}_3 =$$
$$= 0.207 - 0.54 + 0.35 + 0.25 = 0.267$$
$$\dot{x}_4 = \breve{x}_4 = 0.002,\ \dot{x}_5 = \breve{x}_5 = 0.15,$$
$$\dot{x}_6 = \breve{x}_6 = 0.001,\ \dot{x}_7 = \breve{x}_7 = 0.04 \quad (38)$$

Finally, after making the substitution using (31), we find the optimal transient probabilities,

$$\dot{p}_1 = \dot{x}_1 = 0.46,\ \dot{p}_2 = \dot{x}_2 = 0.08,$$
$$\dot{p}_7 = \dot{x}_3 = 0.267,\ \dot{p}_3 = \dot{x}_4 = 0.002,$$
$$\dot{p}_5 = \dot{x}_5 = 0.15,\ \dot{p}_4 = \dot{x}_6 = 0.001,$$
$$\dot{p}_6 = \dot{x}_7 = 0.04 \quad (39)$$

that maximize the port oil terminal system mean lifetime, $\boldsymbol{\mu}(1)$, as expressed by the linear form of equation (28).

#### Optimal reliability indicators

Considering the expressions, (18), (28), and (39), the optimal value of the port oil terminal lifetime, $\boldsymbol{\mu}(1)$, is:

$$\dot{\boldsymbol{\mu}}(1) = 7.85 \cdot \dot{p}_1 + 7.85 \cdot \dot{p}_2 + 7.19 \cdot \dot{p}_3 + 6.64 \cdot \dot{p}_4 + $$
$$+ 7.19 \cdot \dot{p}_5 + 6.64 \cdot \dot{p}_6 + 7.85 \cdot \dot{p}_7 \cong 7.70 \text{ years} \quad (40)$$

Moreover, the corresponding optimal unconditional reliability function defining the port oil terminal critical infrastructure takes the form:

$$\dot{\boldsymbol{R}}(t,1) = 0.46\exp[-0.1274603t] +$$
$$+ 0.08\exp[-0.1274603t] + 0.002\exp[-0.1390476t] +$$
$$+ 0.001\exp[-0.1506349t] + 0.15\exp[-0.1390476t] +$$
$$+ 0.04\exp[-0.1506349t] + 0.267\exp[-0.1274603t]$$
$$\text{for } t \geq 0 \quad (41)$$

Further, considering (40) and (41), the optimal standard deviations of the port oil terminal critical infrastructure unconditional lifetime, in the state subset, is

$$\dot{\boldsymbol{\sigma}}(1) \cong 7.71 \text{ years} \quad (42)$$

Since the port oil terminal system's critical safety state is $r = 1$, its optimal system risk function, according to equation (23) and considering (41), is given by:

$$\dot{\boldsymbol{r}}(t) = 1 - \dot{\boldsymbol{R}}(t,1),\ t \geq 0 \quad (43)$$

Considering expression (24), the moment when the optimal system risk function exceeds a permissible level (for instance, $\delta = 0.05$), is:

$$\dot{\tau} = \dot{\boldsymbol{r}}^{-1}(\delta) \cong 0.39 \text{ years} \quad (44)$$

Based on (40), the port oil terminal critical infrastructure optimal intensities of ageing are defined by:

$$\dot{\lambda}(t,1) = \frac{1}{\dot{\boldsymbol{\mu}}(1)} \cong 0.1299 \quad (45)$$

Considering these intensities of ageing, the optimal coefficient defining the operation process's impact on the oil terminal critical infrastructure intensities of ageing is given as:

$$\dot{\rho}(t,1) = \frac{\lambda^1(t)}{\lambda^0(t)} = \frac{0.1299}{0.1159} \cong 1.1208 \quad (46)$$

Finally, the port oil terminal critical infrastructure resilience indicator, i.e. the coefficient of the port oil terminal critical infrastructure's resilience to influence by the operation process, is

$$\dot{\boldsymbol{RI}}(t,1) = \frac{1}{\dot{\rho}(t,1)} \cong \frac{1}{1.1208} \cong 0.8922 = 89.22\% \quad (47)$$

Comparing the optimal values of the safety indicators given by (40), (42), (44), (45), (46), (47), with their values before optimization (reported by Magryta, 2019) shows:

$$\boldsymbol{\mu}(1) = 7.64 \text{ years}, \boldsymbol{\sigma}(1) = 7.66 \text{ years}, \tau = 0.37 \text{ years},$$
$$\lambda(t,1) = 0.1309, \boldsymbol{\rho}(t,1) = 1.1294, \boldsymbol{RI}(t) = 88.54\%,$$

and these values justify the sensibility of the performed optimization process.

#### New operation strategy proposal

Assuming a system operation time of $\theta = 1$ year $= 365$ days, we can obtain the optimal mean values of the total sojourn times at the particular operation states during this period of operation using (26) and (39):

$$\dot{\hat{M}}_1 = 167.9,\ \dot{\hat{M}}_2 = 29.2,\ \dot{\hat{M}}_3 = 0.73,\ \dot{\hat{M}}_4 = 0.365,$$
$$\dot{\hat{M}}_5 = 54.75,\ \dot{\hat{M}}_6 = 14.6,\ \dot{\hat{M}}_7 = 97.46 \tag{48}$$

The easiest way to change the port oil terminal operational process is to consider the optimal values of the total sojourn times at each operation state (as given by (48)), and attempt to reorganize the process by approaching the real total sojourn times. Essentially, this requires reorganizing the operation process by replacing the total sojourn times, $\hat{M}_b$, of the system at the particular operation states before the optimization determined according to (25) and (27), by their optimal values, $\dot{\hat{M}}_b$, after the optimization, given by (48):

$$\hat{M}_1 = 144.175, \hat{M}_2 = 21.9,\ \hat{M}_3 = 1.095,$$
$$\hat{M}_4 = 0.73,\ \hat{M}_5 = 73,\ \hat{M}_6 = 21.17,\ \hat{M}_7 = 102.93.$$

## Conclusions

The optimization procedure applied to reliability and resilience variables relevant for a port oil terminal critical infrastructure that is influenced by its operational process, provides a practical evaluation of its reliability and supports improvement through developing a new operation strategy. The proposed optimization can be used to improve the operation and reliability of various real critical infrastructures. Further research related to other influencing factors (Torbicki, 2019) and studies focused on solving the problems of critical infrastructure reliability are critical in order to find optimal values of reliability and resilience indicators. These results can also help mitigate accident consequences related to critical infrastructure operations and to enhance its functional resilience in the face of various impacts (Bogalecka, 2019).

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