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APPLICATION OF FIELD MATHEMATICAL MODELS FOR DESIGNING OF WINDINGS OF BRUSHLESS PERMANENT MAGNET MOTORS

ABSTRACT *The article presents a method for the electromotive force (EMF) determination and for correction of a connection diagram of a brushless DC (BLDC) motor winding. An appropriate mathematical field model has been proposed for this purpose. Calculation results are confirmed by experimental researches of the BLDC motor ДБ100-90-1-Р42 with an external rotor.*

Keywords: *brushless DC electric motor, motor with external rotor, mathematical field model, electromotive force, harmonic analysis.*

1. INTRODUCTION

Development of special electric machines (EM) with non-standard active parts as well as designing the EMs that work under extreme operational modes (e.g. at asymmetry of supply voltage) require an advanced analysis of their magnetic fields. The question concerning a correctness of winding connection diagram to ensure an appropriate form (sinusoidal or trapezoidal) and symmetry of phase electromotive force (EMF) arises as well.

The most difficult problem appears during designing process of the EM with winding of a fractional number of slots per the pole and phase:

$$q = \frac{z}{2mp} \neq \text{integer}$$

where z – number of stator slots, m – number of winding phases, p – number of pole pairs.

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Such a type of winding found a wide application in BLDC motors with permanent magnets (PM) [1] as well as in magnetoelectric generators [2].

Analysis of the electromagnetic field in the EM with permanent magnets is the most typical and prospective when the solution is obtained for the real geometry of the EM magnet system, actual current distribution of the winding system and nonlinear properties of ferromagnetic cores.

Calculations of the magnetic field of the BLDC motor with the high-coercitivity PMs are frequently more convenient applying the vector magnetic potential $\boldsymbol{A}$.

For the magnetostatic field, the vector magnetic potential satisfies Poisson's (Laplace's) partial differential equation. For each of the considered domains, the above Poisson's equation has its own particular form [3].

For a linear and anisotropic medium, in the rectangular coordinate system, it has the following generalized form:

$$\frac{1}{\mu_y}\frac{\partial^2 A_z}{\partial x^2}+\frac{1}{\mu_x}\frac{\partial^2 A_z}{\partial y^2}=-j_z \tag{1}$$

The vector of the magnetic flux density $\boldsymbol{B}$ has two components in the plane-parallel field model. The vector of the extrinsic current density $\boldsymbol{j}$ and potential $\boldsymbol{A}$ are perpendicular to the model plane. For the PM, the governing equation in the rectangular coordinate system has the following form:

$$\frac{1}{\mu_y}\frac{\partial^2 A_z}{\partial x^2}+\frac{1}{\mu_x}\frac{\partial^2 A_z}{\partial y^2}=-j_z+\left(\frac{\partial H_{cy}}{\partial x}-\frac{\partial H_{cx}}{\partial y}\right) \tag{2}$$

where μ_x, μ_y – components of the relative magnetic permeability tensor $\boldsymbol{\mu}$; j_z – the axial component of the current density vector.

In an area of non-magnetic gap $\boldsymbol{\mu} = \boldsymbol{\mu}_0$ = const, the equation for vector potential $\boldsymbol{A}$ is simplified and can be written as follows:

$$\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}=0 \tag{3}$$

In non-magnetic areas with currents of stator winding, the equation for $\boldsymbol{A}$ becomes inhomogeneous and assumes the following form:

$$\frac{\partial^2 A_z}{\partial x^2}+\frac{\partial^2 A_z}{\partial y^2}=-\mu_0 j_z(x,y) \tag{4}$$

where $j_z(x,y)$ – the calculated density of the extrinsic current of the stator winding.

In the areas of ferromagnetic cores with nonlinear dependence of the magnetic permeability on the magnetic field strength, the differential equation for $\boldsymbol{A}$ can be expressed as follows:

$$\frac{1}{\mu_y}\frac{\partial^2 A_z}{\partial x^2}+\frac{1}{\mu_x}\frac{\partial^2 A_z}{\partial y^2}=0 \qquad (5)$$

where $\mu = f(H)$.

Boundary conditions that are required for solving the equations (2) – (5) result from physical conditions of the magnetic field existence on the outer boundaries of the area of calculations. The vector magnetic potential at the outer surface of the ferromagnetic stator core is equal to zero ($\boldsymbol{A} = 0$).

Peculiarities of the EM with rare-earth PM such as a linearity of the demagnetization curve, a relatively minor saturation of the magnetic circuit areas, allow to separate magnetic fields created by magnet and armature winding, to study them separately and then to use the superposition method.

Research methods of electromagnetic fields based on the theory of electromagnetic field, are the most prospective ones at designing and optimization of the BLDC motor constructions [4].

Development of computer technologies and application of mathematical tools of different numerical calculation methods have promoted the designing of specialized software (for the magnetic field modelling) such as ELCUT, FEMM or MAXWELL (ANSYS). The application of the above-stated programs shortens considerably the terms of the electromagnetic device designing with simultaneous increasing of calculation accuracy. It also allows to perform optimization of magnet systems at the design stage.

Modelling of the EMF of the winding with fractional q has been performed with the program ELCUT developed by NPKK "TOP", St. Petersburg.

2. MODELLING OF THE EMF OF THE WINDING WITH FRACTIONAL NUMBER OF SLOTS FOR POLE AND PHASE

There are analytical calculation methods of allocation and connections of polyphase windings which are given in [5–7]. At calculation of winding by analytical methods, it is required to make the sufficiently large scope of calculations for determining the EMF harmonic content and winding coefficients.

The modelling algorithm of the winding EMF with the fractional q is made through the example of the winding of the BLDC motor with an external rotor.

The considered electric motor ДБ100-90-1-Р42 has the three-phase tooth winding with number of poles $2p$ = 16, number of slots z = 18, number of phases m = 3 and number of slots per pole and phase:

$$q=\frac{z}{2mp}=\frac{18}{16\cdot 3}=0.375 \qquad (6)$$

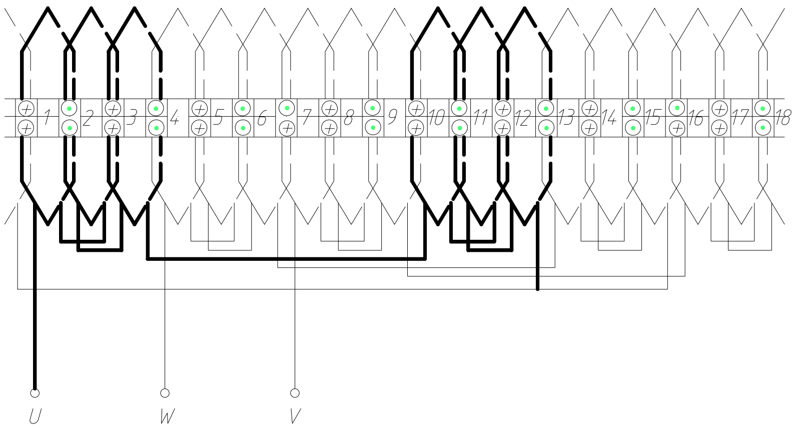

Fig. 1. Winding diagram of electric motor ДБ100-90-1-Р42

A winding diagram of the electric motor ДБ100-90-1-Р42 is shown in Fig. 1. For calculation of the EMF using the field mathematical model, it is required to determine the flux linkage of corresponded phase blocks.

A generalized algorithm for calculations of the phase flux linkages by means of the program ELCUT [8] is as follows:

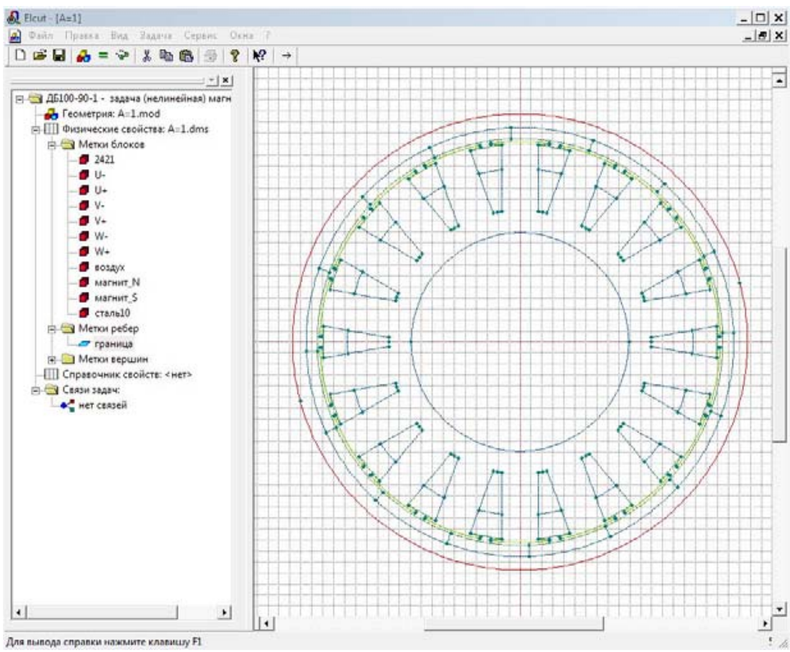

Fig. 2. Setting of field mathematical model in ELCUT

a) field mathematical model of the cross-section of the electric motor magnetic system (with setting the basic units, their characteristics and boundary conditions) is set (Fig. 2);
b) elements of phases *U+, U•, V+, V•, W+, W•* are set accordingly to the winding diagram and current directions;
c) magnetic field sources are set - permanent magnets N and S of polarity;
d) calculation of model is done;
e) on completing the calculation, on pattern of magnetic field, the integration contour by each phase separation (*U+, U•, V+, V•, W+, W•*) with evaluation of each phase flux linkage (ψ_{U+}, $\psi_{U\bullet}$, ψ_{V+}, $\psi_{V\bullet}$, ψ_{W+}, $\psi_{W\bullet}$) in integrated calculator is set;
f) for getting the relationships $\psi = f(\varphi)$ (where φ – turning angle of rotor), it is required to set the rotor spinning in the program LabelMover.

The transient values of the phase EMF are calculated by formulas (7) – (9) [9].

$$e_U = (\psi_{U+} - \psi_{U\bullet}) \cdot \frac{l_{cp} \cdot n \cdot W \cdot p}{9{,}55} \tag{7}$$

$$e_V = (\psi_{V\bullet} - \psi_{V+}) \cdot \frac{l_{cp} \cdot n \cdot W \cdot p}{9{,}55} \tag{8}$$

$$e_W = (\psi_{W\bullet} - \psi_{W+}) \cdot \frac{l_{cp} \cdot n \cdot W \cdot p}{9{,}55} \tag{9}$$

where l_{cp} – average length of the electric motor active parts, *m*; *n* – speed of rotation, rpm; *W* – number of phase turns; *p* – number of pole pairs.

Diagrams of the phase EMF of the electric motor ДБ100-90-1-Р42 depending on the turning angle of the rotor in electric degrees are shown in Fig. 3.

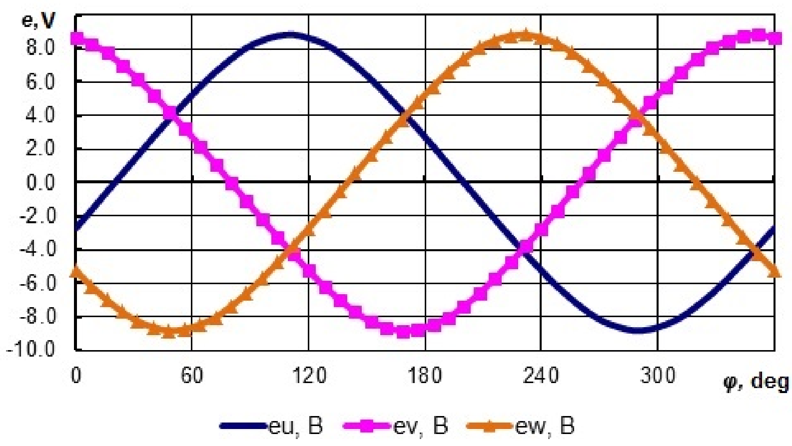

Fig. 3. Phase EMF of electric motor ДБ100-90-1-Р42 depending on turning angle of rotor

Consistency of curves as well as a displacement angle of the phase EMF which is equal to 120 electric degrees indicates the correctness of the section connections and the winding symmetry.

The maximum value of the transient value of the phase EMF is its peak value. The RMS value of the phase EMF is:

$$E_{ph} = \frac{E}{\sqrt{2}} \tag{10}$$

The RMS value of the line EMF is:

$$E_{lin} = \frac{\sqrt{3}E}{\sqrt{2}} = \sqrt{3}E_{ph} \tag{11}$$

The harmonic analysis of the EMF curves has been done by means of Fourier's series expansion [10] by formula:

$$e(\varphi) = \sum_{k=1}^{\infty}\left(e_c \cos\left(\frac{k\pi\varphi}{180\tau}\right) + e_s \sin\left(\frac{k\pi\varphi}{180\tau}\right)\right) \tag{12}$$

where k – serial number of harmonic; τ – pole pitch.

Amplitudes of cosine and sine harmonics e_c and e_s are determined as:

$$\begin{aligned} e_c &= \frac{1}{\tau}\int_0^{2\tau} e(\varphi)\cos\left(\frac{k\pi\varphi}{180\tau}\right) d\varphi \\ e_s &= \frac{1}{\tau}\int_0^{2\tau} e(\varphi)\sin\left(\frac{k\pi\varphi}{180\tau}\right) d\varphi \end{aligned} \tag{13}$$

As a result of the harmonic analysis of the EMF curves, the basic harmonic of the first order and amplitudes of harmonics of high order have been identified. Results of the above calculations are given in Table 1.

The amplitude of the third order harmonic of the EMF is 3 % of the amplitude of the first order harmonic. The amplitudes of the rest harmonics of high order are of 0.2% and less. As a result of the above harmonic analysis, it is possible to make conclusion on non-expediency of using the meaningful measures for suppression of high-order harmonics in the EMF curve of the electric motor ДБ100-90-1-Р42.

TABLE 1
Harmonic analysis of the EMF curve

Number of harmonic	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Amplitude of harmonic, %	100	0,004	3,067	0,001	0,206	0,002	0,044	0,003	0,002	0,006
Number of harmonic	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Amplitude of harmonic, %	0,034	0,003	0,011	0,002	0,069	0,002	0,108	0,002	0,065	0,001
Number of harmonic	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**
Amplitude of harmonic, %	0,017	0,001	0,001	0,009	0,001	0,019	0,001	0,014	0,001	0,004

3. EXPERIMENTAL RESEARCHES

Adequacy of the field mathematical model was confirmed at experimental researches of the BLDC motor ДБ100-90-1-Р42 with the external rotor.

Oscillograms of the phase EMF of the above motor winding are shown in Fig. 4. These oscillograms indicate the correspondence of experimental parameters with calculated ones.

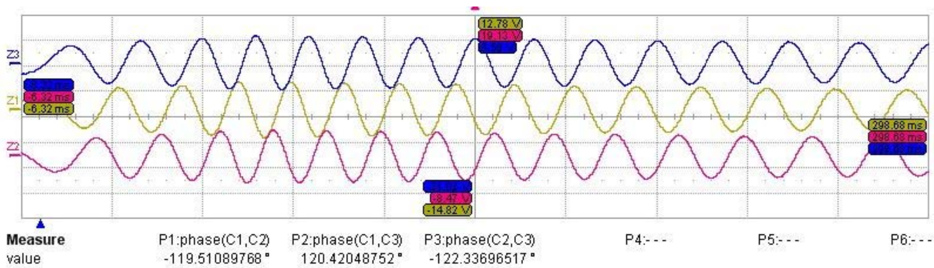

Fig. 4. Oscillograms of phase EMF of electric motor ДБ100-90-1-Р42 winding

4. CONCLUSIONS

- The field mathematical model allows to evaluate (in the process of designing the new electric motor) the correctness of plotting the diagram of the polyphase winding and to evaluate the form and symmetry of the phase EMF.
- As a result of the harmonic analysis, it was revealed that the conducting of meaningful measures for suppression of the EMF high-order harmonics in the electric motor ДБ100-90-1-Р42 is unnecessary.
- The calculation results have been confirmed by the experimental researches concerning the motor under consideration.

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ZASTOSOWANIE MATEMATYCZNYCH MODELI POLA W PROJEKTOWANIU UZWOJEŃ BEZSZCZOTKOWYCH SILNIKÓW Z MAGNESAMI TRWAŁYMI

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STRESZCZENIE *W artykule przedstawiono metodę wyznaczania siły elektromotorycznej bezszczotkowego silnika prądu stałego, dla sprawdzenia poprawności połączeń w jego schemacie uzwojenia. W tym celu zaproponowano odpowiedni model matematyczny pola. Wyniki obliczeń numerycznych potwierdzono badaniami eksperymentalnymi bezszczotkowego silnika z wirnikiem zewnętrznym typu ДБ100-90-1-P42.*

Słowa kluczowe: *bezszczotkowy silnik elektryczny prądu stałego, silnik elektryczny z zewnętrznym wirnikiem, model matematyczny pola, siła elektromotoryczna, analiza harmoniczna*