

Mateusz Jocz*, Marek Lefik

Correlation between Cone Penetration Test parameters, soil type, and soil liquidity index using long short-term memory neural network

<https://doi.org/10.2478/sgem-2023-0023>

received March 3, 2023; accepted October 4, 2023.

Abstract: Accuracy and quality of recognizing soil properties are crucial for optimal building design and for ensuring safety in the construction and exploitation stages. This article proposes use of long short-term memory (LSTM) neural network to establish a correlation between Cone Penetration Test (CPTU) results, the soil type, and the soil liquidity index I_L . LSTM artificial neural network belongs to the class of networks requiring deep machine learning and is qualitatively different from artificial neural networks of the multilayer perceptron type, which have long been widely used to interpret the results of geotechnical experiments. The article outlines the methodology of CPTU testing and laboratory testing of the liquidity index, as well as construction and preparation of data for the network. The proposed network achieved good results when considering a database consisting of the parameters of eight CPTU soundings, soil stratifications, and laboratory test results.

Keywords: geotechnical parameters; Cone Penetration Test (CPTU); liquidity index; Long Short-Term Memory (LSTM) neural network.

1 Introduction

Ground investigations play a crucial role in the process of designing and constructing safe and durable structures such as buildings, bridges, roads, and other infrastructure

elements. Knowledge of the ground conditions allows for the assessment of the interaction between the foundations and the soil, enabling the designer to adopt the optimal structural solution. Knowledge of the ground conditions is also necessary during the construction phase of the object, as it enables the prediction and prevention of adverse phenomena that may occur in the soil, such as excessive settling, landslides, or liquefaction. Preventing these phenomena ensures the safety and efficiency of the work being carried out.

Soil is a complex engineering material that has been formed by a combination of various geological, environmental, and physicochemical processes [5]. As a result of these processes, the soil exhibits anisotropic, nonhomogeneous geotechnical properties, which additionally change with depths and over time. Complexity of the soil structure makes recognition of ground conditions a complicated task. Obtaining information about the soil is carried out by conducting tests in both field and laboratory. Currently, one of the most commonly performed field tests is the Cone Penetration Test (CPTU). Geotechnical experiments usually measure physical quantities that are possibly well correlated with the values of geotechnical soil parameters needed in the design process. This is also the case with the CPTU test. Here, the measured physical quantities are: resistance on the cone, the value of the pore pressure in the cone filter and the friction on the side of the sleeve located above the cone. These data depend not only on the type of soil, but also on the location of tested points at which the cone registers the data during its movement into the ground. There are numerous techniques that allow to correlate the values of these measured quantities with the values of geotechnical parameters, such as oedometric modules, cohesion, angle of internal friction, and many others needed in the design of the foundation, but they provide underfitted results or the ones that are fitted only locally. Correlations can be established through approximations made using mathematical functions and another method

*Corresponding author: **Mateusz Jocz**, student at Interdisciplinary Doctoral School, Lodz University of Technology, Żeromskiego 116, 90-924 Łódź; Division of Geotechnics and Engineering Structures Department of Concrete Structures Lodz University of Technology, Al. Politechniki 6, 90-924 Łódź, E-mail: mateusz.jocz@dokt.p.lodz.pl
Marek Lefik, Division of Geotechnics and Engineering Structures Department of Concrete Structures Lodz University of Technology, Al. Politechniki 6, 90-924 Łódź

involving graphical and tabular interpretations. The article proposes an approximation method using artificial neural networks (ANNs), which has the advantage of incorporating all measurements recorded by the CPTU probe, as well as other additional information such as soil type. ANN functionalities are well suited to solve the problem of establishing a complex relationship of many variables, which cannot be written in a simple way. In this article, using an ANN, based on the measurement data obtained as a result of the use of CPTU, we determine the type of soil and its state characterized by the liquidity index I_L . The choice of parameters is justified mainly by the fact that the appropriate amount of data needed to establish the correlations has been collected for them.

The ANN we adopt here is known as the Long Short-Term Memory (LSTM) neural network (for more about LSTM, see [10], [11], [19]). It is an ANN that is qualitatively different from that usually used in geotechnical applications layered networks trained using the error back propagation method (multilayer perceptrons [MLPs]). Instead of the input and output layers in MLP, associated with the input data and their image at the output of the network, in the LSTM network, an input data stream is distinguished, which, after applying one of the deep learning algorithms, is transformed into an output data stream. Each element of the output stream depends both on the value of the corresponding element of the input stream, as well as on the values of parameters in its narrow or wide neighborhood. The input sequence here is a sequence of pressure values on the measuring cone and friction on the measuring sleeve at each of the successive cone penetration depths. The output stream is the value of the liquidity index and the number assigned to the type of soil in this point of the ground. The LSTM network is usually used for automatic text translation. In our approach, a well-trained LSTM network “translates” the sequence of triples of measured values into the sequences interpreted as the value of the liquidity index and the type of soil at corresponding depths.

The paper is organized as follows: in the second section, LSTM network is shortly characterized; in the third section, we describe the process of experimental data acquisition; in the fourth section, the data preparation procedure is explained. The fifth section presents the application of the LSTM network to calculate the liquidity index and to determine the type of soil, based on the CPTU probe tests. The work ends with conclusions summarizing the presented method of interpretation of measurement data.

2 LSTM network

ANN is a computational tool that is part of a field of computer science called machine learning. ANN works in an algorithmic way on the database which contains a description of a physical phenomenon at hand. The database splits into two sets: input data and output data, related to the input data by the relationships usually not known. The result of an ANN’s activity is a multivariate approximation function that approximates this unknown relation and is able to predict new output for any newly entered input. It is to be highlighted that ANN itself discovers the dependencies between the input and output data during the learning process (shaping of the internal parameters – weights and biases) in training. ANNs with the structure of an MLP have been used in geotechnics for more than 30 years. A review of various applications of MLP in geotechnics can be found in [1]. The success of this neural network is based on the fact that it is a very good approximator of a function or an operator. The best approximation of the values of this functional relation can be found by training based on examples of approximate functional dependence (see [2]). In [4], CPTU test parameters with auxiliary data were used to detect engineering parameters, for example, overconsolidation ratio (OCR), K_o , M , c_u . In [9], CPTU soundings data were used to predict OCR. In this paper, Sulewska used neural networks also for the determination of potential soil liquefaction, prediction of foundation settlement, evaluation of bearing capacity of piles, prediction of compaction parameters for cohesive soil, and compaction control of embankments built of noncohesive soils. Some examples of ANN applications in geotechnics can be found in [12], [16], [18], and [20]. In these works, as in most geotechnical applications, ANNs are used as a tool for discovering and recording various constitutive relationships formulated in frame of the mechanics of fragmented media.

Unfortunately, a classical ANN MPL cannot be used for interpretation of continuous series of measured data coming from CPTU or similar devices, since it cannot interpret sequences of the data with variable length, for which the order of the data is important. Moreover, for interpretation of the current measurement point, the input data far from it should be less important than the data taken from its close neighborhood.

One of the solutions that is not susceptible to the above limitations is the LSTM network, first presented in [10]. LSTM is a recurrent ANN characterized by the ability to find patterns in the input sequence of data, commonly used for analysis of long sequences of data, in particular,

in automatic translation of one sequence of data into another data sequence (sequence2sequence problems). The most common application of LSTM ANN is translating of a string of words in one language into a string of words in another language. LSTM simply replaces the words in one language with the words that appeared in the given context in the training pattern – a set of examples of correct translations furnished in the training process. In the application proposed in this paper, the first string (the input string) is the sequence of pairs of data registered by CPTU: resistance pressure under the cone and friction on the sleeve on each consecutive depth. The output sequence (result of the “translation”) is the number identifying the soil type and the value of liquidity index I_L .

The activity of the LSTM network is assured by three internal subnets of simple structure of sigmoid layers. The first subnetwork has to decide what information we are going to throw away from the current cell state. This sigmoid layer is called the “forget gate layer.” The next sigmoid layer has to decide what new information we are going to store in the cell state. This sigmoid layer, called the “input gate layer,” decides which values will be updated. Finally, we need to decide what we are going to output. This output will be based on the current cell state, which is influenced by the short and long memory states of the previous elements of the sequence. The third sigmoid layer decides which parts of the cell state we are going to output. This process is corrected by comparison of the current output value with the needed target value in the process of training. The weights and biases of the sigmoid layers are changed according to this error by an assumed minimization procedure.

Architecture of LSTM type networks, significance, and selection of hyperparameters such as batch size, learning rate, type of neurons activation functions, number of neurons and layers, and number of epochs are described in [15] and [19]. The interested reader will find there some other references for works with varying levels of detail, theoretical or related to different practical applications of LSTM. Detailed analysis of the structure and actions of LSTM can also be found in many internet pages, for example, [11]. To our knowledge, this kind of ANN has not been applied yet in geotechnical problems.

The advantage of LSTM is that it takes into account the time factor. This means that the network, when performing computational operations on current data, takes into account the state from the previous step. The network thus understands the data as a sequence, where order matters.

In LSTM networks, an important advantage is that the problem of vanishing and exponential gradients has been

eliminated. The problem is that during backpropagation, that is, in the network learning process, fixed weights between neurons may become practically insensitive to change or their values may start to grow exponentially. The expiration of network weight changes results in the fact that a given computational step will be forgotten and will not affect the later one (this applies to longer-input sequences). To overcome this problem, LSTM networks use a gating system called long-term memory that has been described roughly above in this section.

3 *In situ* and laboratory data acquisition methodology

Static CPTU probing involves pushing a cone into the ground (Figure 1) under the influence of machine pressure at a constant speed (approximately 2 cm/s). During the test, information about the resistance on the cone q_c , and the sleeve friction f_s , is transmitted to the computer. The probe also has a sensor for the pore water pressure in the soil u_2 . Basic parameters such as q_c , f_s , and u_2 are recorded from the surface every 1 cm to the planned depth of the sounding, which typically ranges from 3 to 20 m.

The widespread use of CPTU testing in geotechnics is due to the advantages that this method provides compared to other field tests. Thanks to the mechanized system of cone penetration and automatic data recording, CPTU probing has high accuracy of measured data and is practically immune to errors arising from the operator’s

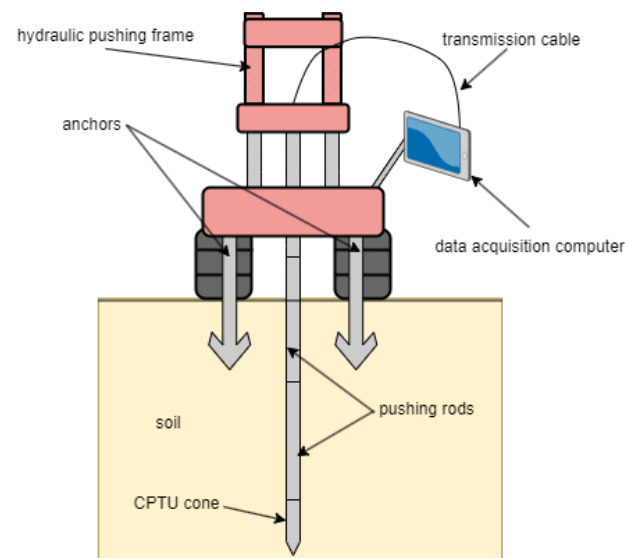


Figure 1: CPTU probe test scheme.

work. The test is conducted *in situ* and it directly tests the ground in the place where the building will be founded. The CPTU test has a continuous character, in contrast to the point data obtained from laboratory tests of single samples. The advantage of CPTU probing is also its relatively short time and low cost.

For the purposes of the presented research, the CPTU test was associated with execution of control geological drillings (control geological drillings are often used in practice, in parallel with the CPTU test). A geologist distinguishes layers of soil and recognizes the type of soil in each of them. This field investigation was completed by laboratory tests for Atterberg limits, moisture content, sieve and oedometer analysis.

The liquidity index is calculated based on laboratory testing of cohesive soil samples taken from the boreholes. It is a dimensionless parameter computed according to the well-known equation (1):

$$I_L = \frac{w_n - w_p}{w_L - w_p} \quad (1)$$

Determination of liquidity index was performed with the same method, in the same laboratory, by two different researchers, under constant conditions, on the same equipment. Liquidity limit was determined using Casagrande apparatus. The change of the operator had no effect on the variability of the results (the same statistical dispersion for both operators).

4 Preparation of data

On the designated area, eight CPTU tests were carried out with an average distance of 50 m between them. The depth of sounding ranged from 5.4 to 17.5 m. At a distance of approximately 2 m from each sounding, a geotechnical borehole was drilled to determine the layers of the soil and to obtain samples for laboratory testing. For each layer, 5–13 samples were taken and their liquidity index was tested. The assumed I_L value for the layer is the arithmetic mean of the results. The test results for one of the research points are shown in Figure 2.

As shown in Figure 2e, the liquidity index graph is constant within the designated geotechnical layer. This is insufficient to build a neural network and inconsistent with engineering intuition. The liquidity index has varying values within the layer. To obtain appropriate data, several assumptions had to be made.

The proposed correlations between the liquidity index and the probing results, including the normative correlation [6], are a function of the cone resistance and the type of soil ($I_L = f(q_c, \text{ground type})$). The relationship is inversely proportional to cone resistance. The lower the q_c value, the higher the liquidity index. This first assumption is commonly seen in the classical literature, for example, in [3] or in [8].

The second assumption involves the statistical data from laboratory elaboration of CPTU results. The average coefficient of variation for determining the liquidity index for all layers was $V_{ILav}=0,093$ (Equation 2)

$$V_{ILav} = \frac{\sum_i^n V_{ILi}}{n} \quad (2)$$

where $V_{ILi} = \frac{\sigma_i}{I_{Lavl}}$ is a coefficient of variability in the layer, σ_i stands for standard deviation in the i^{th} layer, and I_{Lavl} is an average liquidity index in the layer.

The average coefficient of variation for cone resistance q_c assigned to the same layers is significantly higher and equals $V_{qcav}=0,1459$.

The assumption was made that within the layer, the liquidity index changes proportionally to the coefficient of variation of a single measurement of the cone resistance $V_{qci} = \frac{\sigma_{qci}}{q_{cavl}} = \frac{q_{cavl} - q_{ci}}{q_{cavl}}$ relative to the average coefficient of variation $V_{qcav}=0,1459$ (description of this proportionality is $\frac{V_{ILav} \cdot V_{qci}}{V_{qcav}}$), but no more than the average coefficient of variation of the liquidity index $V_{ILav}=0,093$. Due to the occurrence of point peaks in q_c values within the layer and its greater variability than I_L , this approach prevents assigning a single q_{ci} value of the liquidity index I_{Li} , which is not physically possible. This also means that for q_{ci} values that differ significantly from the average, the value of I_L has a constant character. This relationship is expressed below:

$$\begin{cases} \text{for } q_{ci} \geq q_{cavl} & I_{Li} = \max[I_{Lavl}(1 - V_{ILav}); I_{Lavl}(1 - \frac{V_{ILav} \cdot V_{qci}}{V_{qcav}})] \\ \text{for } q_{ci} < q_{cavl} & I_{Li} = \min[I_{Lavl}(1 + V_{ILav}); I_{Lavl}(1 + \frac{V_{ILav} \cdot V_{qci}}{V_{qcav}})] \end{cases} \quad (3)$$

where: I_{Lavl} - average liquidity index for the layer; q_{cavl} - average cone resistance for the layer.

The above relationship was applied to the results of all eight field research points. Finally, the liquidity index charts were transformed, as seen in the example shown in Figure 3.

The transformation procedure was performed for all research points.

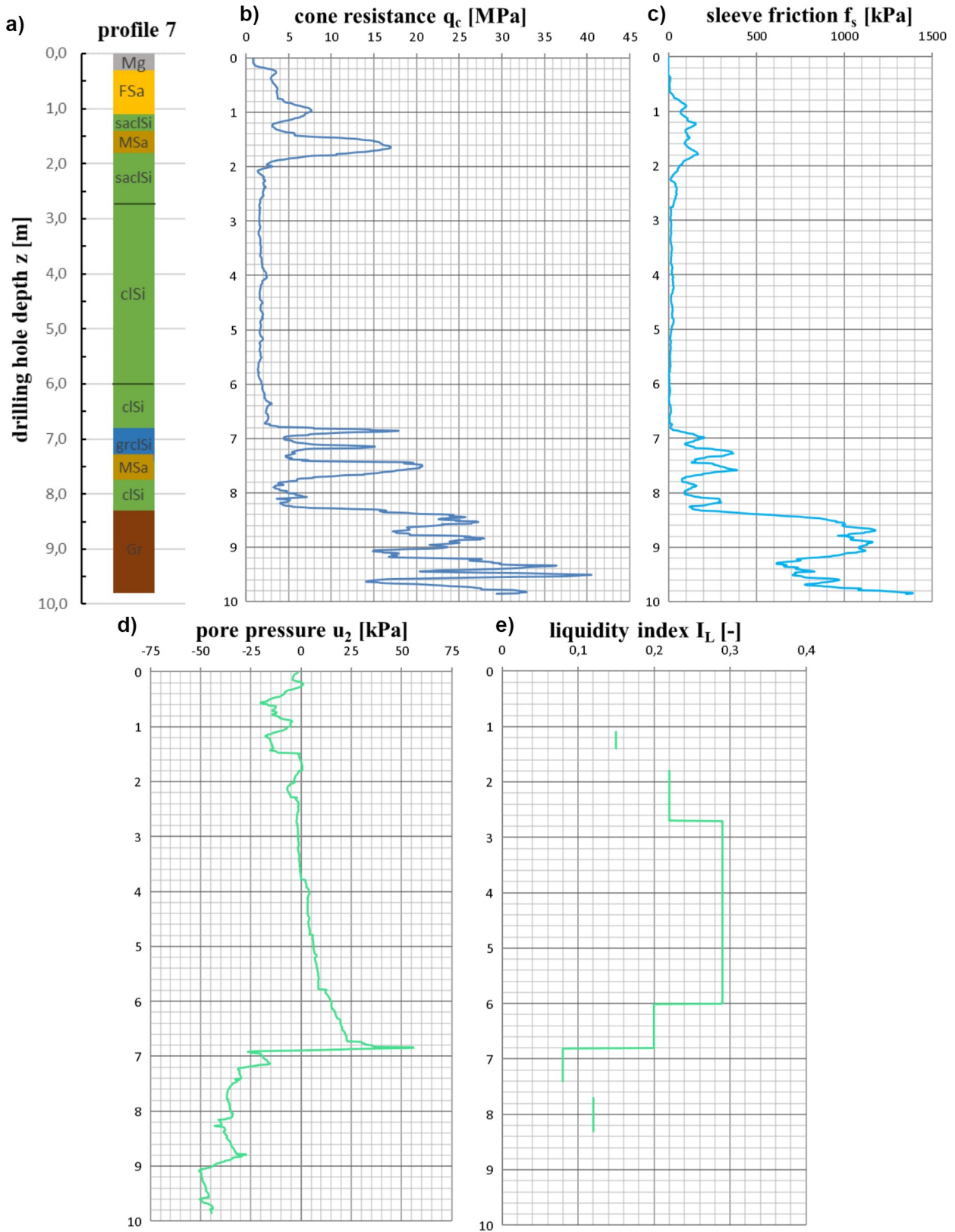


Figure 2: An example of a research point consisting of: a) a cross section of the borehole, where Sa – sand, Si – silt, Cl – clay, Mg – made ground, Gr – gravel; b–d) graphs of the basic parameters of CPTU probing, such as q_c , f_s , and u_2 ; e) a graph of the calculated laboratory values of the liquidity index I_L for the selected layers.

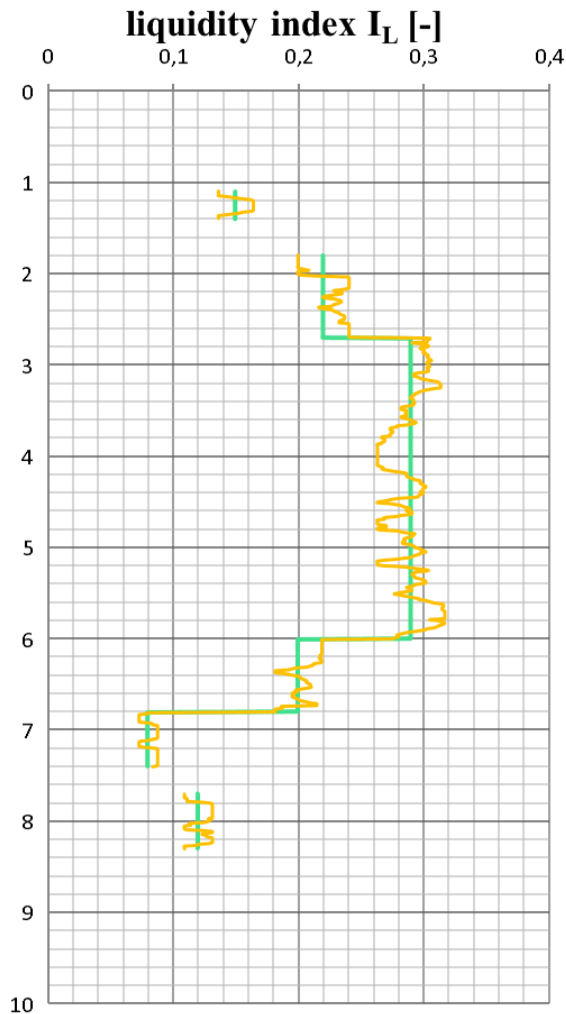


Figure 3: Liquidity index I_L before (green line) and after (orange line) transformation for the example research point.

5 Application of the LSTM network for interpretation of *in situ* geotechnical measurements

5.1 Prediction of values of liquidity index

As a tool for establishing the correlation between I_L and CPTU test results, a LSTM neural network was adopted using the PyTorch environment. It provided better results than a simpler perceptron network. The data used to build the network were the results of eight CPTU tests, the types of layers determined within the tests, and eight liquidity index charts determined according to the model in the previous section. The highest accuracy of the network was achieved by taking the basic CPTU test parameters

Table 1: Division of soil types into categories.

Soil types	Symbol	Category
Made ground	Mg	1
Fine sand	FSa	2
Medium sand	MSa	3
Glacial till	siSa, clSa, sasiCl	4
Settled deposits	clSi, saSi, Si,	5
Settled deposits with gravel	grsaSi, grclSi	6
Gravel	Gr	7

$x_{1i} = q_{ci}$, $x_{2i} = f_{sp}$ and $x_{3i} = z_i$ (depth) as input data, along with the recognized type of cohesive soil $x_{4i} = r_i$, which was classified according to Table 1 (categories 4, 5, and 6). The output value of the network is the liquidity index I_{Li} .

The results of the eight research points were combined while maintaining the order of CPTU measurements according to the depth of the test. The designed network consists of 5297 sets of input and output data. Of these, 4500 initial sets were chosen for network training and the remaining sets were used for testing the network. The network was trained until the mean squared error of predicted values relative to the output data reached the lowest value. Mean Square Error (MSE) of the result obtained was 0.00156.

Figure 4 shows the results of the designed LSTM network. The green dashed line represents the beginning of the data used for testing the network. The red dashed line represents the introduced output data - the liquidity index, and the blue dashed line represents the results of the trained network. Small oscillations of predicted values are natural because there are often interlayers or admixtures of other soils that differ from the parameters of the determined layer in the soil.

5.2 Comparison of obtained results with existing correlations

Below, we present a preliminary comparison of the prediction obtained using LSTM ANN with the one resulting from correlation proposed by the standard document PN-B-04452:2002. The blue line on the graph represents the laboratory results. It can be seen that the prediction results are better than those proposed in the standard document PN-B-04452:2002 given by the equation

$$I_L = 0.581 - 0.0653 \log q_c \quad (4)$$



Figure 4: Results of identification of liquidity index with the developed LSTM network in comparison with known values of the liquidity index. Explanation in the text.

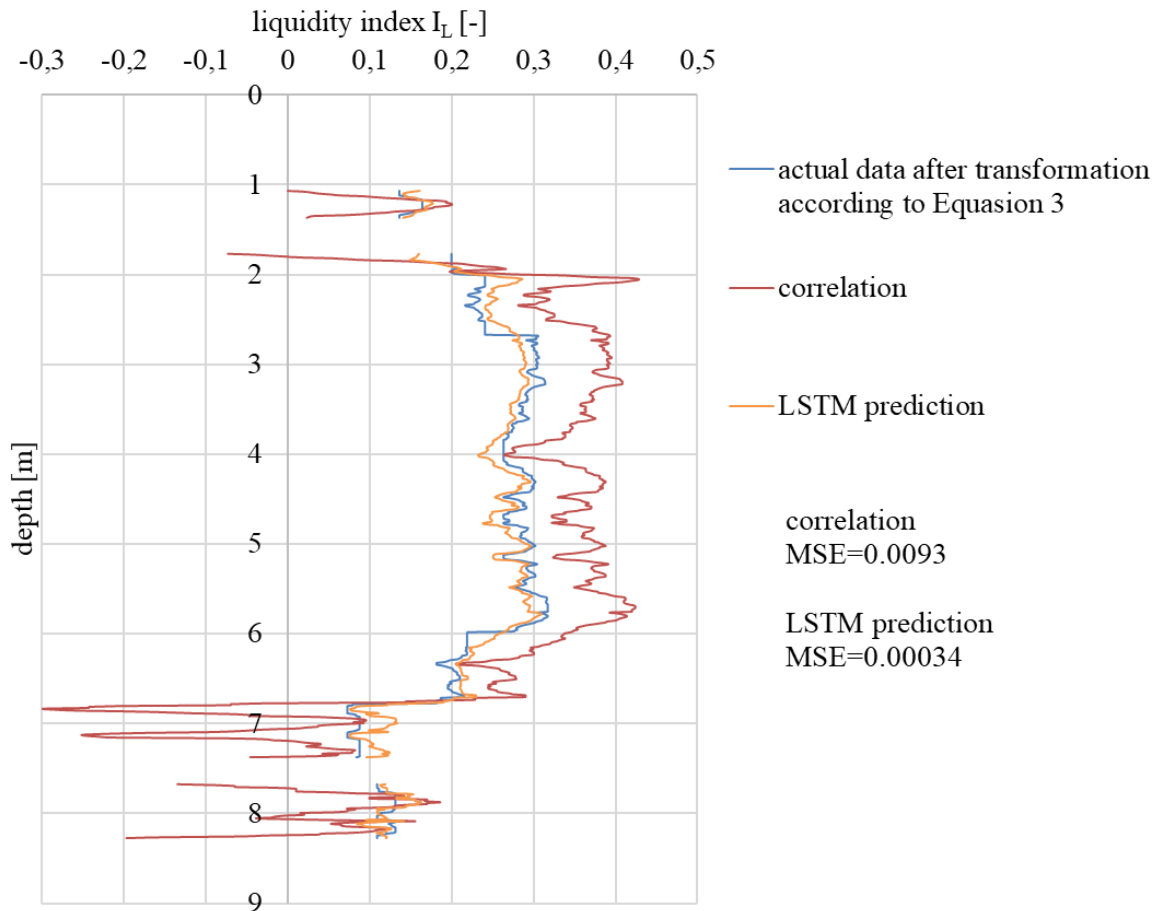


Figure 5: Comparison of the obtained results with the correlation proposed by PN-B-04452:2002 and with laboratory results transformed by Equation 3 for profile number 7.

(for soils with fraction of clays $f_i=10\%-30\%$, but only such soils appear in all probing).

The prediction accuracy of designed LSTM is similar for the remaining profiles. It is presented in Figure 4.

5.3 Identification of the soil type

The basic parameters $x_{1i} = q_{ti}$, $x_{2i} = f_{si}$, $x_{3i} = z_p$, which were the results of the eight CPTU tests, were adopted as input values for designed LSTM network, while the output values were the soil type recognized in parallel boreholes (Figure 5). The soil type was divided into categories according to Table 1.

The designed LSTM network consists of 8060 input and output data sets. For network training, 6200 initial sets were selected, while the remaining sets were used for testing the network. The data were entered chronologically to borehole profiles. Figure 6 shows the results of the designed LSTM network. The mean squared error of

predicted values relative to the output data reached $MSE = 0.0104$.

Network has difficulty in recognizing some layers, but it eventually assigns a similar type of soil, as, for example, in the third borehole, where gravels at a depth of 5.7 m were classified as medium sands. In some points, the network assigns a non-integer number, but rounding these values results in a well-predicted layer. Minor oscillations in the predicted values may result from naturally occurring small geological differences within the layers.

6 Conclusions

Two classical LSTM recurrent ANNs have been created to interpret the results of CPTU. The first one has been trained to establish the correlations between the recognized type of cohesive soil, measured pressure at the cone, and friction on the sleeve for consecutive depths (r_c , q_c , f_s , and

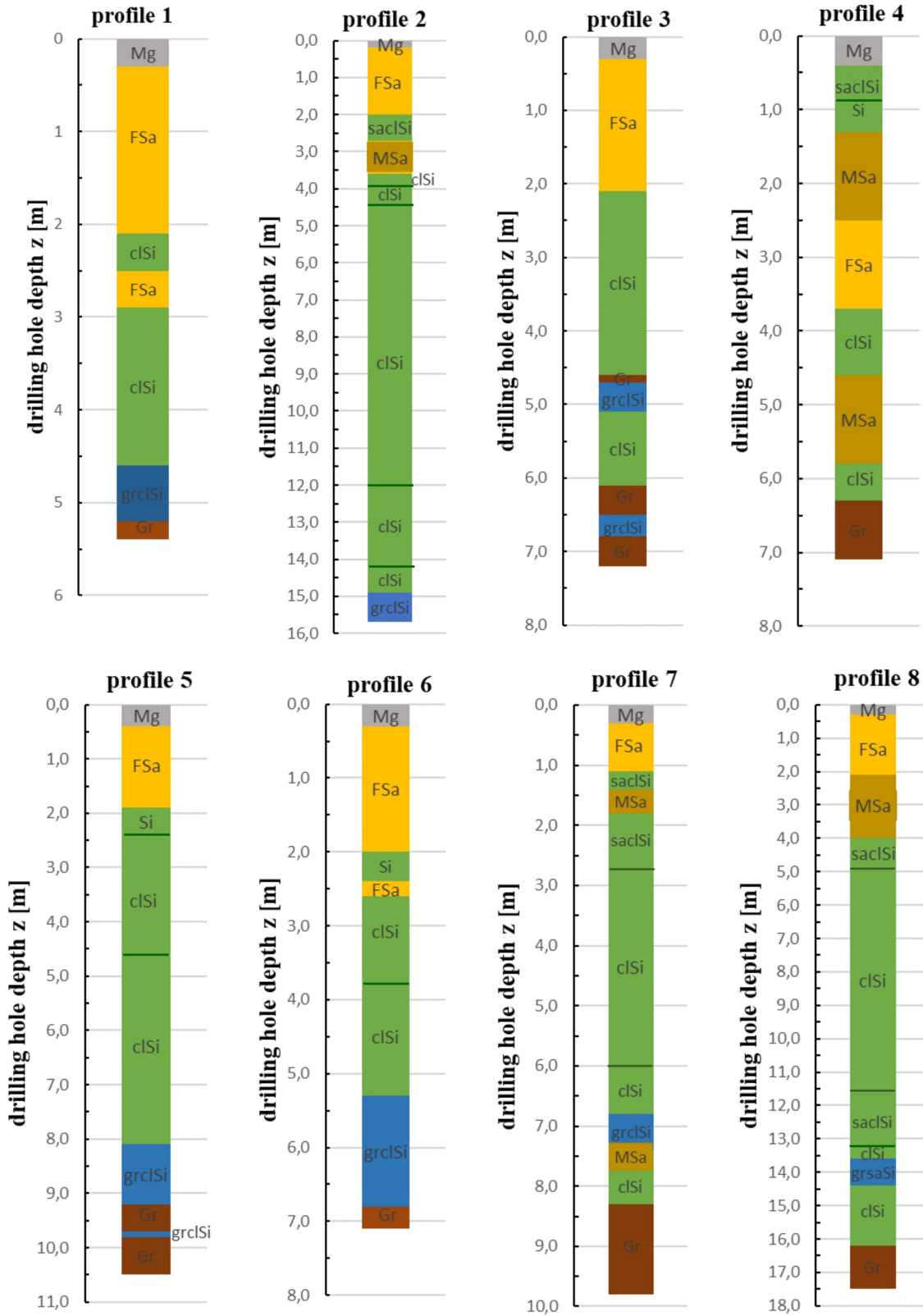


Figure 5: Borehole profiles.

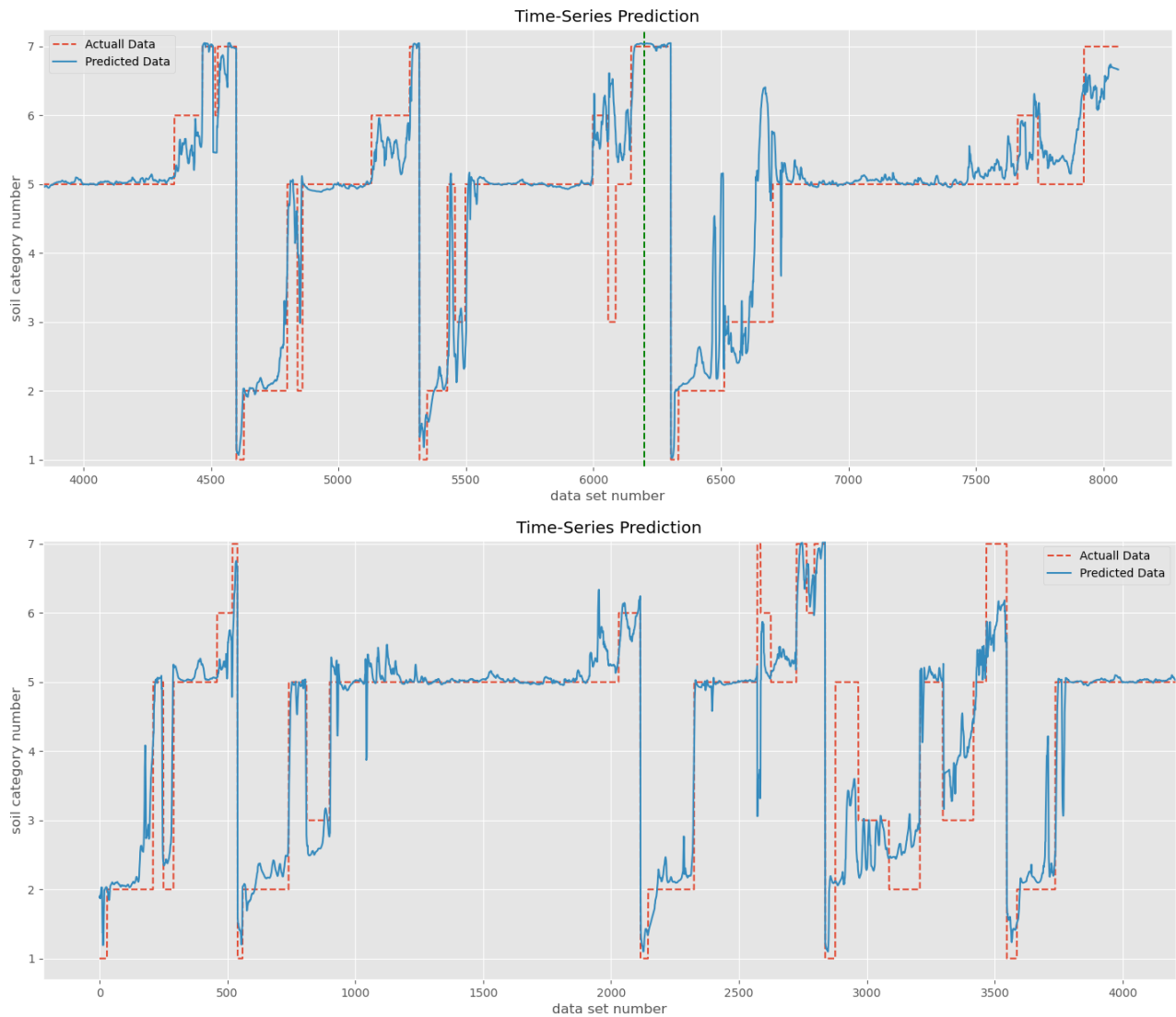


Figure 6: Results of identification of soil type with the developed LSTM network (predicted data) in comparison with known borehole profiles (actual data). The profile includes the following data numbers: profile 1 (0–540), profile 2 (540–2115), profile 3 (2115–2837), profile 4 (2837–3547), profile 5 (3547–4600), profile 6 (4600–5318), profile 7 (5318–6304), profile 8 (6304–8060).

z), with the liquidity index I_L . The second one is intended to recognize the soil type having as the input the same triple of data issued from cone probe (q_c , f_s and z). The network has been trained using the data collected by the first author from his own field investigations.

We believe that the choice of the LSTM type network is appropriate for the interpretation of measurement data strings obtained as a result of probing with the CPTU probe, because during the training phase, the network takes into account the “context” of each single measurement. Therefore, the results of the interpretation show the correct division of the ground area into layers with similar properties. The use of the LSTM type neural network, qualitatively different from the MLP type ANN,

is an element of novelty in the application of ANNs in geotechnics. We observe that the quality of correlation modeled in the LSTM network is good. The final results perform well when considering the database consisting of the parameters of eight CPTU tests and laboratory test results. The accuracy of the network can be improved by increasing the set of input and output data. The LSTM network trained with results of a representative number of surveys will allow for a quick interpretation of the survey results, obtained immediately after completion of the probing.

The use of a transformation equation (Equation 3) for the output data of the LSTM network (liquidity index), which is intended to attempt to reproduce the natural

variability of a geotechnical parameter in the soil, resulted in an improvement in the effectiveness of the network by 20% compared to the same output data, but without their transformation (constant liquidity index in the layer).

It should be mentioned that, apart from the selection of input data, the adopted network architecture also has a significant impact on the quality of network prediction. Because this is a complicated IT issue, it was discussed only superficially in the article.

We are sure that the idea of application of LSTM type ANN is promising in solving many other engineering problems in geotechnics.

References

- [1] Ghaboussi J., Garrett J.H., Wu X.: Knowledge-Based Modeling of Material Behavior with Neural Networks, *Journal of Engineering Mechanics*, 117, s. 132-151 (1991).
- [2] Lefik M., Some aspects of application of artificial neural network for numerical modeling in civil engineering, *Bulletin of the Polish Academy of Sciences. Technical Sciences*, pp. 39-50. (2013).
- [3] Mayne, P.W. (2014). KN2: Interpretation of geotechnical parameters from seismic piezocone tests. *Proceedings, 3rd International Symposium on Cone Penetration Testing (CPT'14, Las Vegas)*, ISSMGE Technical Committee TC 102, Edited by P.K. Robertson and K.I. Cabal: p 47-73 (2014).
- [4] Obrzud R.F., Truty A., and Vulliet L., Numerical modeling and neural networks to identify model parameters from piezocone tests: II. Multi-parameter identification from piezocone data, *Int. J. Numerical and Analytical Methods in Geomechanics* 36 (6), 743-779 (2012).
- [5] Phoon K-K., Kulhawy F.H., Characterization of geotechnical variability, *Canadian geotechnical journal* 36 (4), 612-624 (1999).
- [6] PN-EN 1997-1:2004: 2004 Eurocode 7: Geotechnical design.
- [7] Puakowski S., Ossowski R., Szarf K.,: Data Mining in Quick Clay Investigation – RCPTU Results Analysis with Neural Networks, *International Journal for Numerical and Analytical Methods in Geomechanics* (2018).
- [8] Robertson P.K., Cone penetration test (CPT)-based soil behavior type (SBT) classification system – an update, *NRC Research Press*, pp. 1910-1926.
- [9] Sulewska M. J., Applying Artificial Neural Networks for analysis of geotechnical problems, *Computer Assisted Mechanics and Engineering Sciences*, 18: 231–241, 2011.
- [10] Hochreiter S., Schmidhuber J., Long Short-term Memory, *Neural Computations* 9(8), 1997, 1735-1780.
- [11] Understanding LSTM Networks, <https://colah.github.io/posts/2015-08-Understanding-LSTMs/index.html>, 27-06-2023.
- [12] Haojie C., Gongxing Y., Jie L., Haiyan C., Xialin Y., Predicting undrained shear strength of soil from cone penetration test data applying optimized RBF approaches, *Journal of Applied Science and Engineering*, Vol 26, No 1, Page 121-130, 2022.
- [13] Lunne T., Robertson P.K. and Powell J.J.M., *Cone Penetration Testing in Geotechnical Practice*, Blackie Academic and Professional, 1997.
- [14] Senneiset K., Sandven R., Janbu N., Evaluation of Soil Parameters from Piezocone Tests, *Transportation Research Record* 1235, 1989.
- [15] Lipton Z.C., Berkowitz J., Elkan C., *A Critical Review of Recurrent Neural Networks for Sequence Learning*, 2015.
- [16] Sulewska M.J., Zabielska-Adamska K., ANN-Based Modelling of Fly Ash Compaction Curve, *Archives of Civil Engineering*, LVIII, 2012.
- [17] Tumay M.T., Karasulu Y. H., Mlynarek Z., Wierzbicki J., Effectiveness of CPT-Based classification methods for identification of subsoil stratigraphy, *Proceedings of the 15th European Conference on Soil Mechanics and Geotechnical Engineering*, IOS Press, 2011.
- [18] Wrzesiński G., Sulewska M.J., Lechowicz Z., Evaluation of the Change in Undrained Shear Strength in Cohesive Soils due to Principal Stress Rotation Using an Artificial Neural Network, *Applied Sciences*, 8, 781, 2018.
- [19] Sherstinsky A., *Fundamentals of Recurrent Neural Network (RNN) and Long Short-Term Memory (LSTM) network*, *Physica D: Nonlinear Phenomena*, Volume 404, 2020.
- [20] Kłos M., Waszczyszyn Z., Sulewska M.J., Neural identification of compaction characteristics for granular soils, *Computer Assisted Mechanics and Engineering Sciences*, 18, 265-273, 2011.
- [21] Mlynarek Z., Wierzbicki J., Wołyński W., Use of functional cluster analysis of CPTU data for assessment of a subsoil rigidity, *Studia Geotechnica et Mechanica*, 40(2), 117-124, 2018.