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Determining the spatial orientation of remote sensing sensors on the basis of incomplete coordinate systems

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Abstract

This article presents a method of determining the spatial orientation of measuring sensors. This method is based on isometric space transformation of a rigid body registered in an oblique coordinate system and is adopted for photogrammetric purposes. The approach is based on incomplete coordinate systems used for determination of transformation parameters. In this publication an incomplete coordinate system is one without one of the axes and in which the matching points connected to primary and secondary coordinate systems are on the two other axes. On the basis of angular momentum, translocation of the beginning of coordinate system is determined first. The next step is to calculate the Euler angles – exterior orientation of sensor. In this method the beginning (the center) of the coordinate system is associated with the sensor itself. This approach, in comparison with the methods known from photogrammetry and remote sensing, allows one to reduce the points needed for transformation. In case of determining the Euler angles two points are indispensable and, in case of moving the beginning of coordinate system, three points are essential. At the end of this paper the analysis of transformation, based on independent control points (ICP), was completed.

Introduction

Determining the spatial position of the elements of the world around us, a set of points defined generally by means of distances and angles, is carried out using various types of reference systems. These can be created with planes, surfaces or defined axes. Geodesy and photogrammetry use polar, spherical and rectangular (Cartesian) coordinate systems. In these systems external orientation elements of the measuring sensors are calculated generally in a simplified manner by using a so-called spatial (small-angles) rotation matrix (Sitek, 1991).

In rigid body mechanics a slightly different approach is used. Calculation of the spatial orientation of the sensor is performed based on the points located on the axis of rotation. Generally, in this way the Euler or Tait-Bryan angles are determined and clearly define the spatial orientation of sensors. This approach is also free from the limitation of rotating coordinate systems by only small angles, to apply the small-angle functions (Baranowski, 2013).

In this publication, the authors set themselves the objective of combining the above-mentioned approaches. The aim is to use spatial transformation of inclined systems (oblique coordinate system) known in rigid body mechanics and their application to the orientation sensors and instruments for surveying, photogrammetry and remote sensing, where a different approach in transformation is used. This approach can be very useful in defining the positions of sensors mounted on Unmanned Aerial Systems (UAS).

Simultaneously, the authors of this thesis show that the orientation of sensors and instruments can be made using incomplete coordinate systems (without one dimension) without a decline of final accuracy.

Spatial orientation of measuring sensors

The obliquity of sensors during measurement means that, in practice, the problem of transformation between the two coordinate systems often occurs. With control points which are known in both coordinate systems, calculation of the transformation parameters and application of a conversion function is possible. As a result, the coordinates of points in the secondary system are calculated on the basis of coordinates of the same points in the primary system. Due to the good legibility and ease of describing to the various transformations, the vector and matrix are applied in calculation (Czarnecki, 2014).

Spatial orientation of two mutually angled systems can be determined on the basis of Euler angles (Figure 1).

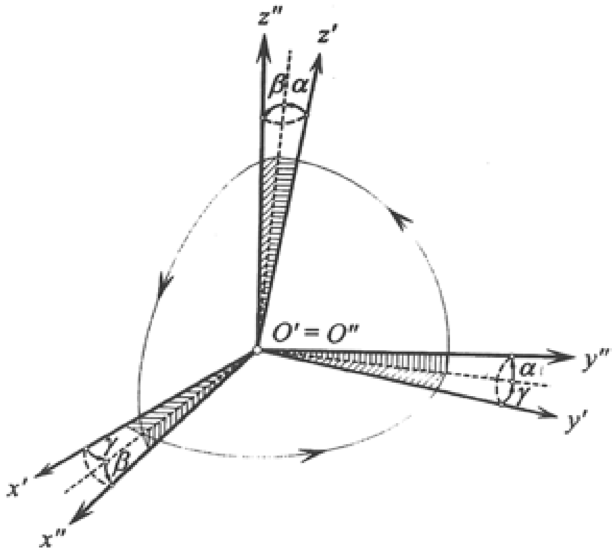

Figure 1. The orientation of spatial systems using Euler angles (Czarnecki, 2014)

Rotations that define the relationship between the axes of systems, may be expressed through cosines of angles which are created together by the various axes (hereafter successive rotations axes are marked with ′ and ″), and represented by a matrix **B**:

$$\mathbf{B} = \begin{bmatrix} \cos(x',x'') & \cos(x',y'') & \cos(x',z'') \\ \cos(y',x'') & \cos(y',y'') & \cos(y',z'') \\ \cos(z',x'') & \cos(z',y'') & \cos(z',z'') \end{bmatrix} \quad (1)$$

In photogrammetry and remote sensing, determination of the spatial orientation of sensor is performed with a combination of three matrix rotations around each axis, which can be written (Kurczyński, 2014):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & \sin\omega \\ 0 & -\sin\omega & \cos\omega \end{bmatrix} \cdot$$
$$\cdot \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix} \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (2)$$

where:
ω – rotation around x-axis;
φ – rotation around the y-axis;
κ – rotation around the z-axis;
X, Y, Z – coordinates in a secondary system (field);
x, y, z – coordinates in a primary (sensor);
X_0, Y_0, Z_0 – vector of translation (shift of the system);
λ – the coefficient of change of scale (in isometric transformation).

Narrowed down to only the angular elements, the transformation is written in the form (Sitek, 1991; Kurczyński, 2014):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad (3)$$

In this matrix a elements contain the angular elements of the external orientation of the sensor in the implicit form. For this reason, these equations are often linearized by expanding the function using the Taylor series and applying a small rotation matrix, which uses the functions of small-angles (Czarnecki, 2014; Kędzierski, Fryśkowska & Wierzbicki, 2015):

$$\mathbf{B} = \begin{bmatrix} 1 & \gamma & -\beta \\ -\gamma & 1 & \alpha \\ \beta & -\alpha & 1 \end{bmatrix} \quad (4)$$

This approach is reasonable in the case of small-angle rotations of systems (inclination of sensor), and in the opposite situation the use of interpolation or iteration algorithms becomes necessary (Preuss, & Kurczyński, 2011; Sanecki et al., 2015).

The application of spatial transformations to describe the absolute displacements of the space structures (for example, the hull structure of a vessel), based on increments of small angles, were presented by Niebylski & Klewski (2015) (Preuss, Kurczyński, 2011):

$$\begin{bmatrix} u_{x_i} \\ u_{y_i} \\ u_{z_i} \end{bmatrix} = \begin{bmatrix} u_{x_{\bar{o}}} \\ u_{y_{\bar{o}}} \\ u_{z_{\bar{o}}} \end{bmatrix} + \begin{bmatrix} 0 & \varphi_{xy} & -\varphi_{zx} \\ -\varphi_{xy} & 0 & \varphi_{yz} \\ \varphi_{zx} & -\varphi_{yz} & 0 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \\ + \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{xy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \tag{5}$$

In the shortened notation:

$$\mathbf{U_i} = \mathbf{U_{\bar{o}}} + \mathbf{\Phi}\ \mathbf{B_i} + \boldsymbol{\varepsilon}\ \mathbf{B_i}$$

where:

$\mathbf{U_i}$ – the displacement vector of the observed point P_i;
$\mathbf{U_{\bar{o}}}$ – vector of translation;
$\mathbf{\Phi}$ – rotation matrix 3×3;
$\boldsymbol{\varepsilon}$ – deformation matrix 3×3;
$\mathbf{B_i}$ – coefficient matrix (of coordinates of P_i points).

A common feature of the listed forms of transformation is that they are based on small angles of rotation. Meanwhile, in the case of the sensors mounted on, for example, UAV (Unmanned Aerial Vehicles), the angles of external orientation may exceed 10°, which leads to the need for additional processing (interpolation or iteration) (Kędzierski, Fryśkowska & Wierzbicki, 2015).

Incomplete coordinate systems

Depending on whether the axes representing the coordinate system are determined on the plane (x, y) or in the space (x, y, z), the system can be flat or three-dimensional. In this case of using a spatial system (without one axis), we would not have their full record, and have to deal with an incomplete coordinate system. Niebylski and Klewski (2015) use the concept of the incomplete system, as a Cartesian system, which overcomes one or two coordinate axes. In practice, the photogrammetric approach to the determination of exterior orientation of sensors using the equation (2) (except the previously described simplifications) eliminates the vertical axis Z, assuming for all points value $Z = 0$. By configuring next the equation the space transformation to the plane is obtained, this is known as a DLT (Direct Linear Transformation) transformation, and can be written:

$$X = \frac{A_1 x + B_1 y + C_1 z + D_1}{A_3 x + B_3 y + C_3 z + 1} \\ Y = \frac{A_2 x + B_2 y + C_2 z + D_2}{A_3 x + B_3 y + C_3 z + 1} \tag{6}$$

The equation (6) is an alternative to isometric transformation (by similarity) and is used for the processing of images which come from the non-metric sensors, for the flat ground. It contains eleven unknowns (A1, B1, C1, ... D2), so to transform coordinates from the original (x, y, z) to the secondary system (X, Y – incomplete) six control points are needed. In practice, in the case of sensors mounted on the platform (UAVs), the incomplete plane orthogonal coordinate system is often used to determine the position and orientation of the sensor (external orientation elements).

The method of determining exterior orientation

The method of determining the angular exterior orientation elements of the measuring sensors consists of using an approach known in rigid body mechanics, in which points are located on the axes (before transformation) (Jeżewski, 1970; Baranowski, 2013).

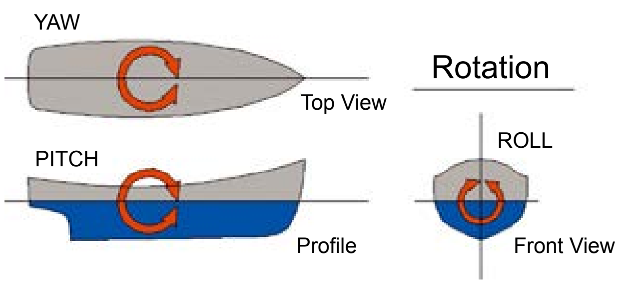

Figure 2. Rotation of coordinate systems: Pitch, Roll, Yaw – Tait-Bryan angles (Wikipedia, 2015)

Defining the next three rotations with Tait-Bryan angles (as in Figure 2). With two points situated on the axes, providing an incomplete coordinate system, the angular orientation of the sensor elements can be determined through the following transformations:

1. Pitch (φ) – Z-axis – plane ZX

$$\varphi = \operatorname{atan}\left(\frac{X_Z}{Z_Z}\right) \tag{7a}$$

The point on the axis Z. After the angle is calculated, the transformation is performed for the – φ angle and then the new coordinates are calculated.

2. Roll (ω) – Z-axis – plane YZ

$$\omega = \operatorname{atan}\left(\frac{Y_Z}{Z_Z}\right) \tag{7b}$$

The point on the axis Z. After the angle is calculated, the transformation is performed for the $-\omega$ angle and then the new coordinates are calculated.

3. Yaw (ω) – X-axis – plane XY

$$\kappa = \operatorname{atan}\left(\frac{Y_X}{-X_X}\right) \tag{7c}$$

The point on the axis X. After the angle is calculated, the transformation is performed for the $-\kappa$ angle and then the new coordinates are calculated.

In this method, the setting of the axis (choice of planes) is free. It is enough to take the motion of the sensor as shown in Figure 2 and apply the order of calculation as shown in the relationships (7). It is connected with the appropriate setting (reorientation) of the axis and subsequently with calculating angles: Pitch, Roll, Yaw. With this approach individual angles are not dependent on each other and there is the possibility of their independent calculation.

The verification of the method

The method of determining exterior orientation of the measuring sensors has been verified in the laboratory and in the field. In the laboratory, the coordinates of the ten points (in [m]) and angles of rotation were assumed, as follows:

$$\mathbf{p_{10}} := \begin{bmatrix} 0 & 2 & 2 & 15 & 3 & 15 & 0 & 4 & 4 & 10 \\ 0 & 2 & 2 & 0 & 3 & 3 & 10 & 4 & 20 & 10 \\ 10 & 2 & 10 & 0 & 3 & 3 & 0 & 4 & 4 & 10 \end{bmatrix} \tag{8}$$

where the first row of the matrix **p** are X, the second Y, and the third Z coordinates;
rotation angles:

$$\omega := -20°,\ \varphi := -15°,\ \kappa := -25° \tag{9}$$

After rotations for the coordinates as adopted above, the matrix took the form (10).

Where **A** is the matrix calculated on the basis of rotation according to (2) and (3) for the assumed values of the angles (9), and takes the form (11).

As the result of reverse transformation, the set of points were found (12).

Comparing the coordinates of the points obtained by the matrix (8) and (12) it can be seen that the differences in the coordinates are less than 10^{-15} m. These differences occur only for points lying on the axes of the coordinate system, and are at a negligible level.

The terrain verification was performed in a similar manner. For this purpose, the control points were founded and the coordinates before and after rotation of the sensor (the total station) were calculated in the same way. Subsequently, the inverse transformation and the origin coordinates were compared. Finally, assuming that the distribution of errors is accidentally determined, the mean error (standard deviation) of measured points and the mean error with which it has been calculated, gave:

$$m_0 = \sqrt{\frac{[VV]}{n-1}} = 0.012\text{ m},\ M_0 = \frac{m_0}{\sqrt{n}} = 0.004\text{ m} \tag{13}$$

where:

V – the difference between the length of the vector (location of points) in the compared systems;

n – the number of points of analyzed space.

$$\mathbf{P_{10}} := \mathbf{A}\cdot\mathbf{p_{10}} = \begin{bmatrix} 2.588 & 3.085 & 5.155 & 13.131 & 4.627 & 15.133 & 4.082 & 6.17 & 12.701 & 15.425 \\ 3.304 & 1.334 & 3.977 & -7.16 & 2.002 & -3.727 & 8.142 & 2.669 & 15.697 & 6.672 \\ 9.077 & 0.838 & 8.099 & -1.138 & 1.257 & 0.347 & -4.128 & 1.676 & -4.928 & 4.19 \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{bmatrix} \cdot \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix} \cdot \begin{bmatrix} \cos\kappa & -\sin\kappa & 0 \\ \sin\kappa & \cos\kappa & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{A} := \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 0.875 & 0.408 & 0.259 \\ -0.477 & 0.814 & 0.33 \\ -0.076 & -0.413 & 0.908 \end{bmatrix} \tag{11}$$

$$\mathbf{M^{-1}}\cdot\mathbf{P_{10}} = \begin{bmatrix} -1.665\cdot10^{-16} & 2 & 2 & 15 & 3 & 15 & 2.914\cdot10^{-16} & 4 & 4 & 10 \\ -4.441\cdot10^{-16} & 2 & 2 & 2.359\cdot10^{-16} & 3 & 3 & 10 & 4 & 20 & 10 \\ 10 & 2 & 10 & 1.11\cdot10^{-16} & 3 & 3 & -4.441\cdot10^{-16} & 4 & 4 & 10 \end{bmatrix} \tag{12}$$

Conclusions

The presented method is based on rigid body transformation of Tait-Bryan angles. This approach reduces the number of control points needed for transformation (in both coordinate systems) to two. These points can be located at any two axes of the coordinate system. The method thus allows determination of the angular exterior orientation of the sensor using an incomplete coordinate system, while simultaneously reducing adjustment points (compared to the transformations commonly used in surveying, photogrammetry and remote sensing). In addition:

- the presented method can be used for any angles and is not limited to small angles and their functions;
- there is no need for transforming function linearization;
- there is no need for an additional interpolation and iteration.

In analyzing the accuracy of this method, it should be emphasized that in the laboratory, the compatibility of points after the inverse transformation was at the level of numerical accuracy of 10^{-15} m. Under field conditions the accuracy was within 0.012 m. Hence, the accuracy is typical for measurements by total station as a result of instrumental, environmental and personal errors. Taking this into account the results should also be recognized as satisfactory. According to the authors, this method may be an alternative to those currently used in photogrammetry and remote sensing for the determination of the spatial orientation of the measuring sensors.

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