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POWER LINE MAGNETIC FIELD MITIGATION
USING A PASSIVE LOOP CONDUCTOR

The paper presents in a tutorial manner methods of the calculation of magnetic fields in vicinity of overhead electric power lines without and with passive mitigation loops. Exact and simplified methods of the determination of the magnetic field of a straight overhead conductor based on the Fourier transform technique are presented. The mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) horizontal located under the power line. The decomposition of the magnetic fields in two components: magnetic field obtained in free space from the Biot-Savart law and the magnetic field produced by earth current shows that in practical cases the effects from earth currents can be neglected as compared with effects from line currents.

1. INTRODUCTION

Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Nowadays of special concern is the possibility of detrimental environmental effects arising from the electrical and magnetic fields formed adjacent to the overhead transmission lines. These fields may affect both operation of near electric and electronic devices and appliances and also various living organisms.

The topic of mitigating the magnetic fields produced by overhead power lines is gaining more significance in recent years. In this context, efforts are continuously being done in order to maximize the utilization of the available line corridors without exceeding the tolerable limits of the lines' magnetic fields.

There are several possibilities to mitigate the field from existing or new overhead lines, e.g.: increasing the height of conductors, phase rearrangement, compaction, splitting of phases, underground cables, gas-insulated lines, passive shields (ferromagnetic and conductive), etc.

During the last decades, the use of conductive shields (passive non-compensated or series capacitor compensated loops) to mitigate extremely low frequency magnetic fields generated from power lines has been proposed [1 - 6].

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Their behavior principle is based on the electromagnetic induction law: time varying, primary magnetic fields, generated by AC sources, induce electromotive forces driving loops currents – additional field sources, which modify and reduce the primary magnetic field.

In the paper exact and simplified methods of the determination of the magnetic field of a straight overhead conductor based on the Fourier transform technique are presented. The mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) horizontal located under the power line.

It is shown that in practical cases the effects from earth currents can be neglected as compared with effects from line currents. The magnetic field under a power line can be obtained from the Biot-Savart law, assuming that the current carrying power line conductors are straight horizontal wires of infinite length located in free space.

2. MAGNETIC FIELD OF A STRAIGHT OVERHEAD CONDUCTOR

2.1. Exact method (Fourier transform technique)

An infinitely long conductor is placed at height h_k above the earth surface, Fig. 1, and carries the current, which flows in direction of the x-axis. The current varies with the time as $\exp(j\omega t)$ where ω is the radian frequency. The x, y plane is considered to be the earth surface. It is assumed that the earth is an isotropic, homogeneous medium of finite conductivity γ. The magnetic permeability of the soil and of the air is μ_0. The displacement currents in both regions: the air and the earth are neglected ($\gamma \gg \omega\varepsilon$).
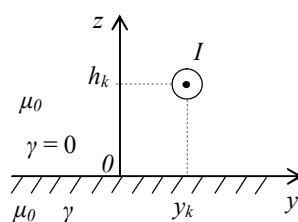

Fig. 1. An infinitely long current carrying conductor above the earth surface

The vector potential of the electromagnetic field has the x-component only denoted $A_x(y,z)$ which satisfies the Poisson equation in the air and the Helmholtz equation in the earth. The vector potential $\vec{A}$ in the air can be obtained if the Fourier transform is used and the boundary conditions in the system considered expressing the continuity of the normal component of the magnetic flux density

and the tangential components of the magnetic as well as electric intensities are taken into account. Hence the *x*-component of the vector potential in the air can be written in the form [8]:

$$A_x(y,z) = \frac{\mu_0 I}{2\pi} \int\limits_0^\infty \left[\frac{e^{-u|z-h_k|}}{u} + \frac{(u-\alpha)e^{-u(z+h_k)}}{u(u+\alpha)} \right] \cos[(y-y_k)u]du \qquad (1)$$

where:

$$\alpha = \sqrt{u^2 + k^2}, \qquad k^2 = j\omega\mu_0\gamma \qquad (2)$$

The magnetic flux density $\vec{B}$ can be obtained from the equation:

$$\vec{B} = rot\,\vec{A} \qquad (3)$$

Thus the magnetic flux density components in the air ($z \geq 0$) become:

$$B_y(y,z) = -\frac{\mu_0 I}{2\pi} \int\limits_0^\infty \left[e^{-u|z-h_k|} + \frac{(u-\alpha)e^{-u(z+h_k)}}{u+\alpha} \right] \cos[(y-y_k)u]du \qquad (4)$$

$$B_z(y,z) = \frac{\mu_0 I}{2\pi} \int\limits_0^\infty \left[e^{-u|z-h_k|} + \frac{(u-\alpha)e^{-u(z+h_k)}}{u+\alpha} \right] \sin[(y-y_k)u]du \qquad (5)$$

Each component of the magnetic flux density can be split in two terms [7]:

$$B_y(y,z) = B_{yu}(y,z) + B_{ye}(y,z) \qquad (6)$$

$$B_z(y,z) = B_{zu}(y,z) + B_{ze}(y,z) \qquad (7)$$

The first term $B_{yu}(y,z)$ given by the relationship

$$B_{yu}(y,z) = -\frac{\mu_0 I}{2\pi} \frac{z - h_k}{(z-h_k)^2 + (y-y_k)^2} \qquad (8)$$

can be interpreted as the *y*-component of the magnetic flux density produced by current carrying infinitely long conductor in a free space environment (air), which can be directly obtained from the Biot-Savart law.

The second term $B_{ye}(y,z)$ denotes the *y*-component of the magnetic flux density produced in the air by currents flowing in the earth, which can be extracted from eqn.(4) and it can be written in algebraic form:

$$B_{ye}(y,z) = \frac{\mu_0 I}{2\pi} \left[\begin{array}{l} \dfrac{z + h_k}{(z+h_k)^2 + (y-y_k)^2} + \\[2ex] -2|k|\int\limits_0^\infty nbe^{-(z+h_k)|k|n} \cos[(y-y_k)|k|n]dn \\[2ex] -j2|k|\int\limits_0^\infty n(n-a)e^{-(z+h_k)|k|n} \cos[(y-y_k)|k|n]dn \end{array} \right] \qquad (9)$$

Similarly, in the relation (7), $B_{zu}(y,z)$ denotes the z-component of the magnetic flux density produced by current carrying infinitely long conductor in homogeneous medium (air):

$$B_{zu}(y,z) = \frac{\mu_0 I}{2\pi} \frac{y - y_k}{(z - h_k)^2 + (y - y_k)^2} \tag{10}$$

whereas $B_{ye}(y,z)$ denotes the z-component of the magnetic flux density produced in the air by currents flowing in the earth, which can be calculated from eqn. (5), and it can be written in algebraic form:

$$B_{ze}(y,z) = \frac{\mu_0 I}{2\pi} \left[\begin{array}{l} -\dfrac{(y - y_k)}{(z + h_k)^2 + (y - y_k)^2} + \\[2mm] + 2|k| \displaystyle\int_0^\infty nbe^{-(z+h_k)|k|n} \sin\left[(y - y_k)|k|n\right] dn + \\[2mm] + j2|k| \displaystyle\int_0^\infty n(n - a)e^{-(z+h_k)|k|n} \sin\left[(y - y_k)|k|n\right] dn \end{array} \right] \tag{11}$$

The formulas (9) and (11) are obtained from eqns (4) and (5), if the relationships are used:

$$u = |k|n \tag{12}$$

$$\sqrt{n^2 + j} = a + jb \tag{13}$$

where:

$$a = \sqrt{\frac{\sqrt{n^4 + 1} + n^2}{2}}, \quad b = \sqrt{\frac{\sqrt{n^4 + 1} - n^2}{2}} \tag{14}$$

2.2. Unmitigated power line magnetic field

Under normal operating condition the phase currents (I_j, $j = 1, 2, 3$) of the power lines are dependent on the load flow and can be treated as known. The currents induced in the earth conductors in the case of two earth conductors μ and v have to be calculated from the relations:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1\mu} & Z_{2\mu} & Z_{3\mu} \\ Z_{1v} & Z_{2v} & Z_{3v} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} + \begin{bmatrix} Z_\mu & Z_{\mu v} \\ Z_{v\mu} & Z_v \end{bmatrix} \begin{bmatrix} I_\mu \\ I_v \end{bmatrix} \tag{15}$$

where Z_μ, Z_v are the unit-length self impedances, $Z_{\mu v}$ is the unit-length mutual impedance between the earth-wires, $Z_{j\mu}$ and Z_{jv} are the unit-length mutual impedances between phase conductor and the earth wire μ and v, respectively. It should be noted that in the calculation of these impedances, the effects of induced

currents in the earth on the electromagnetic field of the power line have to be taken into account.

The total y and z components of the magnetic flux density in the vicinity of the overhead 3-phase power line with phase conductors (n) and earth wires (m) can be obtained by superposition, according to the equations (16):

$$B_y = \sum_{i=1}^{n+m} B_{yi} \quad , \quad B_z = \sum_{i=1}^{n+m} B_{zi} \qquad (16)$$

It should be pointed out, that when the line currents are changing sinusoidally with time and have a relative phase displacement with respect to each other (as in the case of the three phase line), the produced in the observation point magnetic field vector varies with time not only in the magnitude but also in its orientation. In general, the tip of the magnetic flux density describes the path of an ellipse. Values and orientations of the two semi-axes are determined by the usual search for extremal points of the equation of the ellipse [1].

3. MITIGATION EFFECTS DUE TO A PASSIVE LOOP

3.1. Current induced in a mitigation loop

The knowledge of the magnetic field density produced by a power line is necessary in calculation of the magnetic flux passing through conductive loop located under the line. The induced in the loop an electromotive force drives the loop-current through the loop impedance. The loop-current creates under the power line a magnetic field, which opposes the one from the line.

Consider the magnetic flux passing through a surface of the horizontal, rectangular loop located underneath an infinitely long overhead conductor with a current, as in Fig. 2.
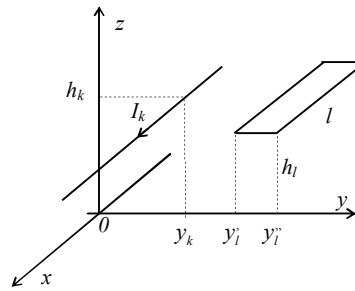

Fig. 2. Current carrying conductor and a mitigation loop

Magnetic flux can be calculated from the formula:

$$\Phi = \int_S \vec{B} \cdot \overrightarrow{dS} \qquad (17)$$

where $\overrightarrow{dS}$ is vectorial surface element.

Taking into account that:

$$\overrightarrow{dS} = \vec{e}_z dS_z \qquad (18)$$

and $dS_z = l dy$, the magnetic flux takes the form:

$$\Phi = l \int_{y_l^{'}}^{y_l^{''}} B_z(y, z = h_l) dy \qquad (19)$$

Inserting relationships (10) and (11) at $z = h_l$ into this integral, the magnetic flux becomes:

$$\Phi = \frac{\mu_0 I_k l}{\pi} \left[\begin{array}{l} \dfrac{1}{4} \ln \dfrac{[(h_k - h_l)^2 + (y_l^{''} - y_k)^2][(h_k + h_l)^2 + (y_l^{'} - y_k)^2]}{[(h_k - h_l)^2 + (y_l^{'} - y_k)^2][(h_k + h_l)^2 + (y_l^{''} - y_k)^2]} + \\[4mm] + \displaystyle\int_0^{\infty} b e^{-(h_k + h_l)|k|n} [\cos[(y_l^{'} - y_k)|k|n] - \cos[(y_l^{''} - y_k)|k|n]] dn + \\[4mm] + j \displaystyle\int_0^{\infty} (n - a) e^{-(h_k + h_l)|k|n} [\cos[(y_l^{'} - y_k)|k|n] - \cos[(y_l^{''} - y_k)|k|n]] dn \end{array} \right] \qquad (20)$$

The electromotive force induced in the loop can be obtained from the relation:

$$E = -j\omega\Phi \qquad (21)$$

The electromotive force drives the loop-current through the impedance of the loop:

$$I_{loop} = \frac{E}{Z_s} = -\frac{j\omega\Phi}{Z_s} \qquad (22)$$

where Z_s is the self-impedance [9] of the loop.

If the effects of the currents induced in the earth on the total magnetic field produced under the power line are negligible as compared with the effects due to the currents flowing in the overhead conductors, the magnetic flux passing the loop can be determined from eqn. (19) with B_{zu} obtained from eqn.(10) at $z = h_l$ so that:

$$\Phi_u = \frac{\mu_0 I_k l}{4\pi} \ln \frac{(h_l - h_k)^2 + (y_l^{''} - y_k)^2}{(h_l - h_k)^2 + (y_l^{'} - y_k)^2} \qquad (23)$$

The approximate value of the loop-current is then:

$$I_{u\,loop} = -\frac{j\omega\mu_0 I_k l}{4\pi Z_s} \ln \frac{(h_l - h_k)^2 + (y_l^{''} - y_k)^2}{(h_l - h_k)^2 + (y_l^{'} - y_k)^2} \qquad (24)$$

The loop current creates under the current carrying overhead conductor a magnetic field, which opposes the one from the conductor.

It should be noted, that the loop impedance Z_s can be changed by insertion of appropriate series capacitance and its phase angle can be adjusted additionally by choice of an appropriate conductor (resistance) [1].

3.2. Mitigated power line magnetic field

The mitigated magnetic field under the power line is the vectorial sum of the local original magnetic field associated with currents in the phase conductors and the earth conductors and of the local auxiliary field caused by the loop current.

The total y and z components of the magnetic flux density in the vicinity of the overhead 3-phase power line with phase conductors (n) and earth wires (m) and auxiliary conductors (k) forming the loop can be obtained by superposition, according to the equations (25):

$$B_y = \sum_{i=1}^{n+m+k} B_{yi} , \quad B_z = \sum_{i=1}^{n+m+k} B_{zi} \tag{25}$$

It should be noted, that the magnetic flux density produced by the k–th auxiliary current has to be obtained according to the eqn. (8 - 10) or according to eqn. (8) and (10) for the exact and approximate method respectively.

3.3. Exemplary calculations

Exemplary calculations of the magnetic field distribution under a power line have been carried out according to the method derived. The geometry of a 750 kV transmission line with mitigation loop located beneath outside phases is shown in Fig. 3.
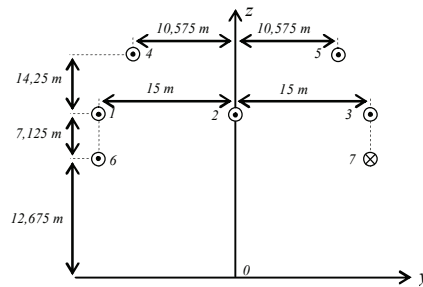

Fig. 3. The assumed tower configuration, adopted from [1]

Assuming balanced three phase currents in phase conductors 1, 2 and 3 $I_1 = 1500e^{j0^\circ} A, I_2 = 1500e^{-j120^\circ} A, I_3 = 1500e^{j120^\circ} A$, span length l = 470 m, f = 60 Hz, γ = 0,01 S/m, first the currents in the earth conductors 4, 5 have been calculated according to eqn.(15), and the unmitigated magnetic field is obtained from eqn.(16). The magnetic flux linked with the loop consisting of closed

conductors 6 and 7 has been determined from the eqn. (20) or (23). The current produced within the mitigation loop according to eqn. (22) is $I_{loop} = 386e^{j180^\circ} A$ if as in [1] the loop impedance (0,173+j1,16) Ω has been changed by insertion of the series capacitor 2,1 mF to the resultant impedance $Z_s = 0.2e^{-j30^\circ} \Omega$. Finally, the resultant mitigated magnetic flux density has been calculated from eqn.(25). Lateral profile at height 1.0 m of the magnetic flux density module under the power line without and with the mitigation loop is presented graphically in Fig. 4. Evident reduction of the magnetic field can be observed.

It should be noted, that the results of the magnetic flux density calculations are nearly identically for the exact and simplified method.
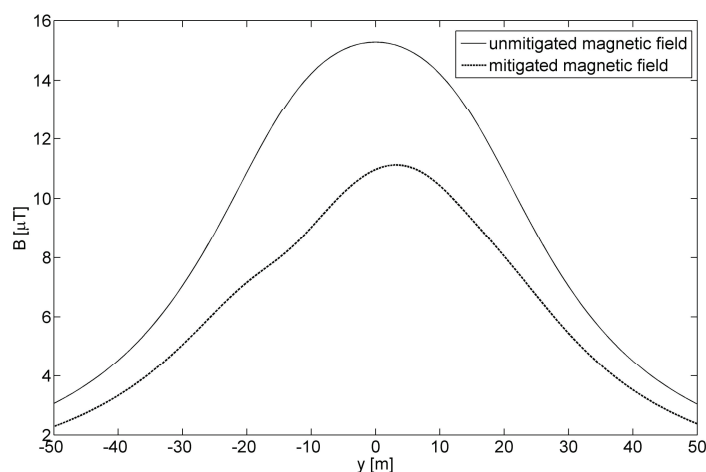

Fig. 4. Lateral profile of the magnetic field under the power line

4. CONCLUSIONS

The design of installation generating low-frequency magnetic and electric fields requires access to effective analytical and computational tools. The paper presents procedures of determining the magnetic flux density of the field produced under power lines.

Exact and approximate methods are developed for analyzing magnetic fields in the vicinity of overhead power lines without and with mitigation passive loops. The exact method of field calculation takes into account the earth currents. The approximate method assumes that effects of earth currents onto magnetic field are negligible. In both methods phase-currents have prescribed values, based on which all of the remaining currents (in earth conductors and in loop-conductors) are computed.

A solution is proposed for modeling magnetic fields in case when phase and earth conductors are straight and infinitely long.

The mitigation effects due to the passive loop are also investigated, whereas the mitigation loop can be treated as a rectangular loop (two-conductor closed mitigation loop) horizontal located under the power line.

The results derived can be used as the foundation for almost every study on the magnetic fields for conditions that are almost always satisfied for power engineering applications.

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