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Conversion Excitation of Intense Sound Fields in Crystals

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Abstract

The resonant excitation of an intense elastic wave in a crystal is described through a special nonspecular reflection close to a conversion when almost all the energy from the incident pump wave falls into the near-surface narrow high-intensity reflected beam. The resonance arises when the excited reflected wave is close to the bulk eigenmode satisfying the condition of free boundary. It is shown that the choice of the crystal surface parallel to a symmetry plane allows simultaneous optimization of reflection geometry when the intensity maximum for the excited wave is accompanied by the intensity minimum for the other (parasite) reflected wave. And the conversion criterion of vanishing of the above minimum is determined by one definite condition on elastic moduli. On this basis the series of real monoclinic, orthorhombic and hexagonal crystals were chosen where the resonant reflection in non-symmetric sagittal planes proves to be very close to conversion.

Keywords: Elastic wave, resonance reflection, conversion, pump wave, anisotropy, diffraction divergence

1. Introduction

Modern acoustics of crystals create new principles of functioning of various instruments and devices based on the use of ultra- and hypersonic waves [1, 2]. Intense ultrasonic beams are widely used in engineering, medicine, scientific instrument technique, etc. The reflection and refraction of such beams at the interfaces between layered isotropic structures are commonly used for their transformation. Crystals open new possibilities of beams transformation. Many acoustic effects arise only due to medium anisotropy [2-4]. For instance, piezoelectricity exists only in crystals and is widely used in acoustic devices. Another spectacular example of a nontrivial role of anisotropy is phonon focusing; the concentration of energy in a crystal along special directions for which the acoustic beam in Poynting vectors is much narrower than that in wave vectors. Here, we will consider another principle of energy concentration in acoustic waves that is also entirely attributable to crystal anisotropy.

In [5, 6] the idea was proposed of resonant energy concentrating in narrow acoustic beams through a nonspecular reflection in the geometry close to a scheme of the total internal reflection (Figure 1). One of the two reflected beams propagates at a small angle β_r to the surface and therefore narrows greatly to width d_r upon the reflection of an incident beam with width D_i. Then the intensity of the narrow reflected beam can exceed considerably the intensity of the pump wave. The beam amplification factor K_2 is estimated by the ratio $\eta D_i/d_r = \eta \sin\alpha_i / \sin\beta_r >> 1$, where η is the fraction of energy falling into the compressed beam from the incident one.

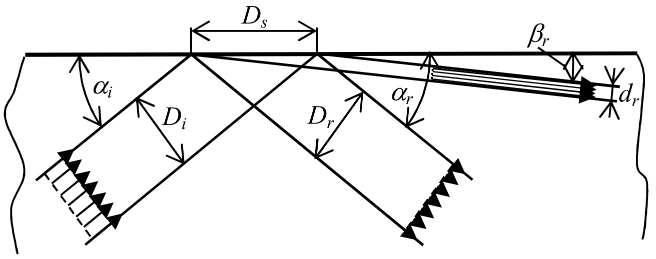

Figure 1. Scheme of resonant excitation of intense acoustic beam; arrows indicate Poynting vectors in the incident and reflected beams

It is essential that we deal here with a purely anisotropic effect. In isotropic media an analogous beam compression for the incidence angles close to the angle of total internal reflection does not result in any amplification. In this case, the fraction η of energy in the reflected beam approaches zero as its width decreases. The same would occur in a crystal as well if the geometry of reflection is not chosen in a specific way. The choice of the plane and angle of incidence is dictated by the requirement that the excited reflected wave be close to the bulk eigenmode with its energy flow along a free boundary.

The fraction η of the pump energy transferred to the excited beam depends on the specific relations between the elastic moduli for specially chosen crystals. The maximum effectiveness (η = 1) of the considered amplification effect is realized when the conversion reflection occurs, i.e. when the amplitude of the parasite quasi-specular reflected wave vanishes. As shown in [5], for unrestricted anisotropy the two basic characteristics of the resonance, K_2 and η, attain their maximums in different geometries of reflection. That is why in [5, 6] we considered more symmetric crystals and reflection geometries related to surfaces and sagittal planes close to symmetry planes. In these cases the extremal conditions for K_2 and η are attained simultaneously for the same reflection geometries. And the maximum effectiveness η = 1 determined by only one condition on elastic moduli may be approximately realized in a series of crystals.

In this paper we shall demonstrate that the minimum symmetry requirement providing a realizable high effectiveness of the resonance is more modest. Namely, for a coincidence of the maximums of K_2 and η it is sufficient to orient the surface of a crystal to be parallel to a symmetry plane. And the condition for a conversion, $\eta = 1$, is again reduced to only one relation between elastic moduli. This means that even for monoclinic system there is a good chance to find crystals providing the resonances of reflection close to conversion. Below we shall choose several monoclinic, orthorhombic and hexagonal crystals where the computations predict the effectiveness close to 100% for geometries related to surfaces parallel to symmetry planes whereas the propagation planes are non-symmetric and even are not close to symmetry planes.

2. Statement of the Problem

Consider an arbitrary half-infinite elastic medium of unrestricted anisotropy with the free surface. Let **n** be the internal unit normal to it. We suppose that on this surface there is such direction $\mathbf{m}_0$ along which an exceptional one-partial bulk eigenwave belonging to the intermediate sheet of the slowness surface may propagate with Poynting vector along the boundary and zero traction at it. There is the existence theorem [7] which guaranties such eigen-solutions occupying the whole lines on the sphere of propagation direction for any anisotropic media. In this sagittal plane $\{\mathbf{m}_0, \mathbf{n}\}$, apart from the exceptional wave, a tree-partial special reflection must also exist [5] for the same tracing velocity v of stationary wave motion along the surface. It includes the incident and reflected waves from the outer sheet of the slowness surface and the localized partial component from the innermost sheet. In the case of a weak perturbation of the initial geometry in which the surface does not change and the exceptional direction $\mathbf{m}_0$ is rotated $\mathbf{m}_0 \to \mathbf{m}$ around the normal **n** through a small angle $\varphi = \angle(\mathbf{m}, \mathbf{m}_0)$, none of the two wave solutions can be retained. In this case, instead of two disappeared solutions, their superposition should appear. The former exceptional bulk wave will enter this superposition as a new reflected component in which the energy flow makes a small angle with the surface. Clearly, a small perturbation of the initial geometry will violate relatively weakly the satisfaction of the boundary condition for a free surface by this near-surface component. This violation is compensated for by the remaining partial components. Therefore, the new near-surface reflected wave should have amplitude exceeding considerably the amplitudes of other partial waves, including the incident one. This process may be considered as the resonant excitation of an intense bulk wave by a weak pump wave incident on the crystal surface at an appropriate angle.

The combined displacement wave field of the perturbed four-partial reflection can be expressed in the form

$$\mathbf{u}(\mathbf{r}, t) = \sum_{\alpha=1}^{4} C_\alpha \mathbf{A}_\alpha \exp\{ik[(\mathbf{m} + p_\alpha \mathbf{n}) \cdot \mathbf{r} - vt]\} \tag{1}$$

where C_α and $\mathbf{A}_\alpha$ are the scalar amplitudes and normalized polarization vectors of the partial waves, respectively, k is the common projection of all wave vectors onto the surface: $k = \mathbf{k}_\alpha \cdot \mathbf{m} = k_x$, $p_\alpha = \mathbf{k}_\alpha \cdot \mathbf{n}/k = k_{\alpha y}/k$ and $v = \omega/k$ is the tracing velocity. Each partial wave $\mathbf{u}_\alpha(\mathbf{r}, t)$ in (1) must satisfy the equation of the dynamical theory of elasticity [3]. And the sum of their tractions at the surface ($y = 0$)

$$\hat{\sigma}_\alpha \mathbf{n}\big|_{y=0} \equiv -ikC_\alpha \mathbf{L}_\alpha \exp[ik(x - vt)] \tag{2}$$

must vanish in accordance with the boundary condition of a free surface. Thus, we obtain

$$\sum_{\alpha=1}^{4} C_\alpha \mathbf{L}_\alpha = 0 . \tag{3}$$

In further considerations we shall choose the numeration so that $\alpha = 4$ would correspond to the incident wave ($C_4 \to C_i$), $\alpha = 1$ to the parasite reflected wave of the same branch as the incident wave ($C_1 \to C_{r1}$), $\alpha = 2$ to the excited reflected wave being the perturbed exceptional wave ($C_2 \to C_{r2}$), and $\alpha = 3$ to the localized partial mode ($C_3 \to C_l$). In these notations equation (3) leads to the following reflection coefficients [5]

$$R_1 = \frac{C_{r1}}{C_i} = -\frac{[\mathbf{L}_4\mathbf{L}_2\mathbf{L}_3]}{[\mathbf{L}_1\mathbf{L}_2\mathbf{L}_3]}, \quad R_2 = \frac{C_{r2}}{C_i} = -\frac{[\mathbf{L}_4\mathbf{L}_1\mathbf{L}_3]}{[\mathbf{L}_1\mathbf{L}_2\mathbf{L}_3]} \tag{4}$$

where [**abc**] means the mixed product of the vectors **a**, **b** and **c**.

3. The Case of Surface Parallel to a Symmetry Plane

In the case when the surface is parallel to a symmetry plane of the crystal the expressions in (4) acquire a more simple structure. Indeed, in this case the vectors $\mathbf{L}_\alpha$ and $\mathbf{L}_{\alpha+3}$ must be symmetric with respect to this plane. This relates both to the real vectors

$$\mathbf{L}_1 = \mathbf{L}_1^s + \mathbf{L}_1^n, \quad \mathbf{L}_4 = \mathbf{L}_1^s - \mathbf{L}_1^n, \tag{5}$$

and to the only complex conjugate pair

$$\mathbf{L}_3 = \mathbf{L}_3^s + i\mathbf{L}_3^n, \quad \mathbf{L}_6 = \mathbf{L}_3^s - i\mathbf{L}_3^n . \tag{6}$$

Here the superscripts s and n indicate the in-plane ($\mathbf{L}_\alpha^s \perp \mathbf{n}$) and out-plane ($\mathbf{L}_\alpha^n \parallel \mathbf{n}$) orthogonal components of the vector $\mathbf{L}_\alpha$.

With (5), (6), the equations in (4) acquire the structure

$$R_1 = -\frac{\phi - \lambda' + i\lambda''}{\phi + \lambda' + i\lambda''}, \qquad R_2 = -\frac{\mu}{\phi + \lambda' + i\lambda''}, \tag{7}$$

where

$$\phi = [\mathbf{L}_1^s\mathbf{L}_2^n\mathbf{L}_3^s], \quad \lambda' = [\mathbf{L}_1^n\mathbf{L}_2^s\mathbf{L}_3^s], \quad \lambda'' = [\mathbf{L}_1^s\mathbf{L}_2^s\mathbf{L}_3^n], \quad \mu = 2[\mathbf{L}_1^s\mathbf{L}_3^s\mathbf{L}_1^n] . \tag{8}$$

One should keep in mind that the vectors $\mathbf{L}_\alpha^{s,n}$ in (8) are determined by the perturbed reflection geometry, i.e. by the orientation of the sagittal plane, $\varphi = \angle(\mathbf{m}, \mathbf{m}_0)$, and by the incidence angle $\delta\alpha$ counted from the angle α_{0i} related to the total internal reflection.

The angle $\delta\alpha$ is directly connected with the shift Δv of the tracing velocity and with the perturbation δp of the parameter $p_2 = k_{2y}/k$ which vanishes at $\delta\alpha = 0$ (Figure 2).

The first equation in (7) transforms the conversion condition $R_1 = 0$ into the following two relations

$$\lambda' = \phi, \qquad \lambda'' = 0 . \tag{9}$$

Here the first equation establishes the relation between the angles φ and $\delta\alpha$ (or δp) for arbitrary crystal moduli. And the second equation determines the condition for those moduli. In fact, this equation is reduced to a requirement of the vanishing component:

$$L_3^n = 0 . \tag{10}$$

Thus, for a medium with the surface parallel to a symmetry plane we have got the simple general criterion for a choice of crystals with the high effectiveness of the resonance.

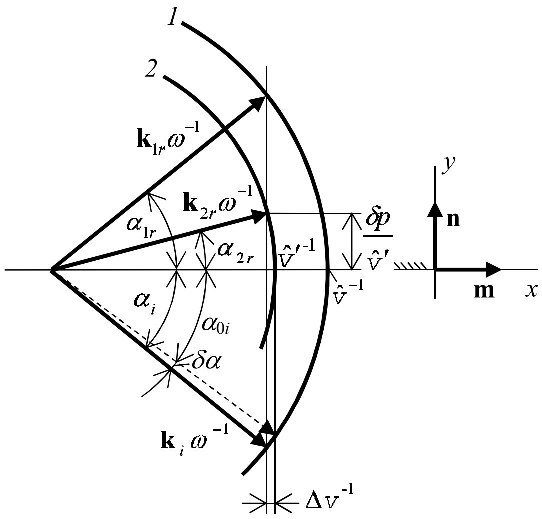

Figure 2. Fragments of external sheets *1* and *2* of the slowness surface in its cut by the sagittal plane and schematic diagram of the reflection

4. Approximate Analytical and Exact Computer Results

Let us give more rigorous definitions to the earlier introduced two basic characteristics of the resonance, the amplification factor K_2 and the effectiveness of excitation η :

$$K_2 = | R_2 |^2 s_2 / s_4 , \qquad \eta = 1 - | R_1 |^2 \tag{11}$$

where $s_{2,4}$ are the group speeds of the excited and incident waves. After the substitution here the relations from (7) with parameters (8) found in the main order in φ and δp [5],

$$\phi \approx \kappa\varphi^2, \quad \lambda' \approx \kappa\lambda_0'\delta p, \quad \lambda'' \approx \kappa\lambda_0''\delta p, \quad \mu \approx \kappa\mu_0\varphi, \tag{12}$$

one obtains the approximate expressions for the functions $K_2(\varphi,\delta p)$ and $\eta(\varphi,\delta p)$:

$$K_2 = \frac{|\mu_0|^2 (s_2/s_4)\varphi^2}{(\varphi^2+\lambda_0'\delta p)^2 + (\lambda_0''\delta p)^2}, \qquad \eta = \frac{4\lambda_0'\delta p\varphi^2}{(\varphi^2+\lambda_0'\delta p)^2 + (\lambda_0''\delta p)^2}. \tag{13}$$

It is easily seen that the both functions in (13) have maximums under the same condition

$$\varphi^2 = \lambda_0'\delta p, \tag{14}$$

which represents an approximate concretization of the first equation in (9). And the second equation in (9) is transformed with the same accuracy to the condition $\lambda_0'' = 0$ which is just the criterion for a choice of crystals close to conversion ($\eta \approx 1$).

The computer analysis was based on the exact formulae (7) and the data [8] for elastic moduli of large number of crystals of various symmetry systems. We were interested in monoclinic, orthorhombic and hexagonal crystals admitting the propagation of exceptional waves in non-symmetric sagittal planes along surfaces parallel to symmetry planes. Many crystals were found where the fraction η of energy in the excited intensive wave exceeded 90% of energy in the incident wave. In some cases the magnitude of η proves to be close to 100% (Table 1).

Table 1. The effectiveness η of the resonance in some crystals for $K_2 = 5$, φ_0 is the azimuth of the exceptional wave normal $\mathbf{m}_0$ in the crystallographic coordinates

Crystals	φ_0, rad	$\delta\alpha$, rad	η
Monoclinic system			
Stilbene	1.5562	0.14	0.996
Tolan $C_{14}H_{10}$	1.0332	0.063	0.968
Triglycine sulphate (TGS)	1.4709	0.084	0.94
Tartaric acid $C_4H_6O_6$	-1.4282	0.02	0.966
Orthorhombic system			
Rochelle salt	0.48	0.015	0.999
Boron-epoxy composite	0.2	0.014	0.9999
Hexagonal system			
AgI	0.1457	0.012	0.993
CeF_3	0.2918	0.011	0.99

Figure 3 demonstrates the dependencies $K_2(\varphi, \delta\alpha)$ and $\eta(\varphi, \delta\alpha)$ for the three crystals of monoclinic (a), orthorhombic (b) and hexagonal (c) systems for similar (monoclinic) symmetry of reflection geometry: the surface was parallel to a symmetry plane while the sagittal plane had non-symmetric orientation. As is seen from the Figure, the sensitivity of the resonance to changes $\delta\alpha$ in the angle of incidence is much sharper than to rotations of the sagittal plane. This manifests itself in different scales of the angles plotted along two axes. The ranges of angles in Figure 3 were chosen so that the amplitude of the amplification factor K_2 remained within the practically acceptable values.

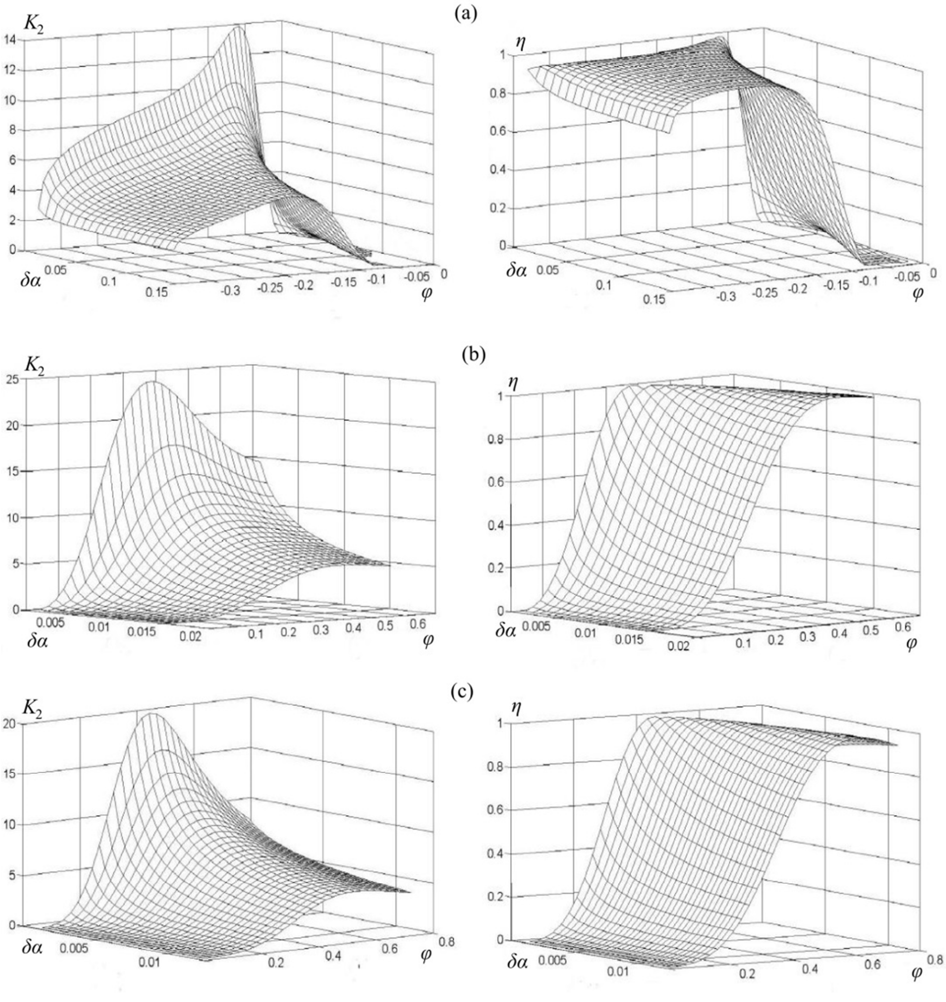

Figure 3. Amplification factor K_2 and fraction of energy in the excited beam η versus angle of incidence $\delta\alpha$ and the deviation angle φ of the sagittal plane in the stilbene (a), Rochelle salt (b) and AgI (c) crystals

We note that the factor K_2 in (13) fast increases with a decrease in perturbation parameters φ, δp and $\delta\alpha$. However, the higher is the resonance peak, the narrower it becomes. Certainly, it is senseless to make the width of the peak in the angles $\delta\alpha$ less than the angle of diffraction divergence of the incident beam. And the divergence of the excited beam is even more limiting with respect to the increase in K_2. Indeed, this critical parameter $\delta\psi_r$, together with K_2, grows inversely proportionally to the width d_r of this beam (Figure 1). Still, according to [5], one can obtain $\delta\psi_r \sim 10^{-2}$ rad keeping $K_2 \sim$ 5-10.

3. Conclusions

As is shown in the above analysis, the discussed effect of resonant reflection in crystals, where a wide incident acoustic beam converts almost all of its energy into a narrow high intensity reflected beam, appears to be quite realizable. By special choice of crystals with a definite relation between elastic moduli the resonance may be optimized up to the effectiveness $\eta \sim 100\%$. Since the resonance region is narrow in angles of incidence, stringent requirements for a weak divergence of the incident beam, $\delta\psi_i \sim 10^{-3}$ rad arise, which can be realized only at sufficiently high ultrasonic frequencies ~100 MHz. For the same reason, the amplitude of the excitation coefficient is also limited to $K_2 \approx$ 5–10. However, in the case of retransformation of the emergent beam through its narrowing in the perpendicular dimension as well, the intensification factor increases many fold, to $\sim 10^2$. In the hypersonic frequency range, the amplification amplitudes can be increased significantly. In this case, however, one might expect additional restrictions due to an increase in the absorption of acoustic waves.

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