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Application of the Cobb-Douglas production function to study the results of the production process and planning under turbulent environment conditions

Introduction

In 1928 C.W. Cobb and P.H. Douglas created and described the production function which has been widely used in research on production in various branches of industry around the world (Mishra and Sudhanshu 2011). The model presented by them is a very good tool to approximate the production process and its results. The purpose of the article is to present the potential of the production function in relation to such a complicated production process as hard coal mining, examine the regularities occurring during the extraction process, verify the validity of the current method of resource allocation, disclose the origin of possible problems, and offer efficient corrective solutions. Polish coal companies are an excellent research subject as a characteristic feature of the mining industry in Poland is high level of variability of its environment. Therefore, it is necessary to look for strategic analysis and planning tools that will be effective in a turbulent environment.

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The article presents the CD production function, which has been assumed here as a universal instrument serving primarily to examine the state of the current production process and the rules governing it. Secondly, there were created scenarios of possible variants of production functions of coal companies in Poland until 2020, considering the aspect of planning in difficult conditions of the changing environment. As a result, it was possible to detect irregularities in the current production process and propose effective remedies. These studies are very important due to the strategic position of the coal industry in Poland. One of the factors determining the possibility of ensuring energy security for Poland is access to energy resources. Almost 50% of primary energy consumed in Poland and as much as 80% of electricity is produced on the basis of coal (Rybak and Manowska 2018). Therefore, it should be possible to provide access to this energy carrier to consumers at a price they may accept, in the desired quantity, quality, location and time. The global economic crisis that began in 2007 has had implications for energy commodity markets. In 2008, there was a sharp decline in oil prices, when they fell to the level of 2003. A strong correlation between prices of energy raw materials caused a drop in hard coal prices around the world (Rybak and Manowska 2017). Taking the cost of mining for one ton of hard coal into account, falling prices led to a situation in which the cost exceeded the price, making it impossible for mining companies to make profits. Mining enterprises generated profits during periods of increased demand for energy raw materials, and this is why they postponed investments in creating new exploitation fronts and modernization increasing production efficiency. As a result of the ongoing restructuring process of the mining industry and the lack of investment, hard coal mining has been systematically reduced year by year since the early 1990s. Currently, part of the demand for hard coal is covered by the import of this raw material – mainly from Russia, which is also justified by sale price of fuel. The remaining energy carriers included in the Poland energy mix are mainly imported – in case of natural gas it is about 70% of demand and in case of crude oil it is almost 100% (Rybak and Manowska 2018). The Cobb-Douglas production function was used To be able to plan the level of coal production and satisfy customers' demand (Cobb and Douglas 1928, Felipe and Adams 2005). Additionally, in order to predict the outcome of the production process in the forthcoming years, forecasts of explanatory variables and a dependent variable were placed into the production function model. As planning in a turbulent environment requires using special methods, the article presents anticipated production function models and three scenarios of trend development for factors determining the level of hard coal production were created. The forecasts acquired thanks to the ARIMA class models were used to build the scenarios. The calculated production function enables describing the relationship between the means of production used and the production level. Therefore, knowing the level of anticipated demand for hard coal in a given period of time (year, quarter, month) it is possible to determine the amount of capital and labor needed. The proposed production function also allows the progress of the restructuring process of hard coal mining to be evaluated. Extremely difficult conditions in the mining companies environment, in which they operate, both in Poland and around the world, require the ability of immediate adaptation of the company's resources to constant changes. The Cobb-Douglas function makes it possible to determine the necessary level of the most appropriate combination of labor and capital (Fraser 2002).

1. Materials and methods

The macroeconomic function of production is the F function, which describes the dependencies between factor inputs, such as physical capital incurred and labor expenditure, and the volume of product Y produced (Saleem et al. 2019). The models were built on the basis of statistical data purchased at the Industrial Development Agency (*Agencja Rozwoju Przemysłu SA*) The costs of hard coal production in 2010–2017 were taken into account. The author conducted similar research in 2006 and 2014. In order to compare the results obtained during previous analyses the same set of explanatory variables was introduced to the model. The choice of variables was statistically valid, which means that the use of additional exogenous variables during modelling resulted in the decrease of model accuracy (reduction in the value of the adjusted determination index). The model includes data on expenditure on depreciation and energy (C_e), as well as on labor (Keen and Ayres 2019; Feldstein 1967). Two types of models were built: the first using data on the volume of annual production, employment, annual energy expenditure and depreciation, the second model, where the endogenous variable was the value of annual hard coal extraction, exogenous variables were the costs of annual employment (labor) and the costs of depreciation and energy (objectified labor).

The production functions models can be described using the following equation (van Elk et al. 2019):

$$Y_t = \beta_2 {W_f}^{\alpha 1} C_e^{\alpha 2}$$

↳ α_1, α_2 , β_2 – parameters of the model.

The OriginPro 8.5.1 program was used to build the model. To simplify the determination of parameters of the model for the time of calculation it was reduced to a linear form (Reynes 2019):

$$\ln Y_t = \ln \beta_1 + \alpha_1 \ln W_f + \alpha_2 \ln C_e, \quad \beta_1 = e^{\beta_1}$$

The following table (Table 1) presents model parameters as well as the coefficient of the determination for each of them. Model validation was based on the determination coefficient and the MAPE error (Gruszczyński and Podgórska 2007; Myttenaere 2016).

2. Results

The analysis began with the creation of 3 models and their parameters are presented in Table 1. Model 1 includes data on the value of production, annual costs of labor and capital (Model 1). The alpha parameter for labor is negative. A parameter value of less than 0 means that outlays on labor have a restrictive effect on shaping the effects of the extraction process. The production increase is possible with the increase of production factor outlays to the limit beyond which the results of such operations will be negative. However, because the assumptions about function parameters are that the alpha value must be greater than zero, it was necessary to find the reasons to be able to build the correct model (Shephard 1970). It was determined that the decrease in hard coal prices had an impact on the occurrence of the negative value of the model parameter. From 2013, production costs exceeded the average coal price by 8% in the period 2013–2016. Therefore, a simulation was carried out to confirm the thesis. Prices were raised to the price level of 2012, which was the last observation before the breakdown of the energy market caused by the economic crisis (Model 3). It has been proven that the thesis is true. The conducted analysis showed that for the assumed price level, the parameters of the model values are accurate. However, their sum is still lower than 1. For this reason Model 2 was created using data on the volume of extraction (eliminating the necessity of taking the price of hard coal PLN/Mg, capital costs and employment in hard coal mining into account. The coefficient of determination of model 2 and 3 is about 0.8, which means that it can be used during extrapolation. Also, the value of the MAPE prediction error ex post, i.e. a relative standard prediction error meter is less than 10%, confirming the high accuracy of the models. The influence of independent variables on the dependent was

Table 1. Parameters of Cobb-Douglas production functions models

Tabela 1. Parametry funkcji Cobba-Douglasa dla utworzonych modeli

Parameter	Production function 1	Production function 2	Production function 3
β_2	30 740	1 775 662	154
α_1	–0.35	0.004	0.56
α_2	0.99	0.32	0.29
R	0.79	0.91	0.87
R^2	0.63	0.82	0.77
MAPE	7.5%	1.3%	8.5%
F	7.14	5.81	5.22
Significance F	0.07	0.03	0.04

Source: own study.

confirmed by analyzing the variance. The analysis based on the F test allowed to conclude that the influence of dependent variables on the production volume is statistically significant (F > significance F) (Pavelescu 2014).

The model parameters are very important regarding production functions. Their values make it possible to determine the relation between the production increase according to the expenditure on it. The highest value of the sum of parameters in the exponent of the model is characterized by Model 3 (0.85). In the case of Model 2 this value is much lower and amounts to 0.32. The α model parameters form production flexibility coefficients relative to the economic factors studied. They determine the ratio of the percentage increase of the production volume to the percentage increase of each independent variable. Due to the fact that these values are dimensionless, it is possible to compare the effects of all the production factors. The higher the value of the α coefficient the stronger the influence of the corresponding exogenic variable on the volume of extraction. The relationship of $\alpha1 + \alpha2 < 1$ occurring in all models created means that there is a decreasing efficiency of production factors in the hard coal mining industry in Poland, i.e. production increases more slowly than the total energy expenditure, depreciation and annual work (Vilcu 2011). Decreasing economies of scale mean that production effects are reduced. An increase in production means the increase in the average cost. This is a phenomenon characteristic of large enterprises where there may be communication problems among employees. In the case of coal companies, an additional factor influencing the size of $\alpha1$ and $\alpha2$ parameters is a significant share of fixed costs of approximately 50% in the structure of the total production. Raising remuneration costs over the years has not translated into an increase in productivity. Figure 1 presents a surface diagram of the Cobb-Douglas function for Model 2. The graph presents production levels that can be achieved for all combinations – the substitution of outlays for the extraction of

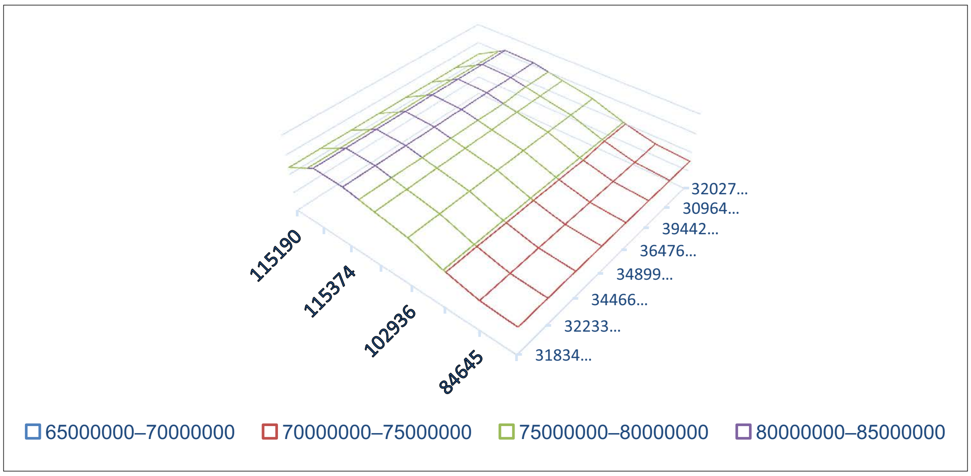


Fig. 1. Cobb Douglas function for Model 2

Rys. 1. Funkcja Cobba Douglasa dla modelu 2 labor and capital. The shape of the presented surface depends on the economies of scale. In the case of disadvantage of the scale, the surface is not steep. If the analyzed process was showed. the occurrence of the economies of scale then the inclination of the obtained surface would be significant – with a proportional increase of outlays production grows rapidly.

In order to obtain detailed information on the process of hard coal extraction in Polish mines, additional parameters were used, such as: substitution of production factors, process productivity and marginal productivity (Ghobadian and Husband 1990). During the extraction of hard coal it is possible to substitute labor factors with the factors of capital and vice versa, assuming that production will remain at the same level. The substitution of production factors (MRS) was determined using the marginal rate of substitution using formulas (Goryl et al. 2007):

$$KSS_{W_f C_e} = -\frac{\alpha_2}{\alpha_1} \cdot \frac{C_e}{W_f}, \quad KSS_{C_e W_f} = -\frac{\alpha_1}{\alpha_2} \cdot \frac{W_f}{C_e}$$

↳ $KSS_{W_f C_e}$ – the marginal rate of substitution of the W_f factor with respect to the C_e factor,
$KSS_{C_e W_f}$ – the marginal rate of substitution of the C_e with respect to the W_f factor.

Substitution determines how much labor should be provided in order to eliminate the effort unit for objectified labor (*ceteris paribus*).

Productivity of the production process was determined using the following formulas (Borkowski et al. 2004):

$$PP_{C_e} = \frac{\hat{y}}{C_e}, \quad PP_{W_f} = \frac{\hat{y}}{W_f}$$

↳ PP_{C_e} – average product with respect to energy and depreciation,
PP_{W_f} – product average for annual work,
$\hat{y}$ – the total product calculated on the basis of the econometric model created.

The productivity of production should be understood as the quotient of the volume of extraction and production factors used to produce it. It is the amount of extraction that was obtained from the unit of a given effort *ceteris paribus*. Marginal productivity determines the increase in production that can be achieved by introducing an additional unit of expenditure of a given production factor, with the assumption that the outlays of other expenditures will remain unchanged (Begg 1992). Productivity is determined as a partial derivative of the production function in relation to a given manufacturing factor (Gruszczyński et al. 2009):

$$PK_{C_e} = \beta \cdot \alpha_1 C_e^{\alpha_1 - 1} \cdot W_f^{\alpha_2}, \quad PK_{W_f} = \beta \cdot b_2 C_e^{\alpha_1} \cdot W_f^{\alpha_2 - 1}$$

PK_{Ce} – marginal productivity in relation to objectified labor outlays,
PK_{W_f} – marginal productivity in relation to labor inputs.

The aforementioned rates were calculated for the Model 3. In the case of the Model 2, the indicators concerning labor outlays are given in relation to one employee, therefore it is impossible to refer them directly to the other models. In Model 3, due to the price freeze it was possible to analyze the organization and financing of the production process and to compare the progress which was made during the restructuring. In addition, the research results were compared with the results obtained by the author in previous years, during an analogous analysis carried out in 2006 and in 2014. Table 2 presents the average results of the obtained rates.

Table 2. The average values characterizing the process of hard coal extraction in Poland in the studied time period

Tabela 2. Wartości średnie charakteryzujące proces wydobycia węgla kamiennego w Polsce w badanym okresie czasowym

Model	$KSS_{W_f C_e}$	$KSS_{C_e W_f}$	PP_{C_e}	PP_{W_f}	PK_{C_e}	PK_{W_f}
Model 3 (2019)	–1.63	–0.62	0.02	0.01	0.002	0.014
2006	–1.45	–0.67	0.04	0.01	0.030	0.017
2014	–5.68	–0.18	0.03	0.01	0.005	0.015

Source: own study.

Calculating the productivity factor made determining the level of coal extraction efficiency in Poland possible and indicated which of the production factors have a stronger impact on the productivity of coal enterprises. The productivity indexes shown in Table 2 indicate that the productivity of objectified labor is higher for each of the presented models. The level of the productivity index is systematically decreasing. Currently, it is two times lower than in 2006. Despite constant declines, it should be noted that PLN 1 spent on objectified labor-oriented factors still returns more than twice as much as the same financial resources allocated for labor. The productivity index in relation to expenditures on employment remains quite stable. One can obtain 0.02 and 0.01 tons of coal from PLN 1 respectively. Therefore, we can conclude that in order to effectively increase the efficiency of hard coal production, we will obtain better results by allocating additional financial resources for investments in machinery and equipment. The marginal productivity index also

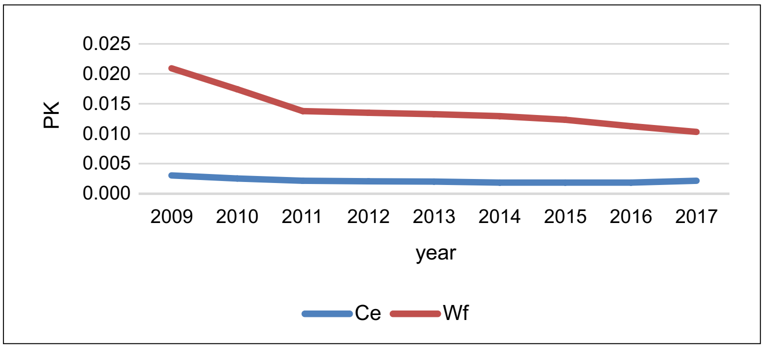


Fig. 2. Marginal productivity in 2009–2017

Rys. 2. Produktywność marginalna w latach 2009–2017

indicates that the increase in work efficiency would be obtained by investing in objectified labor. For Model 3, the value of the index for objectified labor was 0.014 Mg/PLN, for labor only 0.002 Mg/PLN (seven times less). Similarly during previous analyses the value of the index was more favorable for objectified work. An unfavorable downward trend in marginal productivity is presented in Figure 2. The possibility of obtaining a cost advantage makes it easier for the company to achieve a competitive advantage (Porter 2006). In turn, the proper allocation of resources has a huge impact on gaining a cost advantage (Szymański 1995). The marginal rate of substitution shows that a reduction in expenditures on employment by PLN 1 will require expenditure of PLN 0.62 on energy and depreciation respectively. On the other hand, in the reverse situation, PLN 1.63 should be spent on labor.

3. Scenarios for hard coal production in Poland by 2020

The approximation of the production function models and the indexes obtained show that in the last several years the parameters of the function which were built on the basis of actual data deteriorated. As coal mining is a branch of industry operating in a turbulent environment, it is very difficult to manage coal companies being exposed to constant changes in the factors determining the demand for fuel. Therefore, during planning the process of hard coal extraction in Poland one should use methods fitted to such specific and difficult conditions. One of the methods recommended in this situation is scenario planning. During scenario planning the researchers usually use the heuristic methods and opinions of expert teams. As this type of procedure often raises doubts as to the validity and accuracy of the obtained results, the author presents a method that allows to obtain many variants of the future based on reliable mathematical models. The scenarios presented were created using the ARIMA class models (Sen et al. 2016; Yaltaa and Jenal 2009). They were used to describe explanatory variables and a dependent variable. The next step was the introduction of forecasts based on the ARIMA models to the Cobb-Douglas production function model. As the forecasts were created together with the determination of the confidence interval, it was possible to create scenarios based on them. Using the forecast confidence interval it is possible to obtain a range is in which, with 95% probability, there is the value for which the forecast is to be determined. Forecasting a single point cannot provide the decision-makers with the necessary information about the future developments they should be prepared for in a specific future. These ranges consist of two lines representing optimistic and pessimistic scenarios. The key factor affecting the financial results of Polish coal companies is the sales price, which is why it was also forecasted and taken into account when determining the variables introduced to the function model. Several models were built for the forecasts of each of the variables. Finally, models that meet the criteria were selected for the minimization of the information criterion, minimization of the standard deviation of the dependent variable, statistical significance of the model components, lack of autocorrelation of model residues, forecast error ex-post below 10%, normal distribution of model residues.

The logarithm of credibility, Akaike's information criterion, Schwarz-Bayesian information criterion and the Hannan-Quinn criterion are analyzed when comparing models created for the same variables, but with different parameters of the model (Piłatowska 2010). They

Table 3. Parameters of validation of forecasting models used to create production function scenarios

Tabela 3. Parametry walidacji modeli prognostycznych wykorzystanych do stworzenia scenariuszy funkcji produkcji

Parametres	WKC23	WKAE7	WKZ12	WKQ19
Log-likelihood function	–54.07	–191.14	–245.67	–183.18
Schwarz Bayesian	115.33	387.13	496.13	371.16
Akaike	114.14	386.74	495.34	370.36
Hannan–Quinn	113.39	385.89	494.83	369.86
Standard deviation of a dependent variable	42.88	$5.63 \cdot 10^8$	$1.37 \cdot 10^9$	5 535 856
Average error	2.23	$1.23 \cdot 10^8$	$2.03 \cdot 10^8$	−1 270 149
Standard deviation error	31.61	$4 \cdot 10^8$	$9.93 \cdot 10^8$	3 072 489
MPE (%)	0.68	–4.14	1.18	–1.17
MAPE (%)	7.8	8.35	8.98	3.19
Theila index (I) (%)	0.68	1.29	1.01	0.99
Ljung-Box Q' Chi-square distribution	0.05; $p = 0.81$	2.38; $p = 0.12$	2.90; $p = 0.8$	3.4; $p = 0.07$
Doornik-Hansen Chi-square distribution	3.62; $p = 0.16$	5.52; $p = 0.07$	2.11; $p = 0.34$	0.62; $p = 0.73$

Source: own study.

enable the selection of the best model for a dependent variable. It is assumed that the best model is the one for which the criterion value is the lowest. The normal distribution of the model residuals was confirmed by the Doornik-Hansen test, i.e. the test using the transformed value of kurtosis and skewness parameters. In turn, the lack of autocorrelation in the residual process was determined on the basis of results of the Ljung-Box test. The probability values p in Table 3 are higher than the maximum allowable probability of error 0.05. This means that there are no grounds for rejecting the H_0 hypothesis, thus the rest of the model is characterized by normal distribution and the lack of autocorrelation. As a result, it was confirmed that the model was correctly selected and thus there are only random fluctuations in the residuals. The residuals should not contain any systematic components. The ex-post errors of the model were *inter alia* determined by means of the mean absolute percentage error and the average percentage error (Glennon et al. 018). It was assumed that extrapolation may be used for a model which error does not exceed 10%. This assumption was met and the error was under 9% for all models.

Both the autoregression model and the moving average were used. An autoregressive process (AR) is called a process in which the next observation is a combination created on the basis of previous observations (Claveria 2019). The AR model can be described by the following equation, AR model:

$$y_t = \varphi_0 + \varphi_1 y_{(t-1)} + \varphi_2 y_{(t-2)} + \ldots + \varphi_p y_{(t-p)} + \varepsilon_t = \varphi_0 + \sum_{(i=1)}^{p} \varphi_i e_{(t-i)} + \varepsilon_t$$

$y_t, y_{t-1}, y_{t-2}, ..., y_{t-p}$ – value of the forecasted variable in a moment of time or during period $t, t-1, ..., t-p,$
p – lag,
$\varphi_0, \varphi_1, ..., \varphi_{t-p}$ – parameters of the model,
ε_t – white noise.

In the case of the moving average process (MA) the value of the series depends on the disturbance occurring in the present and past. The amount of disturbance from previous periods is determined by the model parameter q (Kot et al. 2007). The MA model can be described by the following relation:

$$y_t = \theta_0 + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \ldots - \theta_q e_{t-q} + \varepsilon_t = \theta_0 + \sum_{j=1}^{q} \theta_j e_{t-j} + \varepsilon_t$$

q – lag,
$e_t, e_{t-1}, ..., e_{t-p}$ – the residuals of the model in periods $t, t-1, ..., t-p$.

As the most reliable models are those with few parameters, the following ARIMA models were created for each variable:

- price – ARIMA(0,0,1),
- cost of objectified labor – ARIMA(1,1,0),
- cost of labor – ARIMA(1,0,0),
- value of production ARIMA(1,0,0).

The necessary technique for objectified labor costs was to bring the series to the stationary form by one-time differentiation. Figure 3 presents the actual data, forecasts and confidence intervals of the price, objectified labor costs, labor and the volume of hard coal extraction.

The obtained forecasts were used to create 25 variants of production functions. The models which are presented are these of created combinations whose parameters were statistically significant and of parameters greater than zero. They were:

- Scenario 1: The most likely model based on the size of forecasted variables.
- Scenario 2: Increase in production volume, prices and production costs.

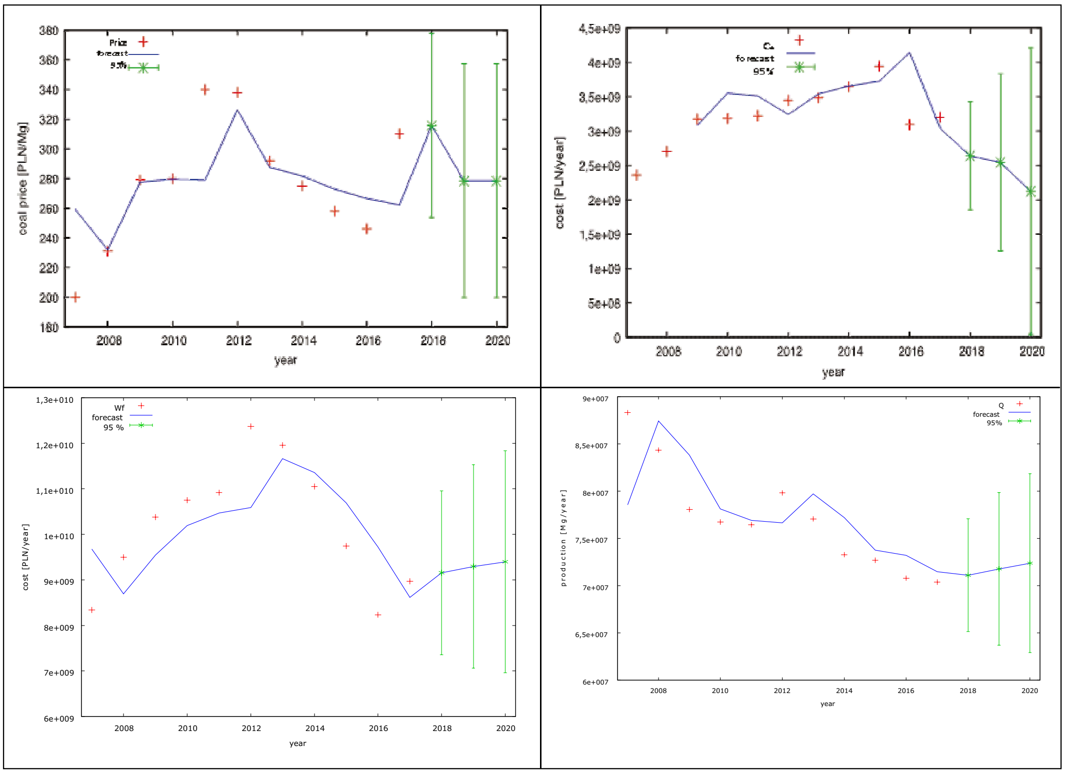


Fig. 3. Price, objectified and labor costs and volume of hard coal extraction with forecast and confidence interval by 2020

Rys. 3. Cena, koszty pracy uprzedmiotowionej i żywej oraz wielkość wydobycia węgla kamiennego wraz z prognozą i przedziałem ufności do roku 2020

- Scenario 3: Increase of extraction, prices, costs of labor and decrease of objectified labor.
- Scenario 4: Decline in production, prices and labor outlays, increase in objectified labor costs.
- Scenario 5: Increase in the extraction volume, prices, decrease in employment costs, increase in investment outlays.

Scenarios enable the evaluation of strategic choices. The created scenarios differ from each other in the way the environment influences the enterprise. Scenarios are used to make a prior assessment of the values of individual strategic choices of the organization, depending on the influence of its environment. The scenarios, with 95% probability, describe how the effects of the future coal production process can take effect.

There were calculated indexes of each of the presented scenarios (Table 4). The economies of scale were achieved only in case of scenario 5 ($\alpha 1 + \alpha 2 = 1.11$). The increasing economies of scale of production mean that a proportional change in the factor of objectified labor causes disproportionate change in production. In that case long-term average costs decrease as the production volume increases.

Table 4. Parameters of production function models for created scenarios

Tabela 4. Parametry modeli funkcji produkcji dla utworzonych scenariuszy

Parameter	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
β	2 261	8 500	3.14	108	0.44
α_1	0.88	0.25	0.92	0.67	0.18
α_2	0.01	0.40	0.06	0.16	0.93
R	0.80	0.93	0.84	0.95	0.78
R^2	0.64	0.87	0.70	0.89	0.61
MAPE	5.10%	9.30%	8.00%	8.30%	10%
F	6.33	16	7.40	19.84	1.80
Significance F	0.03	0.01	0.02	0.01	0.24

Source: own study.

Also in the case of scenario 5, the highest increase in the marginal productivity index was observed (Table 5, Fig. 4). The determined coefficients clearly indicate that the main mistake made during the organization of the extraction process is the lack of sufficient financial resources allocated for investments. The second mistake is the incorrect planning of employment.

Table 5. The average values characterizing the process of hard coal extraction in Poland in the examined time period for scenarios created

Tabela 5. Wartości średnie charakteryzujące proces wydobycia węgla kamiennego w Polsce w badanym okresie czasowym dla utworzonych scenariuszy

Model	$KSS_{W_fC_e}$	$KSS_{C_eW_f}$	PP_{C_e}	PP_{W_f}	PK_{C_e}	PK_{W_f}
Scenario 1	0.91	1.69	0.024	0.007	0.006	0.003
Scenario 2	0.81	1.50	0.002	0.007	0.005	0.004
Scenario 3	0.61	1.70	0.020	0.007	0.005	0.008
Scenario 4	0.60	1.75	0.025	0.007	0.003	0.006
Scenario 5	0.72	1.55	0.020	0.007	0.001	0.020

Source: own study.

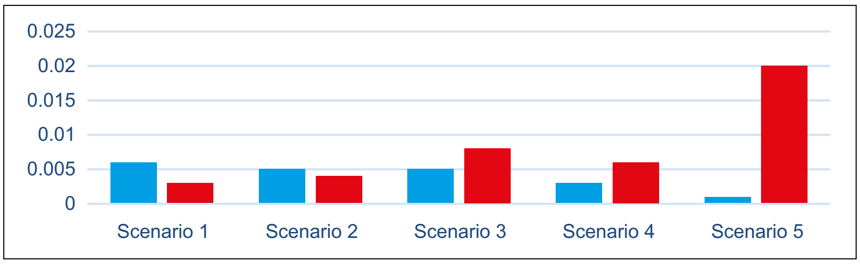


Fig. 4. Marginal productivity in the scenarios created

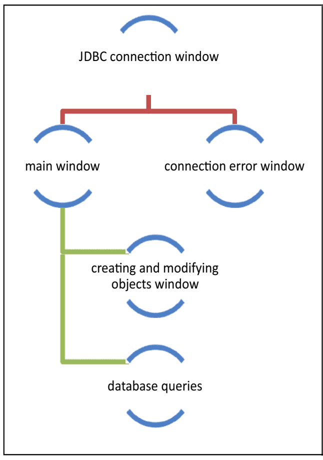
Rys. 4. Produktywność marginalna w poszczególnych scenariuszach

Due to seasonality phenomenon which occurs in hard coal mining there are periods of increased coal production and downtimes in the mines during the year, with spring production decreasing up to 12% in relation to the average value. During this period of time the workload of underground workers is below the average. Therefore, it is appropriate to adjust the employment rate to the current level of extraction. It would be possible thanks to application of one of the hard coal production strategies proposed by the author and presented in previous publications (Rybak A. and Rybak A. 2016):

$$Q_{strategy} = 0.02 \cdot AE^{0.81} \cdot P_R^{0.19}$$

This strategy is in line with the Just in Time principle of production management. The main assumption of the strategy is to adjust the employment level to the volume of extraction. As part of the strategy, reducing the employment to the level required in periods of reduced production was proposed. The level of employment should be raised by outsourcing in the months during which the production volume increases. This would enable reduction of employment and opportunity costs as well as a flexible response to market requirements. The main problem of using outsourcing in hard coal mines were conflicts and lack of cooperation between full-time employees and those who were outsourced. What is more, the nature of the work requires employees to have extensive experience and qualifications. Generally, the use of outsourcing had a negative impact on work efficiency of hard coal mines. This is why the author proposed using the so-called capital outsourcing. It depends on the possibility of transferring employees within the coal company to the mine, where the production level is increased. As a result, the work of miners employed in the same mining company is used. It is necessary to use the right tool in order to be able to plan and optimize the deployment of employees. The solution to this problem are an SQL database and Java application created by the author.

The database contains information on underground workers of the entire coal company. Thanks to this it is possible to redirect employees to this mine where it is necessary to increase the volume of extraction. Thus, it is simultaneously possible to eliminate wastes in mines where extraction has been reduced. Created interface in Java – EMT 2.0 (Employee Management Tool) allows the database to be operated by employees of a coal company, including those who do not use SQL (Fig. 5). The application allows users to establish a connection to the database, as well as display its resources, add and delete records and search and filter information about employees. The database contains such information (arguments) as ordinal number assigned to the employee, the name of the employee, the identification number of the mine where the person is employed, employee status. The most important information in the database is the employee status. It shows whether, at any given time,




```
else if(pole.equals("lp, status"))
{

    try {


     zapytanie= "Select lp, status from pracownik where "+poletekstowe.getText();
   rs = db.statement.executeQuery(zapytanie);
    for (int i =0;i<TbaZap.getRowCount();i++)
    {TbaZap.setValueAt(null, i, kolumna);

   }for (int i=0;rs.next();i++)
   {
      TbaZap.setValueAt(rs.getString("lp"), i, kolumna-1);
      TbaZap.setValueAt(rs.getString("status"), i, kolumna+2);

   }
     } catch (SQLException e) {
          e.printStackTrace();
      }}
```

Fig. 5. EMT 2.0 application scheme and code example

Rys. 5. Schemat aplikacji EMT 2.0 oraz przykładowy fragment kodu the employee is performing any specific tasks related directly to the key activity of a given mine, i.e. extraction, or if there is a possibility of delegating this person to work related to extraction at another mine of the company. This secondment happens in a form of capital outsourcing. It is assumed that an employee is available (status = 1) if the expected period of inactivity at the parent mine is at least two weeks. If the employee's status is 0 it is not possible to transfer this person to another mine.

Discussion

The Cobb-Douglas production function has been used by the author for two reasons. First, it enables the analysis of condition of the current production processes in Polish coal companies.

Thus, the Cobb-Douglas function provides information on expected results of used production strategy. The calculated elasticity coefficients give the decision makers of the coal company information about the marginal productivity of individual capital and labor outsets. It was not possible to build a correct model of production function basing on data pertaining to the outsets on labor and objectified labor, as well as taking into account prices per ton of extracted coal. For the real data, a negative relationship was found between work outsets and the production volume. The analysis which was carried out showed that the reason of this situation are costs exceeding the sale price of a ton of hard coal. The elimination of prices from the model and taking only the production volume, not its value into account would make it impossible to compare the productivity and substitution rates with the results of analyses carried out in previous years. In order to make it possible, during the construction of the model, the prices were set at the level of 2012 and model 3 was created on the basis of which further calculations were made. Over the years 2006–2017 the marginal productivity fell twice considering objectified labor outlays and in the case of labor outlays the decline was 150%. This means that it is possible to achieve a lower and lower increase in production from any additional unit dedicated to the means of production. Therefore, it should be stated that despite the continuous reduction of the volume of extraction the level of costs did not decrease proportionally. This is related to a large share of fixed costs in total production costs. Fixed costs are mainly employment-related costs. This means that the volume of hard coal extraction was systematically reduced while, at the same period, the costs were increasing, not expressing in production growth. Therefore, it is necessary to increase investment capital, reduce employment costs which are nearly 50% of total production costs. Productivity and marginal productivity are in both cases more beneficial and almost twice as high regarding objectified labor factors. Therefore, it is more efficient in terms of production growth to increase expenditure on material investments. They are necessary in order to maintain adequate production capacity, restore mining fronts and create further exploitation levels. Employment since 2010 has decreased in hard coal mining by around 30%. At the same time the wage costs increased. The largest decrease in employment was recorded in 2014–2017, however, the share of wage costs remained virtually unchanged. This translated into a drop in productivity. The reason for this is the lack of knowledge and experience of new employees which has negative impact on performance time of undertaken tasks and the number of mistakes they make. Despite the abovementioned problems, an important issue is deterioration of the level of occupational safety. On the basis of the obtained results, it can be concluded that the substitution of labor outlays with capital should happen, as this leads to the reduction of production costs. Similar conclusions can be drawn by analyzing the marginal substitution rate. An expenditure decline for employment of PLN 1 means that 40% less objectified labor outsets should be spent to keep mining at the same level. It is therefore clearly stated that the conducted restructuring of hard coal mining does not bring positive results, which means that it is necessary to verify the activities which have been carried out in recent years and take appropriate corrective actions. In turbulent environment individual business management functions should permeate creating an endless cycle of improvement tasks. In order to be able to generate profits, also in times of economic downturn and a drop in prices for manufactured goods, it is necessary to adjust the level of production means to the market needs with a great deal of flexibility. Therefore, the second reason for using the production function by the author was to conduct a scenario analysis of the hard coal production process and to indicate the right way to further control the coal mining process. There were 25 scenarios created but only these models were presented whose explanatory variables showed statistical significance and whose parameters met the assumptions consistent with the theory of the production function. The scenarios, determined with 95% probability, contain combinations of different variants of the analyzed variables. They show what the nearest future of the mining process in Polish mining companies may look like. Economies of scale were obtained only for one case, where employment costs were lowered and investment outlays increased. The model also assumes an increase in production level and in the price of 1 Mg of coal. In this case (Scenario 5) it can be expected that production will grow faster than expenditures on it. However, this is an optimistic scenario. In the remaining scenarios, the disadvantages of the current production model in coal companies are still present or even increasing. The scenarios created up to 2020 indicate that appropriate steps should be taken that shall lead to optimization of the employment level in coal companies and increased investment outlays as soon as possible. Only the correct and confirmed identification of the causes of irregularities in the production process can allow the introduction of effective remedies. The effectiveness of the solutions proposed by the author has been confirmed thanks to the simulation during which the impact of the proposed production strategy on the parameters of the CD function was examined. Providing financial resources for investments is an extremely difficult task, which is not dependent on the coal companies themselves, and mines suffer shortages of miners with the necessary experience, which affects the productivity of the extraction process. The author has proposed and presented a computer program that will allow for the adjustment of employment in individual mines of a coal company thanks to capital outsourcing. The EMT program makes it possible to find miners with appropriate competences who are not burdened with duties in their home mine for the next two weeks. Information on this subject is stored in the database in the status of an employee. Thanks to this, such an employee can be temporarily shifted to work at the mine, where exploitation requires additional labor resources. The simulation showed that by optimizing the employment size and adjusting it to the size of demand for hard coal in a given month at least constant economies of scale can be obtained. At this point, the Cobb-Douglas function can help in determining the most beneficial combination of living and objectified work during a given period of the year. The main advantage of the presented solution is that it does not require financial resources because it is based on the resources of a coal company already owned. Summing up, the use of the Cobb-Douglas production function is a great way to test the results of the production process, it can be used to identify process irregularities and indicate effective remedies, the implementation of which will allow the company to grow and, above all, survive on the turbulent market. The presented example is characteristic for enterprises with a very rigid structure, strategy and resources. The proposed solutions are universal and can be used by all companies operating in similar environmental conditions. The next stage of the research will be searching for additional possibilities to reduce the costs of coal extraction and transport. For this purpose, energy clusters will be determined on the basis of spatial analysis. This will enable the supplies to the recipients of coal having appropriate parameters, in the required quantity and time. In addition, the cluster will consist of units that will enable the management of waste from coal combustion, which will be an additional source of added value of the extraction.

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APPLICATION OF THE COBB-DOUGLAS PRODUCTION FUNCTION TO STUDY THE RESULTS OF THE PRODUCTION PROCESS AND PLANNING UNDER TURBULENT ENVIRONMENT CONDITIONS

Keywords

coal production, production function, turbulent environment, ARIMA models

Abstract

The article presents the possibility of using the Cobb-Douglas production function for planning in a turbulent environment. A case study was carried out – the Cobb-Douglas function was used to examine the condition of the Polish hard coal mining industry and the progress which has been made after undertaking certain activities aimed at increasing the competitiveness of coal companies over recent years. Only the correct and confirmed identification of the causes of irregularities in the production process can allow for the introduction of effective remedies. The effectiveness of the solutions proposed by the author has been confirmed thanks to the simulation during which the impact of the proposed production strategy on the parameters of the CD function was examined. Three variants of production functions models were created and production productivity rates and marginal substitution rates were determined. The results enabled the verification of the progress of restructuring as well as identification of the origin of the observed problems and comparison of the current state with the results of analyses carried out in previous years. Scenarios of possible trend developments for the factors introduced into the function model in order to present remedial measures that could improve the process of hard coal extraction were created. The scenarios were created using the ARIMA class models. Which scenario is the most favourable was determined. A computer program, created by the author, for optimising the level and use of labor resources at the level of the entire coal company has been presented.

ZASTOSOWANIE FUNKCJI PRODUKCJI COBBA-DOUGLASA DO BADANIA REZULTATÓW PROCESU PRODUKCYJNEGO I PLANOWANIA W WARUNKACH TURBULENTNEGO OTOCZENIA

Słowa kluczowe

wydobycie węgla, funkcja produkcji, turbulentne otocznie, modele ARIMA

Streszczenie

W artykule zaprezentowano możliwość zastosowania funkcji produkcji Cobba-Douglasa do planowania w warunkach turbulentnego otoczenia. Przeprowadzono studium przypadku – funkcja Cobba-Douglasa wykorzystana została do zbadania stanu polskiego górnictwa węgla kamiennego oraz postępów podejmowanych w ostatnich latach działań mających na celu zwiększenie konkurencyjności spółek węglowych. Utworzono modele funkcji produkcji w 3 wariantach, wyznaczono wskaźniki produktywności produkcji oraz krańcową stopę substytucji. Pozyskane rezultaty umożliwiły zweryfikowanie postępów restrukturyzacji, określenie głównych przyczyn zidentyfikowanych problemów

oraz porównanie obecnego stanu z wynikami analiz prowadzonych w ubiegłych latach. Tylko prawidłowa i potwierdzona identyfikacja przyczyn nieprawidłowości w procesie produkcji umożliwić może wprowadzenie właściwych środków zaradczych. Skuteczność zaproponowanych przez autorkę rozwiązań została potwierdzona dzięki symulacji, podczas której zbadano wpływ proponowanej strategii produkcji na parametry funkcji CD. W celu wskazania środków zaradczych mogących usprawnić proces wydobycia węgla kamiennego, utworzono scenariusze możliwego rozwoju trendów czynników wprowadzonych do modelu funkcji. Scenariusze utworzono z wykorzystaniem modeli klasy ARIMA. Określono, który scenariusz jest najbardziej korzystny. Zaprezentowano także stworzony przez autorkę program komputerowy, który ma za zadanie zoptymalizowanie poziomu i wykorzystania środków pracy żywej na poziomie całej spółki węglowej.