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INVITED PAPER

Dedicated to Professor Jan Dorosz
on the occasion of his 65th birthday

# Instrumentation optical fibres for wave transformation, signal processing, sensors, and photonic functional components, manufactured at Białystok University of Technology in Dorosz Fibre Optics Laboratory

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**Abstract.** Tailored, specialty optical fibres, made of complex glasses, called collectively as a non-telecommunications or instrumentation family, serve for various optical wave transformations for particular functional purposes and optical signal processing, rather than for long distance lossless and dispersionless, undistorted transmission. Research work on these fibres started during the late seventies of the last century in ITME/Warsaw and in Białystok University of Technology at the Faculty of Electrical Engineering. The initiator of this research at Glass Works Białystok [39] and Białystok University of Technology [4] was, then a very young engineer, Jan Dorosz. Over 35 years of development of the technological team, under his skilful management, resulted in a top laboratory which today does research at the cutting edge of the photonics science. The Białystok Optical Fibre Technology Laboratory (OFTL) is now a pearl in the crown of his Alma Mater. The paper opens this special issue of the PAS Bulletin on Technical Sciences, devoted to professor Jan Dorosz, and shows some of the developments in the area of optical fibre photonics, which were carried out at his active laboratory.

**Key words:** optical fibres, specialty optical fibres, soft glass optical fibres, nontelecom optical fibres, optical fibre photonics, multicore optical fibres, optical fibre microoptics, optical capillaries, photonic fibres, structural optical fibres, signal processing in optical fibres, Białystok University of Technology.

## 1. Introduction

The Laboratory of Optical Fibre Technology at Białystok University of Technology, established and run for many years by Jan Dorosz, has a long tradition, now of over 35 years, of intensive and very creative work [1–73]. The Laboratory specialization were (and are) soft, multicomponent glass optical fibres made by modified multi-crucible (MMT) and rod-intube (RiT) technologies [24, 28]. Now, the Laboratory enters also the advanced modified CVD technologies. During this long period, the laboratory provided countless samples of specialty optical fibres to domestic and international optoelectronic and photonic laboratories - university and industry based. Out of these finely designed fibres, numerable and useful functional components, devices, sensors, and optical sources have been made. Some of the designed novel fibres, like multicore constructions, were really if not the first, then very close to the first ones worldwide [5]. Some pilot applications were striking at that time [7, 16]. The results from this Laboratory, and a few tens of other ones which used Białystok fibres, have been published worldwide in a few hundred of well positioned and well cited papers [17, 18, 39, 41]. Some major families of the specialty fibres researched, manufactured and characterized at Białystok Optical Fibre Technology Laboratory and next used in many practical application laboratory experiments included:

- circular isotropic optical fibres of various materials, and tailored dispersion and refraction parameters [20],
- circular optical fibres with ultimately complex refractive index profiles [10],
- polarizing and polarization maintaining optical fibres [6],
- optical fibres with complex cross sections of the core [14],
- complex material, IR and nonlinear optical fibres [11], and
- multicore optical fibres [22, 26].

The paper is a sort of a short tutorial on these, listed above, specialty optical fibres, showing their beautiful transmission and interesting wave transformation properties, but illustrated and supported richly with the specimens designed, manufactured and characterized skilfully by prof. Jan Dorosz and his excellent Photonics Technological Team at the Faculty of Electrical Engineering, Białystok University of Technology.

## 2. Isotropic optical fibers

Classical eigenequation is used to describe the HE/EH$_{\mathrm{ml}}$ modes of the EM wave in an isotropic optical fibre of arbitrary geometry [52]:

$$\begin{aligned}(\mu_1 F_1'/uF_1 + \mu_2 F_2'/wF_2)(n_1^2 F_1'/uF_1 + n_2^2 F_2'/wF_2)\\ = m^2(\mu_1/u^2 + \mu_2/w^2)(n_1^2/u^2 + n_2^2/w^2),\end{aligned} \tag{1}$$

where $F_1(u)$ – a periodic function describing the core field, $F_2(w)$ – an evanescent function describing the out-of-core

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(cladding) field, $F_1(u)' = dF_1/du$, $dF_2(w)' = dF_2/dw$ – derivatives of wave functions, $u^2 = a^2(k_1^2 - \beta^2)$, $w^2 = a^2(\beta^2 - k_2^2)$ – arguments of wave functions, $k_1 = n_1 k_o > \beta > k_2 = n_2 k_o$, $k_o = 2\pi/\lambda_o$ – wave numbers, respectively in the core, cladding and outside the fibre – in vacuum, $\beta = 2\pi/\lambda$ – modal constant of propagation, $\lambda$ – modal wavelength in optical fibre, $\beta_n = \beta/k = n_{eff}$ – normalized propagation constant, which is an effective modal refractive index, $\omega = 2\pi f$, $V = u^2 + w^2 = a^2 k_o^2(n_1^2 - n_2^2) = (\text{akNA})^2$ – normalized frequency, $(n_1^2 - n_2^2)^{1/2} = \text{NA}$ – numerical aperture of optical fibre, $\mu_i$ – relative magnetic permeability of $i$-th region, $n_1$, $n_2$ – refractive indices of the core and the cladding in the fibre, generally they are not constant but are functions of radius $n_i(r)$ or radius and angle $n_i(r,\Theta)$, $m$ – azimuthal modal number in circular and elliptical fibres and transverse modal number in square and rectangular core fibre, $l$ – radial modal number.

For $m = 1$, and a fibre with circular or strip core, the ideal $E_{ol}$ and $H_{ol}$ modes are not coupled and have only the mentioned in their names field components in the direction of propagation. Particular form of Eq. (1), and the kind of wave functions $F$ depend on the fibre geometry, mainly the core. We assume that the fibre supports hybrid modes HE, EH or, in approximation, LP modes of linear polarization [3]. Two expressions in parentheses on the left side of Eq. (1) represent modes of characteristics combined with magnetic permeability (without charges on the boundary, $H_{o1}$ modes in cylindrical fibre) and core/cladding refractions. Right side of the equation couples the modes into a hybrid form of both fields E and H in the direction of propagation. Wave functions $F$ have to possess a property of mutual transformation between various kinds of tailored optical fibres. Strip and cylindrical cores are special cases of elliptical cores, etc. Part of modal power of $m$-th mode $P_m$ is carried in the core $P_1^m$, and the other part is in the cladding $P_2^m$. The expression $\eta(r) = P_1/P_m$, where $P_m = P_1 + P_2$ is a modal power profile $\eta = (n_{eff} n_g - n_2^2)/\text{NA}^2$ [51].

In the angular coordinates – longitudinal axial, radial and azimuthal $(z, r, \phi)$ for a quasi-ideal cylindrical fibre, the wave functions $F$ are quasi-periodic and quasi-exponential Bessel functions $J_m(u)$ and $K_m(w)$, and the eigenequation has the following classical form ($\mu_1 = \mu_2 = 1$):

$$(J_m'/uJ_m + K_m'/wK_m)(n_1^2 J_m'/uJ_n + n_2^2 K_m'/wK_m) = m^2(u^{-2} + w^{-2})(n_1^2 u^{-2} + n_2^2 w^{-2}). \tag{2}$$

Left side of the equation, in a product form of two components, represents the modes dependent and nondependent of the refractive profile of the fibre. For azimuthal modal number $m = 0$ (lack of azimuthal dependence) the right side of equation is equal to zero and the modes are not coupled. Each factor equalled to zero gives $E_z = 0$ for the nondependent mode on the profile $H_{o1}$ and $H_z = 0$ for the mode dependent on the profile $E_{o1}$:

$$\begin{gathered}(J_m'/uJ_m + K_m'/wK_m) = 0,\\ (n_1^2 J_m'/uJ_n + n_2^2 K_m'/wK_m) = 0.\end{gathered} \tag{3}$$

For nonzero azimuthal number $m > 0$, assuming small refraction difference $n_1 \approx n_2$, which is a condition of the weak propagation, and using expressions for Bessel functions derivatives $J_m' = -mJ_m/u + J_{m-1} = mJ_m/u - J_{m+1}$, $K_m' = -mK_m/w - K_{m-1} = mK_m/w - K_{m+1}$ one obtains:

$$(J_m'/uJ_m + K_m'/wK_m) = m(u^{-2} + w^{-2}), \tag{4}$$

$$(J_{m-1}'/uJ_m + K_{m+1}'/wK_m) = 0, \quad \text{for EH}, \tag{5}$$

$$(J_{m+1}'/uJ_m + K_{m-1}'/wK_m) = 0, \quad \text{for HE}. \tag{6}$$

Solutions of these eigenequations for the isotropic fibre are presented schematically in Fig. 1. The wave propagates in the fibre with the phase velocity $v = \omega/\beta$, confined between two ultimate boundaries $v_1 = c/n_1$ and $v_2 = c/n_2$, and with the group velocity $v_g = d\omega/d\beta$. The group refraction index determines this kind of refraction which is combined with the rate of energy transfer along the fibre $n_g = c/v_g$. The fundamental mode is $\text{HE}_{11}$ with the cut-off frequency equal to zero. Power distribution in the fibre cross section, propagation constant, and arguments of wave functions may be all calculated from approximate expressions valid for various ranges of the normalized frequency (assuming $n_1 \approx n_2$) [51]. For small values of $V$, the following equations are valid: $w \approx 2\exp - [e + J_m(V)/VJ_m(V)]$, $e$-Euler constant, $u \approx (1 + 2^{1/2})V/[1 + (4 + V^4)^{1/4}]$. Modal cut-off is for $w = 0$ and $u = V$, and the modal wave propagates with the velocity of an unconfined free wave in the space filled with the cladding material. The cut-off condition for both kinds of modes $E_{ol}$ and $H_{ol}$ is $J_o(u) = J_o(V_c) = 0$, where $V_c$ is a value of $V$ for cut-off. The cut-off for modes which fields are constant azimuthally does not depend on $n_1$ and $n_2$ refractions (which is counter-intuitive). For the modes with $m > 1$, using the asymptotic value of function $K_m(w \to 0) \to (m-1)!2^{m-1}/w^m$, the cut-off condition is:

$$\begin{gathered}J_{m-1}/uJ_m = n_2^2/(m-1)(n_1^2 + n_2^2),\\ \text{for } n_1 \approx n_2 J_{m-1}/uJ_m = 1/2(m-1).\end{gathered} \tag{7}$$

The group velocity, for a certain range of the normalized frequency, is smaller than the value for a free wave in the core medium, or the group refraction has a value bigger than $n_1$. For small values of absolute differential refraction $\Delta n = n_1 - n_2$, the profile of modal power is $\eta = (n_{eff} n_g - n_2^2)/\text{NA}^2 \approx 1 - (u^2/V^2)(1 - K_o^2/K_1^2)$. Normalized power distribution in the far field, in a weakly propagating cylindrical fibre is [52]:

$$\begin{gathered}P/P_o = \{[u^2 w^2/(u^2 - \alpha^2)(w^2 + \alpha^2)]\\ \cdot [J_o(\alpha) - \alpha J_1(\alpha) J_o(u)/uJ_1(u)]\}^2,\\ \alpha = \text{ka}\sin\Theta = \text{VNA}\sin\Theta.\end{gathered} \tag{8}$$

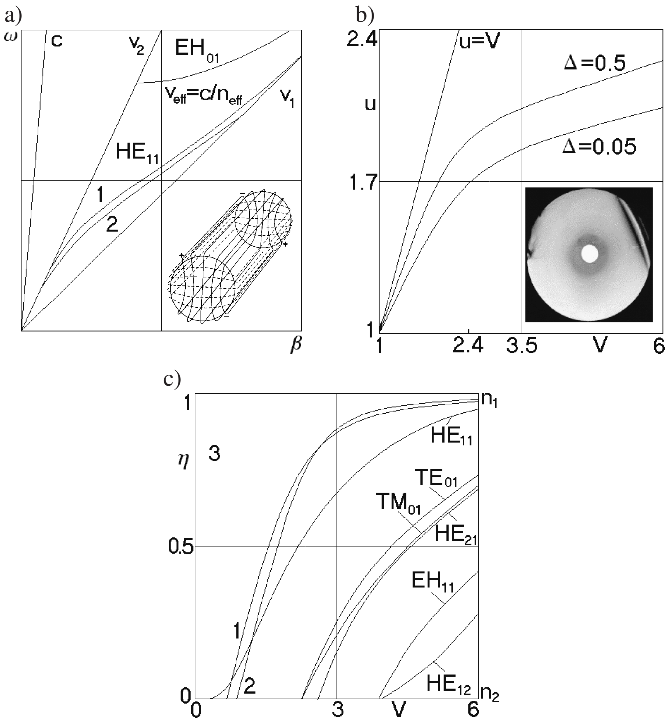



Fig. 1. Schematic characteristics of a cylindrical isotropic optical fibre calculated from the eigenequation (2) and its particular forms (3-8), a) phase space of the fibre $v = \omega/\beta$, $v_g = \partial\omega/\partial\beta$, $v_2 = c/n_2 > v_{eff} > v_1 = c/n_1$, 1,2– $\mathrm{NA}_1 < NA_2$; insert shows field distribution for $\mathrm{HE}_{11}$ mode, b) value of the u argument of the Bessel function for small and big relative differential refraction calculated from $u \approx (1+2^{1/2})V/[1+(4+V^4)^{1/4}]$, $\Delta = (n_1^2 - n_2^2)/n_1^2$, insert shows real pulled fibre by the Białystok OFTL, c) normalized distribution of modal fields as a function of normalized frequency $\eta = (n_{eff} n_g - n_2^2)/\mathrm{NA}^2 \approx 1 - (u^2/V^2)(1 - K_o^2/K_1^2)$, $1 - n_1 = 1,6$, $n_2 = 1,55$; $2 - n_1 = 1,6$, $n_2 = 1$

Optical fibres of the above characteristics were manufactured at the Białystok OFTL by the modified multicrucible technology MMT [24]. Fibre core/cladding proportions were determined using the Poisuille flow law:

$$a_r/a_p = \sqrt{Q_r/Q_p}, \quad \text{where} \quad Q = \pi \Pr{}^4/8\eta l, \tag{9}$$

where $Q(r,p)$ – volume flow of core/cladding glasses, $P$ – pressure difference in the crucible nozzle cross section, $\eta$ – viscosity, $r$, $l$ – nozzle radius and length. External fibre diameter was 125 $\mu$m to fit standard measurement systems, and stems from the flow continuity equation $v_i Q_i$ = const, and depends on diameter of external nozzle and rate of fibre pulling.

## 3. Optical fibres with complex refractive profiles

MMT technology developed at Białystok OFTL [21] allowed to shape complex refractions in fibre cross sections. Three groups of refractions were researched and then the fibres were manufactured and characterized: multi-step, ring and double W. Pairs of relevant glasses were chosen to enable efficient refraction shaping by mobile high-index ion diffusion. Necessary condition for diffusion is existence of gradient of chemical potential. The diffusion was modelled using the second Fick's law for axial symmetry:

$$\left(\partial^2 c/\partial r^2\right) + \partial c/r\partial r - (1/D)\left(\partial c/\partial t\right) = 0, \tag{10}$$

where $c = c(r,t)$ – distribution of ions modifiers, $D$ – diffusion constant, $t$ – diffusion time, $r$ - radius. For boundary conditions: $t = 0$, $c = (r,t) = C_o$ when $0 < r < a$ and $c(r,t) = 0$ for $r > a$, where a-fibre core radius, $C_o$ – concentration of modifying ions in the source glass, the diffusion equation has a solution in a form of concentration distribution of the diffusing ion (thus, changed refraction):

$$c(r,t) = C_0 \int_0^\infty \exp\left(\frac{Dt}{R^2}u^2\right) J_0\left(-\frac{r}{R}u\right) J_1(u)du, \tag{11}$$

where $C_o$ – initial concentration, $D$, $t$, a as above, $J$ – Bessel function, $2R_r$ – diameter of core nozzle, $2R_p$ – diameter of cladding nozzle, $L_d$ – diffusion length. Normalized diffusion (ion exchange) coefficient is defined $K = Dt/R^2$. It is possible to combine $K$ with the parameters of technological process. The flow continuity equation is $R^2 V = a^2 v$, where $R$ – nozzle diameter, $V$ – rate of glass flow, $a$ – fibre radius, $v$ – fibre pulling rate. $K$ is combined with volume glass flow via the relation $K = D\pi l/Q_r$. Diffusion length and time are related $t_d = L_d/V = L_d R^2/a^2 v$, thus $K = DL_d/a^2 v_{tr}$, where $v_{tr}$ – glass flow rate from core crucible. This simplification assumes that the resulting refractive profile of the MMT fibre is approximately of $\alpha$ power type. Calculations of complex profiles for particular ions Pb, Ba, K, and Na lead via determination of the localization of the Matano plane using modified Boltzman-Matano method:

$$\begin{aligned} D_{ion} = -(1/2t_d)(dx/dC)_{|x} \int_{C_r}^{C_x} (X - X_M)dC, K_{ion} \\ = Dl_d/r_c^2 v, \end{aligned} \tag{12}$$

where $X_M$ – localization of Matano plane. Figure 2 shows some of manufactured fibers in Białystok OFTL with very complex refractive index profiles.

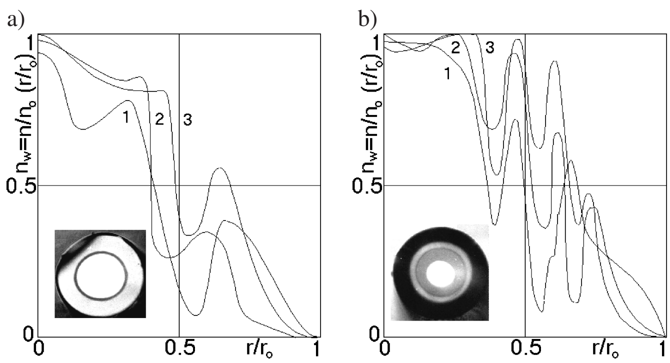



Fig. 2. Optical fibers of complex refractive profiles manufactured at Białystok OFTL, cross-section photographs and measured refractive index profiles: a) glasses F2-S6-S7-S8, b) glasses BF8-BLF2-F2-S6 (after Ref. 24). Curves 1, 2, and 3 show changes of refraction depending on the thermal process parameters

## 4. Ring core optical fibers

Refractive index profile in a ring-core step-index optical fibre is $n(r) = n_1$ for $a \leq r \leq b$ and $n_2$ outside this region, i.e in the central refractive depression and outside the ring. Normalized thickness of the ring core is $\eta = b/a$. We assume TEM wave in this fibre and weak propagation $\Delta \approx 1 - n_2/n_1 \ll 1$. Electrical field in three fibre regions is:

$$\begin{gathered} E_m^y = C_1 I_m(W)\cos(m\theta) \\ \text{or} \quad E_m^y = C_1 I_m(W)\sin(m\theta), \\ W = wr/\eta a = (w/\eta)(r/a), \quad \text{for} \quad 0 \leq r \leq a, \end{gathered} \tag{13}$$

$$\begin{gathered} E_m^y = [C_2 J_m(U) + C_3 N_m(U)]\cos(m\theta), \\ \text{or} \quad E_m^y = [C_2 J_m(U) + C_3 N_m(U)]\sin(m\theta), \\ U = ur/\eta a, \quad \text{for} \quad a \leq r \leq b, \end{gathered} \tag{14}$$

$$\begin{gathered} E_m^y = C_4 K_m(W\eta)\cos(m\theta) \\ \text{or} \quad E_m^y = C_4 K_m(W\eta)\sin(m\theta) \\ \text{for} \quad b \leq r, \end{gathered} \tag{15}$$

where $C_i$ are amplitude constants, $I_m$, $J_m$, $N_m$, $K_m$ are Bessel functions of the first and second kind of the $m$-th order, $u$, $w$ – are wave arguments $u^2 = b^2k^2(n_1^2 - \beta^2k^{-2})$, $w^2 = b^2k^2(\beta^2k^{-2} - n_2^2)$, $V^2 = b^2k^2(n_1^2 - n_2^2)$ – is normalized frequency, $k = 2\pi/\lambda$, $m$ – azimuthal modal number, $\eta = b/a$, $\beta$ – propagation constant. Applying boundary conditions for equations (13–15) one obtains eigenequation for ring-core fibre:

$$\begin{gathered} wK_{m+1}(w)/uK_m(w) \\ = \{uI_m(w/\eta)[J_{m+1}(u)N_{m+1}(u/\eta) \\ -J_{m+1}(u/\eta)N_{m+1}(u)] + wI_{m+1}(w/\eta)[J_{m+1}(u)N_m(u/\eta) \\ -J_m(u/\eta)N_{m+1}(u)]\}/\{uI_m(w/\eta)[J_{m+1}(u/\eta)N_m(u) \\ -J_m(u)N_{m+1}(u/\eta)] + wI_{m+1}(w/\eta)[J_m(u/\eta)N_m(u) \\ -J_m(u)N_m(u/\eta)]\}. \end{gathered} \tag{16}$$

This equation, for $\eta \to \infty$, is reduced to classical eigenequation of a cylindrical weakly propagating fibre. Figure 3 presents numerical solutions of this equation for different $\eta$ ring-core fibres.

The lowest order mode in ring-core fibre is $LP_{01}$, which is equivalent to $HE_{11}$. The next mode is $LP_{11}$, equivalent to $TE_{01}$, $TM_{01}$ or $HE_{21}$. Increasing $\eta$, for constant $V$, from 1 to infinity, ring-index fibre is closer to classical fibre. With decreasing $\eta$ to unity, for $V =$ const, the number of modes is smaller. Ring-index fibres can be divided to two classes: with central depression small - comparable to the wavelength $2a \approx \lambda$, and big $2a \gg \lambda$. The second group, for big 2a, may propagate planar modes, as in slightly curved planar optical waveguide. A standing planar wave builds at the circumference, while the propagation is along the axis. The first group works in two wave conditions: 1 – choice of $V$ and $\eta$ values for singlemode propagation of $LP_{01}$, 2 – via introduction of sufficiently big optical losses in the axial depression and discrimination of $LP_{01}$, due to different penetration depths of $LP_{01}$ and $LP_{11}$ in the depression. Ring-index fibre may be single mode for $LP_{01}$, but also quasi single mode for $LP_{11}$. In the latter case the ring-index fibre works as single-mode at the values of $V$ bigger than for 2,405, Fig. 3c. The value of cut-off normalized frequency for $LP_{11}$, $V_c(LP_{11})$ is obtained from (16) assuming $u = V$ (or $n_{eff}(LP_{11}) = n_2$) for modal numbers and treating $\eta$ as a parameter of the solutions. Figure 3c shows $V_c(LP_{11}) = f(\eta)$. The fibre is singlemode for $LP_{01}$ below the curve $V(\eta)$. When $\eta$ goes to unity then V may be considerably bigger than the cut-off of $LP_{01}$ in classical fibre.

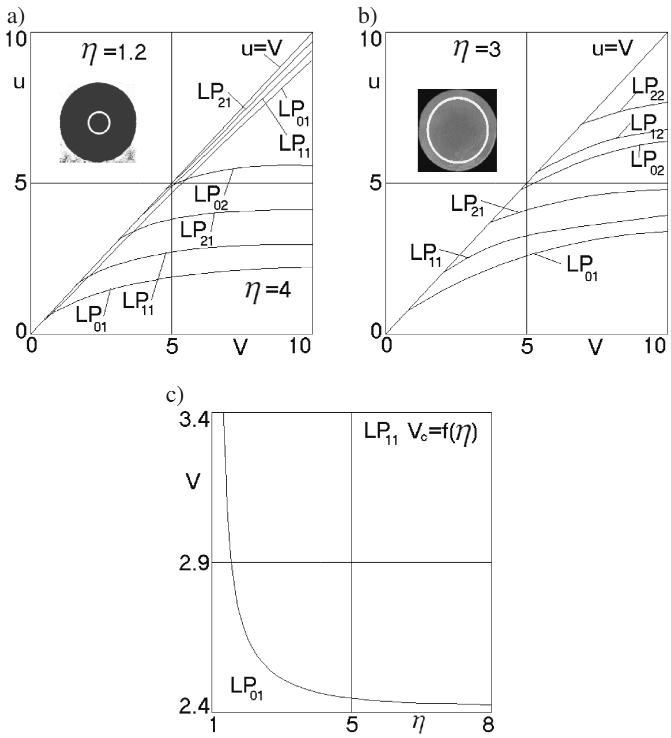



Fig. 3. Optical fibers of ring cores and their modal characteristics. Inserts show cross sections of manufactured optical fibers at Białystok OFTL, a-b) modal cut-off characteristics for various values of $\eta$ parameter, c) calculated condition of single mode work of ring-index fiber as a function of $\eta$

## 5. Elliptical core optical fibres

The wave equation in elliptical coordinates for $E_z$ field component is

$$\begin{gathered} \partial^2E_z/\partial\xi^2 + \partial^2E_z/\partial\eta^2 \\ +[q^2(\varepsilon k_o^2 - \beta^2)(\sinh^2\xi + \sin^2\eta)]E_z = 0, \end{gathered} \tag{17}$$

where $k_o = 2\pi/\lambda_o$ – vacuum wave number, $\lambda_o$ – wavelength in vacuum, $\beta$ – propagation constant, $\varepsilon$ – dielectric constant equal $\varepsilon_1$ in core ($\xi < \xi_o$) and $\varepsilon_2$ in cladding ($\xi > \xi_o$) for step index fibre. Separation of azimuthal $\Theta(\eta)$ and radial components $R(\xi)$ of EM field is done assuming product form of $E$ and $H$ functions: $E(\xi,\eta) = \Theta(\eta)R(\xi)$. Wave equation for azimuthal and radial components in elliptical coordinates has the Mathieu form for $\Theta(\eta)$ and modified Mathieu for $R(\xi)$:

$$\begin{aligned}&\partial^2\Theta/\partial\eta^2+[c-0,5(\varepsilon k_o^2-\beta^2)q^2\cos 2\eta]\Theta=0\\&\partial^2R/\partial\xi^2-[c-(0,5(\varepsilon k_o^2-\beta^2)q^2\cosh 2\xi]R=0,\end{aligned}\tag{18}$$

where $c$ – is a separation constant. Solutions to Mathieu equations are azimuthal Mathieu functions of sinus $s_{en}$ and cosinus type $c_{en}$, equally for the core and cladding of elliptical fibre, but with the change of the value of argument $\gamma$ from $\gamma_{core}$ to $\gamma_{clad}$: $\Theta(\eta)=[s_{en}(\eta,\gamma^2)$ or $c_{en}(\eta,\gamma^2)]$. These functions are equivalents to field changes along the circumference of the circle (here an ellipse) of type $\cos n\phi$ and $\sin n\phi$ for cylindrical fibre. The $\gamma$ parameter is:

$$\begin{aligned}\gamma^2&=(\varepsilon k_o^2-\beta^2)(q^2/4)\\&=[(a/b)^2-1](b^2/4)(\varepsilon k_o^2-\beta^2)\\&=(a^2/b^2-1)(u^2/4),\end{aligned}\tag{19}$$

where $q^2=a^2-b^2$, $a$, $b$ – ellipse semi axes, bigger and smaller, $u=b^2(\varepsilon k_o^2-\beta^2)$ – equivalent to $u$ argument of Bessel function in cylindrical fibre. Periodic solutions of Mathieu wave equations exist only for characteristic values of separation parameter $c$. These values are: $a_n(\gamma^2)$ even and $b_n(\gamma^2)$ odd. For circular core fibre $q=0$ and $\gamma=0$ and Mathieu solutions for azimuthal components are $\cos n\Theta$ and $\sin n\Theta$. Solutions of modified Mathieu equation of the core are radial Mathieu functions: $R(\xi)=[S_{en}(\xi_o,\gamma^2_{core})$ or $C_{en}(\xi_o,\gamma^2_{core})]$, sinusoidal $S_e$ and cosinusoidal $C_e$ type. These functions are similar to J Bessel functions. Solution of modified equation in the cladding are evanescent radial Matthieu functions $F_{ek}$ and $G_{ek}$, similar to $K$ Bessel functions: $R(\xi)=[F_{ek}(\xi_o,\gamma^2_{clad})$ or $G_{ek}(\xi_o,\gamma^2_{clad})]$. At the core/cladding boundary, the periodic Mathieu functions transform smoothly into evanescent ones. Evolution of elliptical core into circular results into transformation of Mathieu functions into Bessel ones. The arguments of azimuthal Mathieu functions are $n_1$ and $n_2$ refractions in step index fibre of $n(r)$ in gradient index fibre. Fitting of fields at the core/cladding boundary requires usage of infinite set of functions opposite to single ones in the case of circular core fibre. This stems from the fact that confocal ellipses do not have the same shape, opposite to concentrical circles. For bigger arguments, the ellipses turn more similar to circles. All modes of elliptical fibre are hybrid, opposite to circular one, where some modes are azimuthally symmetric, transverse TM without longitudinal components of the field. The degenerated fundamental mode $\mathrm{HE}_{11}$ of circular fibre splits to two modes when the core turns weakly elliptical. These modes are odd and even ${}_o\mathrm{HE}_{11}$ and ${}_e\mathrm{HE}_{11}$, where axial magnetic fields are odd and even Mathieu functions. Analogously to $\mathrm{HE}_{11}$, modes ${}_o\mathrm{HE}_{11}$ and ${}_e\mathrm{HE}_{11}$ do not have cut-off and electrical fields are transverse, along the major axis of ellipse for ${}_o\mathrm{HE}_{11}$ and along minor axis for ${}_e\mathrm{HE}_{11}$. When the ellipse turns to a strip, the mode ${}_o\mathrm{HE}_{11}$ turns to $\mathrm{H}_{10}$ and the mode ${}_e\mathrm{HE}_{11}$ turns to $\mathrm{E}_{10}$. To simplify the solutions, a single function description is assumed for core and cladding. Simplification holds only for $\mathrm{HE}_{1m}$ modes, small ellipticity $a/b<2,5$, and weakly guiding condition $\Delta n=n_1-n_2\approx 0$. Other simplifications assume multifunction description in core and single function in cladding or vice versa. The eigenequation for single function description is, respectively, for odd and even modes:

$$\begin{aligned}&[S'_{en}/u^2S_{en}+G'_{ekn}/w^2G_{ekn}]\\&[\varepsilon_1C'_{en}/u^2C_{en}+\varepsilon_2F'_{ekn}/w^2F_{ekn}]\\&=n^2(1/u^2+1/w^2)(\varepsilon_1/u^2+\varepsilon_2/w^2),\end{aligned}\tag{20}$$

$$\begin{aligned}&[C'_{en}/u^2C_{en}+F'_{ekn}/w^2F_{ekn}]\\&[\varepsilon_1S'_{en}/u^2S_{en}+\varepsilon_2G'_{ekn}/w^2G_{ekn}]\\&=n^2(1/u^2+1/w^2)(\varepsilon_1/u^2+\varepsilon_2/w^2).\end{aligned}\tag{21}$$

Arguments of Mathieu functions are $\xi_o$ and $\gamma_1^2(core)=q^2(k_1^2-\beta^2)/4=u^2(a^2/b^2-1)/4$, $\gamma_2^2(clad)=q^2(\beta^2-k_2^2)/4=w^2(a^2/b^2-1)/4$, where (similarly to cylindrical and planar core fibre) $k_1=n_1k_o$, $k_2=n_2k_o$, $u^2=b^2(k_1^2-\beta^2)$, $w^2=b^2(\beta^2-k_2^2)$, $V_b^2=u^2+w^2=k_o^2b^2(n_1^2-n_2^2)$. When the ellipse turns to a circle then $C'_{en}/C_{en}$ and $S'_{en}/S_{en}$ go to $uJ'_n/J_n$ and $F'_{ekn}/F_{ekn}$ and $G'_{ekn}/G_{ekn}$ turn to $wK'_n/K_n$. When ellipse turns to strip then $C_{en}$ goes to $\cos u$, and $S_{en}$ goes to $\sin u$, and $F_{ekn}$ and $G_{ekn}$ go to $e^{-w}$. The mode ${}_o\mathrm{HE}_{11}$ of transverse $E_t$ field along the longer axis, has bigger propagation constant than ${}_e\mathrm{HE}_{11}$ ${}_o\beta>{}_e\beta$, has smaller phase velocity, is more resistant to fibre bending losses and is called slow mode. The most important feature of singlemode elliptical fibre is birefringence $B=\Delta\beta={}_o\beta-{}_e\beta$ or $\Delta n_e={}_on_e-{}_en_e$. $B$ is function of $V$ and $\Delta n=n_1-n_2$. Applying some geometrical and refractive simplifications like small ellipticity and weak propagation several useful analytical forms of eigenequation may be derived:

$$a\Delta\beta/e^2(1-(n_2/n_1)^2)^{3/2}=3\pi V^2/(V+2)^4,\tag{22}$$

$$a\Delta\beta/e^2(1-(n_2/n_1)^2)^{3/2}=u^2w^2/8V^3,\tag{23}$$

$$\begin{aligned}&a\Delta\beta/e^2(1-(n_2/n_1)^2)^{3/2}\\&=[u^4w^3/8V^5J_1^2][K_o/K_1+1/w],\end{aligned}\tag{24}$$

$$\begin{aligned}&a\Delta\beta/e^2(1-(n_2/n_1)^2)^{3/2}\\&=[u^2w^2/8V^3][1+uK_o^2J_2/K_1^2J_1],\end{aligned}\tag{25}$$

$$\begin{aligned}&a\Delta\beta/e^2(1-(n_2/n_1)^2)^{3/2}\\&=(u^2w^2/8V^5)\{(J_o/J_1)^3[(u^2-w^2)w^2/u]\\&+[J_o/J_1]^2[(w^4+u^4\}/u^2]\\&+(J_o/J_1)2u(4+w^2)-(8+w^2-u^2)\}.\end{aligned}\tag{26}$$

Elliptical and strip core fibres exhibit zero birefringence for certain $V$ and ellipticity $\Delta n_g={}^on_g-{}^en_g=0$. For smaller $V$, there is ${}^on_g>{}^en_g$ and $\Delta n_g>0$. For bigger $V$, the sign of ellipticity changes ${}^on_g<{}^en_g$ and $\Delta n_g<0$. Group velocity of the slow (odd, peculiar) mode ${}^o\mathrm{HE}_{11}$ turns bigger than the fast mode (even, regular) ${}^e\mathrm{HE}_{11}$. Phase velocities remain unchanged ${}^ov_p<{}^ev_p$. Rectangular symmetry of elliptical fibre cause that all modes have two orientations: ${}^e\mathrm{HE}_{ml}$, ${}^o\mathrm{HE}_{ml}$, ${}^e\mathrm{EH}_{ml}$, ${}^o\mathrm{EH}_{ml}$, where modal numbers denote: $m$ – field periodicity for $\eta$ coordinate, $l$ – $l$-th root of the eigenequation. Fundamental modes ${}^e\mathrm{HE}_{11}$,

${}^{o}$HE$_{11}$ are not degenerated and have cut-offs for $V = 0$. For bigger ellipticities, modes of higher order ${}^{e}$HE$_{ml}$, ${}^{o}$HE$_{ml}$ have respectively bigger and smaller cut-off frequencies. For bigger ellipticity, the mode ${}^{o}$HE$_{11}$ is more confined in the core than ${}^{e}$HE$_{11}$. Chosen modal characteristics of elliptical fibres manufactured at Białystok OFTL are presented in Fig. 4.

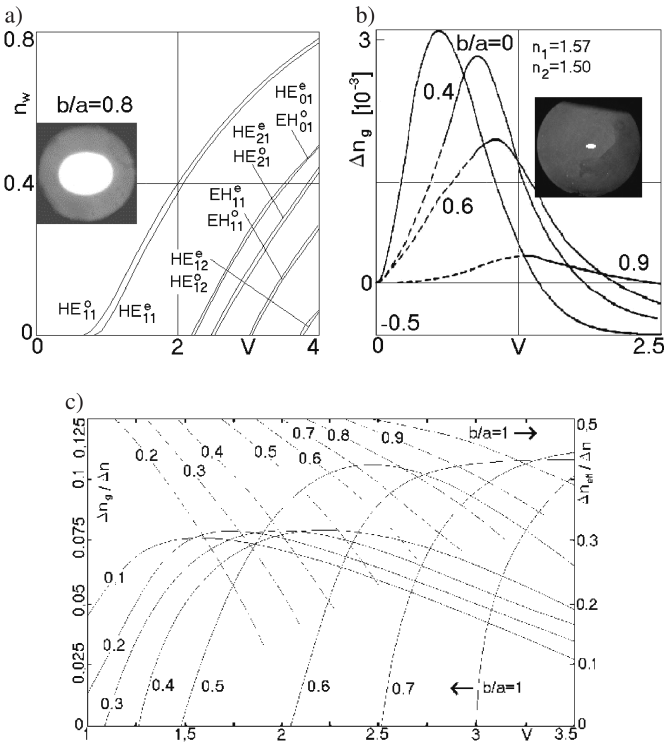



Fig. 4. Optical fibers with elliptical cores, multimode and single-mode [50]. Inserts show elliptical fiber cross-sections, Fibers manufactured at Białystok OFTL, a) propagation constants (modal eigenrefraction) as function of normalized frequency $V$, b) function of differential group refraction, c) normalized birefringence for HE$^{e}_{01}$ and HE$^{o}_{11}$: phase (left axis) and group (right axis) for fundamental modes. $n_w = (n_{eff}^2 - n_2^2)/(n_1^2 - n_2^2)$, $n_{w1} = \Delta n_{eff}/\Delta n^2$, $\Delta n_{eff} = n^o_{eff} - n^e_{eff}$, $\Delta n_g = n^o_g - n^e_g$, $n_g = c/v_g$, $P_w = \Delta\eta/\eta\Delta n$, $\Delta\eta = \eta^o - \eta^e$

## 6. Strip core optical fibres

A single mode optical fibre with perfect strip core, is hypothetically an extremely attractive transmission medium between integrated circuits of planar optics. Low-loss and non-distorting interconnections between photonic integrated circuits (PICs) require precision and selective etching of aligning $V$-grooves for classical circular core optical fibres. Coupling losses between circular cores and planar optical waveguides are a considerable part of the power balance. Let us assume a single mode strip core 2r thick ($x$-direction) and 20r wide ($y$-direction) of $n_1$ refraction, embedded in $n_2$ cladding. Large core width allows one to assume field constancy in this direction. Optical wave propagates along fibre axis in two modes E$_{m0}$, H$_{m0}$ of non-coupled E and H field components in the direction of propagation. A single transverse modal number in planar geometry shows field changeability only in direction of the core thickness. The fundamental modes are E$_{10}$ and H$_{10}$. Eigenequation (1) in general form is here still valid. Periodic and evanescent functions are in this case very simple $F_1 = \sin(u)$, and $\cos(u)$; $F_2 = \exp(-w)$. For such functions, the approximate eigenequation of strip core optical fibre is:

$$[\cos(u)/u\sin(u) - \exp(-w)/w\exp(-w)]$$
$$[n_1^2\cos(u)/u\sin(u) - n_2^2\exp(-w)/w\exp(-w)] = 0, \tag{27}$$

where modal number $l = 0$ denotes field constancy in y direction. Equation (27) holds for even modes with field maximum in the core strip centre. For odd modes with field minimum in core strip centre, the eigenequation is:

$$[-\sin(u)/u\cos(u) - \exp(-w)/w\exp(-w)]$$
$$[-n_1^2\sin(u)/u\cos(u) - n_2^2\exp(-w)/w\exp(-w)] = 0. \tag{28}$$

Equalizing product factors in Eq. (27) to zero, one obtains the eigenequation for H$^e$ modes, which are nondependent of the refractive index profile $\mathrm{tg}(u) = w/u$, and for modes E$^e$ dependent on the profile (of electrical charges on the media boundary) $\mathrm{tg}(u) = n_1^2 w/n_2^2 u$. Equation (28) gives solutions with minimal field in the middle of the strip core thickness, for odd modes H$^o$ $\mathrm{tg}(u) = -u/w$, and for modes E$^o$ $\mathrm{tg}(u) = -n_1^2 u/n_2^2 w$. Regular mode cut-off is when the effective modal refraction $n_{eff}$ is equal to cladding refraction $n_2$. When $\beta_n = n_{eff} = n_2$, then $w = 0$, $u = V = V_c$, $\sin V_c = 0$, $V_c = 0, \pi, 2\pi$,... For peculiar modes is $\cos V_c = 0$, $V_c = \pi/2, 3\pi/2, 5\pi/2$,... Figure 5 shows schematically modal characteristics of strip core optical fibres. Such fibres were manufactured at Białystok OFTL [63]. Fundamental modes H$_{10}$ (slow) and E$_{10}$ (fast) have slightly different propagation constants. Simultaneous excitation of both modes causes the strip core fibre to be birefringent. The modal birefringence $\Delta n_{eff}$ (eigenbirefringence) is approximately proportional to the square of differential refraction of core and cladding $\Delta n$, and holds for a wide range of this value $\Delta n = 0,1$–$10\%$. Normalized birefringence for the strip core is presented in the form $\Delta n_{eff}/(\Delta n)^2 = f(V)$, similarly to elliptical cores. Strip core may be treated approximately as one of the solutions for slim elliptical cores. Phase and group velocities are expressed as previously $v = \omega/\beta$, $\omega = 2\pi f$, $v_g = \partial\omega/\partial\beta$ – is curve inclination in phase coordinates $\omega - \beta$, $v_{gn} = v_g/c = V\partial n_{eff}/\partial V + n_{eff}$, $n_{eff} = \beta/k = \beta c/\omega$, $V = \omega$rNA/c, $c/v_g = n_g$. For small normalized frequencies $V$, both velocities $v$ and $v_g$ of fundamental fast mode E$_{10}$ are bigger than for the slow fundamental mode H$_{10}$. For larger values of $V$, the value of $v_g$ for fast mode is smaller than for slow mode, and the differentia value $\Delta v_g$ is equal to zero for $V = V_{go}$.

The confinement coefficient for modal field is defined as $\eta = P_1/(P_1 + P_2)$. For strip core it has the following analytical form of a function solely of wave arguments and normalized frequency $u^2 + w^2 = V^2$, $\eta = 1 - u^2/V^2(1 + w)$ for the mode nondependent of refraction H$_{10}$, and $\eta = 1 - n_1^2 n_2^2 u^2/(n_1^2 n_2^2 V^2 + n_1^4 w^3 + n_2^4 w u^2)$ for the mode dependent of refraction $E_{10}$. For all shapes of core, the following relation is valid: $c^2/vv_g = n_1^2\eta + n_2^2(1-\eta)$ [51]. For $H_{10}$ mode, the power distribution is not changing in the function of $\Delta n$. For mode $E_{10}$, at pre-set value of $V$, thickness 2r of the strip core gets bigger, when the value of $\Delta n$ diminishes. When 2r→ ∞, $\Delta n \rightarrow 0$ then the whole optical power is contained in the core. Far field of optical fibre with strip core is determined, as in other cases, by the geometry of radiating core aperture, and is a sum of all amplitudes and phases from all points of the aperture. For $H_{10}$ mode, the y direction field component, normalized to one against the maximum value on the core/cladding boundary changes trigonometrically $E_y = \cos(ux/r)/\cos(u)$. Outside the strip core, the field diminishes exponentially to zero $E_y = \exp(-wx/r)/\exp(-w)$. Assuming that the finite width of the strip core does not influence the field too much, and summing over the aperture using Fourier transform and applying $\text{tg}(u) = w/u$ as a solution of the eigenequation for regular modes $H^e$, the dependence on normalized optical power distribution $P(\Theta)$ as function of angle $\Theta$ measured from fibre axis is:

$$\begin{aligned} P_n &= P(\Theta)/P_o \\ &= [u^2w^2(\cos\alpha - \alpha\sin\alpha\cos(u)/u\sin(u))/ \\ &\quad (u^2-\alpha^2)(w^2+\alpha^2)]^2, \\ \alpha &= kr\sin\Theta = V\sin\Theta/NA. \end{aligned} \tag{29}$$

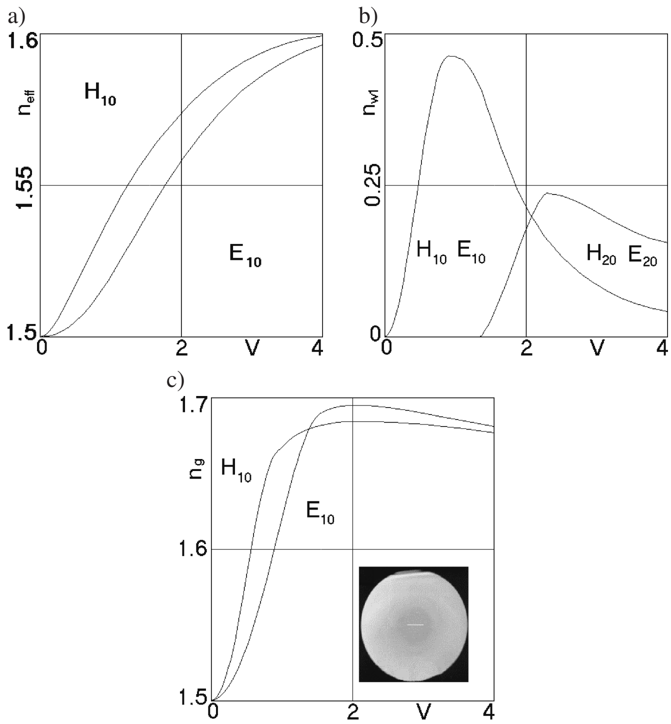



Fig. 5. Modal characteristics of optical fibers with strip cores; $2b/2a = 2 \times 25$ $\mu$m, $b/a = 0,08$, $\Delta = 2\%$; a) effective refraction as function of $V$ for fundamental modes, b) normalized birefringence of strip core optical fiber $n_{w1} = \Delta n_{eff}/(\Delta n)^2 = f(V)$, c) group refraction for fundamental modes. Insert shows strip core optical fiber manufactured at Białystok OFTL

For large $V$, the aperture is wide, measured by the parameter NA/$\lambda_o$, and optical power is totally confined to the core strip $\eta$ ≈1 and the field outside the core, in cladding, falls to zero abruptly. Radiation characteristic of a system with large aperture and strongly defined refraction boundaries of large contrast has a main lobe and a large number of side lobes of relatively big intensities. For smaller $V$, the aperture gets narrower, the main lobe widens, reaching the maximum value. Further decreasing of $V$ causes again narrowing of the main radiation lobe, because the modal field propagates deeper from core to cladding, and the effective radiative aperture increases. Aperture boundaries are now not well (strongly) defined, and the boundary effective refraction contrast fades, causing smaller field changes there. Field distribution in the aperture is, in approximation, Gaussian, and its Fourier transform, or the radiation characteristic, is also Gaussian, without side lobes.

## 7. Polarization optical fibres

Optical fibres with strong birefringence, some of them with additional discrimination of the second polarization component of the fundamental mode, are indispensable in interefrometric applications, where there is needed polarization conformance of the interfering beams of light transmitted long distance along signal optical fibres from sensing optical fibres. In some instruments, the polarization separation between two orthogonally polarized components of the fundamental mode is required to be as high as $-60$ bB or more [54]. Inside a broad family of anisotropic optical fibres, there are birefringent fibres and polarizing ones. Polarizing fibres may be split to several kinds: fibres of strong linear birefringence – maintaining the input state of wave polarization, polarizing fibres – behaving like a distributed polarizer, fibres with circular and elliptical birefringence, and fibres of modulated birefringence. The most widely used polarization maintaining optical fibres are Panda and Bow-tie. The birefringence is inbuilt in the internal fibre structure, via inserts from borosilicate or aluminosilicate glass of higher linear expansion coefficients than the host glass, and located in the cladding symmetrically from both sides of the core. The core is compressed thermally/mechanically only along a single transverse axis/plane, what induces internal birefringence [14]. The induced stress level in the core is of the order of 10 kg/mm$^2$ [51]. This birefringence is a strong function of temperature, thus the fibre exhibits thermal dispersion of birefringence. Technologically induced birefringence, in the range of $10^{-4}$–$10^{-3}$ is much stronger than the accidentally induced birefringence by such factors as: external fibre bending – macro and micro, torques, residual core ellipticity, statistical fluctuations of core diameter, etc. Panda like fibres were manufactured at Białystok OFTL by various hybrid methods, including MMC and RiT. The glass stress sectors/members should be positioned as close to the core as possible. The ultimate confinement is the increase in fibre attenuation, when the core evanescent field begins to feel the lossy members.

The theoretical area of confinements for polarization isolation between orthogonal members of the fundamental mode in a strongly birefringent, polarizing fibre, with stimulated polarization discrimination, is determined by the following factors: Rayleigh scattering, differential polarization attenuation, and natural polarization crosstalk in birefringent fibre. Such a theoretical area is influenced by real life conditions like equipment, lab conditions of measurements, fibre parameters, etc. Polarization separation in real fibre depends on internally inbuilt birefringence, fibre construction – Bow-tie, Panda, D-type, elliptical core; type of cladding and fibre jacket, method of fibre cabling, kind of stress members, kind of introduced external fibre excitation or enforced geometry – like fibre coiling.

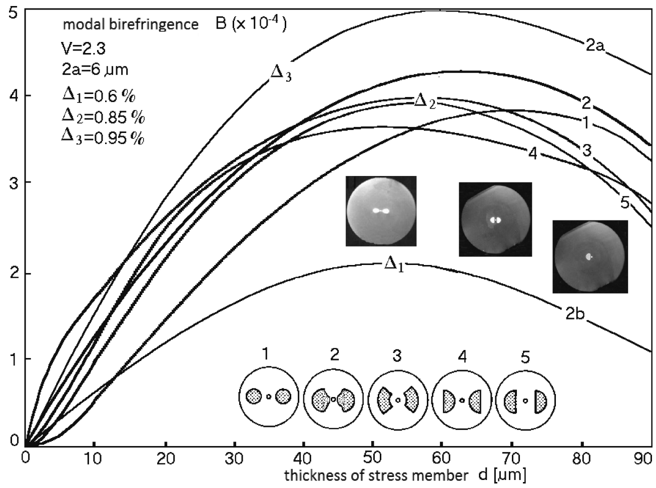



Fig. 6. Polarization maintaining optical fibers, cross sections and birefringence characteristics. Manufactured in Białystok OFTL

## 8. Multi core optical fibres

Analysing single mode multicore optical fibres we assume the following simplifications: inter-core separation is comparable to the core diameter if not smaller, optical fibre is weakly propagating, refraction is step-index, core/cladding geometry is perfect, one of the cores is optically excited, the field of fundamental mode extends to the neighbouring core. Optical power flow of periodic character between neighbouring cores is just the effect of coupling the evanescent fields and relevant additive phase relations of coupled waves. Coupling coefficients between neighbouring cores are [17]:

$$C_{ij} = C_o \int\limits_{A_j} n_j^2 \Psi_i^{clad} \Psi_j^{core} dA,$$

$$i \neq j, \quad i,j = 1,2, \quad C_o = \frac{k\sqrt{\varepsilon_o}}{4\sqrt{\mu_o N_i N_j}}, \tag{30}$$

$$N = \left(\pi a^2 n_{core}/2\right) \sqrt{\frac{\varepsilon_o}{\mu_o}} \frac{V^2}{U^2} \frac{K_1^2(W)}{K_o^2(W)},$$

where $A_j$ – cross section of $j$-th core. By assuming further simplifications: neglecting self-coupling, use known extensions of $K$ functions and substitutes for integrals of Bessel functions, one may obtain coupling coefficients in analytical forms. Optical fibre with M-cores, to obtain coupling coefficients, requires summing of the integrals over all M-1 cores:

$$C_{ij}^M = C_{ij} + C_o \sum_{\substack{core=1 \\ c \neq i, c \neq j}}^{M} \int\limits_{A_c} n_{core}^2 \Psi_i^{clad} \Psi_j^{clad} dA. \tag{31}$$

The second component in Eq. (31) is a coupling between $i$-th and $j$-th cores via mutual coupling in a series of adjacent cores. We neglect this component. This is equivalent to the assumption that the coupling between immediately adjacent cores dominates. Only in such a case the coupling coefficients have analytical form. In opposite case, optical power is a complex superposition of modal power from different cores. Phase relations in arbitrary multicore optical fibre never assume the values 0 or 1, as it is in a twin core optical fibre. Analytical form for inter-core coupling coefficients, in our simplified case is:

$$C_{ij} = C_o \Theta_{ij},$$

$$\Theta_{ij} = \frac{a_j K_o\left(W_i d/a_i\right)}{K_o\left(W_i\right) J_o\left(U_j\right)}$$

$$\frac{\left(W_i/a_i\right) I_1\left(W_i a_j/a_i\right) J_o\left(U_j\right) + \left(U_j/a_j\right) I_o\left(W_i a_j/a_i\right) J_1\left(U_j\right)}{\left(W_i/a_i\right)^2 + \left(U_j/a_j\right)^2}, \tag{32}$$

where $\Theta_{ij}$ is an algebraic relations between $I$, $J$ and $K$ functions. Coupling analysis operates on useful expressions like: average coupling coefficient $-C = \sqrt{C_{12}C_{21}}$, a number of normalized propagation constants $\beta^s = \left(\beta_1 + \beta_2\right)/2$, $\Delta\beta = \beta_1 - \beta_2$, $\Delta\beta^r = \left(\beta_1 - \beta_2\right)/\beta_1$; $\Delta\beta_{AS}^r = \left(\beta_{AS} - \beta_{SA}\right)/\beta_{AS}$, $V_p^{AS} = \varpi/\beta_{AS}$ phase velocity; $\beta_N = \left(\beta^2/k_o^2 - \varepsilon_{cclad}\right)/\left(\varepsilon_{core} - \varepsilon_{clad}\right)$; power transfer efficiency $\mathrm{P}^{te} = \left(1 + \left(\Delta\beta/2C\right)^2\right)^{-1/2}$; and inter-core contrast $P_{ij}^c = \left(P_i - P_j\right)/\left(P_i + P_j\right)|_{z=mZ_b} = -\cos(2mZ_b C_{ij})$. $AS$ symbol is a description of indexed modes in a double core optical fibre, taking into account their symmetric – antisymmetric field distribution against fibre axis. Differential value of propagation constant in twin-core optical fibre is a measure of its inter-core birefringence. Modal beating length is defined as a function of coupling coefficient $Z_b = \pi P^{te}/2C$. The values of C and $P^{te}$ behave as additional propagation constants, because the coupled modes may be treated as new $AS$ super-modes propagated in double-core optical fibre. The fundamental mode $\mathrm{HE}_{11}^x$ is fully coupled from one core to the other, only if the total phase shift $\Delta\Phi$ fulfils the condition $\Delta\Phi = (2m+1)\pi$. For SA modes this is equivalent to the integral relation on $\Delta\Phi$, and the resulting condition of full power transfer:

$$\Delta\Phi = \varpi \int\limits_{-1}^{1} [(V_p^{AS_1})^{-1} - (V_p^{AA_1})^{-1}] ds,$$

$$k_o\sqrt{\varepsilon_{rdzen}} \int\limits_{-1}^{1} \left[\left(1 - \Delta\left(1 - \beta_N^{AS_1}\right)\right)^{1/2} \right. \tag{33}$$

$$\left. - \left(1 - \Delta\left(1 - \beta_N^{AA_1}\right)\right)^{1/2}\right] ds = (2n+1)\pi,$$

where $V_p^{AS}$ is phase velocity of particular SA mode. For small values of $\Delta$, the expression under the integral is simplified to $\Delta\beta_N^{xx} = \beta_N^{AS1} - \beta_N^{AA1}$. When mode $\mathrm{HE}_{11}^x$ is excited in a single core, this is equivalent to excitation by superposition of SA super-modes $AS_1 + AA_1$. The output should be again mode $\mathrm{HE}_{11}^x$, but from the other core, what is a combination of SA super-modes $AS_1 - AA_1$. The coupling coefficient in the double-core or twin-core optical fibre depends on all fibre parameters: geometrical, optical, material and technological. Choice of technology leads to the fibre of strictly assumed coupling characteristics. The parameters that were subject to optimization were: choice of glass set for core/cladding, modification method of crucible technology including separation of crucibles, diaphragming or aperturing of glass nozzles, crucibles of special construction, temperature and rate of fibre pulling. Glass choice influences contrast between cores [18]. When the cores are nearly identical and initially weakly coupled, then propagation constants of fundamental modes $\mathrm{HE}_{11}^x$ in each core are different $\beta_1$ and $\beta_2$. Direct transfer of optical power between fundamental modes is not possible, when the modes are not perfectly fit in phase. At some point of the dispersion characteristic of double core fibre $\Delta\beta = 0$, for $\lambda = \lambda_o^{dysp}$. Figure 7 presents normalized dispersion characteristics $\beta_N(\lambda)$ of double and quadruple core singlemode optical fibres, with examples of such fibres manufactured at Białystok OFTL.

When the wave is coupled to a single core, then the normalized power in the second core is $P(z) = \sin^2(Cz)$, where $C$ – is core coupling coefficient. In twin core optical fibre both self-coupling coefficients are equal. Then the coupling coefficient for twin core optical fibre is:

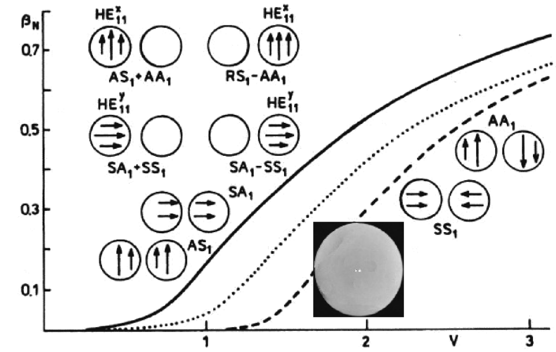
$$C = \frac{(2\Delta)^{1/2}}{a}\frac{U^2}{V^3}\frac{K_o\,(Wd/a)}{K_1^2\,(W)}, \tag{34}$$

or beating length

$$L_B = Z_B = \pi/C,$$

where $U = ka(n_r^2 - \beta^2/k^2)^{1/2}$, $W = ka(\beta^2/k^2 - n_p^2)^{1/2}$ – arguments of Bessel function, $V = kan_r(2\Delta)^{1/2}$ – normalized frequency, $\Delta = (n_r^2 - n_p^2)/2n_r^2$ – refractive profile coefficient. The wavelength $\lambda_o^{dysp}$ for double core optical fibre increases with decreasing difference in core diameters and increasing contrast between the cores. The mutual coupling coefficients $C_{12}$ and $C_{21}$ are not equal in double core optical fibre. The coupling may be stronger in a single direction, resulting in more average power in one core. A measure of coupling symmetry, and optical power transfer efficiency (or detuning from $\lambda_o^{dysp}$) is a relative coefficient of mutual coupling $C_{ij}^r = C_{ij}/C_{ji}$. Calculations of functions $C_{21}^r(\lambda)P_{12}^{te}(\lambda)$, for various kinds of multicore optical fibres were presented elsewhere [51]. There are also calculated and measured dispersion characteristics of: inter-core coupling for twin core optical fibre as a function of normalized core separation, inter-core contrast $P_{12}^c$, efficiency of power transfer, optical power from the coupled core, etc.

a)




b)

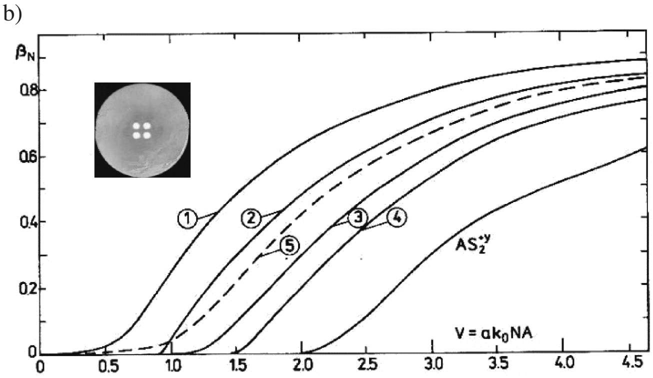



Fig. 7. Twin and quadruple core optical fibers and their modal dispersion characteristics for SA (symmetric-antisymmetric) modes of the lowest order. Inserts show photographs of multicore, singlemode fiber cross sections, a) propagation constant as a function of normalized frequency $\beta(V)$ in twin core optical fiber for modes SA1, AS1, HE11 and SS1, AA1, b) Quadruple core optical fiber and its modal characteristics for SA modes of the lowest order. Normalized propagation constant $\beta_N = (\beta^2/k_o^2 - \varepsilon_5)(\varepsilon_1 - \varepsilon_5)$; $\varepsilon_i$ $(i = 1,2,3,4)$ – core refractions; $\varepsilon_i$ $(i = 5)$ – cladding refraction; homo-core condition $n_1 = n_2 = n_3 = n_4 = n_r = (\varepsilon_1)^{1/2} > n_5 = n_p$, $a_1 = a_2 = a_3 = a_4 = a$; Exemplary fiber data: core diameter $2a = 2,5$ $\mu$m, core separation $d = 0,5$ $\mu$m, $NA = 0,05 \approx 0$, core refraction $n_1 = n_r = 1,6$, step index profile, curve 1 – four degenerated modes in ideally isotropic fiber $\mathrm{SA}^{+x}$, $\mathrm{SA}^{-x}$, $\mathrm{AS}^{+y}$, $\mathrm{AS}^{-y}$; curve 2 – three index mode family $\mathrm{SAA}^{+x}$, $\mathrm{ASA}^{-y}$, etc.; curve 3 – single index mode family $\mathrm{S}^{\pm x}$, $\mathrm{S}^{\pm x*}$, $\mathrm{S}^{\pm y}$, $\mathrm{S}^{\pm y*}$, $\mathrm{A}^{\pm x}$, $\mathrm{A}^{\pm x*}$, $\mathrm{A}^{\pm y}$, $\mathrm{A}^{\pm y*}$, where A*=A± with reversed sequence of signs + and −, ie. instead of ± there is ∓; curve 4 – fundamental two-index modes $\mathrm{SA}^{\pm y}$, $\mathrm{SA}^{\pm y*}$, $\mathrm{AS}^{\pm x}$, $\mathrm{AS}^{\pm x*}$; curve 5 – reference to the fundamental mode $\mathrm{HE}_{11}$ of a single core singlemode fiber of analogous parameters; curve $\mathrm{AS}_2^{+y}$ – single mode cut off point and dispersion curve for the first second order mode in quadruple core optical fiber; Field distributions of SA modes are: a – symmetrical mode S; b – symmetric antisymmetric mode SA; c – antisymmetric mode A; d – symmetric SA mode SAS; e – coupled antisymmetric SA mode ASA = SAS + ASS

## 9. Conclusions

Optical fibre technology may be divided into opto-telecommunications, and opto-instrumentation [51]. Optical fibre communications deals with new generations of linear, ultra-low loss and ultralow dispersion optical fibres transmitting undistorted signals at ultimate rates for very long distances. Non-telecommunication, tailored optical fibres serve for instrumentation purposes, building of photonic functional devices, and are usually manufactured by more complex and less standardized methods. Instrumentation fibres usually are of short lengths and have bigger losses. They serve for building passive and active, simple and complex, optical fibre components, sensors, hybrid circuits for optical wave transformation and optical signal processing, functional circuits of planar (integrated) optics, etc. Tailored optical fibres are manufactured by MMC, RiT, and modified CVD methods. The aim of technology is to manufacture an optical fibre of assumed properties – geometrical, chemical, mechanical and thermal, physical, optical, refractive, dispersive, energy carrying, etc.

Fundamental parameters of instrumentation optical fibres are: numerical aperture NA, absolute differential refraction $\Delta n$, relative differential refraction $\Delta$, refractive index profile function $n(r)$, or $n(r, \Theta)$, physical properties of fibre glass. Refractive profile decides of nearly all signal properties of optical fibre and its sensitivities to external reactions. Thus, one of the fundamental issues of technology of tailored fibres is the ability to precisely form the designed/assumed refractive profile, at the same time keeping the rest of fibre parameters at the best acceptable level. We distinguish several profile classes in tailored optical fibres: step-index, multi-step index, monotonous and non-monotonous profiles, W and M class profiles, ring and multi-ring profiles, capillary, isotropic and anisotropic profiles, birefringent and polarizing, single argument $n(r)$ and two argument $n(r, \Theta)$ depending on the azimuth. Optical fibres of distant profiles have completely different properties: diameter and character of fundamental mode, ability to couple with other fibres, sensitivity to micro-bending, modal structure, modal cut-off. Generally, the first propagated mode may not be $HE_{11}$. The aims of complex refractive profiles in tailored optical fibres are: dispersion optimization, single-point or multipoint dispersion zeroing/flattening/shifting in fibre spectral work band, achromatic and apochromatic fibres, shaping of higher dispersion derivatives, introduction of modal changes/transformation, shaping of modal field – distribution and dimensions, immunizing of optical fibre to certain reactions like temperature changes, micro-bending, acoustic wave, etc., apodization of photonic component – introduction of refractive pedestals in profiles, multiple clad fibres, refraction fitting to planar circuits of integrated optics, shaping of fibre modal structure, modal characteristics dependence on normalized frequency, precise localization of modal cut-off points, shaping of far field radiation characteristic, coupling characteristics with other tailored fibres, etc.

One of the fundamental classes of tailored optical fibres are anisotropic ones. Inside this class are fibres of strong linear birefringence. Such fibres were manufactured in Białystok OFTL using hybrid RiT and MMC technologies. The fibres had the following parameters: normalized birefringence $B \approx 5*10^{-4}$, birefringence susceptibility to external reactions – thermal and mechanical dB/dT≈7 [rad/m/°C], dB/d$\varepsilon$ ≈100 [rad/mm]. These parameters are comparable to the fibres manufactured by other standard methods. Birefringent MMC optical fibres were used for building all optical fibre polarizers, by permanent discrimination of one component of the fundamental mode [24]. Linearly birefringent, high quality MMC fibre displayed 60dB of polarization discrimination.

One of fundamental classes of tailored optical fibres are the ones with non-cylindrical cores. Well defined, geometry based linear birefringence is present in single mode optical fibres with elliptical cores [51]. Additional birefringence may be added to these fibres also by the stress factor [32]. Elliptical core optical fibres of changing ellipticities were manufactured at Białystok OFTL using MMC technology [35] and the results were compared with classical HB type fibres. For large cores the natural geometry-based birefringence was of the order of $B \approx 10^{-5}$, while for small cores was bigger $B \approx 10^{-4}$. Other possibility to increase the birefringence was tested by introduction of heterogeneous, anizotropic refractive index profile. In a fibre with such a profile, the optical wave sees different values of the wave evanescent w argument of Bessel function K(w) in different perpendicular directions of fibre transverse cross-section.

One of Białystok OFTL specialities are multicore optical fibres [33–38]. A large number of various types of single-mode and multimode multicore optical fibres were designed, manufactured, tested, characterized and used in experimental applications [44, 52]. Twin core optical fibres were compared to double mode optical fibres working with $HE_{11}$ and $EH_{01}$ or $LP_{11}$ and $LP_{01}$ modes. Availability of research samples of multicore optical fibres gained a considerable interest in them and resulted in many pilot applications. In a double core optical fibre, and especially in twin-core fibre there was demonstrated efficient transmission of strongly coupled fundamental super-mode waves [41]. Generation of a complex super-mode was obtained in a few types of multicore fibres with three, four and more strongly coupled cores [39]. Some types of very complex optical fibres [46] are structures which cannot be obtained by MCVD method. Hybrid methods RiT/MMC allow to manufacture fibres of ultimately complex internal structure [32], and/or of complex signal and susceptibility characteristics [30].

The current development of Białystok OFTL is concentrated on research on active optical fibres for narrowband and wideband sources and sensors as well as opening MCVD manufacturing facility [67–73]. The references listed below are intentionally confined to the works published in cooperation with J.Dorosz. They are arranged chronologically with time. They show a history of more than 35 years of hard, focused and efficient efforts concerning the development of instrumentation optical fibres. These efforts were concentrated not only on technology, but also on attracting and opening this interesting field of photonics technology to young adepts of science and research. Prof. Jan Dorosz, has recently passed the baton to young generation of Białystok researchers. Now, prof. Dominik Dorosz takes the helm, and he is surrounded by a group of gifted and extremely devoted colleagues. This paper, prepared for the Jubilee Issue of *the Bull. Pol. Ac.: Tech*., bases on a wider description of non-telecom optical fibres published in Polish elsewhere [66]. The bibliography quotations in the text are only exemplary, referring to chosen examples of work done on particular instrumentation fibres. Wider choice of examples may be found in the extensive list of references below.

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