Acta of Bioengineering and Biomechanics
Vol. 16, No. 2, 2014

Original paper
DOI: 10.5277/abb140214

Modeling of human circle of Willis with and without aneurisms

DMITRY IVANOV[1]*, ALEXANDER DOL[1], OLGA PAVLOVA[2], ASEL ARISTAMBEKOVA[2]

[1] Mathematical Modeling Subdepartment, Educational-Research Institute of Nanostructures and Biosystems, Saratov State University, Saratov, Russian Federation.
[2] Biomechanics Subdepartment, Educational-Research Institute of Nanostructures and Biosystems, Saratov State University, Saratov, Russian Federation.

Purpose: This paper includes results of the first stage of research aimed at the development of recommendations for physicians in order to help them to choose a particular type of cerebral arteries aneurysms treatment.

Methods: Recent studies show that the majority of aneurysms develop as a result of hemodynamic and degenerative lesions of the vascular wall. Obviously, such wall damage can be studied using the methods of continuum mechanics and numerical simulations. Biomechanical modelling allows us to study hemodynamic parameters and stress-strain state of these arteries in health and disease, and to formulate practical recommendations for the necessity and reasonable selection of a particular type of cerebral arteries aneurysm treatment.

Results: At this stage the realistic geometric models of arterial circle of Willis were built for its normal state and in the presence of aneurysms. The ultrasound analysis of circle of Willis was conducted in order to obtain blood flow parameters and the boundary conditions for carotid and vertebral arteries. Also, the mechanical properties of these arteries were investigated and constants of the Mooney–Rivlin strain energy function were obtained.

Conclusions: Thus, the boundary problem describing the behaviour of human Willis circle arteries was stated. Further, this problem will be solved numerically using the finite element method. The numerical results will be analyzed from the point of view of the influence of the mechanical factors on the emergence, growth and rupture of circle of Willis aneurysms.

Key words: *3D design, aneurysm, circle of Willis, hyperelastic material, mathematical modelling, mechanical tests*

1. Introduction

Cerebral aneurysm is a disease which does not have pronounced or stable clinic symptoms, but quickly leads to dramatic consequences in case of development of complications. Despite the social and economic value and a long history of studying the issue from the medicine point of view, the mechanisms of aneurysm development and factors affecting their appearance are not yet fully understood. According to the clinical data, up to 80–90% cases of non-traumatic subarachnoid haemorrhage in Russia develop because of intracranial aneurysm rupture. Aneurysm rupture leads either to neurological disorders of varying severity caused by damage of the brain tissue or to death. It is especially important that the subarachnoid haemorrhage disables the working age population (40–60 years).

The reasons of aneurism development can be acquired or congenital. Recent studies show that the majority of aneurysms develop as a result of hemodynamic and degenerative lesions of the vascular wall. Obviously, such wall damage can be studied using the methods of continuum mechanics and numerical simulations. Biomechanical modelling allows us to study hemodynamic parameters and stress-strain state of these arteries in health and disease, and to formu-

* Corresponding author: Dmitry Ivanov, Saratov State University, 83 Astrakhansaya street, 410012 Saratov, Saratov, Russian Federation. Tel. +79173146807, e-mail: ivanovdv@gmail.com

Received: April 9th, 2013
Accepted for publication: February 26th, 2014

late practical recommendations for the necessity and reasonable selection of a particular type of cerebral arteries aneurysm treatment.

There are a lot of studies devoted to the appearance and development of cerebral aneurysms.

Many researchers believe that aneurysms are forming due to hemodynamic wall damage and, therefore, pay attention to the values of shear stresses on the arteries and aneurysms walls. Authors of [5], [12], [15], [14], [22] noted the presence of high values of shear stresses (more than 20 N/m^2) in arterial bifurcations. High shear stresses cause damage to the vessel wall and, consequently, lead to the formation of aneurysms. A series of papers [23], [24] written under the leadership of Professor G.A. Holzapfel are devoted to the study of cerebral aneurism growth processes from the point of view of continuum mechanics. Authors believe that the development of an aneurysm is accompanied by the loss of media layer, and the entire load is applied only to the collagen fiber.

Although three-dimensional reconstruction of the arteries today can be performed even in clinic with the help of built-in software of scanners these images are not three-dimensional models and can only be used for visual examination of the blood vessels geometry. Therefore, the problem of reconstructing a realistic spatial geometry of blood vessels and aneurisms, which is suitable for numerical calculations of their behavior is relevant.

Much attention in the modern scientific community is paid to restoring the geometry from the CT and MRI scanning [1], [25], [7], [17], [18], [22]. These authors are trying to work with tomograms of individual patients, allowing escape from generalizations and to compare the results of calculations with the parameters of the specific person blood flow. These models often have realistic geometry of the arteries. But unfortunately such approach sometimes leads to certain difficulties in the development of computer simulations and problems with numerical calculations. So it is necessary to optimize the geometry [10]. Some authors use their own software to transfer MRI data from STL file to finite element program [3]. Many scientists use specialized software Mimics TM for the initial work with tomograms [1], [20], [26]. This leads to application of third-party programs and grid generators to the geometric images. This greatly complicates the process.

Some attention in current literature is paid to numerical methods for the reconstruction of the geometry. For example, the Monte Carlo method, calibrated in a certain way was used to simulate the human brain based on MRI [0]. However, this approach is quite complex and labour intensive. Obviously, the three-dimensional realistic model of vessels allows describing their behaviour more accurately and making more adequate conclusions on the arising, growth and rupture of aneurysms. However, some authors still use one-dimensional models [11], [6].

In our judgement, it is necessary to find a “middle ground” among the proposed methods of constructing geometric models based on topographic data. It is important not only to create realistic models of vessels. It is well known that the model should be smooth for the numerical computations. It should not have any defects. The model should be covered by a limited number of surfaces. Moreover, the method of constructing such models should not be overly time-consuming.

Researchers paid special attention to boundary conditions, which are set at the inlets and outlets of the vessels during finite-element analysis. Most authors prefer to set inflow velocity at the inlets [13], [21], but some authors set volume blood flow at the inlet [2].

If we consider the patient-specific model of the circle of Willis of individual patient, it is necessary to take into account not only the geometry of vessels. Blood flow velocity at the inlets as well as mechanical properties of the walls are a very important issue in the computer simulation.

Situation with the first two items is quite simple. We can construct a realistic patient-specific geometry from the CT and MRI data. We can use ultrasound examination of the vessels of the same patient to find boundary conditions for the velocity and pressure. The problem is to obtain the mechanical characteristics of the individual patient. At present it is not possible to determine the mechanical properties of blood vessels of a living person. However, it is possible to conduct the study of the mechanical properties under conditions close to physiological ones [19]. Therefore, we have developed the following method: on the basis of mechanical tests of more than 100 samples of human arteries of the circle of Willis [8] which were conducted in 2008–2009, we obtained average mechanical characteristics. The average data corresponding to sex and age group of the patients under examination were used in the numerical calculations.

Though there exist a numerous amount of studies on aneurisms of the arteries of the circle of Willis, there has not been found any practice-applicable research aimed at comprehensive study which includes not only patient-specific problem statement of arterial behaviour simulation but also allowing the mechanical factors leading to aneurism development to be estimated.

In this paper, we introduce the original methods of constructing the 3D patient-specific smooth geometric models of healthy arteries of the circle of Willis and arteries with aneurisms.

In order to define boundary conditions for linear velocity of blood flow at the entries to the circle of Willis we processed ultrasound data for carotid and vertebral arteries.

Finally, the procedure of acquiring material coefficients in the model of circle of Willis arteries' walls based on the data from mechanical experiments is described.

2. Materials and methods

2.1. 3D modeling of human Willis circle arteries

Mathematical modeling of the behavior of blood vessels requires knowledge of equations of blood and wall motion, construction of geometrical model and acquisition of boundary conditions. Up-to-date computer science and numerical methods allow solving a three-dimensional system of partial differential equations in complex spatial domains. Therefore scientists try to reconstruct the geometry of objects under investigation as accurate as possible. This is true for modern research in biomechanics.

Today most accurate geometric models of vessels can be created based on tomography data. There are three methods of diagnosis of cerebral aneurisms in a human being: contrast enhanced computer tomography, magnetic resonance imaging and cerebral closed angiography.

Cerebral closed angiography is considered to be the most accurate method of diagnosis and investigation of the intracranial aneurism properties. But taking into account its complexity and certain harm of X-rays and contrast agent to human body, cerebral closed angiography is usually used for people after subarachnoid hemorrhage. In our study we investigate healthy cerebral vessels and aneurisms before rupture.

According to the Federal Centre of Neurosurgery, the computer tomography (CT) shows good results in the diagnosis of aneurisms. However, this method generally requires an injection of radio contrast agents. In the case of visualization of a healthy circle of Willis the injection is not reasonable, that is why we did not use CT.

Magnetic resonance imaging (MRI) is the least traumatic method of diagnosis. During this procedure, the patient is not subjected to X-rays and the injection of radio contrast agents. At the same time most aneurisms are quite well visible as circumscribed formation. Thus, MRI is an ideal method of examination of healthy Willis circle arteries and aneurisms before subarachnoid hemorrhage.

The construction of human circle of Willis model was conducted based on MRI. Data for investigation were provided by Diagnostic and Treatment Center of International Institute of Biological Systems named after S.M. Berezin in Saratov (Russia).

For the reconstruction of vessel geometry we employed Mimics software, which is used for processing MRI data and converting the latter to 3D models. Stacks of 2D image data of human head were imported in Mimics TM software. Then we selected the range of the shades of gray corresponding to blood density and 3D surface model was created automatically (Fig. 1).

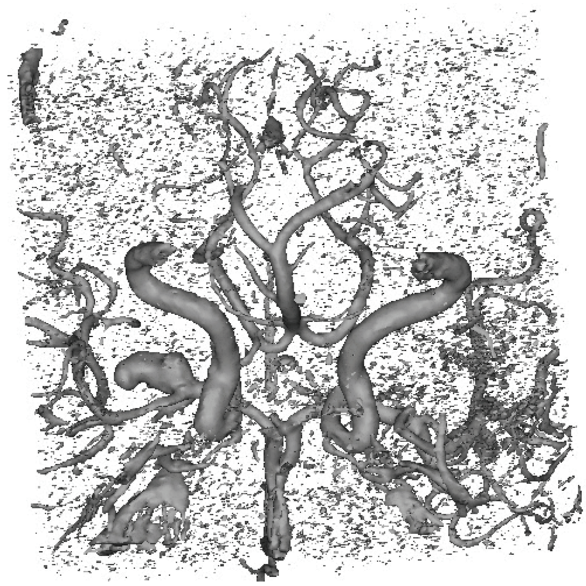

Fig. 1. 3D vessel representation in Mimics TM

All items of vascular bed that are out of our interest and meaningless data whose density is close to the blood density were deleted by means of Mimics TM software tools.

The model obtained was saved in STL format (Fig. 2) and imported into CAD-system to smooth the geometry and create a new model without irregularities and distortions.

Further construction of 3D model of the circle of Willis was conducted in the specialized program package SolidWorks TM. This step was necessary due to many flaws caused by methods of the reconstruction of 3D objects from images in Mimics that would not allow carrying out finite-element experiments.

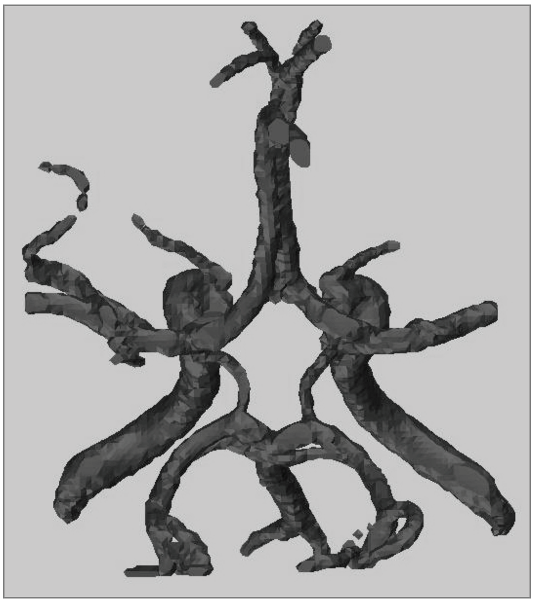

Fig. 2. Geometric model, imported from Mimics TM

It was supposed that all vessel cross-sections were circular. Part of the internal carotid artery (Fig. 4) was constructed using Lofted Boss/Base tool based on the circles shown in Fig. 3.

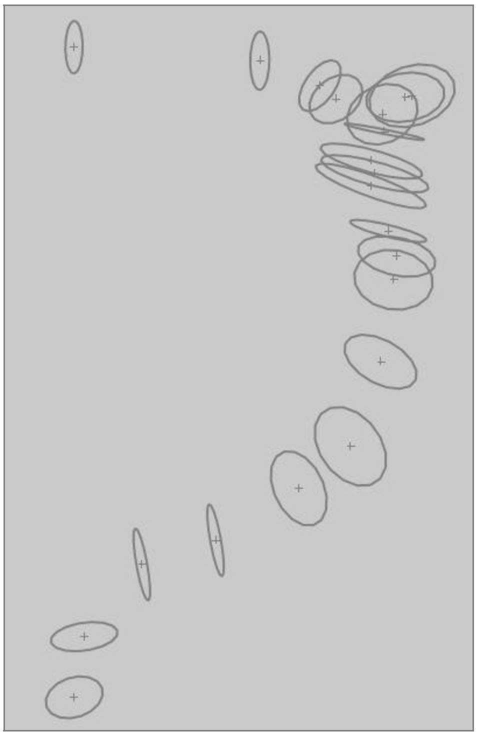

Fig. 3. Circles used for construction of the model of the internal carotid artery

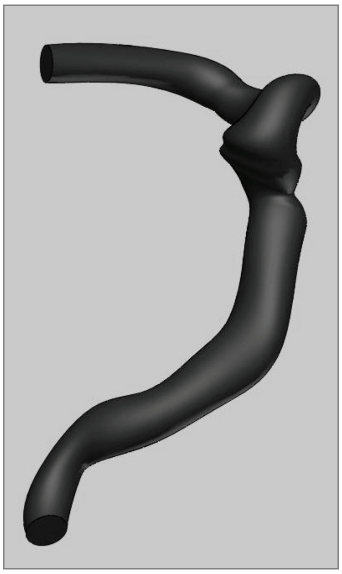

Fig. 4. The model of the internal carotid artery

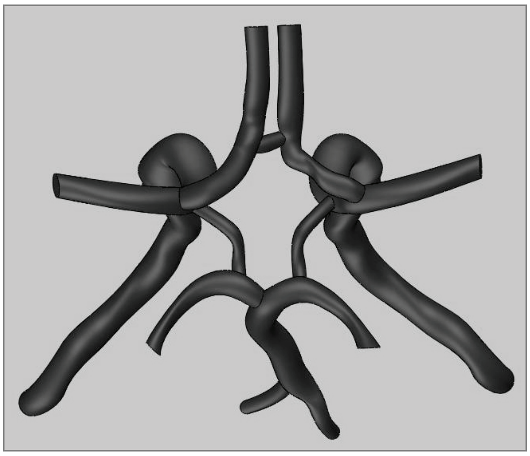

Fig. 5. The model of the circle of Willis

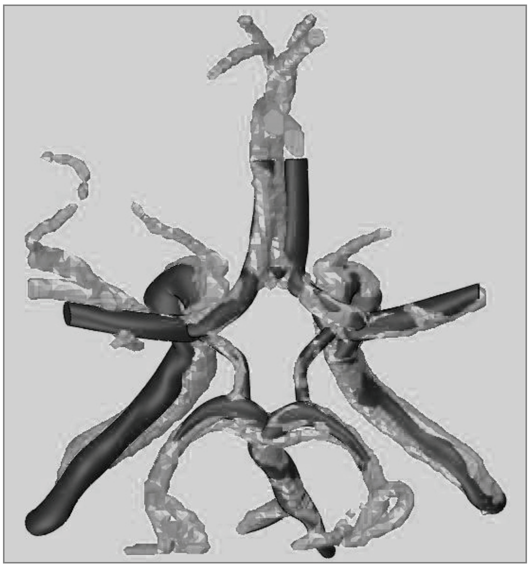

Fig. 6. Correspondence between Mimics TM and SolidWorks TM models

Models of the other arteries of the circle of Willis were created similarly (Fig. 5).

Figure 6 represents the correspondence between the model from SolidWorks and the model obtained in Mimics. It is obvious that there is maximal resemblance in most parts.

It should be noted that there were no vessel walls in the geometry imported from Mimics because their density is higher than blood density, i.e., it corresponds to the other range (of the shades) of gray in tomography.

Thereby vessel walls were constructed in SolidWorks TM. The thickness of the vessel wall in each part of the vascular bed was defined based on morphologic data provided by anatomy department of Saratov State Medical University named after V.I. Razumovsky (see Table 1).

Table 1. Average thickness of the circle of Willis arteries

	Artery	Thickness, mm
1	Carotid artery	0.35
2	Vertebral artery	0.35
3	Basilar artery	0.35
4	Anterior cerebral artery	0.25
5	Middle cerebral artery	0.35
6	Posterior cerebral artery	0.3
7	Posterior communicating artery	0.15
8	Anterior communicating artery	0.15

The model of incomplete circle of Willis with aneurism of medial cerebral artery was constructed similarly (Fig. 7).

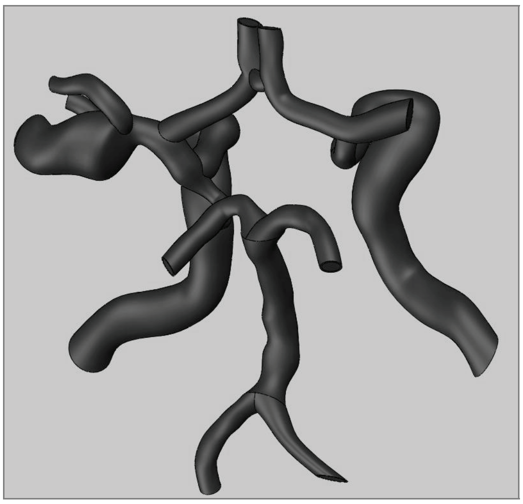

Fig. 7. The model of incomplete circle of Willis with aneurism of medial cerebral artery

The method described above allows three-dimensional patient-specific geometric models of vessels to be created based on tomography data. Moreover, such models have a number of indispensable properties for numerical computation realization.

First of all, constructed models have smooth walls providing computational mesh of high quality. Secondly, we can specify necessary quantity of bounding surfaces that is important for definition of boundary conditions at entries, exits and lateral walls. In the third place, the geometry can be reconstructed when needed, for example, some pathology can be added, or shape and (or) size of cross-section can be changed.

2.2. Boundary conditions for the flow velocity at the entries of the circle of Willis

Let us discuss the definition of boundary conditions for linear velocity of blood flow at the entries to internal carotid and vertebral arteries. These two pairs of main arteries feed the brain and they are responsible for the filling of the circle of Willis with blood.

For the boundary conditions at the entrances of the circle of Willis it is enough to know the relationship between linear blood flow velocity and time during the cardiac cycle. There is no need to complicate the velocity profile giving it a parabolic form by introducing the expression dependence on the coordinates. Numerical calculations showed that at a distance of several vessel diameters from the entrance the profile takes physiological parabolic shape.

To obtain the boundary conditions at the inlets, ultrasound data of the carotid and vertebral arteries were used. At the outlet of the anterior, middle, and posterior cerebral arteries, a zero pressure was applied.

Let us focus on the analysis of the ultrasound data.

Ultrasound (US) is a mandatory procedure conducted for each patient before surgery.

US is widespread due to its low cost, high informativity, non-invasiveness, safety and feasibility of repeated investigations.

Ultrasound data are represented in the form of the spectrum with its envelope curve (peak velocity loop). Typically, the measurement of the speed is conducted in the middle of the stream, where maximum values are observed. The horizontal axis represents time and the vertical one represents flow velocity in sm/s (Fig. 8). In addition, maximum blood velocities in systole and diastole are presented.

There are several ways to obtain input boundary conditions for speed. For example, Liu and Yamaguchi [12] described the velocity curve with analytical function.

In this work, we used a different approach of obtaining the boundary conditions for speed at inlets. It allows getting more accurate and realistic speed data. For this purpose the following steps were taken:

(1) plot of the blood velocity during the cardiac cycle produced by dopplerograms was digitized in a raster graphics editor (e.g., Microsoft Paint), through its delineation (see Fig. 9);

(2) scaling of these values to the values of the velocity in systole and diastole, defined by ultrasound dopplerogram;

(3) cardiac cycle length is scaled to the unit interval.

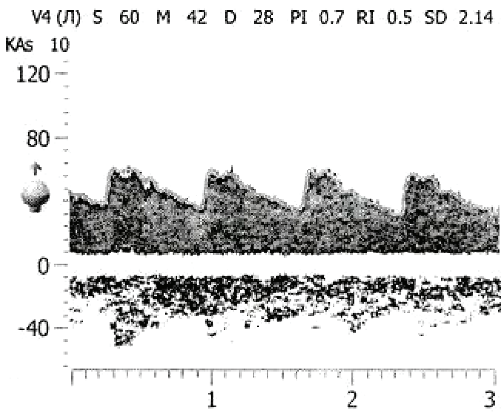

Fig. 8. Doppler ultrasound data for vertebral artery [4]

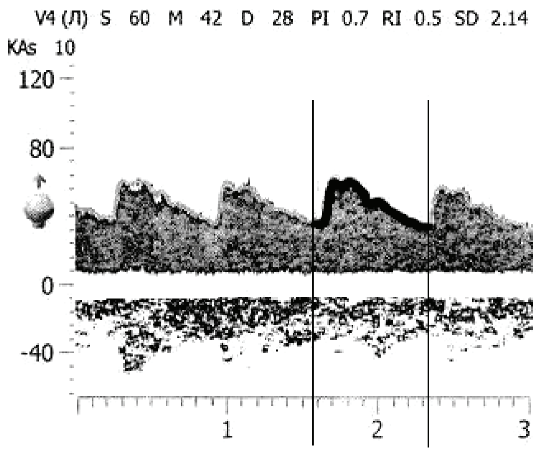

Fig. 9. Delineation of the velocity curve using MS Paint TM

As a result a set of values for the velocity of the internal carotid and vertebral arteries was found. Velocity plots are presented in Fig. 10.

The resulting data can be used for the boundary conditions at the entrances to the circle of Willis vertebral and internal carotid arteries.

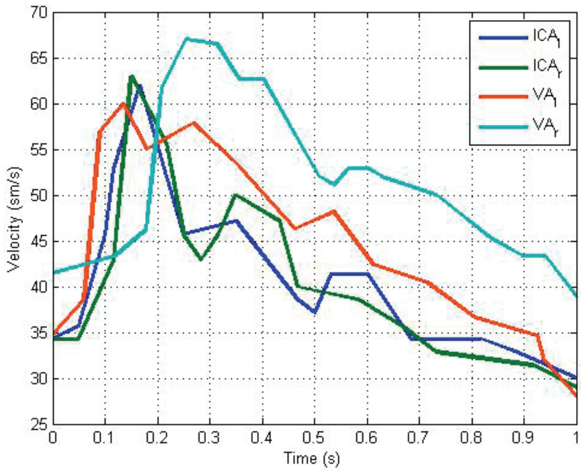

Fig. 10. Graph of the velocity of the left and right internal carotid artery, the left and the right VA

2.3. Obtaining material parameters of the arteries

In mechanical experiments we used a uniaxial tensile testing machine Tiratest 28005 (registration number 23512-02 in the State Register of the Russian Federation) with 100 N loadcell. The values of the applied force (N) and stretch (mm) in the application of force direction were recorded in the course of the experiment.

For the experiments in longitudinal direction we had 112 COW taken post-morten, for the experiments in transversal direction we had 12 samples of human vessels. All the samples were taken from people who died at the age between 21 and 87 years. Before the experiments all the samples were preserved in saline at a temperature of about 20 °C. Experiments were conducted at a temperature of about 20 °C.

All the samples of vessels were divided into two gender groups and into four age groups (see Table 1, age of death 22–90 corresponds to four age groups for male and female).

Table 2. Age and gender groups

Age group number	Age, years	
	Male	Female
1	22–35	21–35
2	36–60	36–55
3	61–74	56–74
4	75–90	75–90

Ultimate tensile strength of male arteries was greater than ultimate tensile strength of female arteries. This was observed for all analyzed arteries corresponding to gender groups. Ultimate tensile strength in longitudinal direction decreases during the lifetime. The greatest difference between the ultimate tensile strength for the first and the fourth age groups is reached for the basilar arteries. And the value of this difference is about 50%. For other arteries this value is about 30–40%. The greatest gender difference of ultimate tensile strength in longitudinal direction is reached for the vertebral arteries.

It is typical that deformability of mostly all arteries is lowered during lifetime. But for some male and female arteries of the fourth age group deformability grows up in comparison with the third age group. Walls of middle cerebral arteries have the greatest deformability in compassion with the other arteries of the circle of Willis.

Ultimate tensile strength in cross direction decreases during lifetime for all tested arteries. The greatest difference between the ultimate tensile strength for the first and the fourth age groups is reached for the basilar arteries.

Now we describe the procedure of obtaining the constants of the Mooney–Rivlin strain energy function, based on the numerical results of the mechanical experiments on investigated arteries.

The Mooney–Rivlin model was used as a model of the wall material

$$W = C_1(I_1 - 3) + C_2(I_2 - 3) + C_3(I_1 - 3)(I_2 - 3) \quad (1)$$

where W – strain energy function of the material, I_1, I_2 – Cauchy–Green strain tensor invariants, C_1, C_2, C_3 – the unknown material parameters.

This type of strain energy function W (1) was obtained by Rivlin and Saunders from experiments on the deformation of a thin sheet.

In order to determine parameters of Mooney–Rivlin hyperelastic rubber-like material model, it is necessary to find the dependence σ on λ (stress–extension rate) for uniaxial tension with the strain energy function of the material. In this paper, we will not present the derivation of this relationship, since it has been done previously [8].

With this Mooney–Rivlin strain energy function the dependence ($\sigma - \lambda$) is as follows

$$\sigma = 2\left(\lambda^2 - \frac{1}{\lambda}\right)\left(C_1 + \frac{1}{\lambda}C_2 + 2C_3\left(\lambda - \frac{1}{\lambda} + \frac{1}{\lambda^2} - 1\right)\right). \quad (2)$$

Selection of unknown constants C_1, C_2, C_3 was performed using the least squares method.

For example, we will show experimental and theoretical diagram ($\sigma - \lambda$) for basilar artery. In Fig. 11 the dotted line presents dependence ($\sigma - \lambda$), based on the interpolated points of the experimental data, solid line presents dependence ($\sigma - \lambda$), obtained by formula (2) with unknown constants C_1, C_2, C_3 found above.

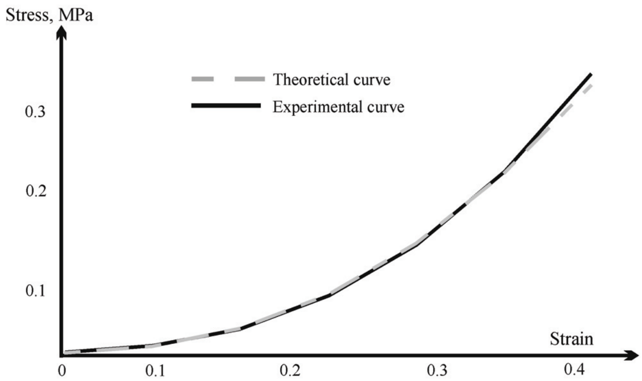

Fig. 11. Solid line presents dependence ($\sigma - \lambda$), based on the interpolated points of experimental data, the dotted line presents relation ($\sigma - \lambda$), obtained from (2)

The graph shows that the theoretical and experimental curves approximately coincide along the whole range of deformation. The selected Mooney–Rivlin strain energy function with the constants found describes the material of the walls of the arteries in the circle of Willis quite good in assumption of isotropy.

3. Discussion

Most modern authors [10], [14], [23], [24] support the hemodynamic theory of human artery aneurysm development. According to this theory the main reason of the aneurism occurrence, growth and rupture can be explained by mechanical (hemodynamic) factors. Authors investigate stresses in the wall, shear stress at the wall, blood pressure and other mechanical factors which influence the vessel wall. This needs to state boundary problem of the blood flow in the arteries. This includes both the selection of appropriate equations and boundary conditions, as well as the construction of realistic geometric models of vessels. Moreover, it is important to choose the adequate model to describe the behavior of the arterial walls.

This paper presents the first stage of research dedicated to the numerical study of the behaviour of the circle of Willis arteries in its normal state and in the presence of aneurysms. The results will be aimed at the development of guidelines for surgeons conducting surgical treatment of Willis artery aneurysms.

This article presents data that are necessary for mathematical definition of coupled elasticity and hydrodynamics problem that describe the blood flow in cerebral vessels. First of all, the images obtained with the magnetic resonance imaging were processed. Three-dimensional highly accurate realistic computer models of Willis artery in its normal state and in the presence of aneurysm of middle cerebral artery were constructed on the basis of these. Reconstruction method for such models based on up-to-date software for tomogram data processing and 3D-design was described. In addition, data of morphological examination carried out in Saratov State Medical University named after V.I. Razumovsky were used for creation of geometric models of arteries' walls. Wall thickness was averaged according to age and gender.

Further, to obtain boundary conditions at vessel inlets and outlets, ultrasound data were processed and dependence of linear flow velocity on time was calculated. Other researchers [13], [21] use similar boundary conditions. There was zero pressure at the outlets.

Mechanical characteristics of the wall of arteries were obtained from data of uniaxial tensile experiments conducted in our laboratory before [8]. Deformation properties of arteries were investigated, some features of their variability with age and gender were revealed.

Figure 11 shows that the tension diagram of cerebral arteries is essentially nonlinear, that is why we used the model of hyperelastic material to model the arterial wall. It accurately approximates the experimental curve. Other authors use similar models of hyperelastic materials in their research, which again confirms that hyperelastic model is suitable for the description of the vascular wall behavior. There are works [19] where authors show the adequacy of such models with the help of biomechanical experiments and numerical modeling.

In contrast to other works [1], [25], [17], the method introduced in this paper allows us to create sufficiently easy and fast not only realistic but also smooth and easy to modify three-dimensional models. It is important to notice that this method is independent of the software used.

Thereby, the research presented in this article is the part of complex project aimed at investigation of the behavior of the circle of Willis arteries in their normal state and in the presence of pathologies.

In future work we intend to take into account such properties of vessel tissue as anisotropy and presence of three layers.

4. Results

The introduced original method of the realistic three-dimensional geometric models of vessel construction based on imaging data allows researchers to obtain highly accurate and suitable models for the numerical calculations. It becomes quite easy to change the configuration of the geometry, which is necessary in performing the numerical experiments.

Boundary conditions for the linear blood velocity at the inlets of the circle of Willis were obtained based on ultrasound investigations. Mechanical properties of the Willis circle arteries were investigated. Mooney–Rivlin strain energy function constants were calculated using the least squares method based on mechanical experiments. They are used to describe the behaviour of the walls of the arteries.

Thus, the boundary problem describing the behaviour of human Willis circle arteries was stated. Further, this problem will be solved numerically using the finite element method. The numerical results will be analyzed from the point of view of the influence of mechanical factors on the emergence, growth and rupture of circle of Willis aneurysms.

Acknowledgments

This work was supported by Russian Foundation for Basic Research (Project No. 12-01-31310).

References

[1] AURICCHIO F., CONTI M., DE BEULE M., DE SANTIS G., VERHEGGHE B., *Carotid artery stenting simulation: from patient-specific images to finite element analysis*, Med. Eng. Phys., 2011, 33(3), 281–291.

[2] BALOSSINO R., PENNATI G., MIGLIAVACCA F., FORMAGGIA L., VENEZIANI A., TUVERI M., DUBINI G., *Computational models to predict stenosis growth in carotid arteries: which is the role of boundary conditions?* Comput. Methods Biomech. Biomed. Engin., 2009, 12(1),113–123.

[3] BLAGOJEVIĆ M., NIKOLIĆ A., ZIVKOVIĆ M., ZIVKOVIĆ M., STANKOVIĆ G., *Influence of blocks' topologies on endothelial shear stress observed in CFD analysis of artery bifurcation*, Acta Bioeng. Biomech., 2013, 15(1), 97–104.

[4] CHING-CHENG CHUANG, YU-TZU LEE, CHUNG-MING CHEN, YAO-SHENG HSIEH, TSAN-CHI LIU, CHIA-WEI SUN, *Patient-oriented simulation based on Monte Carlo algorithm by using MRI data*, Biomed. Eng. Online, 2012, 11(21), DOI: 10.1186/1475-925X-11-21.

[5] DEMPERE-MARCO L., OUBEL E., CASTRO M., PUTMAN C., FRANGI A., CEBRAL J., *CFD analysis incorporating the influence of wall motion: application to intracranial aneurysms*, Med. Image Comput. Comput. Assist. Interv., 2006, 9(Pt 2), 438–445.

[6] GRINBERG L., CHEEVER E., ANOR T., MADSEN J.R., KARNIADAKIS G.E., *Modeling blood flow circulation in intracranial arterial networks: a comparative 3D/1D simulation study*, Ann. Biomed. Eng., 2011, 39(1), 297–309.

[7] HO H., COOLING M.T., HUNTER P., *Towards a Multiscale Integrative Model of WSS-Induced Signaling Pathways in Cerebral Aneurysms*, IFMBE Proceedings, 2010, 31, 1159–1162.

[8] IVANOV D.V., FOMKINA O.A., *The mechanical properties of arteries of Willis polygon*, Russian Journal of Biomechanics, 2008, 12(4), 75–84.

[9] JOU L.D., MAWAD M.E., *Timing and size of flow impingement in a giant intracranial aneurysm at the internal carotid artery*, Med. Biol. Eng. Comput., 2011, 49(8), 891–899.

[10] KIM C.S., KIRIS C., KWAK D., DAVID T., *Numerical simulation of local blood flow in the carotid and cerebral arteries under altered gravity*, J. Biomech. Eng., 2006, 128(2), 194–202.

[11] LIANG F., FUKASAKU K., LIU H., TAKAGI S., *A computational model study of the influence of the anatomy of the circle of willis on cerebral hyperperfusion following carotid artery surgery*, Biomed. Eng. Online, 2011, 10(84), DOI: 10.1186/1475-925X-10-84.

[12] LIU H., YAMAGUCHI T., *Waveform dependence of pulsatile flow in a stenosed channel*, J. Biomech. Eng., 2001, 123(1), 88–96.

[13] MARZO A., SINGH P., REYMOND P., STERGIOPULOS N., PATEL U., HOSE R., *Influence of inlet boundary conditions on the local haemodynamics of intracranial aneurysms*, Comput. Methods Biomech. Biomed. Engin., 2009, 12(4), 431–444.

[14] OUBEL E., DE CRAENE M., PUTMAN C.M., CEBRAL J.R., FRANGI A.F., *Analysis of intracranial aneurysm wall motion and its effects on hemodynamic patterns*, Proc SPIE, 2007, DOI: 10.1117/12.708937.

[15] SHOJIMA M., OSHIMA M., TAKAGI K., TORII R., HAYAKAWA M., KATADA K., MORITA A., KIRINO T., *Magnitude and Role of Wall Shear Stress on Cerebral Aneurysm. Computational Fluid Dynamic Study of 20 Middle Cerebral Artery Aneurysms*, Stroke, 2004, 35, 2500–2505.

[16] TORII R., MARIE OSHIMA M., KOBAYASHI T., TAKAGI K., TEZDUYAR T.E., *Fluid–structure Interaction Modeling of Aneurysmal Conditions with High and Normal Blood Pressures*, Computational Mechanics, 2006, 38(4–5), 482–490.

[17] TORII R., OSHIMA M., *An integrated geometric modelling framework for patient-specific computational haemodynamic study on wide-ranged vascular network*, Comput. Methods Biomech. Biomed. Engin., 2012, 15(6), 615–625.

[18] TORII R., OSHIMA M., KOBAYASHI T., TAKAGI K., TEZDUYAR T.E., *Fluid–structure interaction modeling of blood flow and cerebral aneurysm: Significance of artery and aneurysm shapes*, Comput. Methods Appl. Mech. Engrg., 2008, 198, 3613–3621.

[19] TOTH B., RAFFAI G., BOJTAR I., *Analysis of the mechanical parameters of human brain aneurysm*, Acta Bioeng. Biomech., 2005, 7(1), 3–22.

[20] TRACHET B., RENARD M., DE SANTIS G., STAELENS S., DE BACKER J., ANTIGA L., LOEYS B., SEGERS P., *An integrated framework to quantitatively link mouse-specific hemodynamics to aneurysm formation in angiotensin II-infused ApoE -/- mice*, Ann. Biomed. Eng., 2011, 39(9), 2430–2444.

[21] VALEN-SENDSTAD K., MARDAL K.A., STEINMAN D.A., *High-resolution CFD detects high-frequency velocity fluctuations in bifurcation, but not sidewall, aneurysms*, J. Biomech., 2013, 18, 46(2), 402–407.

[22] WANG X.H., LI X.Y., ZHANG X.J., *A Computational Study on Biomechanical Differences between Cerebral Aneurysm and Normal Cerebral Artery Employing Fluid-Structure Interaction Analysis*, IFMBE Proceedings, 2010, 31, 1503–1506.

[23] WATTON P.N., RABERGER N.B., HOLZAPFEL G.A., VENTIKOS Y., *Coupling the hemodynamic environment to the evolution of cerebral aneurysms: computational framework and numerical examples*, J. Biomech. Eng., 2009, 131(10), 101003, DOI: 10.1115/1.3192141.

[24] WATTON P.N., SELIMOVIC A., RABERGER N.B., HUANG P., HOLZAPFEL G.A., VENTIKOS Y., *Modeling evolution and the evolving mechanical environment of saccular cerebral aneurysms*, Biomech. Model Mechanobiol., 2011, 10, 109–132.

[25] WENYU FU, AIKE QIAO, *Fluid Structure Interaction of Patient Specific Internal Carotid Aneurysms: A Comparison with Solid Stress Models*, IFMBE Proceedings, 2010, 31, 422–425.

[26] XENOS M., RAMBHIA S.H., ALEMU Y., EINAV S., LABROPOULOS N., TASSIOPOULOS A., RICOTTA J.J., BLUESTEIN D., *Patient-based abdominal aortic aneurysm rupture risk prediction with fluid structure interaction modeling*, Ann. Biomed. Eng., 2010, 38(11), 3323–3337.