Tanguy Christian
Orange Labs, Issy-les-Moulineaux, France

On a few reliability issues in telecommunication networks

Keywords

network reliability, availability, failure frequency, uncertainty, large systems

Abstract

The paper proposes a short survey on a few issues currently addressed in telecommunication networks. We show on an example that the *k*-terminal reliability of recursive families of graphs can also be expressed in terms of products of matrices, leading to a simple asymptotic result. The uncertainty on equipment failure rates, which are not always easy to assess, and the possible occurrence of common-cause failures combine to possibly make the overall connection availability and failure frequency different from their expected values assuming independent failures. We finally discuss a source of impairment in long-haul optical networks, and other current issues where improvements would lead to a reduction of costs and to a better quality of service.

1. Introduction

Network availability and reliability have long been a practical issue in telecommunication networks. Quality of Service (QoS) requirements imply high availabilities A, but also a good knowledge of the failure frequency ν of point-to-point connections, when the system can be repaired. Because of the operational cost, most networks have usually a meshed architecture. Not surprisingly, the study of network reliability has led to a huge body of literature, starting with the work of Moore and Shannon [23], and including textbooks and surveys [4], [9], [15], [22], [26].

The sheer number of possible system states clearly precludes the use of an enumeration of states strategy for realistic networks, and shows that the final expression may be extremely cumbersome. Consequently, most studies have considered graphs with perfect nodes and edges of identical reliability p. It was shown early on (see for instance the discussion in [9]) that the calculation of k-terminal reliability is #P-hard in the general case, even with the following simplifying and restricting assumptions that (i) the graph is planar (ii) all nodes are perfectly reliable (iii) all edges have the same reliability p. All reliabilities are then expressed as a polynomial in p, called the reliability polynomial. The difficulty of the problem has stimulated many approaches: partitioning techniques, sum of disjoint products [1], [3], [10], [25], [28], graph simplifications (series-parallel reductions [23], delta-wye transformations [12], factoring [18]), Monte-Carlo simulations [11], [17], and ordered binary decision diagram (OBDD) algorithms [24], [27], [37].

In recent years, the tremendous growth of Internet traffic has called for a better evaluation of the reliability of connections in optical networks. This, of course, strongly depends on the connection under consideration. Actual failure rates and maintenance data show that a proper evaluation of two-terminal reliabilities must put node and edge equipments on an equal footing, i.e., both edge (fiber links, optical amplifiers) and node (optical cross-connects, routers) failures must be taken into account.

It has been recently shown that the two-terminal reliability of recursive networks may be written as a product of matrices [31], where each edge and node reliability is arbitrary. The solution for generic architectures can be applied to particular cases. One such example is the simplified Arpanet network, which is a particular instance of the solvable Beichelt-Spross configuration [5], [32], as shown in *Figure 1*. From the exact, analytical expression of the reliability, many performance measures – the sensitivity, the failure frequency, etc. – can also be derived quite simply from partial derivatives [14], [29], [30].

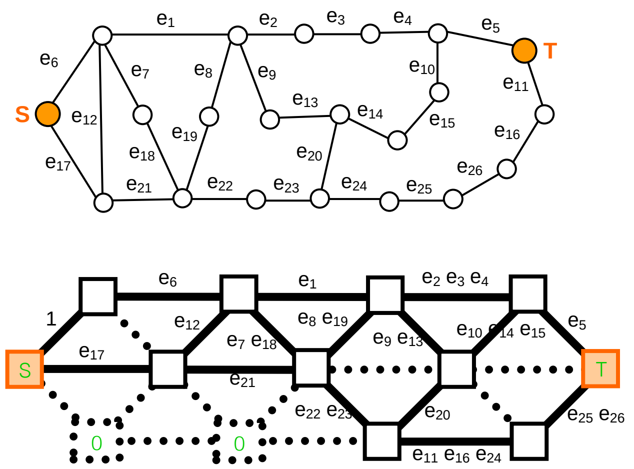

Figure 1. Translation of the simplified Arpanet network (top) in terms of a Beichelt-Spross ladder (bottom). Dotted links are absent: their availabilities should be set to zero.

Because the most general expressions for extended networks are quite large, it is often assumed that many elements are identical, or even that they never fail. While this is an oversimplification we must be careful about, it may give some insight to the general behaviour of large systems, which are of considerable interest [20]. For instance, if each link of the simplified Arpanet network has an availability p, we can show that the associated reliability polynomial is given by

$$\frac{\mathrm{Rel}_2(p)}{p^6} = 1 + p + p^2 + 6\,p^3 - p^4 + 3\,p^5 - 3\,p^6 - 20\,p^7 - 23\,p^8 + 17\,p^9 + 42\,p^{10} - 14\,p^{11} - 18\,p^{12} + 42\,p^{13} + 6\,p^{14} - 80\,p^{15} - 19\,p^{16} + 87\,p^{17} + 9\,p^{18} - 54\,p^{19} + 18\,p^{20};$$

this expression will be used in various following sections.

This paper is organized as follows. We first look at the specificity of k-terminal reliability for a simple, recursive architecture by considering the 4-terminal reliability of its end terminals. The exact solution shows that in the large system limit, the main conclusion can be obtained from the simpler two-terminal reliability. We then turn to the thorny issue of uncertainty in reliability calculations, which can originate with the lack of accuracy of field data and by the possible existence of common-cause failures. While this topic has been the subject of many studies in various fields, with major breakthroughs coming from the nuclear reactor industry [24], among other disciplines, the systems that have been dealt with are of moderate size. We finally enumerate several topics of current interest – the influence of optical impairments in long-haul fiber links, data collection of repair times, and expected lifetimes of equipments undergoing variable load – and briefly sketch approaches to tackle the issues.

2. Three- and four-terminal reliability for a simple architecture

We mentioned in the introduction that for recursive families of graphs, the two- and all-terminal reliabilities may be expressed by products of matrices. It is therefore reasonable to assume that it should also be the case for the k-terminal reliabilities. But then, how does the size of the matrix vary with k? What happens for large systems?

2.1. Simple ladder architecture

We shall consider here a simple network configuration that describes typical long-haul optical networks [35]. We have represented in *Figure 2* a typical point-to-point connection, which corresponds to primary plus backup path between the source S and the destination T, with several links between intermediate nodes, so as to improve the reliability of the whole system. This configuration has been solved recently for Rel_2 [31].

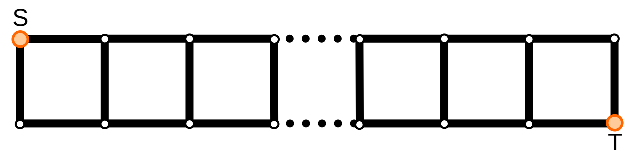

Figure 2. Two-terminal connection for a simple ladder architecture, with source S, and destination T.

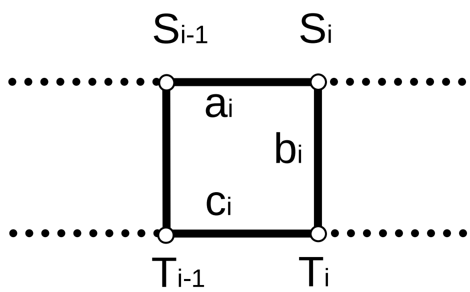

Figure 3. Close-up on the *i*th building block of the simple ladder network: a_i, b_i, and c_i are the edges' availabilities, S_i and T_i the nodes' availabilities.

Let us briefly review the main results of [31]. To keep the problem as general as possible, we

attribute to each link and node of the system a specific availability, as shown in *Figure 3*.
The two-terminal reliability $Rel_2(S_0,T_n)$ can be written as a product of 3x3 matrices, while the all-terminal reliability is given by 2x2 matrices. When the edge and node availabilities are p and ρ respectively as in [6], the generating function $G_2(z)$ defined by

$$G_2(z)=\sum_n \mathrm{Rel}_2(S_0,T_n)\,z^n = \frac{p\rho^2\left(1-p^2\,(1-p)\rho^2 z\right)}{(1-\varsigma_0\,z)(1-\varsigma_+\,z)(1-\varsigma_-\,z)}$$

with

$$\varsigma_0 = p\rho(1-p\rho)$$
$$\varsigma_\pm = \frac{p\rho}{2}\left(1+2p\rho(1-p)\pm\sqrt{1+4p^2\rho(1-p\rho(2-p))}\right)$$

A partial fraction decomposition of $G_2(z)$ leads therefore to an expression of the form

$$\mathrm{Rel}_2(S_0,T_n)=\alpha_+\varsigma_+{}^n+\alpha_-\varsigma_-{}^n+\alpha_0\varsigma_0{}^n \approx \alpha_+\varsigma_+{}^n \qquad (n>>1)$$

because one of the eigenvalues is larger than the other two (see *Figures 5* and *Figure 6*). The asymptotic large-size limit is therefore controlled by ζ_+.

2.2. 3- and 4-terminal reliability

We now wish to address a different situation, in which we are interested in the connection probability of a few sites (the customers) located at the outskirts of the network. These terminals will be S_0, T_0, S_n, T_n (see *Figure 4*).

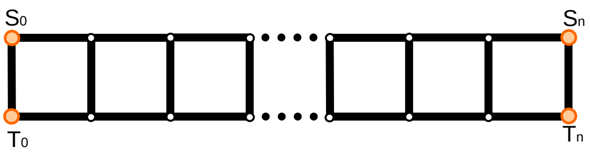

Figure 4. Four-terminal connection for the same architecture as in Fig. 3. S_0, T_0, S_n, and T_n must be connected.

The pivotal decomposition – taking possibly imperfect nodes into account and already used for the derivation of $Rel_2(S_0,T_n)$ – is helpful again when calculating $Rel_4(S_0,T_0, S_n,T_n)$. However, it is easy to see that this decomposition gives rise to terms similar to $Rel_3(S_0,T_0, S_{n-1})$, $Rel_3(S_0,T_0, T_{n-1})$, and $Rel_4(S_0,T_0, S_{n-1},T_{n-1})$. If we want to establish a proper recursion, we also have to find relationships between the two $Rel_3(.)$ above. This can be done smoothly because these quantities constitute a 'closed' system. The knowledge of $Rel_3(S_0,T_0, S_1)$, $Rel_3(S_0,T_0, T_1)$ and $Rel_4(S_0,T_0, S_1,T_1)$ gives the necessary initial conditions of the recursion. Finally, the 3-terminal reliabilities can be obtained by of product of 4x4 matrices, whereas $Rel_4(S_0,T_0, S_n,T_n)$ requires 6x6 matrices. Let us recall that such expressions are valid for arbitrary values of the edge and node reliabilities.

As in the case of $Rel_2(S_0,T_n)$, we may wonder about the large-*n* dependence of the $Rel_3(.)$ and $Rel_4(.)$. Here again, we can compute the generating functions

$$G_3(z)=\sum_n \mathrm{Rel}_2(S_0,T_0,T_n)\,z^n = -p\rho^2+\frac{p\rho^2}{(1-\varsigma_+\,z)(1-\varsigma_-\,z)}$$

$$G_4(z)=\sum_n \mathrm{Rel}_4(S_0,T_0,S_n,T_n)\,z^n = -p\rho^2+\frac{(1-p)\rho^2}{1-\xi_0 z} - \rho^2\,\frac{1-2p-p\rho(1-2p+p\rho)z}{(1-\varsigma_+ z)(1-\varsigma_- z)}$$

where $\xi_0 = p^2(1-p)\rho^2$. We note the presence of ζ_+ and ζ_-. *Figures 5* and *Figure 6* display the variations of ζ_+, ζ_-, ζ_0, and ξ_0 for perfect and imperfect nodes.

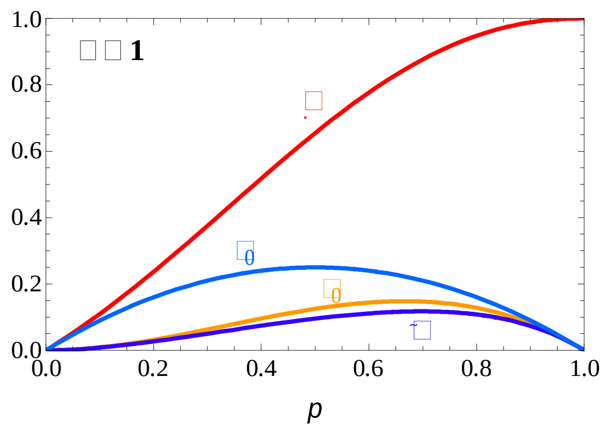

Figure 5. Variation with p of the different eigenvalues ζ_+, ζ_-, ζ_0, and ξ_0 appearing in the 4-

terminal reliability $Rel_4(S_0,T_0, S_n,T_n)$, for perfect nodes.

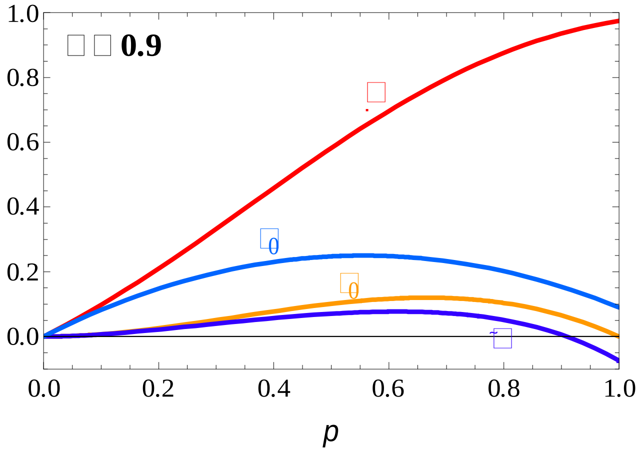

Figure 6. Same as *Figure 5*, with ρ=0.9.

The main result is that when the network is large, i.e., $n >> 1$, all two-, three- and four-terminal reliabilities essentially behave as power-laws, with the same scaling factor ζ_+. The reliabilities are given by the probability of connecting the mot distant end-terminals.

3. Uncertainty on the availability and the failure frequency

An important question is: can we trust calculations for a connection, given the uncertainty on the probabilities that each network's component is properly functioning? This issue has already been considered in the literature [7], [8], [16], [32]. We will consider here two sources of uncertainty, and see their consequences on the two-terminal reliability of the Arpanet network of Figure 1.

3.1. Uncertainty on the data

If the availability p of each element is not perfectly known – the failure rate is not so easy to determine accurately – this must have an influence on the total availability A of a connection. Telecommunication networks are built with quite reliable components so that it is reasonable to assume that the uncertainty on p is a fraction of the unavailability

$$\Delta p = \gamma(1-p) = \gamma q$$

Provided that q is small, the uncertainty on A, ΔA, is approximately given by

$$\Delta A \approx \frac{\partial A}{\partial p}\Delta p$$

The relative uncertainty $\Delta A/A$ is small if

$$\frac{\Delta A}{A} \approx \frac{1}{A}\frac{\partial A}{\partial p}\Delta p = \frac{1}{A}\frac{\partial A}{\partial p}\gamma q << 1$$

In the case of the Arpanet architecture, we find in the $q \to 0$ limit

$$\frac{\Delta A}{A} \approx \gamma(12 q^2 + 168 q^3 + ...) << 1$$

This means that the availability calculation is meaningful only when $12\,\gamma\, q^2 << 1$. If q is about 10^{-3}, this condition will be fulfilled. A general treatment for arbitrary recursive architectures has been given in [32]. Another way to present the result is to consider the relative uncertainty of the unavailability $U = 1 - A$. It reads

$$\frac{\Delta U}{U} \approx 2\gamma \quad (q \to 0)$$

This should urge caution make statement about the accuracy of connection unavailabilities.

3.2. Influence of common-cause failures

In most network reliability studies, equipment failures are assumed to be independent. However, we cannot always discard the possibility of common-cause failures. Admittedly, the corresponding failure rates are even more difficult to assess than their single failure counterparts. The purpose of this subsection is to show how the true availability of a connection differs from that computed assuming independent failures and using the *observed* availability of each component, defined as $<p>$.

The general expressions for the true availability have been given in [34]. We consider in the following the Arpanet configuration, with perfect nodes and identical edges, the number of which is N. We call $\lambda^{(i)}$ the failure rate of an event affecting jointly i elements, and assume a uniform repair rate μ for all elements of the network. It is then easy to show that the average availability $<p>$ of an element is

$$< p > = \frac{\mu}{\mu + \hat{\lambda}_1}$$

and more generally that

$$< p^m > = \prod_{k=1}^{m} \frac{k\,\mu}{k\,\mu + \hat{\lambda}_k}$$

$$\hat{\lambda}_k = \sum_{i=1}^{n} \left[\binom{N}{i} - \binom{N-k}{i} \right] \lambda^{(i)}$$

Because of the $\lambda^{(i)}$'s ($i \geq 2$), we cannot write $<p^m> = <p>^m$ anymore. For a reliability polynomial $R(p) = \Sigma_m a_m p^m$, we have

$$< R > = \sum_{m=1}^{N} a_m < p^m > \neq R(< p >)$$

As shown in the above equations, the final result depends on the $\lambda^{(i)}$'s. A first result for $\lambda^{(1)} = \lambda$, $\lambda^{(N)} = \Lambda$, and all other coefficients set to zero (the β-factor model where $\beta = \lambda/(\lambda+\Lambda)$) is that the true availability may be either greater or smaller than $R(<p>)$, depending on the architecture and size of the network, along with the unavailability of each of its elements [34].
We present here what happens for the Arpanet configuration if we consider, instead of the β-factor model, another well-known model of common-cause failures, namely the binomial failure rate model [2, [36]. Let us set $\lambda^{(i)} = \lambda\, \xi^{(i-1)}$; this leads to

$$\hat{\lambda}_k = \lambda (1+\xi)^N \frac{1-(1+\xi)^{-k}}{\xi},$$

so that $\hat{\lambda}_1 = \lambda (1+\xi)^{N-1} = \lambda_{\text{eff}}$ and

$$< p > = \frac{\mu}{\mu + \lambda_{\text{eff}}}.$$

We can therefore define the *effective* beta-factor by

$$\tilde{\beta} = 1 - \frac{\lambda}{\lambda_{\text{eff}}},$$

which is nothing but the ratio of all the non-single failure rates over the effective observed failure rate.
Figures 7 shows the behavior of the true availability for the Arpanet configuration, for both models (the independent-failures limit is obtained for $\beta = 0$ or $\tilde{\beta} = 0$). We observe a substantial variation of both the availability and the failure frequency as these parameters increase. While the general behavior of the availability is approximately the same for both models (see *Figure 7*), the slope at the origin is slightly positive for the β-factor model, $<R>_{\text{beta}}$, while it is negative for the combinatorial one, $<R>_{\text{combi}}$. The true availability of a connection may be either greater or smaller than the simple 'independent' calculation of $R(<p>)$ with the assumption $<p> = 0.9$. In telecommunication networks, we expect the amount of common-cause failures to be rather small. Knowing whether we underestimate the true unavailability is important.
If we turn to the true failure frequency $<\nu>$ (see *Figure 8*), we see that the two models lead to qualitatively different behaviors. It is worthwhile noting that the vicinity of $\tilde{\beta} \to 1$ may lead to strong variations for both $<R>$ and $<\nu>$. Furthermore, for $<p> = 0.8$, the variations of $<R>$ are quite different. A detailed study of the two models and their consequences will be presented elsewhere. Caution is therefore recommended even when only trying to predict the qualitative change brought by taking common-cause failures into account.

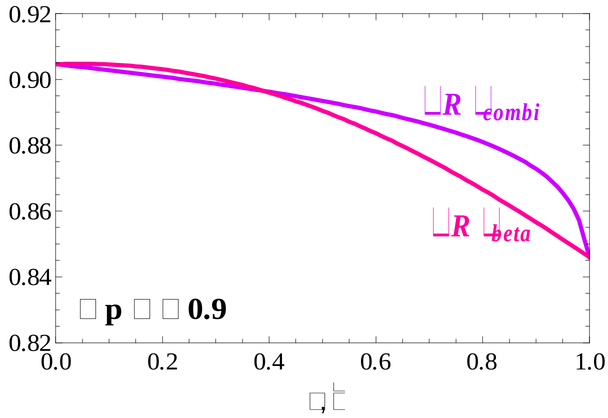

Figure 7. True availability of the two-terminal connection S-T for the Arpanet architecture given in *Figure 1*, for two models of common-cause failures. The *observed* availability of each link is <p> = 0.9.

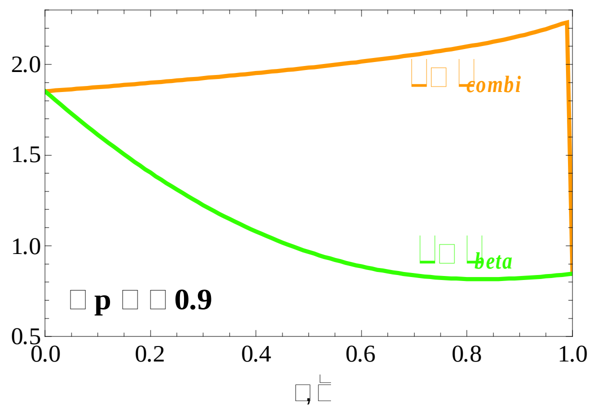

Figure 8. True failure frequency of the two-terminal connection for the Arpanet architecture given in *Figure 1*, for both models of common-cause failures.

4. Further issues in the quality of connections in optical networks

Quality of Service (QoS) is a crucial issue for telecommunication operators. The unavailability of connections in optical networks is therefore an important performance measure, which must be determined. This measure encompasses various topics, of which we mention a few in the following.

4.1. Polarization mode dispersion in optical transmission

In network reliability studies, hardware failures are usually considered (servers may be down, fiber links accidentally cut, etc.). The overall availability of a connection is then calculated – using probabilistic methods – as a function of all the availabilities of the equipments likely to be used between its source and its destination. The contribution of software failures is also assessed.

These failures are, however, not the only causes of impairment for optical connections. Several physical phenomena occurring in the propagation of light pulses in fibers, such as polarization mode dispersion (PMD), crossed-phase modulation, and other nonlinear effects, may lead to some "blurring" of the signal, making sometimes the '0' and '1' of the transferred data unrecognizable. These phenomena should play an even larger role in the next few years, all the more so as higher transmission data rates, 40 and 100 Gbps, are now required in order to satisfy an ever increasing traffic demand.

We focus here on the influence of one of these phenomena, namely the polarization mode dispersion, on a connection's unavailability. Amongst the main characteristics of PMD relevant to our objective, its stochastic nature is the trickiest. The delay τ between the two polarization modes leads to a broadening of the light pulse that carries the digital information [13], [19] (see *Figure 9*).

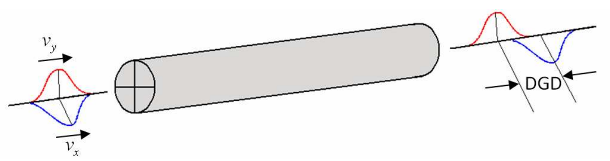

Figure 9. Different propagation velocities for polarization modes leads to a smearing out of the optical signal.

In the simplest statistical description of the PMD, the delay τ obeys a Maxwellian distribution given by

$$P(\tau)=\frac{32\tau^2}{\pi^2<\tau>^3}\exp\left(-\frac{4\tau^2}{\pi<\tau>^2}\right),$$

where the mean differential group delay (DGD) $<\tau>$ is proportional to the *square root* of the length L of the fiber link (as in a random walk). If at a given time t, τ is larger than a value depending on the data transfer rate and engineering constraints, the optical signal will be too blurred to be decoded; this leads to a de facto unavailability of the otherwise perfectly operational fiber link (note that maybe a few seconds later, τ may be small enough again for the optical transmission to be fully functional again, without any repair being made). One current goal is therefore to evaluate the contribution of the PMD to the total unavailability of an optical connection.

4.2. Repair time distribution and maintenance policy

The availability of each network element is obviously a value on which a connection's quality of service relies heavily. This parameter depends of course on the failure rate of the element, but also on the equivalent repair rate. The determination of a realistic description of the repair time distribution is something telecommunication companies cannot afford to disregard. To attain such a goal, data collection is a crucial issue. It may be possible to use simple laws: while the exponential distribution is well-known for *not* being the best of candidates, the lognormal or the inverse Gaussian distribution are usually among the favored ones [26]. Still, the difficulty – as in other fields – is to get enough field data to make the choice of a given model sufficiently meaningful. Another problem is the input of approximate or rounded off values, which somehow 'pollute' the whole data set. Once a repair-time distribution is adopted, it can be used in the proper assessment of the weak parts of a network and in the optimization of resources, such as the scheduling of tasks for repair people.

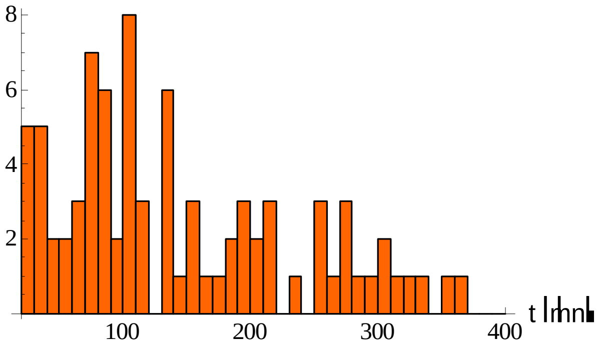

Figure 10. One week-worth of unprocessed repair times of a network equipment.

4.2. Expected lifetime of 'real' equipments

In many deployed telecommunication networks, equipments undergo variable loads. Figure 11 represents the typical weekly energy consumption of a network element. This should translate to a varying failure rate, and therefore to a mean time before failure that cannot be calculated assumed a constant failure rate. A first simple approach has been proposed in [33]. Refinements could be helpful too.

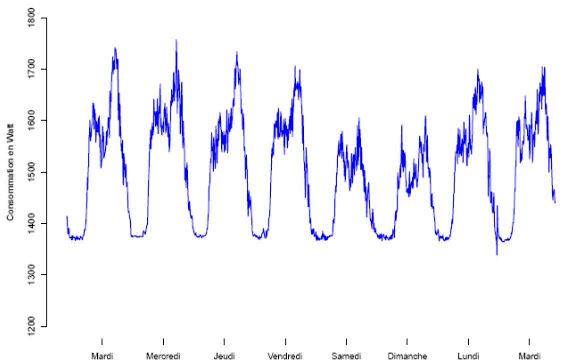

Figure 11. Weekly variation of energy consumption by a network equipment. The influence of week-ends is clearly observable.

5. Conclusion

We have tried to give a short overview on several issues that are currently of crucial importance in optical networks, and for which various reliability and system safety techniques can be usefully and successfully applied.

Acknowledgments

We would like to thank my colleagues Esther Le Rouzic, Yousri Slaoui, Erwan Pincemin, Olivier Klopfenstein, Matthieu Chardy, and Karol Grelewicz for useful discussions.

References

[1] Abraham, J. A. (1979). An improved algorithm for network reliability. *IEEE Transactions on Reliability* 28 (1), 58–61.

[2] Atwood, C. L. (1986). The binomial failure rate common cause model. *Technometrics 28*(2), 139–148.

[3] Balan, A. O. & Traldi, L. (2003). Preprocessing minpaths for sum of disjoint products, *IEEE Transactions on Reliability* 52 (3), 289–295.

[4] Ball, M.O. , Colbourn, C.J. & Scott Provan, J. (1995). *Network reliability*, in Handbooks in operations research and management science, volume 7: Network Models, editors M. O. Ball, T.L. Magnanti, C.L. Monma, and G.L. Nemhauser, Elsevier, Amsterdam, pp. 673–762 (with over 400 references).

[5] Beichelt, F. & Spross, L. (1989). Bounds on the reliability of binary coherent systems. *IEEE Trans. Reliability* 38, 425–427.

[6] Bulka D. & Dugan, J.B. (1994). Network s-t reliability bounds using a 2-dimensional reliability polynomial. *IEEE Transactions on Reliability* 43 (1), 39–45.

[7] Coit, D.W. (1997). System-reliability confidence-intervals for complex-systems with estimated component-reliability. *IEEE Transactions on Reliability* 46 (4) (1997) 487–493.

[8] Coit, D.W., Jin, T. & Wattanapongsakorn, N. (2004). System optimization with component reliability estimation uncertainty: a multi-criteria approach, *IEEE Transactions on Reliability* 53 (3), 369–380.

[9] Colbourn, C.J. (1987). *The combinatorics of network reliability*. Oxford University Press, Oxford.

[10] Dotson, W. P. & Gobien, J. O. (1979). A new analysis technique for probabilistic graphs. *IEEE Transactions on Circuits and Systems* 26 (10), 855–865.

[11] Fishman, G.S. (1986). A comparison of four Monte Carlo methods for estimating the probability of s-t connectedness. *IEEE Transactions on Reliability* 35 (2), 145–155.

[12] Gadani, J.P. (1981). System effectiveness evaluation using star and delta transformations. *IEEE Transactions on Reliability* 30 (1), 43–47.

[13] Gordon, J.P., & Kogelnik, H. (2000). PMD fundamentals: Polarization mode dispersion in optical fibers**.** *PNAS* 97 (9), 4541–4550.

[14] Hayashi, M. (1991). System failure-frequency analysis using a differential operator. *IEEE Transactions on Reliability* 40 (5), 601–609, 614.

[15] Henley, E.J. & Kumamoto, H. (1991). *Probabilistic Risk Assessment: Reliability Engineering, Design and Analysis*. IEEE Press, Piscataway, chapter 10.

[16] Jin, T. & Coit, D.W. (2001). Variance of system-reliability estimates with arbitrarily repeated components. *IEEE Transactions on Reliability* 50 (4), 409–413.

[17] Karger, D.R. (2001). A randomized fully polynomial time approximation scheme for the all terminal reliability problem. *SIAM Review* 43 (3), 499–522.

[18] Kevin Wood, R. (1985). A factoring algorithm using polygon-to-chain reductions for computing K-terminal network reliability. *Networks* 15, 173–190.

[19] H. Kogelnik, H., Nelson, L.E. & Jonson, R.M. (2002). Polarization-Mode Dispersion, in *Optical Fiber Telecommunications IV B*, editors I. P. Kaminow & T. Li, Academic Press, San Diego, 725–861.

[20] Kołowrocki, K. (2004). *Reliability of Large Systems*. Elsevier, Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney - Tokyo.

[21] Kuo, S., Lu, S. & Yeh, F. (1999). Determining terminal pair reliability based on edge expansion diagrams using OBDD. *IEEE Transactions on Reliability* 48 (3), 234–246.

[22] Kuo, W. & Zuo, M.J. (2003). *Optimal Reliability Modeling*. John Wiley and Sons, Inc., Hoboken.

[23] Moore, E.F. & Shannon, C.E. (1956). Reliable circuits using less reliable relays. *J. Franklin Institute*, 262 (September), 191–208; 262 (October), 281–297.

[24] Mosleh, A., Fleming, K.N., Parry, G.W., Paula, H.M., Worledge, D.H. & Rasmuson, D.M. (1988). *Procedures for treating common cause failures in safety and reliability studies: procedural framework and examples*. Technical Report NUREG/CR-4780, U.S. Nuclear Regulatory Commission.

[25] Rai, S., Veeraraghavan, M. & Trivedi, K.S. (1995). A survey of efficient reliability computation using disjoint products approach. *Networks* 25, 147–163, and references therein.

[26] Rausand, M. & Høyland, A. (2004). *System Reliability Theory*, 2nd edition. John Wiley and Sons, Inc., Hoboken.

[27] Rauzy, A. (2003). A new methodology to handle Boolean models with loops. *IEEE Transactions on Reliability* 52 (1), 96–105.

[28] Rauzy, A., Châtelet, E., Dutuit, Y. & Bérenguer, C. (2003). A practical comparison of methods to assess sum-of-products. *Reliability Engineering and System Safety* 79, 33–42.

[29] Shi, D.H. (1981). General formulas for calculating the steady-state frequency of system failure. *IEEE Transactions on Reliability* R-30 (5), 444–447.

[30] Singh, C. & Billinton, R. A new method to determine the failure-frequency of a complex system. *IEEE Transactions on Reliability* R-23 (4), 231–234.

[31] Tanguy, C. (2007). What is the probability of connecting two points? *J. Phys. A: Math. Theor.* 40, 14099–14116.

[32] Tanguy, C. (2008). Relative uncertainty of availability and failure frequency for a few large, recursive networks. *Proceedings of the 14th ISSAT International Conference on Reliability and Quality in Design*, Orlando, USA, August 7–9, 2008, editors H. Pham and T. Nagakawa, 213–218.

[33] Tanguy, C. (2009). Mean Time To Failure for periodic failure rate. *Proceedings of the Third Summer Safety and Reliability Seminars*, Gdansk-Sopot, Poland, July 20–25, 2009, editors K. Kołowrocki, J. Soszyńska, and E. Zio, 347–353.

[34] Tanguy, C. (2009). A solvable common-cause failure model for coherent system. *Proceedings of the ESREL 2009 Conference*, Prague, Czech Republic, September 7–10, 2009, editors R. Briš, C. Guedes Soares, and S. Martorell, 1541–1547.

[35] To, M. & Neusy, P. (1994). Unavailability analysis of long-haul networks. *IEEE Journal on Selected Area in Communications* 12 (1), 100–109.

[36] Vesely, W. E. (1977). Estimating common cause failure probabilities in reliability and risk analyses: Marshall-Olkin specialization. *Nuclear Systems Reliability Engineering and Risk Assessment*, editors J. B. Fussell and G. R. Burdick, SIAM, Philadelphia, 314–341.

[37] Yeh, F. M., Lu, S.K. & Kuo, S.Y. (2002). OBDD-based evaluation of *k*-terminal network reliability. *IEEE Transactions on Reliability* 51 (4), 443–451.