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NUMERICAL SIMULATION OF MOVING TRUCK EFFECT ON CONCRETE PAVEMENTS

Abstract

Concrete pavements are widely used construction in road engineering. Today possibilities of computer technique enable to solve various problems by numerical way. The goal of this contribution is to introduce the computing model of the moving truck and the computing model of the concrete pavement and by numerical way to simulate the time histories of pavement vertical deflections, the time histories of vertical component of the tire forces and to follow the influence of various parameters.

Keywords: concrete pavements, numerical methods, computing models, dynamic effects

1. Introduction

Concrete pavements are widely used road constructions in these latter days. One possibility how to obtain the information about the dynamic effect of moving vehicles on the pavement is to use the numerical simulation methods. This approach demands to create the vehicle and pavement computing models and to solve the mathematical apparatus by numerical way. The finite element method is commonly used. But there are also other possibilities of solution. In this contribution the computing model of the concrete pavement based on the theory of the thin slab on elastic foundation and plane computing model of the truck are introduced. The goal of the calculation is to obtain the time history of vertical deflection in one point of the pavement during the movement of the vehicle Tatra 815 and the time histories of vertical components of the tire forces. The equations of motion are derived in the form of differential equations. The assumption about the shape of the slab deflection area is adopted. It is assumed the validity of Maxwell theorem about mutuality of deflections. The equations of motion are solved numerically in the environment of program system MATLAB. The results following the influence of various parameters (speed of vehicle motion, stiffness characteristics, road profile, …) on the pavement vertical deflections and the vertical tire forces are introduced. The outputs from numerical solution in time domain can be transformed into frequency domain and subsequently employ for the solution of further tasks.

2. Computing model of a vehicle

For the purpose of this contribution the plane computing model of the truck Tatra 815 is adopted, Figure 1. The computing model of the truck has 8 degrees of freedom – 5 mass and 3 massless. The massless degrees of freedom correspond to the vertical movements of the contact points of the model with the surface of the roadway. The vibration of the mass objects of the model is described by the 5 functions of time $r_i(t)$, (i = 1, 2, 3, 4, 5). The massless degrees of freedom are associated with the tire forces $F_j(t)$, (j = 6, 7, 8,) acting at the contact points. The equations of motions and the expressions for tire forcers have the following form:

$$\begin{aligned}
&\ddot{r}_1(t) = -\{+k_1 \cdot d_1(t) + b_1 \cdot \dot{d}_1(t) + \\
&+k_2 \cdot d_2(t) + b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dot{d}_2(t) / d_{cv}\} / m_1 \\
&\ddot{r}_2(t) = -\{-a \cdot k_1 \cdot d_1(t) - a \cdot b_1 \cdot \dot{d}_1(t) + \\
&+b \cdot k_2 \cdot d_2(t) + b \cdot b_2 \cdot \dot{d}_2(t) + f_2 \cdot \dot{d}_2(t) / d_{cv}\} / I_{y_1} \\
&\ddot{r}_3(t) = -\{-k_1 \cdot d_1(t) - b_1 \cdot \dot{d}_1(t) + \\
&+k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t)\} / m_2 \\
&\ddot{r}_4(t) = -\{-k_2 \cdot d_2(t) - b_2 \cdot \dot{d}_2(t) - \\
&- f_2 \cdot \dot{d}_2(t) / d_{cv} + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t) + \\
&+k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t)\} / m_3 \\
&\ddot{r}_5(t) = -\{-c \cdot k_4 \cdot d_4(t) - c \cdot b_4 \cdot \dot{d}_4(t) + \\
&+c \cdot k_5 \cdot d_5(t) + c \cdot b_5 \cdot \dot{d}_5(t)\} / I_{y_3}
\end{aligned} \qquad (1)$$

$$F_6(t) = -G_6 + k_3 \cdot d_3(t) + b_3 \cdot \dot{d}_3(t)$$
$$F_7(t) = -G_7 + k_4 \cdot d_4(t) + b_4 \cdot \dot{d}_4(t) \qquad (2)$$
$$F_8(t) = -G_8 + k_5 \cdot d_5(t) + b_5 \cdot \dot{d}_5(t)$$

The meaning of the used symbols is as follows: k_i, b_i, f_i are the stiffness, damping and friction characteristics of the model, m_i, I_{yi} are the mass and inertia characteristics, a, b, c, s are the length characteristic of the model, g = 9,81 m×s^{-2}, G_i are the gravity forces acting at the contact points with the surface of the roadway. The deformations of the spring elements are $d_i(t)$ and the derivation with respect to time is denoted by the dot over the symbol.

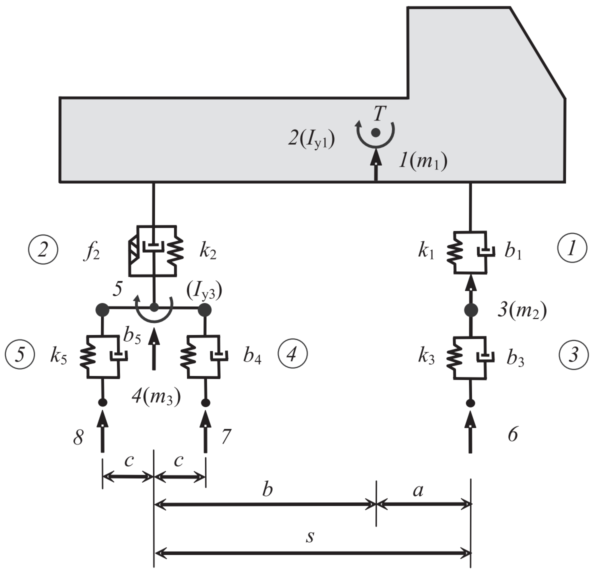

Fig. 1. Truck computing model

3. Computing model of a concrete pavement

The computing model of the concrete pavement is based on the Kirchhoff theory of thin slabs on elastic foundation [1].

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + $$
$$+K \cdot w + \mu\frac{\partial^2 w}{\partial t^2} + 2\mu\omega_b\frac{\partial w}{\partial t} = p(x,y,t) \qquad (3)$$

The wanted function $w(x, y, t)$ describing the slab vertical deflections will be expressed as the product of two functions

$$w(x,y,t) = w_0(x,y) \cdot q(t) \qquad (4)$$

The function $w_0(x, y)$ figures as known function and it is dependent on the coordinates x, y only and the function $q(t)$ figures as unknown function and it is dependent on the time t. The function $q(t)$ has the meaning of generalized Lagrange coordinate. The assumption about the shape of the function $w_0(x, y)$ was introduced as in Figure 2.

$$w_0(x,y) = \frac{1}{2}\left(1 - \cos\frac{2\pi x}{l_x}\right) \cdot \left(\sin\frac{\pi y}{l_y}\right) \qquad (5)$$

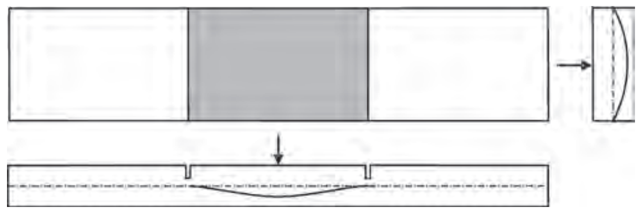

Fig. 2. Assumption about the shape of the slab deflection area

The meaning of remaining symbols is as follows: D slab stiffness N×m^2/m, K modulus of foundation [N×m^{-3}], μ mass intensity kg×m^{-2}, ω_b angular frequency of damping rad×s^{-1}.

The intensity of the dynamic load is $p(x, y, t)$. In the case of moving vehicles the discrete load due to tire forces $F_j(t)$ must be transformed on continuous load by the procedure proposed by Dirac [1]:

$$p(x,y,t) = \sum_j \varepsilon_j \cdot F_j(t) \cdot \delta(x - x_j) \cdot \delta(y - y_j) \qquad (6)$$

$$p(x,y,t) = \sum_j \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_{mn,j}(t) \cdot \frac{1}{2}\left(1 - \cos\frac{m2\pi x}{l_x}\right) \cdot \left(\sin\frac{n\pi y}{l_y}\right) \qquad (7)$$

where

$$p_{mn,j}(t) = \frac{2}{l_x} \cdot \frac{2}{l_y} \int_0^{l_x}\int_0^{l_y} \varepsilon_j \cdot F_j(t) \cdot \delta(x - x_j) \cdot \delta(y - y_j) \cdot \frac{1}{2}\left(1 - \cos\frac{m2\pi x}{l_x}\right)\left(\sin\frac{n\pi y}{l_y}\right) dxdy =$$
$$= \varepsilon_j \cdot F_j(t) \cdot \frac{4}{l_x l_y} \cdot \frac{1}{2}\left(1 - \cos\frac{m2\pi x_j}{l_x}\right) \cdot \left(\sin\frac{n\pi y_j}{l_y}\right) \qquad (8)$$

Then

$$p(x,y,t) = \sum_j \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \varepsilon_j \cdot F_j(t) \cdot \frac{1}{l_x l_y} \cdot \left(1 - \cos\frac{m2\pi x_j}{l_x}\right) \cdot \left(\sin\frac{n\pi y_j}{l_y}\right) \cdot \left(1 - \cos\frac{m2\pi x}{l_x}\right) \cdot \left(\sin\frac{n\pi y}{l_y}\right) \qquad (9)$$

Regarding to the convergence of the series in equation (9) we can take in to account the 1st member of the series only. Then the equation (9) can be rewritten as

$$p(x,y,t)=\sum_j \varepsilon_j \cdot F_j(t)\cdot\frac{1}{l_x l_y}\cdot\left(1-\cos\frac{2\pi x_j}{l_x}\right)\cdot \left(\sin\frac{\pi y_j}{l_y}\right)\cdot\left(1-\cos\frac{2\pi x}{l_x}\right)\cdot\left(\sin\frac{\pi y}{l_y}\right) \qquad (10)$$

For the plane truck computing model

$$F_j(t)=-G_j+k_j\cdot d_j(t)+b_j\cdot \dot{d}_j(t), \quad (j=6,7,8) \qquad (11)$$

where G_j is the gravity force of j-th axis, k_j and b_j stiffness and damping of the j-th tire and $d_j(t)$ is the tire deformation. Derivation with respect to the time t is denoted by the dot over the symbol.

Substituting the assumption (4) and (5) into equation (3) the left side of the equation (3) will change into

$$\ddot{q}(t)\cdot\left[\frac{1}{2}\mu\left(1-\cos\frac{2\pi x}{l_x}\right)\cdot\sin\frac{\pi y}{l_y}\right]+\dot{q}(t)\cdot\left[\frac{1}{2}2\mu\cdot\omega_b\left(1-\cos\frac{2\pi x}{l_x}\right)\cdot\sin\frac{\pi y}{l_y}\right]+q(t)\cdot\left[-\frac{D}{2}\left(\frac{2\pi}{l_x}\right)^4\cdot\cos\frac{2\pi x}{l_x}\cdot\sin\frac{\pi y}{l_y}-\frac{2D}{2}\left(\frac{2\pi}{l_x}\right)^2\left(\frac{\pi}{l_y}\right)^2\cdot\cos\frac{2\pi x}{l_x}\cdot\sin\frac{\pi y}{l_y}+\frac{D}{2}\left(\frac{\pi}{l_y}\right)^4\left(1-\cos\frac{2\pi x}{l_x}\right)\cdot\sin\frac{\pi y}{l_y}+\frac{K}{2}\left(1-\cos\frac{2\pi x}{l_x}\right)\cdot\sin\frac{\pi y}{l_y}\right]=p(x,y,t) \qquad (12)$$

When we will follow the deflections in one point of the slab only, for example in the middle of the slab, than $x = l_x/2$ and $y = l_y/2$ and the equation (3) comes by the definitive form

$$\ddot{q}(t)\cdot\mu+\dot{q}(t)\cdot 2\mu\cdot\omega_b+q(t)\cdot D\cdot\left[\frac{1}{2}\left(\frac{2\pi}{l_x}\right)^4+\left(\frac{2\pi}{l_x}\right)^2\left(\frac{\pi}{l_y}\right)^2+\left(\frac{\pi}{l_y}\right)^4+\frac{K}{D}\right]=\sum_j \varepsilon_j\cdot F_j(t)\cdot\frac{2}{l_x l_y}\cdot\left(1-\cos\frac{2\pi x_j}{l_x}\right)\cdot\left(\sin\frac{\pi y_j}{l_y}\right) \qquad (13)$$

4. Numerical analysis

4.1. Parameters of the computing model

For the purpose of numerical analysis the following pavement construction was considered as in Figure 3.

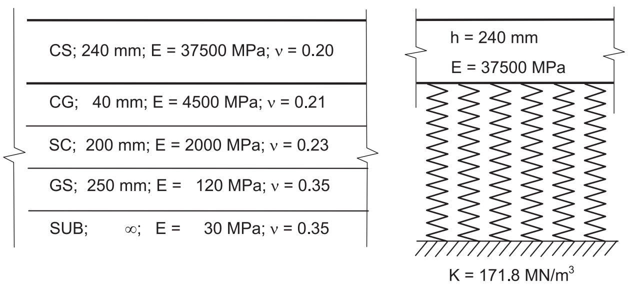

Fig. 3. Pavement computing model, CS – concrete slab, CG – coated gravel, SC – soil cement, GS – gravel sand, SUB – subgrade

The 1st layer of the pavement construction is considered as the concrete slab with the height h = 0.24 m, length in longitudinal direction l_X = 6.0 m, width in transverse direction l_Y = 3.75 m. Modulus of elasticity E = 37500 MPa, Poisson's number υ = 0.20. The layers 2-5 are taken into calculation as Winkler elastic foundation. The modulus of compressibility K = 171.8 MN×m^{-3} was calculated by the use of the program LAYMED [2]. The mass intensity of the slab $\mu = \rho\times h$ = 2500×0.24 = 600.0 kg×m^{-1}. The damping circular frequency is taken as ω_b = 0.1 rad×s^{-1}.

Parameters of truck computing model are as follows: k_1 = 143716,5 N/m, k_2 = 761256 N/m, k_3 = 1275300 N/m, k_4 = 2511360 N/m, k_5 = 2511360 N/m, b_1 = 9614 kg/s, b_2 = 130098.5 kg/s, b_3 = 1373 kg/s, b_4 = 2747 kg/s, b_5 = 2747 kg/s, m_1 = 11475 kg, m_2 = 455 kg, m_3 = 1070 kg, I_{y1} = 31149 kg×m^2, I_{y3} = 466 kg×m^2, a = 3.135 m, b = 1.075 m, c = 0.660 m, s = 4.210 m. The parameters of the truck computing model correspond to the truck TATRA 815.

4.2. Influence of the speed of truck motion

For the numerical solution of the mathematical apparatus the computer program in the programming language MATLAB was created. The program enables to calculate the time courses of all kinematical values of the truck (deflection, speed, acceleration), kinematical values at 1 point of the pavement and tire forces under the individual axles. The illustrations of the form of the obtained results are in Figure 4, 5.

The results of solution are influenced by various parameters of the considered system (speed of truck motion, stiffness of subgrade, modulus of elasticity of the slab, thickness of the slab, road profile, …).

The influence of the speed of the vehicle motion was analyzed in the interval of speeds V = 0–120 km/h with the step of 5 km/h. The maximums of vertical deflections at the monitored point of the pavement versus speed of the truck motion are plotted in the Figure 6. The results are obtained for the smooth road surface.

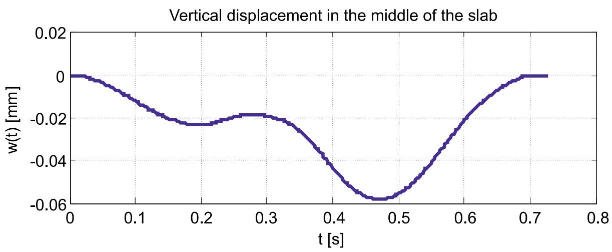

Fig. 4. Vertical displacement in the middle of the slab, speed V = 55 km/h

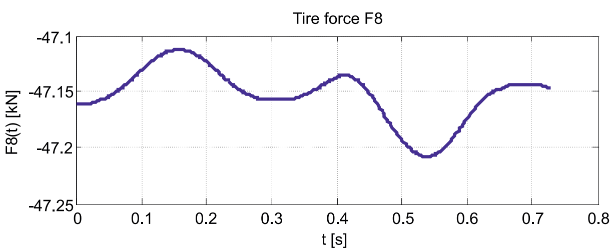

Fig. 5. Tire force $F_8(t)$ under rear axle, speed V = 55 km/h

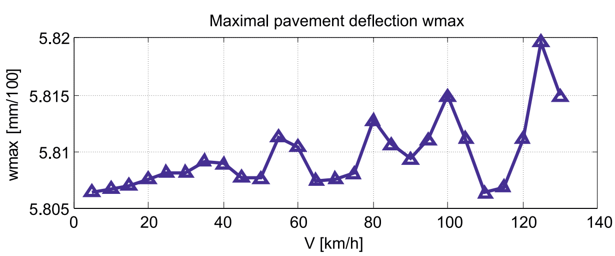

Fig. 6. Maximal deflection in the middle of the slab versus truck speed

4.3. Influence of the subgrade stiffness

The influence of the modulus of foundation K was analyzed in the interval 50–200 MPa with the step of 25 MPa and in the interval 200–500 MPa with the step of 50 MPa.

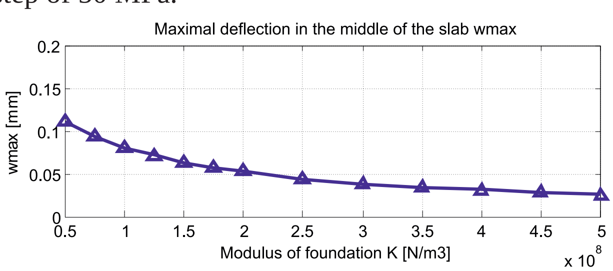

Fig. 7. Maximal deflection in the middle of the slab versus modulus of foundation K

The speed of vehicle motion was V = 55 km/h. The maximums of vertical deflections at the monitored point of the pavement versus modulus of foundation are plotted in the Figure 7. The extremes (maximum, minimum) of tire force under rear axle versus modulus of foundation are plotted in Figure 8.

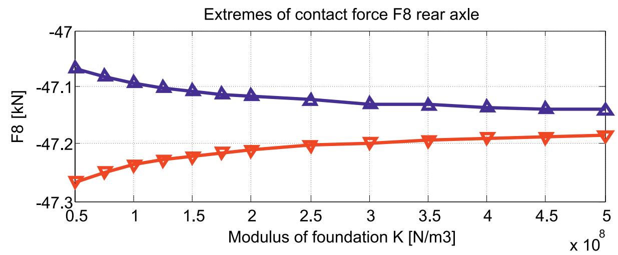

Fig. 8. Extremes of tire force $F_8(t)$ under rear axis versus modulus of foundation K

Similarly as the influence of the subgrade stiffness there is possible to follow also the influence of the slab modulus of elasticity and influence of the slab thickness on the changes of slab deflections and the changes of tire forces. There is possible to carry out the general deduction. When the stiffness of the structure is increased from any reason the vertical slab deflection are decreased and the range between the maximal and minimal value of the tire force is decreased.

4.4. Influence of the road profile

The above-mentioned results were obtained for the smooth road profile. The real road profile has random character and it represents the dominant source of kinematical excitation of vehicle. Also the vehicle response has random character. Random road profile can be approximated by a Power Spectral Density (PSD) in the form of

$$S(\Omega) = S(1).\Omega^{-k} \qquad (14)$$

where $\Omega = 2\pi/L$ in [rad/m] denotes the wavenumber and $S(1) = S(\Omega_0)$ in [m²/(rad/m)] describes the value of the PSD at reference wavenumber Ω_0 = 1 rad/m. According to the international directive ISO 8608 [3], typical road profiles can be grouped into classes from A to E. By setting the waviness to k = 2, each class is simply defined by its reference value $S(1)$. Class A with $S(1) = 1 \cdot 10^{-6}$ m²/(rad/m), class E $S(1) = 256 \cdot 10^{-6}$ m²/(rad/m). In the next the random road profile of average quality (class D, $S(1) = 64 \cdot 10^{-6}$ m²/(rad/m)) was numerically generated by the equation (15), Figure 9.

$$h(x) = \sum_{j=1}^{N} \sqrt{2 \cdot S(\Omega_j) \cdot \Delta\Omega} \cdot \cos(\Omega_j \cdot x + \varphi_j) \qquad (15)$$

In the equation (15) the φ_j is the uniformly distributed phase angle in the range between 0 and 2π. The results of solution can be presented in time or in frequency domain. As an example the tire force under front axle $F_6(t)$ and its PSD evaluated for average profile are presented in Figures 10 and 11.

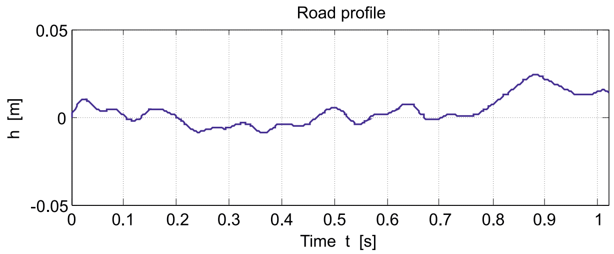

Fig. 9. Random road profile of the average quality pavement

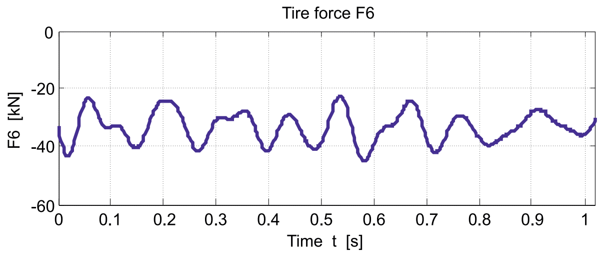

Fig. 10. Time history of tire force $F_6(t)$ under front axle

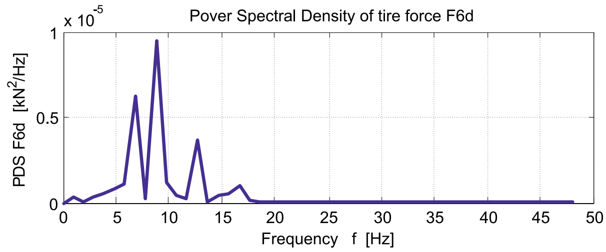

Fig. 11. PSD of the dynamic component of tire force $F_6(t)$ under front axle

5. Conclusions

Computing model of the concrete pavement based on the theory of thin slab on elastic foundation with adopting the assumption about the shape of the slab deflection area provides the effective tool for the solution of many dynamic problems in time domain. Numerical solution can be realised in the environment of the program system MATLAB. The outputs from numerical solution in time domain can be transformed into frequency domain and subsequently employed for the solution of further tasks.

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